ON THE CONVERSION OF
NEUTRON STARS INTO STRANGE STARS*

Angela V. Olinto

Center for Theoretical Physics
Laboratory for Nuclear Science
and Department of Physics
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139 U.S.A.

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ABSTRACT

Strange matter is quark matter that is assumed to be absolutely stable. A seed of strange matter in a neutron star will convert the star into a strange star. The speed at which this conversion occurs is calculated. The calculation takes into account the rate at which the down and strange quarks Fermi seas equilibrate via weak interactions and the diffusion of strange quarks towards the front. The speed is found to be a function of the temperature of the star, and the minimum strangeness necessary for strange matter stability. The result is discussed for different stages of a neutron star evolution.

I. INTRODUCTION

Strange matter is bulk quark matter that is conjectured to be stable and, therefore, to be the ground state of hadronic matter.¹ It consists of roughly the same number of up, down, and strange quarks, plus a small fraction of electrons that guarantees charge neutrality. The existence of strange matter was shown to be plausible within the uncertainties inherent to a strong-interactions calculation.⁴ Strange matter objects can vary from ~ 100 to 2.5×10^8 in baryon number. The lower limit is due to shell effects; strange "light" baryons are not stable. The upper limit is determined by gravitational collapse and corresponds to a $\sim 2 M_\odot$ object. The heavier objects resemble neutron stars and are called strange stars.³

If a stable strange matter seed (baryon number > 100) is formed inside a neutron star, the star will convert into a strange star which is, by hypothesis, a lower energy configuration.¹⁻³ Isolated nucleons do not convert spontaneously into strange matter since the latter is a higher energy state for low baryon numbers. But a nucleon that penetrates a region of stable strange matter will lose tens of MeV in energy by converting; it will disassemble into its quark components that will become part of the strange matter quark sea. Neutrons easily penetrate regions of strange matter, while protons have to overcome the strange matter Coulomb barrier.

Once a strange matter seed is formed inside a neutron star all nuclear matter with densities higher than the neutron drip density ($\sim 4 \times 10^{11} \text{ g cm}^{-3}$) will be converted. Neutron drip occurs when free neutrons are energetically favored in nuclear matter, so they start to "drip out of" the nuclei. The Coulomb barrier prevents nuclear matter of densities below neutron drip from penetrating the strange matter region.³ In neutron stars, the outermost layer of the crust has densities below neutron drip and can, therefore, survive the conversion. The history of formation and, in particular, the speed of conversion, will determine whether the final strange star has a bare strange matter

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surface or a thin crust. Some of the mechanisms by which a seed of strange matter can be formed were discussed in Ref. [3]: via two-flavor quark matter, clustering of lambdas, burning of neutron matter, neutrino sparking, and seeding from the outside. The present letter, starting with the assumption that seeding has occurred, studies the consequent conversion of the rest of the star.

II. THE CONVERSION SPEED

The strange matter region is electrically neutral and has a constant baryon number density, n , if small gravitational effects are ignored. We can then write:

$$3n = n_u + n_d + n_s \quad (1)$$

$$2n_u = n_d + n_s + 3n_e, \quad (2)$$

where n_i = number density of particle i (u, d, s, and e stand for up, down, strange quarks, and electrons).

If we neglect interactions the number density for each quark is $n_i = \frac{\mu_i^3}{\pi^2}$, where μ_i is the chemical potential of quark i . In equilibrium strange matter, $\mu_i \sim 300$ MeV. The up and down quark masses can be neglected, while the strange quark mass, m_s , introduces corrections of second order in $(\frac{m_s}{\mu_s})$. For simplicity we will neglect m_s and with it the small electron fraction. Then (2) becomes:

$$2n_u = n_d + n_s, \quad (3)$$

and, therefore:

$$n_u = n \quad (4)$$

$$n_d + n_s = 2n. \quad (5)$$

In equilibrium, $\mu_d = \mu_s$ and the three number densities are equal, i.e. $n_u = n_d = n_s$. Out of equilibrium, constraints (4) and (5) leave only one free parameter. (In this picture μ_u is constant, so we will use $\mu_u \equiv \mu \approx 300$ MeV.)

The problem of finding the speed at which neutron matter converts into strange matter is simpler when viewed from the rest frame of the conversion front. The volume in which strange matter equilibrates is assumed to be much smaller than that of the total strange matter region, so that the problem can be treated one-dimensionally. In this frame the front is at $x = 0$, neutron matter is at $x < 0$, strange matter at $x > 0$, and asymptotically ($x \rightarrow \infty$) strange matter is in equilibrium. The small positive x region has an excess of down quarks relative to strange quarks given that neutrons (udd) are swallowed at $x = 0$. The excess down quarks will convert into strange quarks via the weak process $d + u \rightarrow s + u$, as long as $\mu_d > \mu_s$ ($n_d > n_s$). For large positive x the system is close to equilibrium, so $\mu_d \approx \mu_s$ and $n_d \approx n_s \approx n_u$.

We can define:

$$a(x) = \frac{n_d(x) - n_s(x)}{2n}, \quad (6)$$

such that, for neutron matter of density n , $a(x < 0) = 1$, and, as $x \rightarrow \infty$, $a(x) \rightarrow 0$. As $x \rightarrow 0$, from the strange matter side ($x > 0$), $a \rightarrow a_0$, where a_0 is the maximum, a for which strange matter is stable. The value of a_0 corresponds to the minimum number density of strange quarks such that strange matter is stable. It is necessary that strange matter next to the conversion front be stable, so that the front can move on swallowing more neutrons. The limit $a_0 \rightarrow 0$ corresponds to the case where only strange matter with $n_d = n_s$ can be stable. Then the front cannot move given that the swallowed neutrons do not have $n_d = n_s$. The case where $a_0 \rightarrow 1$ corresponds to two-flavor quark matter being stable; the neutrons can all convert instantaneously and the front moves with infinite speed (i.e., there is really no front). Two-flavor quark matter is known to be unstable; nuclei with baryon number A are made of A nucleons and not of $3A$ quarks in a single bag. Therefore we expect $0 < a_0 < 1$.

The process can be thought of as a fluid of excess down quarks coming from the left at a velocity equal in magnitude to that of the front and becoming, asymptotically, equilibrium strange matter ($a \rightarrow 0$). This transformation occurs mainly via two processes:

1. "decay" of down quarks into strange quarks via $d + u \rightarrow s + u$ ($\mu_d > \mu_s$): so

$$\frac{da}{dt} = -R(a), \quad (7)$$

where $nR(a)$ is the rate at which down quarks convert into strange quarks, i.e. $nR(a) = \frac{dn_d}{dt} = -\frac{dn_s}{dt}$.

2. diffusion of the strange quarks from the right to the left

$$\frac{da}{dt} = D \frac{d^2 a}{dx^2}, \quad (8)$$

where D is the diffusion coefficient.

The total derivative corresponds to the change in time of $a(x)$ with respect to a point moving along with the fluid. In general, $\frac{da}{dt} = \frac{\partial a}{\partial t} + \vec{v} \cdot \nabla a$ so, for the unidimensional steady state solution, $\frac{da}{dt} = v \frac{da}{dx}$. Together with (7) and (8), we get:

$$Da'' - va' - R(a) = 0, \quad (9)$$

where $a' \equiv \frac{da}{dx}$, with boundary conditions

$$a(0) = a_0 \quad (10)$$

and

$$a(x \rightarrow \infty) \rightarrow 0. \quad (11)$$

Conservation of baryon number at $x = 0$ gives rise to another boundary condition. The flux of neutrons coming from the left, $n_N v_N$, has to equal the flux of baryon number arriving at the right, i.e.,

$$n_N v_N = n v, \quad (12)$$

where n_N = density of neutron matter, and v_N = velocity at which neutrons arrive at the front in its rest frame. Integrating (9) over the volume of a small cylinder whose axis coincides with z , and letting its axis go to zero, we find:

$$Dna'|_{0^+} = vna|_{0^+} - vna|_{0^-} \quad (13)$$

If $n_N = n$, then $v_N = v$ and (13) implies:

$$a'(0) = -\frac{v}{D}(1 - a_0) \quad (14)$$

In this case v is determined by the solution of the differential equation (9) which can satisfy the overdetermined boundary conditions (10), (11), and (14); this leaves only a special value for v .

The density of neutrons can be less than that of strange matter. In this case the front's speed is given by baryon number conservation:

$$v = n_N v_N / n \quad (15)$$

Then $a(0^+)$ will be determined by the solution of (9) that satisfies (10), (11), and (14). i.e., $a(0^+)$ will be a function of the flux of neutrons. If $a(0^+) > a_0$, then this way is greater than the maximum allowed for stability of strange matter (i.e., if $a(0^+) > a_0$), neutrons will not be absorbed as fast as they arrive at the front. There will be an accumulation of neutrons next to the front that will tend to make $n_N \rightarrow n$. As $n_N \rightarrow n$, v is again determined by the solution of (9), (10), (11), and (14). $v = v(a_0)$. Maximum $a(0^+)$ (i.e., a_0), corresponds to maximum v ; the speed determined when $n_N = n$ is the fastest the front can move.

The decay rate can be calculated by integrating the square of the matrix element for the process $d + u \rightarrow s + u$ over the phase space of the four particles with the appropriate fermi factors. Explicitly:

$$nR(a) = \int \frac{dp_d dp_u dp_s}{(2\pi)^{12} 2^4 E_d E_u E_s E_s} (2\pi)^4 \delta^4(P_d + P_u - P_{u_2} - P_s) \times |M_{d \rightarrow s}|^2 f(E_d) f(E_u) (1 - f(E_{u_2})) (1 - f(E_s)) \quad (16)$$

where p_i , E_i , P_i are the momentum, energy and four-momentum for the quark i , respectively, and the fermi factor $f(E_i) = (1 + e^{-\beta(E_i - \mu_i)})^{-1}$. The matrix element is given by:

$$|M_{d \rightarrow s}|^2 = 2^7 G_F^2 \cos^2 \theta_c \sin^2 \theta_c (P_d \cdot P_{u_2}) (P_s \cdot P_{u_1}) \quad (17)$$

where G_F is the fermi constant, and θ_c the Cabbibo angle. Substituting (17) into (16) and integrating in the zero temperature limit (i.e., $f(E_i) \rightarrow \theta(\mu_i - E_i)$), we get:

$$R(a) = \frac{16}{9\pi^4} G_F^2 \cos^2 \theta_c \sin^2 \theta_c \mu^5 \operatorname{sgn}(a) a^2 \quad (18)$$

Therefore we can write:

$$R(a) = \operatorname{sgn}(a) \frac{a^2}{\tau} \quad (19)$$

where

$$\tau \simeq 4.7 \times 10^{-9} \text{ sec} \quad (20)$$

To solve (9) with (10), (11), and (14) boundary conditions we can rewrite (9) by defining $x = \eta t$ and choosing $\eta = D/v$. Then (10) reduces to:

$$\ddot{a} - \dot{a} - \operatorname{sgn}(a) g a^2 = 0 \quad (21)$$

where $\dot{a} \equiv \frac{da}{dt}$ and g is defined to be:

$$g \equiv \frac{D}{\tau v^2} \quad (22)$$

Equation (14) now reads:

$$\dot{a}(0) \equiv \dot{a}_0 = -(1 - a_0) \quad (23)$$

This equation is analogous to a classical mechanics problem in which the acceleration, $\ddot{a} = \dot{a} + \operatorname{sgn}(a) g a^2$, is equivalent to a negative friction force plus a conservative force. The conservative force can be derived from the potential:

$$V(a) = -\frac{\operatorname{sgn}(a) g a^3}{a_0^3} \quad (24)$$

and the solution corresponds to the unit mass particle that starts at a_0 with speed $\dot{a}_0 = -(1 - a_0)$ and arrives at $a = 0$ at $t \rightarrow \infty$. For a given a_0 and $\dot{a}_0$, small g overshoots, while large g undershoots. Numerically, we can find a curve for $g(a_0)$ by narrowing down the region in which a small variation in g takes us from undershooting to overshooting or vice-versa. The numerical result (plotted in Fig. 1) is very well fit by the following estimate.

We expect the solution $a(x)$ to be monotonically decreasing, and $\dot{a}(a)$ to vary monotonically from $\dot{a}(a_0) = -(1 - a_0)$ to $\dot{a}(0) = 0$. We can estimate the work done by the friction force, W_f , by approximating the integral of $\dot{a}$ over a by the integral over a straight line, i.e.:

$$W_f = \int \vec{F} \cdot d\vec{r} = \int_{a_0}^0 \dot{a} da \simeq \frac{\dot{a}_0 a_0}{2}$$

Energetics then would give us

$$\begin{aligned} \frac{\dot{a}_0^2}{2} - \frac{g a_0^3}{3} - \frac{\dot{a}_0 a_0}{2} &= 0 \\ g &= \frac{3}{2} \frac{(1 - a_0)}{a_0^3} \end{aligned} \quad (25)$$

which implies:

III. DISCUSSION

The uncertainties present in strong-interaction calculations leave the precise value of a_0 undetermined. Reasonable values for a_0 lie in the range $0 < a_0 \leq 0.5$,⁴ where larger values of a_0 correspond to stability of strange matter with fewer strange quarks. An optimistic guess for a_0 would be $a_0 = 0.5$, a moderate one $a_0 = 0.1$, and a conservative one $a_0 = 0.01$. For these three values of a_0 , $\frac{1}{\sqrt{g}}$ is equal to 0.41 , 2.7×10^{-3} , and 8.2×10^{-4} , respectively.

Neutron stars are born in supernovae with temperatures around $10 - 20$ MeV and cool via neutrino and photon emission. The conversion to strange stars is qualitatively different in different stages of neutron stars' evolution.

SUPERNOVAE

The birth of neutron stars is associated with the collapse of massive stars that undergo supernova explosions. The mechanism by which these explosions take place is not fully understood. Calculations of the evolution of nonrotating stars with masses between $10 \leq \frac{M}{M_\odot} \leq 70$ exhibit the development of an unstable core, containing roughly $1.4M_\odot$ of iron, which collapses dynamically.⁶ This suggests that the collapsed core becomes an $\sim 1.4M_\odot$ neutron star (of radius ~ 10 km), or a black hole, while the rest of the massive star explodes away. The time scale for the collapse is $\sim 4 \times 10^{-3}$ s (free fall).

At this stage, the conversion speed, given by (28), ranges from $\sim 57m/s$ to $\sim 11cm/s$, where we used $T = 10$ MeV and $a_0 = 0.5$ and 0.01 , respectively. The conversion of a 10 km radius star is too slow to influence the explosion. If the seeding of strange matter occurs in a small region, the effects of the conversion will be felt over a period that ranges from 3 minutes to a day. The conversion liberates $\sim 3 \times 10^{58}$ MeV of energy if we assume that 20 MeV is liberated when a neutron converts to strange matter. This energy will be radiated mainly as neutrinos, photons and e^+e^- pairs.

On the other hand, it is reasonable to expect that the seeding of strange matter occurs globally in the collapsed core. Given the extremely high densities attained at core-bounce, a phase transition to two-flavor quark matter is likely to occur in a large volume of the star. In this case our conversion speed is less relevant; the conversion occurs globally in a weak interaction timescale $\tau \sim 5 \times 10^{-9}$ s and the energy liberated will influence the explosion.

HOT NEUTRON STARS

X-ray satellite observations from the Einstein Observatory determined upper limits on the surface temperatures of about 50 supernova remnants. Seven of them are historical remnants with known ages; for stars of $\sim 10^2 - 10^4$ years, upper limits to the surface temperature range from $\sim 10^5 - 10^6$ K. Models for the temperature profile of neutron stars show an approximately isothermal behavior up to the outermost layer, where there is a drop of two orders of magnitude. Therefore, the observed remnants have neutron stars with internal temperatures up to $\sim 10^8$ K.

The conversion speed for these stars ranges from 66 km/s ($a_0 = 0.5$) to 130 m/s ($a_0 = 0.01$), for $T = 10^8$ K. The conversion generates heat at the front that

Fig.1: A plot of $\log g$ versus $\log a_0$.

and finally:

$$v \approx \sqrt{\frac{D}{\tau}} \frac{2a_0^3}{3(1-a_0)} \quad (26)$$

The diffusion coefficient is roughly equal to a third of the average speed of the quarks times their mean free path, λ , in equilibrium strange matter. The quarks move close to the speed of light and their mean free path can be estimated by finding the rate at which they scatter off each other. This rate has a temperature dependence that is mainly determined by the phase space integral of the fermi factors and the energy-momentum conservation delta-function. It is found to be proportional to $\left(\frac{T}{\mu}\right)^2$ for T significantly smaller than μ . A reasonable estimate of the mean free path is $\lambda \approx 1$ fm $\left(\frac{\mu}{T}\right)^2$. Therefore,

$$D = \frac{\bar{v}\lambda}{3} \approx 10^{-3} \frac{cm^2}{s} \left(\frac{\mu}{T}\right)^2 \quad (27)$$

Finally, the speed at which the conversion front moves is found to be:^{*}

$$v \approx \frac{1}{\sqrt{g}} \left(\frac{\mu}{T}\right)^{4.6} \frac{m}{s} \quad (28)$$

* Baym *et al.*⁵ estimated this conversion speed; the results are formally similar but numerically distinct. They also state that the front will move faster in lower-density regions; we found that the opposite is true.

increases the temperature around it. So the temperature and, therefore, the diffusion coefficient are really functions of the distance to the front. We expect the conduction of heat in both strange and nuclear matters to be extremely fast compared to the front's speed, so that the heat generated at the front is distributed throughout the star and the increase in temperature is small. (If the conduction is not fast enough the conversion will be slower, somewhere between the above estimates and the estimates for supernovae temperatures.)

Some of these measured supernovae remnants have associated pulsars. Pulsars are believed to be rotating neutron stars whose strong magnetic fields create electromagnetic pulses. Their periods range from ~ 4.3 s to 1.6 ms. If they are (not yet converted) neutron stars, the conversion to strange stars can be detected as a change in their periods, a glitch. The conversion will take from 0.15 s to 77 s and will change the density profile of the star; consequently, the star's moment of inertia, and, therefore, period of rotation will change. To determine the change in moment of inertia due to conversion, it is necessary to know the equation of state that describes neutron stars.

If neutron stars are described by a stiff equation of state, the moment of inertia decreases with the conversion, while with a soft equation of state it increases. For example: the stiff equation of state referred to as T1 (tensor interaction) in Ref [6] implies that a $1.4M_{\odot}$ neutron star has a moment of inertia of $\sim 2.2 \times 10^{45}$ g cm²; for the soft Reid equation of state (R in Ref.[6]), the moment of inertia of a $1.4 M_{\odot}$ star is $\sim .99 \times 10^{45}$ g cm².⁶ Finally, for a $1.4 M_{\odot}$ strange star the moment of inertia is $\sim 1.9 \times 10^{45}$ g cm².³ For the above examples the changes on the angular speed over its initial value are $\frac{\Delta\Omega}{\Omega} = -0.14$ and $+1.01$ (for the stiff and soft equations of state respectively). Models for neutron stars like the "Bethe-Johnson model" and the "three-nucleon interaction model" (as in Ref.[6]) give equations of state that imply smaller changes on $\frac{\Delta\Omega}{\Omega}$ due to conversion for a $1.4 M_{\odot}$ star. In any case, these changes are considerably bigger than the observed glitches in pulsars (that range from $\sim 10^{-6}$ to 10^{-8}). A glitch due to conversion will be much stronger and will occur only once (unlike the observed ones that reoccur). If such a glitch is observed, it will be evidence for the existence of strange matter and its magnitude will decide on the equation of state for the original neutron star.

The conversion of hot neutron stars is very slow compared to the speed of sound in either neutron or strange stars. Therefore, the process is mild process (in fluid dynamics terms), the star can readjust rapidly to the change of density, and regions of density lower than neutron drip will survive after conversion. The material at these densities will form the thin crust of the final strange star.³

COLD NEUTRON STARS

Neutron stars older than 10^7 years have temperatures $\leq 10^5$ K, and cool faster from this age on, since photon emission becomes more efficient than neutrino emission.⁶ This is also the age at which pulsars are expected to have "decayed": the energy lost in electromagnetic radiation slows down the star. Therefore these objects are very hard to detect.

These cold and usually quiet objects can convert to strange matter with speeds ranging from 130 km/s to 6.6×10^4 km/s $= .22 c$ (for $a_0 = 0.01$, and 0.5 respectively). The estimate with $a_0 = 0.5$ will lead to a conversion violent enough to destroy the outer crust and convert the material that did not escape from the star into strange matter. A bare strange star can be formed in this case. The outcome of such an event will be the emission of as much as 10^{58} MeV of energy in .15ms as radiation with a typical temperature of tens of MeV. This conversion can be observed as a gamma-ray burst.

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