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QUANTIZATION OF THE N=1,2 SUPERPARTICLE WITH IRREDUCIBLE CONSTRAINTS

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ABSTRACT

An action for the $D=10$ superparticle with $N=1,2$ supersymmetry, containing irreducible (functionally independent) Lorentz-covariant constraints only, is proposed. The key ingredient is the enlargement of the usual superspace to include additional bosonic (Lorentz spinors) harmonic co-ordinates which are pure gauge degrees of freedom. Upon quantization the extended superparticle provides a Lorentz covariant on-shell description of the linearized $D=10$ type IIB supergravity after imposing an appropriate reality condition on the superfield wave function. The first-quantized BRST charge contains a multighost interaction term which is inevitable when the functional dependence of the reparametrization and the local fermionic symmetry constraints is correctly taken into account.

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1. Recently there has been much interest in analyzing Siegel's superparticle¹⁾⁻³⁾, especially in view of the fact that, representing a "zero-mode" approximation to the covariant Green-Schwarz superstring⁴⁾, it shares some of the fundamental features of the latter, in particular the local fermionic symmetry.

Siegel's superparticle action¹⁾:

$$S = \int d\tau [p_\mu \dot{x}^\mu + p_\theta^\alpha \dot{\theta}_\alpha - \lambda p^2 - \psi^\alpha \not{p}_{\alpha\beta} d^\beta],$$

where λ and ψ^α are Lagrange multipliers and the Grassmann spinor θ_α is Majorana or Majorana-Weyl (MW) (in D=10; we shall concentrate on this case here), possesses two first-class constraints^{*)}:

$$p^2=0 \text{ and } \not{p}_{\alpha\beta} d^\beta \equiv \not{p}_{\alpha\beta} (p_\theta^\beta - \tilde{p}^{\beta\gamma} \theta_\gamma) = 0.$$

However, a systematically overlooked property of these constraints is that they are functionally dependent, i.e. they form a reducible set in the sense of ref. 5)^{**)} .

Then one is left with the following two alternatives:

- i) either one must separate the half-independent constraints from $\not{p}_{\alpha\beta} d^\beta$ which cannot be performed in a Lorentz-covariant manner, since $\not{p}_{\alpha\beta}$ is a D=10 MW spinor (there are no Lorentz-covariant objects with half the number of the components of a MW spinor);

*) The D=10 and D=8 spinor conventions are explained in the Appendix.

**) The set of the first-class constraints $\{\Phi_{\alpha_0}\}$ possesses a degree of reducibility L if the following relations hold:

$$\Phi_{\alpha_0} \prod_{(1)}^{\alpha_0} Z_{\alpha_1} = 0, \quad \prod_{(1)}^{\alpha_0} Z_{\alpha_1} \prod_{(2)}^{\alpha_1} Z_{\alpha_2} \Big|_{\Phi=0} = 0, \quad \dots, \quad \prod_{(L-1)}^{\alpha_{L-2}} Z_{\alpha_{L-1}} \prod_{(L)}^{\alpha_{L-1}} Z_{\alpha_L} \Big|_{\Phi=0} = 0,$$

$$\alpha_k = 1, \dots, m_k,$$

where $Z_{(1)\alpha_1}^{\alpha_0}, \dots, Z_{(L)\alpha_L}^{\alpha_{L-1}}$ are functions of the canonical variables which do not identically vanish on the constraint surface $\Phi_{\alpha_0}=0$.

ii) or one has to employ the general scheme of Batalin-Fradkin-Vilkovisky (BFV)⁵⁾ for systems with reducible constraints. In this case, the price for working with reducible constraints is the introduction, for each functional dependence among the latter, of additional ghosts which enter the BRST⁶⁾ charge and the BFV Hamiltonian⁷⁾. However, one can easily deduce that, in this case as well, due to the MW nature of the constraint $\mathcal{P}_{\alpha\beta} d^\beta$, it is impossible to retain manifest Lorentz invariance maintaining simultaneously a finite degree of reducibility L of the system.

Thus, consistent Lorentz-covariant canonical quantization of Siegel's superparticle action, properly accounting for the reducibility of the constraints, is impossible without some further modification.

In this letter we propose a formalism which overcomes the above-mentioned difficulties. The key ingredient is the introduction of bosonic Lorentz-spinor harmonics $v_\alpha^A, \tilde{v}_A^\alpha$ subject to the constraint $v_\alpha^A \tilde{v}_B^\alpha = \delta_B^A$ as additional dynamical variables. The enlarged $N=1,2$ superspace co-ordinates are now labelled by $(x^\mu, \theta_\alpha^{1,2}, v_\alpha^A, \tilde{v}_A^\alpha)$. (In what follows we shall directly consider the more general case of the point particle with $N=2$ supersymmetry.) The spinor harmonics serve as "bridges" converting the $SO(1,9)$ MW spinor indices α into a pair of $SO(8)$ spinor indices $A=(a, \dot{a})$ corresponding to the $(s), (c)$ representations respectively. The inclusion of $v_\alpha^A, \tilde{v}_A^\alpha$ is crucial in order to separate in a Lorentz-covariant way the 8 independent components of the MW spinor constraint $\mathcal{P}_{\alpha\beta} d^\beta$. We also include additional first-class constraints into the superparticle action to guarantee that $v_\alpha^A, \tilde{v}_A^\alpha$ are pure gauge degrees of freedom.

The present formulation is very much inspired by the harmonic superspace approach to extended supersymmetric (gauge) theories in $D=4$ ⁸⁾ and by the recently developed covariant light-cone superspace in $D=10$ ³⁾. However, the latter differs substantially from the present one in two respects: (a) the additional harmonics introduced in ref. 3) are Lorentz vectors instead of Lorentz spinors; (b) the physical content of the theory in ref. 3) is different. Moreover, the constraints of the modified superparticle action proposed there once again form a reducible set.

In Section 2 an action with irreducible constraints for the extended superparticle is proposed. In Section 3 the covariant first quantization à la

Dirac is accomplished. After the imposing on-shell of a suitable covariant reality condition on the superfield wave function it is demonstrated that the theory coincides with the linearized D=10 IIB supergravity known in the light-cone formalism⁹⁾ [cf. also the second ref. 2)]. In Section 4 the first-quantized BRST charge is computed and found to contain a multighost term, i.e. the N=1,2 superparticle is a constrained system of rank two in the sense of refs. 7).

2. Let us consider the following classical action for the D=10 extended superparticle described by co-ordinates $(x^\mu, \theta_\alpha^1, \theta_\alpha^2, V_\alpha^A, \tilde{V}_A^\alpha)$:

$$S_{N=2} = \int d\tau \left[p_\mu \dot{x}^\mu + \sum_{k=1,2} p_\theta^{k\alpha} \dot{\theta}_\alpha^k + p_{V,A}^\alpha \dot{V}_\alpha^A + \right. \\ \left. + \dot{\tilde{V}}_A^\alpha p_{\tilde{V}\alpha}^A - H \right], \quad (1)$$

$$H = \lambda p^2 + \rho^\alpha \mathcal{Q}_\alpha + \mathcal{Q}^\alpha \Psi_\alpha + \Lambda_{AB} \mathcal{Q}^{AB}, \quad (2)$$

$$\Phi_\alpha^\beta \equiv V_\alpha^A \tilde{V}_A^\beta - \delta_\alpha^\beta = 0, \quad \chi_\alpha^\beta \equiv V_\alpha^A p_{V,A}^\beta - p_{\tilde{V}\alpha}^A \tilde{V}_A^\beta = 0. \quad (3)$$

Here $\theta_\alpha^{1,2}$ are D=10 MW (left-handed) Grassmann spinors. The bosonic Lorentz-spinor harmonics $v_\alpha^{(a,\dot{a})}$, $v_{\dot{\alpha}(a,\dot{a})} = (C_8^{-1})_{\dot{\alpha}\beta} \tilde{v}^\beta_{(a,\dot{a})}$ transform as D=10 MW left- and right-handed spinors with respect to the indices $\alpha, \dot{\alpha}$ and transform as SO(8) (s),(c)-spinors with respect to the indices $(a,\dot{a}) = A$. The Hamiltonian is taken as a linear combination of irreducible first-class constraints with the following notations:

$$\mathcal{Q}_\alpha \equiv \tilde{V}_\alpha^\gamma p_{\gamma\beta} (p_\theta^{1\beta} - \tilde{p}^{\beta\gamma} \theta_\gamma^2), \quad \mathcal{Q}^\alpha \equiv p_\theta^{2\alpha} - \tilde{p}^{\alpha\beta} \theta_\beta^1, \quad (4)$$

$$\mathcal{Q}^{AB} \equiv p_V^A V_\alpha^B - \tilde{V}^{\alpha A} p_{\tilde{V}\alpha}^B.$$

Let us emphasize that with the help of $V_\alpha^A, \tilde{V}_A^\alpha$ we have covariantly separated Siegel's fermionic constraint into two parts

$$p_{\alpha\beta}(p_\theta^{1\beta} - \tilde{p}^{\beta\gamma}\theta_\gamma^2) = v_\alpha^a \mathcal{D}_a + v_\alpha^{\dot{a}} \mathcal{D}_{\dot{a}},$$

and the second part $\mathcal{D}_{\dot{a}}$:

$$\mathcal{D}_{\dot{a}} \equiv \tilde{v}_{\dot{a}}^\alpha p_{\alpha\beta}(p_\theta^{1\beta} - \tilde{p}^{\beta\gamma}\theta_\gamma^2) = - \left[(\tilde{S}^{-1})_{\dot{a}b} v_\alpha^b (p_\theta^{1\alpha} - \tilde{p}^{\alpha\beta}\theta_\beta^2) \right] p^2 - (\tilde{S}^{-1}\tilde{R})_{\dot{a}}^{\dot{b}} \mathcal{D}_{\dot{b}}, \quad (5)$$

$$(\tilde{R}^{ab} \equiv v_\alpha^a \tilde{p}^{\alpha\beta} v_\beta^b, \tilde{S}^{ab} \equiv v_\alpha^a \tilde{p}^{\alpha\beta} v_\beta^{\dot{b}}),$$

whose functional dependence on the remaining constraints p^2 and $\mathcal{D}_{\dot{a}}$ is explicitly displayed in (5), is excluded from (1) and (2).

The action (1) is manifestly invariant under N=2 supersymmetry transformations:

$$\delta_{SS} x^\mu = -i \left[\varepsilon_\alpha^1 (\tilde{\theta}^\mu)^{\alpha\beta} \theta_\beta^2 + \varepsilon_\alpha^2 (\tilde{\theta}^\mu)^{\alpha\beta} \theta_\beta^1 \right],$$

$$\delta_{SS} \theta_\alpha^{1,2} = \varepsilon_\alpha^{1,2}, \quad \delta_{SS} \tilde{V}_A^\alpha = \delta_{SS} V_\alpha^A = 0,$$

where $\varepsilon_\alpha^{1,2}$ are D=10 MW spinors.

For N=1 supersymmetry the action (1) and the Hamiltonian reduce to:

$$S_{N=1} = \int d\tau \left[p_\mu \partial_\tau x^\mu + p_\theta^\alpha \partial_\tau \theta_\alpha + p_{V,A}^\alpha \partial_\tau V_\alpha^A + \partial_\tau \tilde{V}_A^\alpha p_{\tilde{V}_\alpha^A} - H \right],$$

$$H = \lambda p^2 + \rho^a \tilde{V}_a^\alpha p_{\alpha\beta} d^\beta + \Lambda_{AB} \mathcal{D}^{AB},$$

Equation (3) remaining the same.

The constraints $\Phi_\alpha^\beta, \chi_\alpha^\beta$ (3) are second class and form a conjugate pair which commutes with all first-class constraints in Eq. (2)*):

*) Except for $a: \{\chi_\alpha^\beta, a\} = -\tilde{V}_\alpha^\beta \mathcal{D}_\beta V_\alpha^B$. The latter, however, has no effect on the Dirac bracket relations (6).

$$\begin{aligned} \{\Phi_\alpha^\beta, \chi_\gamma^\delta\} \big|_{\Phi, \chi=0} &= 2 \delta_\alpha^\delta \delta_\gamma^\beta ; \\ \{\mathcal{Q}^{AB}, \Phi_\alpha^\beta\} &= \{\mathcal{Q}^{AB}, \chi_\alpha^\beta\} = 0 \quad \text{etc.} \end{aligned}$$

Thus, any Poisson bracket relations involving the first-class constraints, in particular their algebra, remain unchanged after introducing the corresponding Dirac brackets:

$$\begin{aligned} \{\mathcal{Q}_a, \mathcal{Q}^\alpha\} &= -2 \tilde{V}_a^\alpha p^2, \quad \{\mathcal{Q}_a^\beta, \mathcal{Q}_c\} = \delta_c^\beta \mathcal{Q}_a, \\ \{\mathcal{Q}_A^B, \mathcal{Q}_C^D\} &= \delta_C^B \mathcal{Q}_A^D - \delta_A^D \mathcal{Q}_C^B, \\ \{\mathcal{Q}_{\dot{a}}^\beta, \mathcal{Q}_c\} &= -\delta_c^\beta \left[(\tilde{S}^{-1})_{\dot{a}d} V_\alpha^d (p_\theta^{1\alpha} - \tilde{p}^{\alpha\beta} \theta_\beta^2) p^2 + \right. \\ &\quad \left. + (\tilde{S}^{-1} \tilde{R})_{\dot{a}}^d \mathcal{Q}_d \right], \end{aligned} \tag{6}$$

rest = 0 .

Now, let us briefly comment on the significance of the Lorentz-spinor harmonics $V_\alpha^A, \tilde{V}_{\dot{A}}$. From the constraint $\phi_\alpha^\beta = 0$ (3) one immediately sees that the objects:

$$\begin{aligned} \Pi_{L\alpha}^\beta &\equiv V_\alpha^a \tilde{V}_a^\beta, \quad \Pi_{R\alpha}^\beta \equiv V_\alpha^{\dot{a}} \tilde{V}_{\dot{a}}^\beta, \\ \Pi_{L\alpha}^\beta + \Pi_{R\alpha}^\beta &= \delta_\alpha^\beta, \end{aligned} \tag{7}$$

are Lorentz-covariant projectors for the D=10 MW spinors. Thus (7) are precisely the covariant analogues of the usual light-cone projectors:

$$-\frac{1}{2}(\mathcal{G}^+ \tilde{\mathcal{G}}^-)_\alpha^\beta = \Pi_{L\alpha}^{(8)\beta}, \quad -\frac{1}{2}(\mathcal{G}^- \tilde{\mathcal{G}}^+)_\alpha^\beta = \Pi_{R\alpha}^{(8)\beta}, \tag{7'}$$

where $\Pi_{L,R}^{(8)}$ are the D=8 chiral projectors.

Under group transformations generated by the $GL(16;R)$ algebra of $\mathcal{D}^{AB}$ (6), V_α^A and $\tilde{V}_A^\alpha$ transform as follows:

$$\exp\{\omega \mathcal{L}_{\mathcal{D}}\} V_\alpha^A = G(\omega)^A_B V_\alpha^B, \exp\{\omega \mathcal{L}_{\mathcal{D}}\} \tilde{V}_A^\alpha = \tilde{V}_B^\alpha G(\omega)^B_A \quad (8)$$

where:

$$G(\omega)^A_B = (\exp \omega)^A_B, \quad (\omega \mathcal{L}_{\mathcal{D}}) F(V, \tilde{V}) \equiv \omega^A_B \{ \mathcal{D}_A^B, F(V, \tilde{V}) \}.$$

Thus, using (8) one can always impose the following Lorentz-non-covariant gauge on $V_\alpha^A, \tilde{V}_A^\alpha$:

$$V_\alpha^a = -\frac{1}{2}(\mathcal{G}^-)_\alpha^a, \quad V_{\dot{\alpha}}^{\dot{a}} = -\frac{1}{2}(\mathcal{G}^+)_{\dot{\alpha}}^{\dot{a}}, \quad \tilde{V}_a^\alpha = (\tilde{\mathcal{G}}^+)_a^\alpha, \quad \tilde{V}_{\dot{a}}^\alpha = (\tilde{\mathcal{G}}^-)_{\dot{a}}^\alpha. \quad (9)$$

In particular, in the gauge (9) the covariant projectors (7) reduce to the usual light-cone ones (7'). This non-covariant gauge will be useful for establishing the on-shell equivalence of the covariantly quantized extended superparticle with the linearized D=10 IIB supergravity in the light-cone formulation⁹⁾.

3. Let us now consider the covariant first quantization of (1) à la Dirac. The superfield wave function $\Phi = \Phi(x, \theta^1, \theta^2, V, \tilde{V})$ must satisfy the equations:

$$p^2 \Phi = 0, \quad (10.a)$$

$$\mathcal{D}_a \Phi = 0, \quad (10.b)$$

$$\mathcal{D}^\alpha \Phi = 0, \quad (10.c)$$

$$\mathcal{D}^{AB} \Phi = 0, \quad (10.d)$$

where now:

$$p_\mu = -i \partial / \partial x^\mu, \quad p_{\theta^{1,2}}^\alpha = -i \partial / \partial \theta_\alpha^{1,2}, \quad p_{V,A}^\alpha = -i \partial / \partial V_\alpha^A, \quad p_{\tilde{V}}^A = -i \partial / \partial \tilde{V}_A^\alpha.$$

Equation (10.c) gives:

$$\hat{\Phi}(x, \theta^1, \theta^2, v, \tilde{v}) = e^{i \theta^2_\alpha \tilde{p}^{\alpha\beta} \theta^1_\beta} \hat{\Phi}(x, \theta^1, v, \tilde{v}). \quad (11)$$

Next, Eq. (10.b) with an account of (10.a), (11), reduces to the form:

$$\tilde{v}^\alpha_a \tilde{p}_{\alpha\beta} \partial_{\theta^1_\beta} \hat{\Phi}(x, \theta^1, v, \tilde{v}) = 0. \quad (12)$$

To solve (12) it is convenient to change variables $\theta^1_\alpha \rightarrow \varphi_a, \psi_{\dot{a}}$ as follows:

$$\begin{aligned} \varphi^a &= [(R^{-1})^b{}_a \tilde{v}^\alpha_b - (S^{-1})^{\dot{b}}{}_a \tilde{v}^\alpha_{\dot{b}}] \theta^1_\alpha, \quad \psi_{\dot{a}} = \tilde{v}^\alpha_{\dot{a}} \theta^1_\alpha, \\ (R_{ab} &\equiv \tilde{v}^\alpha_a \tilde{p}_{\alpha\beta} \tilde{v}^\beta_b, \quad S_{a\dot{b}} \equiv \tilde{v}^\alpha_a \tilde{p}_{\alpha\beta} \tilde{v}^\beta_{\dot{b}}). \end{aligned} \quad (13)$$

With (13), Eq. (12) reduces to:

$$S_{a\dot{b}} \partial_{\psi_{\dot{b}}} \hat{\Phi}(x, \varphi, \psi, v, \tilde{v}) = 0, \text{ i.e. } \hat{\Phi} = \hat{\Phi}(x, \varphi, v, \tilde{v}). \quad (14)$$

In terms of the new co-ordinates (13), the expressions for $\mathbb{D}^{AB}$ (4) look more complicated:

$$\begin{aligned} i \mathcal{D}_a{}^b &= v^\alpha_b \partial_{v^\alpha_a} - \tilde{v}^\alpha_a \partial_{\tilde{v}^\alpha_b} - \varphi^b \partial_{\varphi^a}, \\ i \mathcal{D}_{\dot{a}}{}^{\dot{b}} &= v^\alpha_{\dot{b}} \partial_{v^\alpha_{\dot{a}}} - \tilde{v}^\alpha_{\dot{a}} \partial_{\tilde{v}^\alpha_{\dot{b}}} + \psi_{\dot{a}} \partial_{\psi_{\dot{b}}}, \\ i \mathcal{D}_a{}^{\dot{b}} &= v^\alpha_{\dot{b}} \partial_{v^\alpha_a} - \tilde{v}^\alpha_a \partial_{\tilde{v}^\alpha_{\dot{b}}} - (R_{ac} \varphi^c) (S^{-1})^{\dot{b}}{}_{\dot{d}} \partial_{\varphi^d} + \\ &\quad + [R_{ac} \varphi^c + (RS^{-1})^{\dot{c}}{}_a \psi_{\dot{c}}] \partial_{\psi_{\dot{b}}}, \\ i \mathcal{D}_{\dot{a}}{}^b &= v^\alpha_b \partial_{v^\alpha_{\dot{a}}} - \tilde{v}^\alpha_{\dot{a}} \partial_{\tilde{v}^\alpha_b} - (R^{-1})^b{}_d S_{\dot{a}\dot{c}} \varphi^c \partial_{\varphi^d} - \\ &\quad - (R^{-1} S)^d{}_{\dot{a}} \varphi^b \partial_{\varphi^d} + p^2 (S^{-1})^{\dot{b}}{}_{\dot{c}} \psi_{\dot{c}} (R^{-1} \tilde{S}^{-1})^d{}_{\dot{a}} \partial_{\varphi^d}. \end{aligned} \quad (15)$$

As a consequence of (15), on-shell ϕ^a transforms with respect to $\mathfrak{D}_A^B$ only through itself:

$$\exp\{i\omega^A{}_B \mathfrak{D}_A^B\} \Big|_{p^2=0} \varphi^a = \left(\exp\{-\Omega(\omega, v, \tilde{v}; p)\} \right)^a{}_b \varphi^b \quad (16)$$

with

$$\begin{aligned} \Omega(\omega, v, \tilde{v}; p)^a{}_b &\equiv \omega^a{}_b + (S^{-1})^{a\dot{c}} \omega_{\dot{c}}^d R_{db} + (R^{-1})^{ad} \omega_{\dot{c}d} S_{\dot{c}b} + \\ &+ (R^{-1}S)^{a\dot{c}} \omega_{\dot{c}d} . \end{aligned}$$

Therefore, Eqs. (10.d) yield:

$$\hat{\Phi}(x, \varphi, v, \tilde{v}) = \hat{\Phi}(x, \exp\{-\Omega\}\varphi, G^{-1}v, \tilde{v}G) , \quad (17)$$

with $G(\omega)$ as in (8) and $\Omega(\omega, v, \tilde{v}; p)$ as in (16). Then we can impose on-shell the following covariant reality condition:

$$\begin{aligned} \hat{\Phi}^*(x, \varphi, v, \tilde{v}) &= \det^{-1/2}(\mathcal{T}(v, \tilde{v}; p)) \times \\ &\times \int d^8\varphi' \exp\{i\varphi^a \mathcal{T}_{ab}(v, \tilde{v}; p) \varphi'^b\} \hat{\Phi}(x, \varphi', v, \tilde{v}) , \end{aligned} \quad (18)$$

where under group transformations generated by $\mathfrak{D}_A^B$:

$$\mathcal{T}_{ab}(G^{-1}(\omega)v, \tilde{v}G(\omega); p) = \mathcal{T}_{cd}(v, \tilde{v}; p) \Omega^c{}_a(\omega, v, \tilde{v}; p) \Omega^d{}_b(\omega, v, \tilde{v}; p) . \quad (19)$$

Eq. (19) guarantees that (18) is consistent with (17).

If we now impose the non-covariant light-cone-type gauge (9) and choose:

$$\mathcal{T}_a{}^b(-\tfrac{1}{2}\sigma^\pm, \tilde{\sigma}^\pm; p) = (p^+)^{-1} \delta_a{}^b , \quad \left(p^+ = \frac{1}{\sqrt{2}}(p^0 + p^9) \right) ,$$

we arrive precisely at the light-cone on-shell description of the linearized D=10 IIB supergravity in terms of an unconstrained superfield $\Phi_{GS}(x, \hat{\phi})$ depending on a single $SO(8)$ (s)-spinor $\hat{\phi}^a$ (9):

$$\Phi_{GS}(x, \hat{\varphi}) \equiv \hat{\Phi}(x, \exp\{-\Omega_0\}\varphi, -\frac{1}{2}\sigma^\pm, \tilde{\sigma}^\pm) ,$$

$$p^2 \Phi_{GS}(x, \hat{\varphi}) = 0 ,$$

$$\Phi_{GS}^*(x, \hat{\varphi}) = (p^+)^4 \int d^8 \hat{\varphi}' \exp\{i \hat{\varphi}^a \hat{\varphi}'_a (p^+)^{-1}\} \Phi_{GS}(x, \hat{\varphi}') ,$$

where $\Omega_0 \equiv \Omega(\omega_0, v, \tilde{v}; p)$ and $G(\omega_0)$ is just the transformation (8) for which the gauge (9) is reached:

$$G^{-1}(\omega_0) v = -\frac{1}{2} \sigma^\pm , \quad \tilde{v} G(\omega_0) = \tilde{\sigma}^\pm .$$

Therefore, we conclude that Eqs. (10.a) and (17)-(19) constitute a Lorentz-covariant on-shell description of D=10 linearized IIB supergravity.

4. To perform the second quantization of the superparticle (1), one needs first to construct the first-quantized BRST charge Q_{BRST} and then proceed e.g. along the lines of refs. 10). It turns out that the $N=1,2$ superparticle is a constraint system of second rank [according to the terminology of refs. 7)], i.e., its BRST charge contains a multighost interaction term due to the non-trivial dependence of the structure "constants" of the constraint algebra (6) on the canonical variables. After straightforward (but lengthy) calculations we find:

$$\begin{aligned} Q_{BRST} = & c p^2 + \eta^a \mathcal{D}_a + \mathcal{D}^\alpha \chi_\alpha + C^A_B \mathcal{D}_A^B + i \frac{\partial}{\partial \lambda} \frac{\partial}{\partial \tilde{c}} + \\ & + i \frac{\partial}{\partial \Lambda^A_B} \frac{\partial}{\partial \tilde{c}_A^B} - \frac{\partial}{\partial p^a} \frac{\partial}{\partial \tilde{\eta}_a} - \frac{\partial}{\partial \psi_\alpha} \frac{\partial}{\partial \tilde{\chi}^\alpha} + i C^B_D C^A_B \frac{\partial}{\partial C^A_D} + \\ & + 2i(\chi_\alpha \tilde{v}_a^\alpha \eta^a) \frac{\partial}{\partial c} - i \eta^b c_b^a \frac{\partial}{\partial \eta_a} + \eta^b c_b^{\dot{a}} \left[i (\tilde{S}^{-1} \tilde{R})_{\dot{a}}^d \frac{\partial}{\partial \eta^d} + \right. \\ & \left. + (\tilde{S}^{-1})_{\dot{a}d} v_\alpha^d (\frac{\partial}{\partial \theta_\alpha^1} - i p^\alpha \theta_\beta^2) \frac{\partial}{\partial c} \right] + \\ & + \frac{1}{4} \eta^b c_b^e c_{\dot{e}}^{\dot{a}} (\tilde{S}^{-1})_{\dot{a}\dot{e}} (\tilde{S}^{-1})^{\dot{d}} \frac{\partial}{\partial \eta^d} \frac{\partial}{\partial c} , \end{aligned} \quad (20)$$

where $(C, \tilde{c}, \lambda)$, $(\eta^a, \tilde{\eta}_a, \rho^a)$, $(\chi_\alpha, \tilde{\chi}^\alpha, \phi_\alpha)$, $(C^A_B, \tilde{c}_A^B, \Lambda^A_B)$ are the ghosts, antighosts and Lagrange multipliers respectively, for the first-class constraints in (2).

For the second quantization it is useful to perform the following unitary transformation on $Q_{\text{BRST}}^{11)}$:

$$\begin{aligned} Q'_{\text{BRST}} &= U Q_{\text{BRST}} U^{-1}, \\ \ln U &= -(\ln \alpha) (c \partial_c + \tilde{c} \partial_{\tilde{c}} - 1), \quad \alpha \equiv (2\lambda)^{1/2}; \\ Q'_{\text{BRST}} &= \alpha^{-1} c (p^2 + i \partial_c \partial_{\tilde{c}}) + i \partial_c \partial_{\tilde{c}} + \left[\text{same as in (20) with} \right. \\ &\quad \left. \partial_c \rightarrow \alpha \partial_c \right]. \end{aligned}$$

Then the BFM Hamiltonian⁷⁾

$$H_{\text{BFV}} = \{ Q'_{\text{BRST}}, \partial_c \} = \alpha^{-1} (p^2 + i \partial_c \partial_{\tilde{c}})$$

has precisely the same form as for the usual bosonic point particle and, accordingly, the second-quantized BRST action reads¹⁰⁾:

$$S = \int d\tau (dz) \Phi^*(\tau, \tilde{z}) [\alpha i \partial_\tau - p^2 - i \partial_c \partial_{\tilde{c}}] \Phi(\tau, \tilde{z}) \quad (21)$$

where $(\tilde{z}) \equiv (x^\mu, \theta^{1,2}, \nu, \tilde{\nu}; \alpha, c, \tilde{c}; \eta^a, \tilde{\eta}_a, \rho^a; \chi_\alpha, \tilde{\chi}^\alpha, \psi_\alpha; C^A_B, \tilde{C}^B_A, \Lambda^A_B)$.

For Eq. (21) to represent the Lorentz-covariant action for the D=10 IIB supergravity in terms of unconstrained N=2 superfields one still needs to extend off-shell the reality condition (18). At present, unfortunately, we have not found a closed form for the latter.

In conclusion, let us once again stress that, as discussed in Section 1 above, Lorentz-covariant treatment of superparticle actions, containing the generator of the local fermionic symmetry as a constraint, is not consistent due to the reducibility of the set of first-class constraints unless one introduces additional (pure gauge) degrees of freedom - the bosonic Lorentz-spinor harmonics, which enable us to select an irreducible subset of covariant constraints. As a result, the rank of the system becomes two and, correspondingly, the BRST charge contains a higher-order ghost-interaction term.

A further more important task is to generalize the present formulation for the covariant canonical quantization of the Green-Schwarz superstring⁴⁾. There, similarly, the problem of reducible constraints does arise. Although a Lorentz-covariant separation of the second- and first-class constraints has been performed¹²⁾, the latter appear as projected MW spinors and thus apparently form a reducible set. Also, in Siegel's modification [second ref. 1)] of the Green-Schwarz superstring the reducibility of the string generalizations of the constraints $p^2=0$ and $p\cdot d=0$ should be taken into account for a consistent BFV-BRST quantization.

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APPENDIX

The following γ -matrices and D=10 charge conjugation matrix are taken in the following representation:

$$\gamma^\mu = \begin{pmatrix} 0 & (\sigma^\mu)_{\alpha\dot{\beta}} \\ (\tilde{\sigma}^\mu)^{\dot{\alpha}\beta} & 0 \end{pmatrix}, \quad C_{10} = \begin{pmatrix} 0 & C_8^{\alpha\dot{\beta}} \\ (-C_8)^{\dot{\alpha}\beta} & 0 \end{pmatrix}, \quad C_8^T = C_8^{-1} = C_8;$$

$$\gamma^\pm = \frac{1}{\sqrt{2}}(\gamma^0 + \gamma^9), \quad (\sigma^\mu)_{\alpha\dot{\beta}}(\tilde{\sigma}^\nu)^{\dot{\beta}\gamma} + (\sigma^\nu)_{\alpha\dot{\beta}}(\tilde{\sigma}^\mu)^{\dot{\beta}\gamma} = 2\eta^{\mu\nu}\delta_\alpha^\gamma,$$

$$\tilde{\not{p}} = \not{p} \tilde{\sigma}_\mu.$$

Indices of D=10 left-(right-)handed MW spinors $\Phi_\alpha, \Psi_{\dot{\alpha}}$ are raised by means of C_8 :

$$\Phi^{\dot{\alpha}} = (-C_8)^{\dot{\alpha}\beta}\Phi_\beta, \quad \Psi^\alpha = C_8^{\alpha\dot{\beta}}\Psi_{\dot{\beta}}.$$

Analogously,

$$(\tilde{\sigma}^\mu)^{\alpha\beta} = C_8^{\alpha\dot{\alpha}}(\tilde{\sigma}^\mu)_{\dot{\alpha}\beta}, \quad (\sigma^\mu)_{\alpha\beta} = (-C_8)_{\beta\dot{\beta}}(\sigma^\mu)_\alpha^{\dot{\beta}}.$$

The D=8 charge conjugation matrix may in turn be taken in the form:

$$C_8 = \begin{pmatrix} C_6^{ab} & 0 \\ 0 & (-C_6)^{\dot{a}\dot{b}} \end{pmatrix}, \quad C_6^T = C_6^{-1} = C_6,$$

and the indices of SO(8) (s),(c)-MW spinors $\phi_a, \psi_{\dot{a}}$ are raised as:

$$\varphi^a = C_6^{ab}\varphi_b, \quad \psi^{\dot{a}} = (-C_6)^{\dot{a}\dot{b}}\psi_{\dot{b}}.$$

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