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Effective Lagrangians For Four Dimensional Superstrings*

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Abstract

We derive effective supergravity lagrangians for the massless sector of new four dimensional superstrings. The untwisted sector is obtained by suitable Z_2 or $Z_3 \otimes Z_2$ truncations of $N = 4$, $D = 4$ lagrangians. For the $N = 1$ "minimal" models, related to the $N = 4$, $SO(44)$ superstrings, we derive the Kähler potential, the superpotential and the f gauge kinetic function. The underlying nonlinear sigma model is based on the Kähler manifold $\frac{SU(1,1)}{U(1)} \otimes \prod_{i=1}^3 \frac{SO(2, n_i)}{SO(2) \otimes SO(n_i)}$. For models without massless twisted sector, this result provides the full superstring low energy effective action. For models with massless twisted sector, the $N = 1$ theory may be obtained once the quaternionic manifold of the corresponding $N = 2$ hypermultiplets is specified.

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Physically relevant four dimensional superstring models can be obtained either from the known ten dimensional superstrings,^{1,2} by compactification on a suitable six dimensional space, e.g. Calabi-Yau manifolds³ or orbifolds,⁴ or by construction of superstring models directly defined in four space-time dimensions.^{5,6,7,8}

Semirealistic models with chiral fermions and one supersymmetry can be obtained in both approaches. It is thus worthwhile to examine the essential properties of the resulting models, by studying the effective theories which describe their massless mode interactions. For the Calabi-Yau and orbifold compactifications, some approximate effective lagrangians have been already suggested in the literature.^{9,10}

The aim of this work is to derive the effective $N = 2$ and $N = 1$ supergravities, which are induced by the recently constructed four dimensional superstring models.^{7,8} The method we use, is based on some general properties of the models given in refs. [7, 8].

A relatively easy way of constructing four dimensional string models, is to introduce two dimensional free fermions as internal string degrees of freedom, carrying a non linear realization of the world-sheet supersymmetry.⁶ The consistency of these models on higher genus surfaces,¹¹ strongly constraints the permitted choices on the fermionic spin structures.^{7,8,12} As it was shown in ref. [8], modular invariance and factorization are sufficient to define a consistent string theory with correct spin-statistic and sensible particle interpretation.¹³

An equivalent bosonic construction of the same models, by means of a toroidal compactification^{14,7} is also known. We examine here the class of models obtained by using free Majorana-Weyl world-sheet fermions, with periodic and antiperiodic boundary conditions.⁸ In this case, modular invariance and factorization constraints define four dimensional models which are characterized by a set of fermion number projections^{7,8} (-generalizations of the GSO¹⁵ projection which is necessary for the consistency of the ten dimensional superstrings).

If we consider an $N = 4, 4 - D$ superstring model, defined by n fermionic number projections (Z_n^f), we can obtain an $N = 2$ model with an extra Z_2 operation with the following characteristics:⁸

- i) There is a sector of the $N = 2$ model which is obtained by a Z_2 truncation of the corresponding $N = 4$ model. We call this sector the "untwisted sector" (UTS). (This in analogy with the orbifold compactification.)

- ii) There is always an additional sector, necessary for the modular invariance, which cannot be obtained by any Z_2 truncation of the corresponding $N = 4$ model. This sector is by itself Z_2 symmetric and we call it "twisted sector" (TS).

Similarly, we can obtain an $N = 1$ model, with two extra $Z_2 : Z_2 \otimes Z_2$, starting from an $N = 4$ model.^{7,8} Here also, there are TS and UTS sectors. We write schematically:

$$\mathcal{L}^{N=4} \xrightarrow{(Z_2)} \mathcal{L}^{N=2} \xrightarrow{(Z_2^{UTS})} \mathcal{L}^{N=1} \xrightarrow{(Z_2^{UTS})} \mathcal{L}^{N=1} \quad (1)$$

In some of the $N = 2$ and $N = 1$ models, the TS sector contains only massive states, and therefore does not contribute to the effective supergravity theory. In those cases, the entire $N = 2$ and $N = 1$ effective theories are given, once the corresponding $N = 4$ ones are known, and once the action of Z_2 and $Z_2 \otimes Z_2$ is defined. Because the $N = 4$ effective theory is entirely specified by the gauge group,¹⁶ it follows that the complete $N = 2$ and $N = 1$ models are given, once the corresponding $N = 4$ effective theory is known.

In the more general situation where a massless twisted sector is also present, the $N = 1$ effective Lagrangian can be obtained by Z_2^f truncation of the $N = 2$ theory. In fact, due to $N = 2$ supersymmetry, the TS is uniquely specified by the quaternionic manifold of the hypermultiplets and has a universal trilinear "superpotential" function dictated by $N = 2$ global supersymmetry. In what follows, we will restrict ourselves to the $N = 2$ and $N = 1$ models which are obtained from the "minimal" $N = 4$, $SO(4,4)$ superstring model. Other models can be similarly treated.

Following the rules of ref. [8], one can easily find that the "minimal" $N = 2$ models are based on a gauge group which is the product of two orthogonal groups.⁸

corresponding $N = 4$, $SO(44)$; $N = 2$, $SO(n_A) \otimes SO(n_B)$; and $N = 1$, $\bigotimes_{i=0}^3 SO(n_i)$

massless (UTS) spectra. In all cases the states are given as a direct product of the left and right moving states (in our conventions the left states satisfy a world sheet supersymmetry while the right ones don't). The $N = 4$, $SO(44)$ massless states are:

$$(\mu > +|I_6 > +|i_4 >_a)_L \otimes (|\bar{\mu} > +|A_{44} >)_R \quad (5)$$

where $|\mu >_L$ and $|\bar{\mu} >_R$ are left (right) four dimensional space time vectors in the light cone gauge $(\mu, \bar{\mu} = 3, 4)$. $|I_6 >$ is a $\mathfrak{g}$ vector of $SO(6) \simeq SU(4)$. $|i_4 >_a$ carries a space-time spinor index α , and is in the $\mathbf{4}$ of $SU(4)$. $|A_{44} >$ is in the adjoint of $SO(44)$. The $N = 2$, $SO(n_A) \times SO(n_B)$ UTS massless states are given by:⁸

$$\begin{aligned} &(|\mu > +|I_2 > +|i_1 >_a)_L \otimes (|\bar{\mu} > +|A_4 > +|A_B >)_R + \\ &(|I_4 > +|i_2 >_a)_L \otimes (|V_A \otimes V_B >)_R \end{aligned} \quad (6)$$

where $I_2 = 1, 2$, $I_4 = 3, 4, 5, 6$ are the split of $I_6 = (I_2, I_4)$ in the decomposition $SO(6) \rightarrow SO(2) \times SO(4)$, $i_1 = 0, 1, i_2 = 2, 3$ are the split of $i_4 = (i_1, i_2)$ ($i_4 \in \mathfrak{g}$ of $SO(6)$). $|V_A \otimes V_B >$ is the vectorial $\otimes$ vectorial of $SO(n_A) \otimes SO(n_B)$; $|A_4 >$ and $|A_B >$ are in the adjoints of $SO(n_A)$ and $SO(n_B)$. Notice the splitting of $|A_{44} > = |A_4 > + |A_B > + |V_A \otimes V_B >$ under the decomposition of $SO(44) \rightarrow SO(n_A) \otimes SO(n_B)$. From (5) and (6) one derives easily the action of Z_2 on the $SO(6)$ and $SO(44)$ representations.

$$\mathbf{4} \quad (|i_1 >, |i_2 >) \xrightarrow{Z_2} (|i_1 >, \alpha|i_2 >) \quad (7a)$$

$$\mathfrak{g} \quad (|I_2 >, |I_4 >) \xrightarrow{Z_2} (|I_2 >, \alpha|I_4 >) \quad (7b)$$

$$\mathbf{44} \quad (|V_A >, |V_B >) \xrightarrow{Z_2} (|V_A >, \alpha|V_B >) \quad (7c)$$

$$\text{adj } SO(44) \quad (|A_4 >, |A_B >, |V_A \otimes V_B >) \xrightarrow{Z_2} (|A_4 >, |A_B >, \alpha|V_A \otimes V_B >) \quad (7d)$$

where $\alpha \mathcal{Z}_2, \alpha^2 = 1$.

Once the Z_2 action is defined, the $N = 2$ effective theory $\mathcal{L}_{UTS}^{N=2}$ is obtained by keeping only the Z_2 invariant fields of the $N = 4$ supergravity of ref. [16]. It is sufficient to study the scalar sector only, because the $N = 2$ theory is entirely determined by them via $N = 2$ supersymmetry transformations.¹⁷ The Z_2 operation becomes even more straightforward, once we use the $\mathcal{L}^{N=4}$ written in terms of the

$$N = 2 \text{ "minimal" : } SO(n_A) \otimes SO(n_B)$$

$$\text{with : } n_A + n_B = 44, \quad n_A = 2r + 8m \quad n_A, n_B \neq 0$$

$$r \in \{0, 1, 2\}, \quad m \in \{0, 1, 2, 3, 4, 5\}, \quad (2)$$

while the "minimal" $N = 1$ models, are based on a product of four orthogonal groups:⁸

$$N = 1 \text{ "minimal" : } SO(n_0) \otimes SO(n_1) \otimes SO(n_2) \otimes SO(n_3)$$

$$\text{with } n_0 = 2 + 4k, \quad n_0 + n_1 = 0 \pmod{8}, \quad n_0 + n_1 + n_2 + n_3 = 44. \quad (3)$$

The $N = 2$ models represent all the possible ones, constructed with Majorana-Weyl world-sheet fermions, and having no more than two factor groups. The $N = 1$ models are those with four factor groups at most, and allow for fields in complex representations.

The restrictions on n_A, n_B and n_0, n_i , are necessary for the consistency of the models and the breakdown: $SO(44) \rightarrow SO(n_A) \otimes SO(n_B) \rightarrow \bigotimes_{i=0}^3 SO(n_i)$; together with the breakdown of supersymmetry $N = 4 \rightarrow N = 2 \rightarrow N = 1$.⁸ Always following ref. [8], the TS sector contains massless states only when:

$$n_A, n_B = 8 \text{ or } 16; \quad N = 2 \text{ case} \quad (4a)$$

$$n_0 + n_1 = 8 \text{ or } 16 \text{ for at least one } i = \{1, 2, 3\} \quad N = 1 \text{ case.} \quad (4b)$$

So, whenever the conditions (4a) and (4b) are not satisfied, the TS does not contribute to the effective theory, and the $N = 2$ or $N = 1$ effective theories are given as Z_2 or $Z_2 \otimes Z_2'$ truncations of the $N = 4$, $SO(44)$ supergravity model. When eq. (4a, b) are verified, the Z_2 or $Z_2 \otimes Z_2'$ truncations give only the UTS of the corresponding theory.

As we already mentioned the TS is also obtainable once the structure of the $N = 2$ quaternionic manifold is known. Keeping this in mind we first give the derivation of $\mathcal{L}_{UTS}^{N=2}$ and $\mathcal{L}_{UTS}^{N=1}$ and we will comment later on the TS. To start with, we need to specify the action of Z_2 and $Z_2 \otimes Z_2'$ on the $N = 4$, $SO(44)$ fields. This action is fixed completely from the modular invariance request.⁸ It can be easily obtained from the

representative fields for the $N = 4$ matter scalar manifold of ref. [16]: Φ_J^I , with $I, J = 1, 2, \dots, (\dim A_4 + 6)$. I and J are then split into $SO(6)$ vectorial, and $SO(44)$ adjoint indices. Φ_J^I is the representative of the coset space:¹⁸

$$\frac{SO(6, \dim A_4)}{SO(6) \otimes SO(\dim A_4)} \quad (8)$$

To define the Z_2 truncation of Φ_J^I , decompose I and J in two groups, O and A :

$$O = \{I_1, A_A, A_B\}, \quad A = \{I_4, V_1 \otimes V_2\} \quad (9)$$

Then, Φ_J^I decomposes as $\Phi_A^O + \Phi_A^A + \Phi_O^A + \Phi_A^A$. Only the Φ_O^O and Φ_A^A survive the Z_2 truncation. Therefore, the scalars of the $N = 2$ models parametrize the cosets:

$$\frac{SO(2, \dim A_A + \dim A_B)}{SO(2) \otimes SO(\dim A_A + \dim A_B)}, \text{ from } \Phi_O^O \quad (10.a)$$

and

$$\frac{SO(4, \dim V_A \otimes V_B)}{SO(4) \otimes SO(\dim V_A \otimes V_B)}, \text{ from } \Phi_A^A \quad (10.b)$$

The first coset space is Kählerian, because $SO(2) \simeq U(1)$, and it is indeed a space of type $A - IV_m$ in the classification of Calabi and Vesentini.¹⁹ It parametrizes the scalars of the $SO(n_A) \otimes SO(n_B)$ $N = 2$ vector multiplets. The second coset space is parameterized by the scalars of the $N = 2$ hypermultiplets. These scalars transform under the gauge group as Vector $\otimes$ Vector of $SO(n_A) \otimes SO(n_B)$. This manifold is quaternionic, because $SO(4) \simeq SU(2) \otimes SU(2)$. It is one of the 3rd family of homogeneous quaternionic manifolds listed by Wolf.²⁰ Of the six graviphotons of $N = 4$, which carry indices in the vectorial of $SO(6)$, only two survive the Z_2 projection. One of them is the graviphoton of the $N = 2$ supergravity multiplet, while the other fits into an $N = 2$ vector multiplet whose scalar component is the spin-zero field of the $N = 4$ gravitational multiplet, parametrizing the space $SU(1,1)/U(1)$ (the pseudoscalar component is supplied by an antisymmetric tensor through a duality transformation). To sum all this up, we find that the UTS part of the $SO(n_A) \otimes SO(n_B)$ "minimal" models contains scalars parametrizing the coset:

$$\frac{SU(1,1)}{U(1)} \otimes \frac{SO(2, \dim A_A + \dim A_B)}{SO(2) \otimes SO(\dim A_A + \dim A_B)} \otimes \frac{SO(4, \dim V_A \otimes V_B)}{SO(4) \otimes SO(\dim V_A \otimes V_B)} \quad (11)$$

For the $N = 2$ superstring models with $n_A, n_B \neq 8, 16$, this manifold defines the full $N = 2$, $SO(n_A) \times SO(n_B)$ low energy effective theory.

Note that in the presence of a massless twisted sector, the $N = 2$ quaternionic manifold needs not to be of the form given by eq.(11); but it must be one of the list given in ref.(20). However, the trilinear $N = 2$ global "superpotential", is completely specified and given by the coupling *Hypermultiplet-Hypermultiplet-Vector multiplet*, as dictated by the $N = 2$ global supersymmetry.¹⁷ As we will see below, this fact completely specifies the $N = 1$ TS superpotential after the Z_2' truncation.

We consider now the $N = 1, \otimes \prod_{i=0}^3 SO(n_i)$ superstring models. Following again the rules of ref. [8] we easily obtain the massless UTS spectrum:

$$\begin{aligned} &(|\mu\rangle + |0\rangle_a)_L \otimes (|\bar{\mu}\rangle + |\bar{0}\rangle_a)_{R+} + |\bar{A}_3\rangle + |\bar{A}_3\rangle_{R+} \\ &(|i_1\rangle + |1\rangle_a)_L \otimes (|V_0 \otimes V_1\rangle + |V_2 \otimes V_3\rangle)_{R+} \\ &(|i_2\rangle + |2\rangle_a)_L \otimes (|V_0 \otimes V_2\rangle + |V_1 \otimes V_3\rangle)_{R+} \\ &(|i_3\rangle + |3\rangle_a)_L \otimes (|V_0 \otimes V_3\rangle + |V_1 \otimes V_2\rangle)_{R+} \end{aligned} \quad (12)$$

where $(|i_1\rangle, |i_2\rangle, |i_3\rangle)_L = |I_a\rangle_L$ is the decomposition of the $\mathfrak{g}$ of $SO(6)$ under $SO(6) \rightarrow SO(2)_1 \otimes SO(2)_2 \otimes SO(2)_3$, $(|0\rangle_a, |1\rangle_a, |2\rangle_a, |3\rangle_a)_L = |i_a\rangle_{a,L}$ are space-time spinors in the $\mathfrak{4}$ of $SU(4)$ under the same decomposition. $|\mu\rangle_L$ and $|\bar{\mu}\rangle_R$ are space-time indices in the light cone gauge. $|V_i\rangle_R$ are vectors of $SO(n_i)$, and $|\bar{A}_i\rangle$ are in the adjoints of the $SO(n_i)$. By comparing eq. (5) and (12) we obtain the action of $Z_2 \otimes Z_2'$ on the representations of $SO(6)$ and $SO(44)$:

$$(\alpha \epsilon Z_1, \beta \epsilon Z_2', \alpha^2 = \beta^2 = 1)$$

4 of	$SU(4)$	$(0\rangle, 1\rangle, 2\rangle, 3\rangle) \xrightarrow{Z_2 \otimes Z_2'} (0\rangle, \alpha 1\rangle, \beta 2\rangle, \alpha\beta 3\rangle)$																					
$\mathfrak{g}$ of	$SU(4)$	$(i_1\rangle, i_2\rangle, i_3\rangle) \xrightarrow{Z_2 \otimes Z_2'} (\alpha i_1\rangle, \beta i_2\rangle, \alpha\beta i_3\rangle)$																					
Vect. of	$SO(4,4)$	$(V_0\rangle, V_1\rangle, V_2\rangle, V_3\rangle) \xrightarrow{Z_2 \otimes Z_2'} (V_0\rangle, \alpha V_1\rangle, \beta V_2\rangle, \alpha\beta V_3\rangle)$																					
A_{44} = adjoint of	$SO(44)$	<table style="margin: auto; border-collapse: collapse;"> <tr> <td style="padding: 0 10px;">A_0</td> <td style="padding: 0 10px;">$V_0 V_1$</td> <td style="padding: 0 10px;">$V_0 V_3$</td> <td style="padding: 0 10px;">1</td> <td style="padding: 0 10px;">α</td> <td style="padding: 0 10px;">β</td> <td style="padding: 0 10px;">$\alpha\beta$</td> </tr> <tr> <td style="padding: 0 10px;">A_1</td> <td style="padding: 0 10px;">$V_1 V_2$</td> <td style="padding: 0 10px;">$V_1 V_3$</td> <td style="padding: 0 10px;">$Z_2 \otimes Z_2'$</td> <td style="padding: 0 10px;">1</td> <td style="padding: 0 10px;">$\alpha\beta$</td> <td style="padding: 0 10px;">β</td> </tr> <tr> <td style="padding: 0 10px;">A_2</td> <td style="padding: 0 10px;">$V_2 V_3$</td> <td style="padding: 0 10px;">A_3</td> <td style="padding: 0 10px;">1</td> <td style="padding: 0 10px;">α</td> <td style="padding: 0 10px;">β</td> <td style="padding: 0 10px;">$\alpha\beta$</td> </tr> </table>	A_0	$V_0 V_1$	$V_0 V_3$	1	α	β	$\alpha\beta$	A_1	$V_1 V_2$	$V_1 V_3$	$Z_2 \otimes Z_2'$	1	$\alpha\beta$	β	A_2	$V_2 V_3$	A_3	1	α	β	$\alpha\beta$
A_0	$V_0 V_1$	$V_0 V_3$	1	α	β	$\alpha\beta$																	
A_1	$V_1 V_2$	$V_1 V_3$	$Z_2 \otimes Z_2'$	1	$\alpha\beta$	β																	
A_2	$V_2 V_3$	A_3	1	α	β	$\alpha\beta$																	

As in the $N = 2$ case we restrict our attention to the bosonic part of the theory and define the $Z_2 \otimes Z'_2$ truncation on the representative fields Φ_I^J (see eq. (8)). Denote with O the set of indices I, J which do not transform under $Z_2 \otimes Z'_2$, by A and B those transforming only under Z_2 and Z'_2 respectively, and by AB those which transform under Z_2 and Z'_2 :

$$\begin{aligned} O &= \{A_1, A_2, A_3, A_4\} \\ A &= \{i_1, V_0 \otimes V_1, V_2 \otimes V_3\} \\ B &= \{i_2, V_0 \otimes V_2, V_1 \otimes V_3\} \\ AB &= \{i_3, V_0 \otimes V_3, V_1 \otimes V_2\} \end{aligned} \quad (14)$$

The fields surviving the $Z_2 \otimes Z'_2$ truncation are:

$$\phi_O^O, \phi_A^A, \phi_B^B, \phi_{AB}^{AB} \quad (15)$$

ϕ_O^O does not describe physical degrees of freedom. It corresponds to compensator fields and can be set equal to 1. $\phi_A^A, \phi_B^B, \phi_{AB}^{AB}$, define three coset spaces of the form¹⁶:

$$\begin{aligned} m_i &= \frac{SO(2, K_i)}{SO(2) \otimes SO(K_i)}, \quad i = A, B, AB \\ \text{with } K_A &= \dim V_0 \otimes V_1 + \dim V_2 \otimes V_3 = n_0 n_1 + n_2 n_3 \\ K_B &= \dim V_0 \otimes V_2 + \dim V_1 \otimes V_3 = n_0 n_2 + n_1 n_3 \\ K_{AB} &= \dim V_0 \otimes V_3 + \dim V_1 \otimes V_2 = n_0 n_3 + n_1 n_2 \end{aligned} \quad (16)$$

Let us examine the representatives of, say, m_A :

$$(\Phi_I^A, \Phi_I^B) \quad r \in \{V_0 \otimes V_1 \text{ or } V_2 \otimes V_3\}.$$

i_1 in terms of the indices (ij) in the $\mathfrak{4}$ of $SU(4)$ is either $(0,1)$ or $(2,3)$. The reality condition of ref. [16],

$$\Phi_I^{(ij)} = (\Phi_{(ij)I})^* = \frac{1}{2} \epsilon^{ijkl} \Phi_{(kl)I} \quad (17a)$$

implies:

$$(\Phi_I^{(3,4)})^* = \Phi_I^{(1,2)} \equiv \Phi_I \quad (17b)$$

with $I = (1,2), (3,4), r$.

Then Φ_I actually splits into two compensators $a = \phi_{(1,2)}, b = \phi_{(3,4)}$, and physical fields $Z_r = \Phi_r$. The Φ_I however, are not independent, because of the constraints on $\Phi_I^{J,16}$

$$\begin{aligned} \Phi_I^* \Phi_J \eta^{IJ} &= -\frac{1}{2}, \quad \Phi_I \Phi_J \eta^{IJ} = 0 \\ \eta^{IJ} &= \text{diag}(-1, -1, 1, \dots, 1) \end{aligned} \quad (18)$$

or:

$$\begin{aligned} |a|^2 + |b|^2 - \sum_r |Z_r|^2 &= \frac{1}{2} \\ a^2 + b^2 - \sum_r Z_r^2 &= 0 \end{aligned} \quad (19)$$

The kinetic terms of the scalars are reduced to:

$$\begin{aligned} &-\frac{1}{2} (\Phi^I \partial_\mu \Phi_{J(i)}) (\Phi^J \partial_\mu \Phi_{i(j)}) \\ &\text{with } \Phi_I^* \Phi_{J,r} = \eta_{IJ} + \Phi_{i(j)}^{(i)} \Phi_{(i)J} \end{aligned} \quad (20)$$

They can then be rewritten as:

$$-(2\eta^{IJ} \partial_\mu \Phi_I \partial^\mu \Phi_J + 4\Phi^I \partial_\mu \Phi_J^* \Phi_i^* \partial^\mu \Phi^{iJ}) \quad (21a)$$

and can be recast in the form:

$$\partial_\mu \Phi_I \mathcal{M}_{IJ} \partial^\mu \Phi_J^*, \quad \mathcal{M}_{IJ} = N_{IJ} - N_{IL} \Phi^L N_{JK} \Phi^{*K} \quad (21b)$$

as in de Wit, Lauwers and Van Proeyen.¹⁷ Here $N_{IJ} = -2\eta_{IJ}$. Defining now unconstrained fields y^r , the a, b , and Z^r are expressed as functions of the y^r as :

$$\begin{aligned} Z^r &= y^r Y^{-\frac{1}{2}} \\ a &= \frac{1}{2} (1 + y^r y^r) Y^{-\frac{1}{2}} \equiv y^{(1,2)} Y^{-\frac{1}{2}} \\ b &= \frac{i}{2} (y^r y^r - 1) Y^{-\frac{1}{2}} \equiv y^{(3,4)} Y^{-\frac{1}{2}} \end{aligned} \quad (22)$$

with $Y = 1 - 2y^r y^{r*} + (y^r y^r)(y^r y^r)^*$. One can easily check that the constraints (19) are solved by the Z^r , a and b given in eqs. (22) for every y^r . The y^r are then

unconstrained physical fields. We can now find the Kähler potential of the product manifold²¹ $J = J_0 + J_1 + J_2 + J_3$.

Notice first that $Y = -2y^I \eta_{IJ} y^J = y^I N_{IJ} y^J$, that $\mathcal{M}_{IJ}$ satisfies the transversality conditions¹⁷ $\Phi^I \mathcal{M}_{IJ} = 0, \mathcal{M}_{IJ} \Phi^{*J} = 0$, and that the kinetic terms of the scalars are:

$$Y^{-1} \mathcal{M}_{IJ} \partial_\mu y^I \partial^\mu y^J \quad (23)$$

From eqs. (22), $y^{(12)}$ and $y^{(3,4)}$ are analytic functions of the independent coordinates y^r , so:

$$Y^{-1} \mathcal{M}_{IJ} \partial_\mu y^I \partial^\mu y^J = \left(\frac{\partial}{\partial y^r} \frac{\partial}{\partial y^s} \log Y \right) \partial_\mu y^r \partial^\mu y^s. \quad (24)$$

It follows from this equation that $J_i = -\log Y_i$ is the Kähler potential of the theory.²¹ J is defined up to a Kähler transformation. To fix it, we have to find also the superpotential g of the theory, and define the G function of ref. [21]. To do this, it is sufficient to isolate the gravitino mass matrix of the $N = 4$ theory.¹⁶

$$\mathcal{M}_{ij} = -\frac{2}{3} i \phi f_{AB}^C \Phi_i^A \Phi_j^B (\Phi_C^D)^* \quad (25)$$

where¹⁶ $\phi = \frac{1-Z^*}{(1-ZZ^*)^{1/2}}$, and Z is the complex scalar field of the $N = 4$ supergravity multiplet. Only the Φ_j^B give contribution to eq.(25). The mass of the gravitino surviving the $Z_2 \otimes Z'_2$ truncation is $m_{3/2} = m_{11}$. From ref. [21] we have:

$$m_{3/2}^2 = e^G, \quad G = J + \log |g|^2 \quad (26)$$

J is the Kähler potential and g the superpotential. After the truncation we find:

$$m_{3/2}^2 = \frac{4}{9} Y_0^{-1} Y_1^{-1} Y_2^{-1} Y_3^{-1} \left| f_{A_1 A_2 A_3 A_4} y_1^{A_1} y_2^{A_2} y_3^{A_3} \right|^2 \quad (27)$$

where:

$$\begin{aligned} Y_0 &= S + S^*, \quad S \equiv \frac{1+Z}{21-Z} \\ Y_i &= 1 - 2y_i^A y_i^{A*} + (y_i^A y_i^A)(y_i^B y_i^B)^* \\ i &= 1, 2, 3, \text{ (no summation on } i \text{)}. \end{aligned} \quad (28)$$

The G function therefore is:

$$G = -\log Y_0 - \log Y_1 - \log Y_2 - \log Y_3 + \log |g|^2$$

The f_{AB}^C in eq. (25) are the structure constants of $SO(44)$ restricted to:

$$A \in \{V_0 \otimes V_1 \otimes V_2 \otimes V_3\}, B \in \{V_0 \otimes V_2 \otimes V_1 \otimes V_3\},$$

$$C \in \{V_0 \otimes V_3 \otimes V_1 \otimes V_2\}.$$

Thus, defining $A_0 \in V_0, A_i \in V_i, A_3 \in V_3$, the superpotential g is:

$$\begin{aligned} g &= C \left(y_1^{A_0 A_1} y_2^{A_1 A_2} y_3^{A_2 A_3} - y_1^{A_0 A_1} y_2^{A_0 A_2} y_3^{A_1 A_3} \right. \\ &\quad \left. + y_1^{A_0 A_2} y_2^{A_1 A_3} y_3^{A_1 A_2} - y_1^{A_0 A_2} y_2^{A_0 A_3} y_3^{A_1 A_2} \right) \end{aligned} \quad (29)$$

where $y_i^{ab} = -y_i^{ba}$ $i = 1, 2, 3$ and C is a constant dependent from the normalization of the structure constants of $SO(44)$.

In order to specify completely the $N = 1$ effective theory, it is sufficient to find the $f_{AB}(Z)$ function²¹, analytic in the fields, which determines the couplings of the gauge supermultiplets. From the $N = 4$ coupling:¹⁶ $-\frac{1}{2} a_{IJ} F_{\mu\nu}^{I+J}$, where the a_{IJ} are given by:

$$\frac{1}{2} a_{IJ} = S \Phi_I^i \Phi_{Jij} + S^* (\eta_{IJ} + \Phi_I^i \Phi_{Jij}), \quad (30)$$

we can easily find its form. Because no graviphoton survives the $Z_2 \otimes Z'_2$ truncation, the I, J indices are in the adjoint of $\prod_{i=0}^3 SO(n_i)$. However, all the remaining scalars are in vectorial representations of the gauge groups; thus: $\frac{1}{2} a_{IJ} = S^* \eta_{IJ}$ and,

$$f_{AB}(S) = S \delta_{AB} \quad (31)$$

Notice that the only field dependence of $f_{AB}(S)$ comes from the S field, as in the case of Calabi-Yau and orbifold compactifications [9, 10].

Let us summarize now the results obtained for the minimal $N = 1$ models:

- i) Gauge group $SO(n_0) \otimes SO(n_1) \otimes SO(n_2) \otimes SO(n_3)$.
- ii) Scalar manifold
$$\frac{SU(1,1)}{U(1)} \otimes \frac{SO(2, K_A)}{SO(2) \otimes SO(K_A)} \otimes \frac{SO(2, K_B)}{SO(2) \otimes SO(K_B)} \otimes \frac{SO(2, K_{AB})}{SO(2) \otimes SO(K_{AB})}.$$

with $K_A = n_0 n_1 + n_2 n_3, K_B = n_0 n_3 + n_1 n_2, K_{AB} = n_0 n_3 + n_1 n_2$.

$$J_0 = -\log(S + S^*)$$

$$J_i = -\log(1 - 2|\tilde{y}_i|^2 + |\tilde{y}_i^*|^2) \quad \tilde{y}_i = y_i^n$$

iii) $G = J_0 + J_1 + J_2 + J_3 + \log|g|^2$ with superpotential g given in formula (29).

iv) Field dependent gauge coupling constant:

$$f_{ab} = \delta_{ab} S, \quad a, b \in \text{adj} \prod_{i=0}^3 SO(n_i).$$

We now comment about the extension of these results for models with massless twisted sector. To be definite we consider as an example the $N = 2$ $SO(16) \otimes SO(28)$ minimal model. A suitable Z_2^t truncation breaks the $N = 2$ to $N = 1$ and the gauge group to $SO(10) \otimes SO(6) \otimes SO(14) \otimes SO(14)$.^{7,8} The $N = 1$ chiral multiplets (in the $\underline{16}$ of $SO(10)$) are given by Z_2^t truncation of the $N = 2$ hypermultiplets members of the spinor representation of $SO(16) \rightarrow SO(10) \otimes SO(6)$. Under the $SU(2) \otimes SU(2)$ global symmetry and $SO(10) \otimes SO(6)$ gauge group, the chiral $N = 2$ multiplets have the following quantum numbers:

$$S_1 = \{2, 1; 16, 4\} \text{ and } S_2 = \{1, 2; 16, \bar{4}\} \quad (32)$$

So, in total, this model contains sixteen $SO(10)$ families. (eight in S_1 and eight in S_2). From the $N = 2$ gravitino mass term and after Z_2^t truncation we obtain the $N = 1$ superpotential of the TS,

$$g_{TS} = S_1 S_1 V_{01} + S_2 S_2 V_{01} \quad (33)$$

where $V_{01} = |V_0, V_1\rangle$ is the scalar multiplet of UTS transforming as $(10, \underline{9})$ under $SO(10) \otimes SO(6)$, singlet under all the other groups. By the way, g_{TS} is the only possible trilinear function, singlet under the global and local symmetries of the $N = 1$ model.

Let us consider now the Kähler manifold of the TS. In this case, contrary to the previous ones, some assumptions are necessary. By neglecting any interference terms among UTS and TS, the natural ansatz is to assume the $N = 2$ symplectic manifold

$\frac{Sp(2, 2n)}{Sp(2) \times Sp(2n)}$. Within this approximation and analyzing the Z_2^t transformation properties of the compensating hypermultiplets, it follows that the $N = 2$ manifold after Z_2^t truncation is reduced to a non compact CP^n Kähler manifold $\frac{SU(1, n)}{U(1) \times SU(n)}$, which corresponds to the following Kähler potential;

$$J_{TS} = -2 \log(1 - S_{TS} S_{TS}^*) \quad (34)$$

where S_{TS}, S_{TS}^* are the scalar of the $N = 1$ TS.

We conclude with a few remarks:

i) The scaling $S \rightarrow \lambda S$, $g_{\mu\nu} \rightarrow \lambda g_{\mu\nu}$ is a symmetry for the classical equations of motion:

$$\sqrt{g} \mathcal{L} \rightarrow \lambda \sqrt{g} \mathcal{L}$$

This symmetry means that, classically, the theory is scale invariant, as it is expected in all the string generated models.⁹

ii) $Im S \rightarrow Im S + b$ is a symmetry for the lagrangian.

This property is due to the equivalence of $Im S$ with an antisymmetric tensor, through a duality transformation. Remind that in string theories one obtains naturally the antisymmetric tensor representation

$$B_{\mu\nu} = |\mu\rangle_L \otimes |\nu\rangle_R - |\nu\rangle_L \otimes |\mu\rangle_R.$$

iii) R-symmetry: the superpotential g is homogeneous of degree three in the y fields. The R-symmetry acts on all the scalar and gauge supermultiplets with the exception of the S field.

iv) Many of the isometries of the Kähler manifold are broken explicitly by the superpotential and by the gauge interactions. Still, there are many flat directions in the theory, and all the minima of the potential preserve supersymmetry.

v) The S field is the only singlet under the gauge group. This is an essential difference between these models and those obtained via Calabi-Yau or orbifold compactifications.^{9,10} There in fact, all the massless fields coming from the metric are gauge singlets.^{9,10} Here, however, these degrees of freedom are replaced by world

sheet fermionic modes⁸ and participate to the gauge interactions. Because of this extra gauging the Calabi-Yau and orbifold modes parametrized by T in ref. [9], and T_{ij} in [10], are replaced by gauge nonsinglets.

vi) The Kähler manifold is no longer a $\otimes \prod_i \frac{U(m_i, n_i)}{U(m_i) \otimes U(n_i)}$ coset space, as in all the cases previously considered^{8,10,22}, it is instead a $\otimes \prod_i \frac{SO(2, n_i)}{SO(2) \otimes SO(n_i)}$ coset which is also a product of Kähler-Einstein spaces.

vii) The complete knowledge of the $N = 2$ quaternionic manifold, in the presence of massless TS states, does not follow from the $N = 4$ truncated theory. This fact forced us to a simplified assumption in order to discuss the $N = 1$ effective lagrangian relevant for the TS. This assumption is well justified only for small field fluctuations or in a flat gravitational background where the possible UTS and TS interferences can be neglected.

Finally, it should be stressed that for all the physically relevant $N = 1$ models, the presence of a massless twisted sector is necessary in order to get a realistic theory with chiral fermions.

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