CHARGE ASYMMETRY OF $\mu^+\mu^-$

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The measurement of the charge asymmetry of the $\mu^+\mu^-$ state on top of the $Z^0$ has long been considered as a fundamental test of the standard model. To contribute useful information, however, it should provide a value of $\sin^2 \theta_W$ with a relative error approaching 1%. Here we show that this is indeed possible.

The asymmetry is given by the general formula,

$$A_{ch} = \frac{3}{2} \frac{A^2 B_1 + 2V^2 A^2 B_2}{1 + 2V^2 B_1 + (V^2 + A^2) B_2},$$

where

$$B_1 = \frac{g m^2 s (s - m^2)}{(s - m^2)^2 + m^2 r^2} \quad \text{and} \quad B_2 = \frac{g m^4 s^2}{(s - m^2)^2 + m^2 r^2}.$$

On top of the $Z^0$ resonance ($s = m^2$), $B_1 = 0$ and $B_2 = R = (g m^2 / r)^2$, where $g = 4.5 \times 10^{-5}$. In the standard model, $A = 1$ and $V = 1 - 4 \sin^2 \theta_W$. We assume that $V_\mu = V_\tau = V$. The total cross-section is

$$\sigma_{tot} = \frac{4\pi \alpha^2}{3s} \left[ 1 + 2V^2 B_1 + (V^2 + A^2) B_2 \right].$$

We assume that a first scan of the $Z^0$ has given its mass and width with the accuracy quoted previously, as well as $\sigma_{tot}$. We come back on top of the resonance and accumulate data during $N$ days. Figure 1, giving $\Delta V/V$ across the

![Fig. 1 Statistical error $\Delta V/V$](image)

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resonance versus the value it takes on top of it, for a given amount of time, shows that we should sit on top of the $Z^0$.

With our assumption regarding the luminosity, we register $\sim 10^3 \mu^+\mu^-$ pairs per day in the full solid angle. The expected integrated asymmetry is, on top of the $Z^0$,

$$A_{\text{ch}} = 3V^2.$$  

Figure 2 shows this as a function of $\sin^2 \theta_w$, which in our case ($\sin^2 \theta_w = 0.23$) amounts to $\sim 2\%$. For our choice of $\theta_w$, the relation giving the error on $\sin^2 \theta_w$ can be written:

$$\frac{d \sin^2 \theta_w}{\sin^2 \theta_w} = \left(1 - 4 \frac{\sin^2 \theta_w}{4 \sin^2 \theta_w}\right) \frac{dV}{V} = 0.09 \frac{dV}{V}.$$  

Differentiating the formulae for $\sigma_{\text{tot}}$ and $A_{\text{ch}}$, we get in full generality,

$$\frac{dV}{V} = \frac{1}{2} \frac{dA_{\text{ch}}}{A_{\text{ch}}} + \frac{1}{4} \frac{d\sigma_{\text{tot}}}{\sigma_{\text{tot}}} + \frac{1}{2} \frac{d\Gamma}{\Gamma} - \frac{1}{4} \frac{1}{\sqrt{s}} \frac{d(s-m^2)}{s}.$$  

For $\sin^2 \theta_w = 0.23$ the last coefficient is numerically equal to $\sim 100$. We note that, as expected, the absolute value of $s$ does not matter here: what counts is the uncertainty on the distance of the measurement from the $Z$ pole. We adopt $\Delta E/E = 2 \times 10^{-4}$ for the uncertainty on the reproducibility and stability of the LEP energy. The corresponding relative error on $\sin^2 \theta_w$ is $< 0.4\%$. Errors due to $\Delta \sigma$ and $\Delta \Gamma$ are much less important.

The main uncertainty will be due to the statistical and the systematic error on $A_{\text{ch}}$. A total of 200 days of exposure will bring the statistical error to what is shown in Fig. 2, i.e. $0.5\%$ for $s^2 = 0.23$. One must demonstrate that the systematic error can reach a similar level.

![Fig. 2 Values of the charge asymmetry and of the relative error on $\sin^2 \theta_w$, as a function of $\sin^2 \theta_w$.](image)
This implies that
\[
\frac{\Delta A_{\text{ch}}}{A_{\text{ch}}}_{\text{syst}} \approx -11\% \quad \text{or} \quad \Delta A_{\text{ch}} = 0.22\%.
\]

It has been shown that a careful treatment of the radiative corrections can in principle bring the corresponding uncertainty to the 0.2% level; there is no fundamental problem, but much work and good co-operation between experiment and theory are needed.

Table 1 gives the published values of the systematic error on \( A_{\text{ch}} \) of the PETRA and PEP experiments [1-4]. Also indicated are the main sources of uncertainty according to the authors. We note that several e\(^+\)e\(^-\) experiments, absent from the table, have not fully pushed their study of systematics (because of their large statistical errors), and confine themselves to safe upper limits.

Several sources of uncertainty present at PETRA and PEP will have disappeared at LEP:

i) Since \( R_{\mu\mu} \approx 180 \) on the \( Z^0 \), the \( \gamma\gamma \) and cosmic-ray backgrounds will be negligible;

ii) since in their respective domains of energy the momentum resolution of LEP detectors will be better than that of PETRA-PEP detectors, charge mis-identification will no longer occur;

iii) confusion between \( Z^0 \to e^+e^- \) and \( Z^0 \to \mu^+\mu^- \) seems to be totally excluded in LEP detectors;

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**Table 1**

The systematic error on \( A_{\text{ch}} \) in four PETRA and PEP experiments, and the main contributions to these errors

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Syst. error (%)</th>
<th>( s ) (GeV)</th>
<th>Source</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAC</td>
<td>0.3</td>
<td>29</td>
<td>Mainly Bhabha</td>
<td>1</td>
</tr>
<tr>
<td>TASSO</td>
<td>0.5</td>
<td>34</td>
<td>Radiative corrections</td>
<td>2</td>
</tr>
<tr>
<td>HRS</td>
<td>0.5</td>
<td>29</td>
<td>Bhabha + possible experimental bias</td>
<td>3</td>
</tr>
<tr>
<td>JADE</td>
<td>( \pm 0.9 )</td>
<td>34</td>
<td>Error in charge determination is dominant</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>( -0.5 )</td>
<td>42</td>
<td></td>
<td>4</td>
</tr>
</tbody>
</table>

| JADE | 1.5 | 42 |         | 4   |
iv) the contamination from $\tau^+\tau^-$, which is also dependent on the momentum resolution, will be very small: furthermore, $\tau^+\tau^-$ pairs have in principle the same asymmetry.

We are therefore left with systematic errors due to the detector itself. At a given $(\theta, \phi)$ the detector may, for various reasons, have a slightly different acceptance or reconstruction efficiency for positive and negative tracks. In principle such an effect could be eliminated by running for half of the time with opposite polarity of the solenoid. However, part of the effect would remain—for instance, a possible effect depending on $E \times B$. Furthermore, the procedure may be cumbersome. But we can argue that the sophistication of LEP detectors, in particular of their trigger system, and the large counting rates available on top of the $Z^0$, would make it possible to measure such a bias (if it exists) with a sufficient accuracy.

For the $\mu$ trigger and detection, let us mention two separate functions:

i) the reconstruction of the $\mu$ in the tracking detectors;

ii) its signature in the calorimeters and $\mu$ identifier.

To check a possible bias in (i), we can use $e^+e^-$ events triggered in a way which does not involve the tracking devices, for instance by requiring large energy depositions in the e.m. calorimeters. For (ii), we can do the same using the $\mu^+\mu^-$ events themselves if, at the trigger level, identification is required on a single leg.

These are the usual procedures in the presently running $e^+e^-$ detectors, and we feel that such trigger modes will be quite possible at LEP.

If $f^+$ and $f^-$ are the efficiencies (integrated over solid angle and time) for positive and negative tracks, let us define

$$\epsilon = \frac{1}{2} (f^--f^+) \text{ (small)},$$

and

$$\overline{\epsilon} = \frac{1}{2} (f^++f^-) \approx 1.$$

It is easily found that $\Delta A_{\text{ch}} = A_{\text{meas}} - A_{\text{true}} = \epsilon / \overline{\epsilon} \approx \epsilon$. To reach $\Delta A_{\text{ch}} = 0.2\%$, it would therefore be necessary to measure a possible difference of acceptance of $0.4\%$.

Let us suppose that we register $10^5 e^+e^-$ as described above. Assuming $\overline{\epsilon} = 0.97$ (typical for existing detectors) we will miss the reconstruction of \sim 3000 tracks of each sign. A difference of 10\% on these numbers is quite noticeable with enough statistical significance.

We do not see that it will be much of a problem to reach the level quoted. Therefore we feel that a long running period, which gives a small statistical error and thereby a small systematic one on $A_{\text{ch}}$, is fully justified: 100 days
would allow us to approach the 1% limit on $\Delta \sin^2 \theta / \sin^2 \theta$, and 200 days would allow us to get below it.

This discussion could be repeated for the case where one beam is longitudinally polarized with polarization $P_e$ (~60%).

The formula giving the asymmetry (see G.Altarelli's paper [5]) is now linear in $V$: $A_{\text{ch}}^\text{pol} = 1.4 V P_e$. The asymmetry is substantially larger than without polarization (~4 times larger on top of the $Z^0$ for $\sin^2 \theta_W = 0.23$). Therefore less time is needed to reach the same relative statistical accuracy.

Since here

$$\frac{\Delta V}{V} = \frac{\Delta A_{\text{ch}}}{A_{\text{ch}}} - \frac{\Delta P_e}{P_e},$$

the same error $\Delta A_{\text{ch}}$ gives a smaller error on $\Delta V/V$ than in the previous case (half the amount for our choice). However, $\Delta P_e/P_e$ should also be kept below the 10% level and possibly reach 5%.

Finally, we recall that longitudinal polarization makes it possible to perform a different type of asymmetry measurement, which consists in measuring the difference in counting rate when the polarization is flipped. Any kind of final state can be used, with no need to distinguish between charges. The statistical and systematic errors (from the detector side) are much smaller, but knowledge of the polarization, of the possible secondary effects related to its manipulation, and of the normalization has to be good.

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REFERENCES

[5] G. Altarelli, Precision tests of the electroweak theory at the $Z^0$, this report.