MEASUREMENT OF ANTIPROTON-PROTON
SMALL-ANGLE ELASTIC SCATTERING AT LOW MOMENTUM

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Abstract

Two high statistics measurements of antiproton-proton small-angle elastic scattering, at $p=233$ MeV/c and $p=272$ MeV/c, are presented. The measurements were carried out at the LEAR-facility at CERN. By the Coulomb-nuclear interference method, values are obtained for the real-to-imaginary ratio ρ of the $\bar{p}p$ forward nuclear scattering amplitude and for its exponential slope b : $\rho=+0.041\pm0.026$ and $b=71.5\pm4.5$ (GeV/c)⁻² at 233 MeV/c and $\rho=-0.014\pm0.027$ and $b=47.7\pm2.7$ (GeV/c)⁻² at 272 MeV/c. The method to derive these values is discussed in detail and so are the uncertainties contributing to their systematic error. The results are compared with predictions from forward dispersion relation calculations and with predictions from $\bar{p}p$ potential models.

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1. Introduction

We have measured the $\bar{p}p$ small-angle differential elastic cross section at two momenta below 300 MeV/c. The experiment was carried out at the Low Energy Antiproton Ring LEAR at CERN, which provides a high intensity cooled antiproton beam free of contaminating particles. For the measurements a 1.148 cm liquid hydrogen target was used. The LEAR beam momentum was $p=310$ MeV/c, giving a momentum of 272 MeV/c at the centre of the target. A second measurement was done at $p=233$ MeV/c. For this purpose a degrader of 3.7 mm carbon was put in the beam at an intermediate focus [1]. The data cover an angular range from 7° to 21° in the lab. system. The measurements at 272 MeV/c and 233 MeV/c lasted 8 hours and 26 hours for a total useful beam of approximately 40×10^6 antiprotons each.

Using the description of the differential elastic cross section in terms of the sum over Coulomb and nuclear amplitudes, the real-to-imaginary ratio ρ of the forward nuclear scattering amplitude and its slope b are determined. The measurement of ρ provides both a direct and an indirect way to look for resonances in the $\bar{p}p$ scattering process. Via dispersion relations the behaviour of ρ at very low momentum is related to the existence of bound states near the $\bar{p}p$ threshold. Dispersion relation calculations require the existence of a pole in the unphysical region close to the $\bar{p}p$ threshold to explain the behaviour of ρ above 400 MeV/c [2] [3] [4]. This additional pole-term would imply fast decreasing values for ρ below 300 MeV/c.

The paper is organized as follows. The experimental apparatus and the data taking procedure are described in section 2. Section 3 deals with the off-line data analysis. In section 4 the measured differential cross section is presented. The folding procedure, which allows to compare the measurements with the theoretical description is described. The results of the fits are presented in section 5. In section 6 our results for ρ and b are compared with the values from other experiments carried out below 0.75 GeV/c, and with predictions from dispersion calculations and $\bar{p}p$ potential models. Finally, a summary is given in section 7.

2. Experimental apparatus and data taking

The set-up is shown in fig. 1. The apparatus is built around a 1.148 ± 0.008 cm long liquid hydrogen target, which can be emptied for background measurements. The target diameter is 4 cm. Along the beam line its assembly contains a total thickness of 650 μ m mylar windows. The incoming beam is defined by two scintillation counters, B0 and B1. B1 has a thickness of 0.5 mm and a diameter of 1 cm and is placed 11 cm before the target. B0 is placed in front of the degrader at an intermediate beam focus 20 metres upstream. The time-of-flight between B0 and B1 eliminates the small ($\leq 0.1\%$) contamination of pions and muons produced in the degrader.

The direction of the incoming beam track is measured with a precision of ± 0.9 mrad by two multiwire chambers PC1 and PC2. They each consist of one horizontal and one vertical multiwire detector with 1 mm wire spacing. A similar chamber is placed near the degrader and is used for beam tuning. The direction of the outgoing track is measured with a precision of ± 1.1 mrad by four multiwire chambers PC3, PC4, PC5 and PC6. They also consist of one horizontal and one vertical multiwire detector, but with 2 mm wire spacing. The high voltages on all wire chambers are set at such values that their detection efficiency for minimum ionising particles is very low, whereas the efficiency for the detection of low momentum $\bar{p}$'s is still 96% per plane.

Behind the last wire chamber, at a distance of 788 mm from the target, four rectangular scintillation counters, R1, R2, R3 and R4, are placed in a partly overlapping configuration, which leaves a square hole of 6×6 cm² for the outgoing beam. These R-counters are used to trigger on elastically scattered events, thus eliminating the very small-angle region ($< 2^\circ$) around the outgoing beam, which is dominated by multiple scattering. The time-of-flight between B1 and R_i ($i=1,2,3,4$) and the pulseheight in R_i are recorded for every event. This information is used in the off-line analysis to reject pions from annihilation events.

During the 233 MeV/c run the liquid hydrogen target was surrounded by a box of counters, the S-counters S1-S5. They cover 90% of the solid angle around the target. The time-of-flight between B1 and S_i was recorded for each event, thus presenting another way to reject annihilation events. At 272 MeV/c the S-counters were not used.

The trigger for elastic events is formed in two steps. For the events accepted by the first level trigger $B0 \cdot B1 \cdot (R1 + R2 + R3 + R4)$, a microprocessor (ESOP) calculates the distance between the coordinates measured in the last multiwire chamber (PC6) and the nominal beam axis. If this distance is smaller than 44 mm, the event is rejected. As a result, events with a scattering angle of 6° have an 80% probability of acceptance (see section 3.4). The trigger accepts $\approx 5\%$ of the events at 272 MeV/c and $\approx 10\%$ at 233 MeV/c, giving a typical trigger rate of 250 events per second.

Apart from the data taken with the elastic trigger, separate measurements are done with a beam trigger $B0 \cdot B1$. For these so-called beam-events the decision of the trigger $B0 \cdot B1 \cdot (R1 + R2 + R3 + R4) \cdot \text{ESOP}_{\text{accept}}$ is recorded on tape. Beam-events are used to determine the angular resolution for full and empty target runs and the acceptance function of the trigger at small angles.

The quality of the incoming beam as measured by PC1 and PC2 is high. The spatial width at the location of the target is ± 3.5 mm in both transverse directions. The angular width of the beam is ± 9 mrad horizontally and ± 15 mrad vertically. The momentum spread, determined from the B0-B1 time-of-flight spectra, is smaller than ± 1.2 MeV/c. The average momentum loss in the liquid hydrogen target is 10 MeV/c at 272 MeV/c and 14 MeV/c at 233 MeV/c. At 233 MeV/c, when the target is full, most antiprotons reach the end of their range in the R-counters and annihilate there. In the other runs most antiprotons traverse the whole set-up. Energy straggling calculations indicate that less than 0.5% of the antiprotons stop before reaching the R-counters at both momenta. This has no influence on the measured angular distributions (see section 3.2).

3. Data analysis

3.1 Event reconstruction

For the reconstruction of the incoming beam trajectory it is required that hits are found in PC1 and PC2 in both projections and that the coordinates, x and y , of the reconstructed track at the centre of the target lie within a distance of ± 12.5 mm from the beam line. For the reconstruction

of the outgoing track at least three aligned hits are demanded in the chambers behind the target in each projection. Then scattering vertices are calculated for all combinations of incoming and outgoing tracks. A 5σ -cut is applied to the vertex position in both transverse directions. If one and only one vertex is found, the event is accepted.

This preselection reduces the data sample by $\approx 40\%$. Half of the reduction originates from the rejection of annihilation events, having no outgoing track, as the wire chamber efficiencies are low for pions. Events with more than one outgoing track are found very rarely ($< 0.5\%$). The second half of the reduction originates from wire chamber inefficiencies for $\bar{p}$'s or spurious hits in the beam region of PC1 or PC2, causing the program to find a second incoming track. These reconstruction efficiencies are taken care of in the computation of the absolute normalization of the results (see section 4.1)

3.2 Rejection of annihilation events

The time-of-flight and pulseheight information provided by the R-counters are used to further reject annihilation events. Fig. 2 shows pulseheight spectra of R1. In the 272 MeV/c runs and at 233 MeV/c with empty target the antiprotons traverse the R-counters, giving narrow pulseheight spectra. At 233 MeV/c with full target most antiprotons stop in the R-counters, yielding a wider pulseheight spectrum. The low pulseheights on the left originate from pions and kaons, produced by antiprotons stopping close to R1, e.g. in one of the overlapping R-counters. The pulseheight information is only used to correct the measured time-of-flight in Ri.

At 272 MeV/c the time-of-flight between B1 and Ri is 11.2 nsec for $\bar{p}$'s and 4.7 nsec for pions produced in an antiproton annihilation in the target. The recorded time-of-flight to Ri is corrected for several effects:

1. travel-time of the light in the scintillator itself,
2. pulseheight in Ri,
3. $\bar{p}$ path length and scattering kinematics.

The corrected time-of-flight distributions have a FWHM of 0.8 nsec at both momenta. Fig. 3 shows the corrected time-of-flight distribution of R1 for the 233 MeV/c full target run. For the selection of elastic scattering all events with a signal earlier than 4 nsec before the nominal time (arbitrarily set to 0 nsec in fig. 3) in any of the R-counters are rejected. Moreover, events are only accepted if the R-counter in the direction of the outgoing $\bar{p}$ has a signal within the limits of ± 4 nsec around the nominal time.

At 233 MeV/c the time-of-flight between B1 and Si is recorded. On these time-of-flight spectra two distinct peaks are visible. The first one originates from prompt annihilations in the target; the second one is formed by backward annihilation products from $\bar{p}$ annihilations in the R-counters. Events with an early signal in one of the S-counters are rejected.

The above cuts eliminate 2% of the events in the Coulomb region at 272 MeV/c, 4% in the interference region and 7% in the nuclear region. At 233 MeV/c they eliminate 4% of the events in the Coulomb region, 13% in the interference region and 19% in the nuclear region. The cuts lead to a loss of at most 1.5% of the elastic scattering events at large angle.

The additional cut on the time-of-flight at 233 MeV/c removes two types of events, which cannot be distinguished at 272 MeV/c:

1. the small fraction of the $\bar{p}$ annihilations in the target region that are not rejected by the time-of-flight cuts on the R-counters,
2. antiproton-proton backward elastic scattering, as a forward going proton is indistinguishable from a forward going antiproton in our set-up. In this case the recoil antiproton will stop and annihilate in the liquid hydrogen target and most likely a prompt signal will be seen in the S-counters.

By combining the pulseheight information of the R-counters and the time-of-flight information of the S-counters and R-counters at 233 MeV/c one can separate the two effects. In this way an estimate is made of the contamination of the forward elastic scattering sample at 272 MeV/c, assuming a 20% decrease of the annihilation cross section [5] and a 60% decrease of the backward elastic cross section [W. Derks and J.J. de Swart, private communication] from 233 MeV/c to 272 MeV/c. The result is that the anti-

proton annihilations constitute 1% of the elastic scattering sample at small angle and 3% at the largest angle (21°) at 272 MeV/c. The backward elastic scattering constitutes 0.5% at small angle and 1.5% at large angle. The data sample is corrected accordingly.

3.3 Experimental resolution.

Due to the low antiproton momenta, the angular resolution of the measurement is dominated by multiple scattering. A fit to the exponential slope of the reconstructed beam-events at very forward angles yields the experimental angular resolution in full and empty target runs as listed in table 1. Fig. 4 shows the fit to the forward going beam-events for the 272 MeV/c full target run, giving a mean multiple scattering angle $\theta_0^F = 25.7$ mrad $= 1.47^\circ$. The error on the measurement of the scattering angle due to the resolution of the wire chambers is ± 1.5 mrad only.

Table 1: Experimental angular resolution.

p (MeV/c)	θ_0^F full target (mrad)	θ_0^E empty target (mrad)
272	25.7	24.4
233	31.2	29.3

In order to determine the resolution $\sigma_z(\theta)$ of the vertex position along the beam axis, full and empty target data are subtracted. For this purpose the empty target data are smeared by a Monte Carlo method simulating multiple scattering in the liquid hydrogen target, to obtain the same angular resolution as for the full target data (see table 1). This allows us to determine $\sigma_z(\theta)$ as a function of the scattering angle θ . A $3\sigma_z(\theta)$ -cut is applied to the data on both ends of the target, varying from ± 84 mm at $\theta = 120$ mrad to ± 27 mm at $\theta = 370$ mrad, to be compared with the actual target length of 11.48 mm.

3.4 Angular acceptance

The angular acceptance of the measurement is shown in fig. 5. It is limited on the small-angle side by the ESOP trigger and on the large-angle side by the size of the detectors. The trigger acceptance is derived from beam-events for which the decision of the elastic trigger $B0 \cdot B1 \cdot (R1 + R2 + R3 + R4) \cdot ESOP_{\text{accept}}$ is recorded. The mean trigger acceptance at angles larger than 10° is 99% at 272 MeV/c and 96.3% at 233 MeV/c. It is 90% for $\theta = 7^\circ$ at 272 MeV/c and for $\theta = 9^\circ$ at 233 MeV/c. The difference between the two momenta is caused by antiproton annihilations in Ri at the lower momentum, where one of the annihilation products crosses PC6 in the veto region. The data are used for final analysis down to the angle where the trigger acceptance is 90%. At this angle the Coulomb differential cross section is approximately equal to the nuclear differential cross section at both momenta.

The large-angle geometrical acceptance is determined by a Monte Carlo calculation. Incoming beam trajectories are simulated according to the distributions measured in PC1 and PC2. Scattering in the liquid hydrogen target is generated according to a flat distribution in θ^2 and in z-position along the target. To include the effect of multiple scattering, the Monte Carlo results are convoluted with the multiple scattering probability function as determined from the full target beam-events (see section 3.3). The data are used up to $\theta^2 = 0.13 \text{ rad}^2$, $\theta = 21^\circ$, where the acceptance is 75%.

4. The differential elastic cross section

4.1 The measured differential elastic cross section

The subtraction of empty from full target data yields the measured differential elastic cross section:

$$\frac{d\sigma^m}{dt} = \frac{1}{\alpha \epsilon \Delta t} \left[\frac{N^F(t)}{N_0^F e^{-\alpha\sigma}} - \frac{N^E(t)}{N_0^E} \right] \quad (\text{mb}/(\text{GeV}/c)^2), \quad (1)$$

- with α = the number of target protons per unit area
 $= (5.16 \pm 0.10) \times 10^{-5} / \text{mb}$,
 ϵ = the transmission probability for antiprotons through the
empty target set-up,
 $N(t)$ = the number of elastic scattering events observed in the
momentum transfer bin with mean value t and width Δt ,
corrected for geometrical acceptance,
 N_0 = the number of incoming antiprotons corrected for recon-
struction efficiency,
 σ = the $\bar{p}p$ inelastic cross section plus the cross section for
large-angle elastic scattering.

The superscripts F and E refer to full and empty target measurements respectively. The term $e^{-\alpha\sigma}$ corrects for antiproton absorption in the liquid hydrogen target; it corresponds to 0.9867 ± 0.0008 at 272 MeV/c and 0.9856 ± 0.0009 at 233 MeV/c. Using an $A^{2/3}$ mass dependence for the $\bar{p}$ absorption cross section on nuclei [6] the correction factor $1/\epsilon$ for the antiproton absorption in the empty target set-up is determined to be 1.020 ± 0.005 at 272 MeV/c and 1.025 ± 0.006 at 233 MeV/c. The reconstruction efficiency for elastic scattering events is around 70% at both momenta, mainly due to wire chamber inefficiencies and spurious hits in the beam region of PC1 and PC2. Table 2 gives the number of incoming $\bar{p}$'s corrected for reconstruction efficiency at both momenta.

Table 2: Total number of incoming antiprotons corrected for reconstruction efficiency.

p (MeV/c)	N_0^F full target ($\times 10^6$)	N_0^E empty target ($\times 10^6$)
272	12.6	13.3
233	22.1	10.8

Table 3 gives $d\sigma^m(\theta)/dt$ at both momenta and the average folding correction on each data point (see section 4.2). The total number of events in the (full minus empty) data sample equals 16218 at 272 MeV/c and 24712 at 233 MeV/c. At the largest angle (21°) the number of events scattered off the liquid hydrogen target is four times larger than the number of events scattered off the empty target set-up. This ratio decreases to 1 at 9° at both momenta. The estimated error on the absolute normalization amounts to $\pm 5\%$ at both momenta.

4.2 The folding procedure

The measured differential elastic cross section cannot be compared with the theoretical differential cross section directly as the measurements are smeared by multiple Coulomb scattering. Given the low momenta at which the measurements are carried out and the high statistical accuracy of the data, the application of an accurate folding procedure, which simulates this smearing effect, is important to determine the parameters of the forward nuclear scattering amplitude precisely. The method generally used to convolute the differential cross section with a truncated Gaussian distribution, e.g. [4], appears to be inadequate. A full derivation of our folding procedure may be found in [7]. Here we confine ourselves to an outline of the method plus the relevant expression.

To derive an expression for the measured differential cross section at angle θ in terms of the pure $\bar{p}p$ differential cross section $d\sigma/dt$ plus instrumental effects, we regard elastic scattering off our full target apparatus as a combination of two consecutive scattering processes: the background scattering off the empty target set-up followed by the scattering off hydrogen. The number of elastic scattering events observed in angular element $d\theta d\phi$ during a full target measurement equals the total number present in that element from background scattering minus the number of events that are scattered out of the element by the hydrogen scattering (out-scattering) plus the number of events that are scattered into the element from elsewhere (in-scattering). In-scattering and out-scattering are considered over angles θ' between 0 and $\pi/6$ rad only, as the elastic cross section decreases rapidly with increasing angle. The geometry of the integral over θ' is shown in fig. 6. The measured differential cross section is therefore described as:

Table 3: The measured $\bar{p}p$ differential elastic cross section

$d\sigma^m(\theta)/dt$ at 233 MeV/c and 272 MeV/c. The term F represents the average folding correction on each data point; $d\sigma(\theta)^m/dt$ has to be divided by F to obtain the pure differential cross section $d\sigma(\theta)/dt$.

233 MeV/c				272 MeV/c		
θ^2	t	$d\sigma^m(\theta)/dt$	F	t	$d\sigma^m(\theta)/dt$	F
(rad ²)	(0.001× (GeV/c) ²)	(mb/(GeV/c) ²)		(0.001× (GeV/c) ²)	(mb/(GeV/c) ²)	
0.0145				1.07	7024±876	1.16
0.0155				1.14	7817±803	1.15
0.0165				1.21	6864±712	1.11
0.0175				1.29	5514±662	1.08
0.0185				1.36	4893±621	1.07
0.0195				1.43	5048±588	1.06
0.0205				1.51	4718±576	1.06
0.0215				1.58	4502±537	1.06
0.0225				1.65	4053±524	1.05
0.0235				1.72	3160±523	1.04
0.0245				1.80	4265±500	1.03
0.0255	1.37	6680±781	1.11	1.87	3733±464	1.03
0.0265	1.43	7297±741	1.10	1.94	3864±474	1.03
0.0275	1.48	6752±725	1.10	2.01	3531±453	1.03
0.0285	1.53	5951±696	1.10	2.09	3462±454	1.03
0.0295	1.59	6203±626	1.09	2.16	3697±430	1.02
0.0310	1.66	6520±438	1.07	2.27	3706±296	1.01
0.0330	1.77	5265±414	1.05	2.41	3284±287	1.01
0.0350	1.88	4489±399	1.04	2.56	3318±285	1.00
0.0370	1.98	4621±369	1.04	2.70	3159±279	1.00
0.0390	2.09	4406±360	1.04	2.85	3240±273	1.00
0.0410	2.19	3878±347	1.03	2.99	2993±262	1.00
0.0430	2.30	3995±340	1.03	3.13	3118±262	1.00
0.0450	2.40	4771±319	1.02	3.28	3232±267	1.00
0.0470	2.51	4556±320	1.02	3.42	3255±261	1.00
0.0490	2.62	4027±329	1.02	3.56	2960±256	1.00
0.0510	2.72	3681±305	1.02	3.71	3147±249	1.00
0.0530	2.82	3728±313	1.02	3.85	2607±241	1.00

0.0550	2.93	4539±296	1.01	3.99	3247±255	1.00
0.0570	3.03	3998±304	1.01	4.13	2958±242	1.00
0.0590	3.14	3855±306	1.01	4.27	2730±241	1.00
0.0610	3.24	4549±281	1.00	4.42	3058±248	1.00
0.0630	3.35	4427±288	1.00	4.56	2774±238	1.00
0.0650	3.45	4307±285	1.00	4.70	3139±240	1.00
0.0670	3.55	4105±271	1.00	4.84	2816±240	1.00
0.0690	3.66	3721±277	1.00	4.98	2589±226	1.00
0.0710	3.76	3786±284	1.00	5.12	2641±231	1.00
0.0730	3.86	3604±283	1.00	5.26	2716±229	1.00
0.0750	3.97	3419±265	1.00	5.40	2734±232	1.00
0.0770	4.07	3580±259	1.00	5.54	2676±224	1.00
0.0790	4.17	3418±266	1.00	5.68	2789±226	1.00
0.0810	4.27	3370±255	1.00	5.82	2688±219	1.00
0.0830	4.38	3647±260	1.00	5.96	2655±213	1.00
0.0850	4.48	3769±247	1.00	6.10	2572±219	1.00
0.0870	4.58	3480±245	1.00	6.24	2836±218	1.00
0.0890	4.68	3792±254	1.00	6.38	2864±226	1.00
0.0910	4.79	2958±256	1.00	6.52	2724±220	1.00
0.0930	4.89	2743±255	1.00	6.66	2379±211	1.00
0.0950	4.99	3425±249	1.00	6.80	2744±215	1.00
0.0970	5.09	3173±253	1.00	6.93	2644±220	1.00
0.0990	5.19	3575±248	1.00	7.07	2307±209	1.00
0.1010	5.29	2979±251	1.00	7.21	2549±218	1.00
0.1030	5.39	2953±234	1.00	7.35	2351±206	1.00
0.1050	5.50	3127±244	1.00	7.48	2149±205	1.00
0.1070	5.60	2827±253	1.00	7.62	2413±210	1.00
0.1090	5.70	2752±250	1.00	7.76	2095±203	1.00
0.1110	5.80	2818±241	1.00	7.90	2394±207	1.00
0.1130	5.90	3448±244	1.00	8.03	2419±211	1.00
0.1150	6.00	3019±243	1.00	8.17	2626±223	1.00
0.1170	6.10	2903±247	1.00	8.30	2719±215	1.00
0.1190	6.20	2875±258	1.00	8.44	2319±216	1.00
0.1210	6.30	2375±262	1.00	8.58	2176±216	1.00
0.1230	6.40	2992±240	1.00	8.71	2167±220	1.00
0.1250	6.50	3052±259	1.00	8.85	2019±225	1.00
0.1270	6.60	2753±255	1.00	8.98	2149±217	1.00

$$\frac{d\sigma(\theta)^f}{dt} = \int_0^{2\pi} \int_0^{\pi/6} \frac{d\sigma(\theta')}{dt} \left[\frac{dP^E(\theta-\theta')}{d\Omega} - \frac{dP^E(\theta)}{d\Omega} \right] \sin\theta' d\theta' d\phi' \left[\frac{\text{mb}}{(\text{GeV}/c)^2} \right], \quad (2)$$

where $d\sigma(\theta')/dt$ represents the pure $\bar{p}$ -hydrogen differential cross section including nuclear scattering, single Coulomb scattering, Coulomb-nuclear interference and multiple Coulomb scattering (see section 4.3). The term $dP^E/d\Omega$ represents the angular probability function for background scattering. It is obtained by combining triggered ($5^\circ < \theta < 21^\circ$) and non-triggered ($0^\circ < \theta < 5^\circ$) empty target data. The measured differential cross section $d\sigma^m(\theta)/dt$ is compared with $d\sigma^f(\theta)/dt$ to determine the parameters of $d\sigma(\theta)/dt$. The average folding correction on each data point is listed in table 3.

4.3 The shape of the $\bar{p}$ -hydrogen differential cross section

To determine the shape of the $\bar{p}$ -hydrogen differential cross section the formulae for the $\bar{p}p$ differential elastic cross section are modified to include multiple scattering in the hydrogen target using the formalism described by Bethe [8] and Øveras [9]. For the $\bar{p}p$ differential elastic cross section the usual description in terms of the sum over Coulomb and nuclear amplitudes is taken [10] [4] [11].

The total Coulomb angular probability distribution $dP_C(\theta)/d\Omega$ (including multiple, plural and single Coulomb scattering) is expressed as a function of the mean multiple scattering angle $\theta_0 = X_C \sqrt{B}$. The parameters X_C and B depend both on the properties of the incoming antiproton and on the medium it passes. They are described in [9]. Using this probability distribution the Coulomb differential cross section is written as:

$$\frac{d\sigma^C(\theta)}{dt} = \frac{2.606 \times 10^{-4}}{\beta^2 p^4} \frac{\pi}{X_C^2} \frac{dP_C(\theta)}{d\Omega} \left(1 + \frac{t_s}{0.71}\right)^{-8} \left[\frac{\text{mb}}{(\text{GeV}/c)^2} \right], \quad (3)$$

where $t_s = t + p^2 X_\alpha^2$, the screened momentum transfer taking into account the effect of the electron moving around the proton in the hydrogen atom. The

screened momentum transfer prevents the Coulomb-nuclear differential cross section from going to infinity at scattering angle $\theta=0$. The formula for calculating the screening angle X_α may also be found in [9]. At 272 MeV/c the term $p^2 X_\alpha^2 = 2.03 \times 10^{-11} \text{ (GeV/c)}^2$.

The nuclear differential cross section is parametrized as:

$$\frac{d\sigma^N(\theta)}{dt} = 0.0511 \sigma_{\text{tot}}^2 (1+\rho^2) (1+\eta^2) e^{-bt} \left[\frac{\text{mb}}{(\text{GeV/c})^2} \right], \quad (4)$$

with $\rho =$ the real-to-imaginary ratio of the forward nuclear scattering amplitude,
 $b =$ the slope of the forward nuclear amplitude $(\text{GeV/c})^{-2}$,
 $\sigma_{\text{tot}} =$ the $\bar{p}p$ total cross section (mb).

The parameter η expresses the spin-dependence of the forward nuclear amplitude. When fitting to experimental differential cross sections, the assumption of spin-independence is usually made. Some models, however, predict values as large as $\eta=0.33$ near $p=250 \text{ MeV/c}$ [13] [14] [15]. The parameters ρ and b are assumed to be constant over the limited t -range of the measurement.

The Coulomb-nuclear interference cross section is written as:

$$\frac{d\sigma^{\text{CN}}(\theta)}{dt} = 0.0073 \frac{1}{\beta t} (1+t_s/0.71)^{-4} \sigma_{\text{tot}} e^{-0.5bt} (\rho \cos \delta(t_s) - \sin \delta(t_s)) \left[\frac{\text{mb}}{(\text{GeV/c})^2} \right]. \quad (5)$$

For $\delta(t)$, the phase of the Coulomb amplitude in the presence of the nuclear amplitude, the expression derived by Locher [12] is used:

$$\delta(t) = \frac{1}{137\beta} (\ln(9.5t) + 0.5772). \quad (6)$$

The expression for $\delta(t)$ is not unique and depends on the range of the nuclear interaction. However, when different expressions for $\delta(t)$ are used the fitted value for p changes at most by 0.01 [11] [B. Moussallam private communication].

To illustrate the influence of the three parameters p , b and σ_{tot} on the shape of the differential cross section $d\sigma(\theta)/dt$, fig. 7 shows $d\sigma(\theta)/dt$ together with the separate contributions $d\sigma^C(\theta)/dt$, $d\sigma^{\text{CN}}(\theta)/dt$ and $d\sigma^N(\theta)/dt$ at $p=272$ MeV/c. To draw this figure the numbers $\sigma_{\text{tot}}=263$ mb, $b=40$ (GeV/c) $^{-2}$, $p=0.5$ and $\eta=0$ were chosen. The total cross section determines the absolute normalization of the nuclear differential cross section and of the Coulomb-nuclear interference. It therefore determines the height of the curve at angles larger than 12° and it influences the location of the bend near $\theta=8^\circ$. The slope b of the nuclear amplitude determines the exponential fall-off of $d\sigma(\theta)/dt$ at angles larger than $\theta=12^\circ$. The value for p is set at +0.5 to obtain a reasonably large positive interference part for the figure. The value of p influences the shape of the bend near $\theta=8^\circ$; it will show a dip for negative p -values.

The parameters p, b and σ_{tot} are very correlated as we will describe below. The parameters p and b can only be determined through measurements of small-angle elastic scattering, whereas σ_{tot} can be measured independently. At LEAR we have also carried out two very precise σ_{tot} -scans as a function of momentum in a transmission experiment. The first scan [1] was carried out in the momentum range from $p=388$ MeV/c to $p=599$ MeV/c in momentum steps of 5-10 MeV/c with a statistical accuracy of better than $\pm 0.4\%$ and an accuracy of $\pm 0.7\%$ in absolute normalization. The results show a smooth behaviour as a function of momentum, corresponding to the momentum dependence $\sigma_{\text{tot}}=65.8+53759/p$ (mb), with p in MeV/c. The absolute normalization of these results agrees to within 3% with other precise measurements of σ_{tot} in the same momentum range. Recently a second scan has been carried out below 413 MeV/c down to approximately 220 MeV/c. As the results are consistent with the above parametrization, we will use it as an input to our fits, allowing an estimated error of $\pm 1.7\%$ on σ_{tot} at 272 MeV/c and $\pm 2.4\%$ at 233 MeV/c.

5. Results of the fits

Figs. 8a and 8b show the measured differential elastic cross sections at 272 MeV/c and 233 MeV/c, together with the curves of the fits. At 272 MeV/c the data are fitted from $t=0.001 \text{ (GeV/c)}^2$ (7°) to $t=0.009 \text{ (GeV/c)}^2$ (21°). At 233 MeV/c the angular range is somewhat smaller, from 9° to 21° , due to the reduced trigger acceptance. The momentum transfer range, from $t=0.0014 \text{ (GeV/c)}^2$ to $t=0.0065 \text{ (GeV/c)}^2$, is however 30% smaller as t is proportional to p^2 .

The fits are done using the input-values $\sigma_{\text{tot}}=263 \text{ mb}$ at 272 MeV/c and $\sigma_{\text{tot}}=296 \text{ mb}$ at 233 MeV/c, derived from the parametrization of our total cross section measurements. Table 4 summarizes the results of the fits, both assuming a spin-dependence $\eta=0$ and also $\eta=0.33$ as predicted by some theoretical models. The errors are statistical. The χ^2 of the fit is 45.3 for 63 degrees of freedom at 272 MeV/c and 70.0 for 52 degrees of freedom at 233 MeV/c for $\eta=0$.

Table 4: Results of the fits at 272 MeV/c and 233 MeV/c.

	p=272 MeV/c $\sigma_{\text{tot}}=263 \text{ mb}$		p=233 MeV/c $\sigma_{\text{tot}}=296 \text{ mb}$	
	$\eta=0.0$	$\eta=0.33$	$\eta=0.0$	$\eta=0.33$
p	$-.014 \pm .027$	$-.107 \pm .033$	$+.041 \pm .026$	$-.038 \pm .030$
b ((GeV/c) $^{-2}$)	47.7 ± 2.7	60.6 ± 2.3	71.5 ± 4.5	84.9 ± 4.0
$\chi^2/\text{d.o.f.}$	45.3/63	49.0/63	70.0/52	69.4/52

Table 5 gives a detailed list of the contributions to the systematic errors on the results of Table 4. The estimated errors of $\pm 1.7\%$ on σ_{tot} at 272 MeV/c and $\pm 2.4\%$ at 233 MeV/c determine the error on the absolute normalization. Variations in the cut-off at small angle principally affect the correctness of the folding procedure. A small incorrectness in the folding procedure would have a large influence on χ^2 in the Coulomb and interfer-

Table 5: Systematic errors on the fits with fixed σ_{tot}

Origin	p=272 MeV/c $\sigma_{\text{tot}}=263$ mb		p=233 MeV/c $\sigma_{\text{tot}}=296$ mb	
	$\Delta\rho$	Δb (GeV/c) ⁻²	$\Delta\rho$	Δb (GeV/c) ⁻²
Absolute normalization	± 0.03	± 4.0	± 0.03	± 6.0
Trigger acceptance	-0.03	-1.1	-0.02	+0.3
Variation in cut-off at small angle	± 0.02	+1.8 -1.6	± 0.01	+1.5 -0.5
Variation in cut-off at large angle	± 0.01	+2.0 -3.0	± 0.01	+2.3 -2.1

ence region and thus on the fitted ρ -value. Variations in the cut-off at large angle principally influence the fitted slope b , which may depend on the t -range.

The results of the fits depend strongly on the value of σ_{tot} . This is illustrated in fig. 9, which shows the correlation between the ρ and b at 272 MeV/c as a function of the input-value for σ_{tot} ($\eta=0$). The contours represent the values of ρ and b corresponding to a χ^2 of 1 above the minimum. Over the large σ_{tot} -range shown in the figure, χ^2 of the fit changes by at most 5.6 for 63 degrees of freedom. This shows that the use of an independent measurement of σ_{tot} is to be preferred above extracting ρ , b and σ_{tot} simultaneously from the measurement of the differential elastic cross section.

The parametrization of the differential elastic cross section used for the fits to our data is based on some approximations, which may in fact only hold at higher energies (see e.g. [16]). We have examined some suggestions, which were put forward to modify the above parametrization. We have applied, for example, a modified formalism based on the Nijmegen an-

tinucleon-nucleon potential model [13] [17]. In this formalism the slope b of the nuclear amplitude is replaced by an average slope b' , used in the nuclear differential cross section, plus a spin-independent slope b^{CN} , used in the Coulomb-nuclear interference. The average slope b' is smaller than b^{CN} , e.g. $b' = 32.7 \text{ (GeV/c)}^{-2}$ and $b^{\text{CN}} = 69.6 \text{ (GeV/c)}^2$ at 272 MeV/c. However, this modified formalism does not influence the results of our fits. If σ_{tot} is entered as a free parameter, the result is consistent with the correlation shown in fig. 9. Moreover we have tried adding a single pion exchange contribution to the differential elastic cross section following a description by Scotti and Wong [18]. This changed p by less than 0.01.

6. Discussion

The results for p from the present experiment are compared with other measurements of $p(\bar{p}p)$ below 0.75 GeV/c in fig. 10. To get a consistent set, all results assuming $\eta=0$ are taken. Table 6 supplies a summary of all $\bar{p}p$ small-angle elastic scattering measurements below 0.75 GeV/c. The measurements of Brückner et al. [11] were also done at LEAR, whilst the others originate from the pre-LEAR era, when high intensity, low momentum pure $\bar{p}$ -beams were not feasible. One should note the high statistical accuracy and large angular range covered by the LEAR experiments. Contrary to this advantage, the low momentum experiments suffer from additional difficulties due to multiple scattering.

As can be seen from table 6, most groups have extracted only two out of three parameters p , b and σ_{tot} from their data, as a result of the highly correlated nature of these parameters. For the parallel LEAR experiment however, all three parameters were fitted simultaneously to the data. At their highest momenta, 505 MeV/c and 590 MeV/c, the σ_{tot} -values can be compared with a series of high accuracy independent σ_{tot} -measurements, showing that the values from Brückner et al. are on average 8% lower. This illustrates that a three-parameter fit is very difficult, even on high quality data.

It should be remarked that the agreement between the p -values shown in fig. 10 is good in spite of the different techniques and fitting procedures used. The general trend of the data can be described in the follow-

Table 6: Summary of the $\bar{p}p$ small-angle elastic scattering experiments

below 0.75 GeV/c. The angular range covered is alternatively denoted in terms of momentum transfer t ((GeV/c)²) or in laboratory angles (degrees). The type BC stands for bubble chamber experiment and ED stands for electronic detector experiment.

Ref.	type	p-range (MeV/c)	number of momenta	angular range (GeV/c) ² or degrees lab.	number of angular bins	stat. error per bin	parameters fitted
[19]	BC	697	1	0.0018-0.032	16	±9%	ρ , b
[20]	BC	353-578	8	0.0005-0.06	80	±20%	ρ , b , σ_{tot}
[4]	ED	413-715	25	3°-10°	7	±10%	ρ
[21]	ED	359-652	11	4°-16°	26	±8%	ρ , b
[11]	ED	505,590 181-287	2 5	5°-23° and 28°-32°	50	±15%	ρ , b , σ_{tot}
This	ED	233,272	2	7°-21°	60	±9%	ρ , b

ing way: between 700 MeV/c and 400 MeV/c ρ decreases with decreasing momentum from $\rho=0.3$ to $\rho=-0.1$, then, between 400 MeV/c and 300 MeV/c, the trend changes and ρ starts to increase with decreasing momentum, crossing zero near 270 MeV/c.

In dispersion relation calculations, a large positive contribution to ρ at low momentum arises from the dispersion integral over the $\bar{p}p$ physical region. If σ_{tot} increases as $1/p$ right down to the $\bar{p}p$ threshold, this integral diverges as $\ln(1/T)$, where T is the lab. kinetic energy. The curve in fig. 10 shows the result of the dispersion relation calculations of Iwasaki et al. [4] using the formalism first applied by Grein [2]. In these calculations the dispersion integral is cancelled with a phenomenological pole at a $\bar{p}p$ mass of 1864 ± 12 MeV/c², in order to explain the behaviour of ρ above

400 MeV/c. The existence of this pole would imply large negative values for p at low momentum, which is in clear disagreement with the results from LEAR as can be seen from the figure. The implication is that the discontinuity in the unphysical region must be more complicated than a simple pole. Physically, values of p close to zero mean that the annihilation dominates over the real part of the potential.

Fig. 11 shows p -results from the same measurements as fig. 10, but now assuming η as suggested by the Paris antinucleon-nucleon potential model, varying from $\eta=0.26$ at 800 MeV/c to $\eta=0.33$ near 250 MeV/c. Therefore the values from Iwasaki et al. [22] [4] are lowered by 0.1 as suggested by the authors. The value from Kaseno et al. [19] is also lowered by 0.1. The results from Ashford et al. [21] assuming η -values from the Paris model are given by the authors. The values for p from Cresti et al. [20] and Brückner et al. [11] remain the same. The two curves in fig. 11 represent the predictions from the Paris [14] [15] and Nijmegen [13] [17] [23] antinucleon-nucleon potential models. The spin-dependence η predicted by the Nijmegen model decreases from $\eta=0.35$ at 200 MeV/c to $\eta=0.13$ at 800 MeV/c. These values agree well with the η -values from the Paris model at low momentum (<400 MeV/c), but they are smaller at high momentum. Both the Nijmegen and the Paris model predict slowly decreasing p -values with decreasing momentum below $p=0.75$ GeV/c. As can be seen from fig. 11, the p -values from these models do not agree with the experimental results. Between 400 MeV/c and 800 MeV/c the predictions from the Nijmegen model are typically 0.15 lower. Below 300 MeV/c this difference becomes as much as 0.5. The Paris model is in reasonable agreement with the data above 400 MeV/c, but below 300 MeV/c the difference is typically 0.3. Some possible sources of this discrepancy, like the influence of the $\bar{p}p \rightarrow \bar{n}n$ threshold at 200 MeV/c and the accuracy of the parametrization of the differential elastic cross section, are studied by Moussallam [16]. The effect of the former appears to be very small, whereas the effect of the latter accounts only partly for the difference.

Fig. 12 shows the measured values for b as a function of momentum below 0.75 GeV/c. To obtain a consistent set the values assuming $\eta=0$ are taken. The figure shows that the agreement between the b -values from the various experiments is worse than the agreement between the p -values. As b may depend on the momentum transfer t , this may be due to the difference in t -range between the various fits. The results for b below 300 MeV/

c from Brückner et al. are lower than our results. This is in accordance with the lower σ_{tot} -values resulting from their fits as is illustrated in fig. 9.

7. Summary

We have presented two high statistics measurements of antiproton-proton small-angle elastic scattering at $p=233$ MeV/c and $p=272$ MeV/c carried out at LEAR. From the data the real-to-imaginary ratio of the $\bar{p}p$ forward scattering amplitude and the slope of the forward scattering amplitude are determined assuming σ_{tot} -values from an independent measurement. They give $\rho=-0.014\pm0.027$ at 272 MeV/c and $\rho=+0.041\pm0.026$ at 233 MeV/c, $b=47.7\pm2.7$ (GeV/c) $^{-2}$ at 272 MeV/c and $b=71.5\pm4.5$ (GeV/c) $^{-2}$ at 233 MeV/c assuming $\eta=0$. The systematic error on ρ is ±0.05 at both momenta. The systematic error on b is ±5 (GeV/c) $^{-2}$ at 272 MeV/c and ±7 (GeV/c) $^{-2}$ at 233 MeV/c. In agreement with a parallel experiment at LEAR we find that ρ is close to zero near 250 MeV/c and increases with decreasing momentum. This behaviour is in contrast with the general trend of the measurements above 400 MeV/c, which show that ρ decreases with decreasing momentum, and it is also in clear disagreement with present dispersion relation calculations and potential model predictions.

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Figure captions

1. The experimental set-up for measuring $\bar{p}p$ small-angle elastic scattering. PC denotes multiwire chambers and R trigger counters.
2. Pulseheight spectra measured in scintillation counter R1 at both momenta with full and empty target.
3. The corrected time-of-flight spectrum of R1 at 233 MeV/c with full target.
4. The angular distribution $dN/d\theta^2$ of beam tracks in the very forward direction at 272 MeV/c with full target, showing the Gaussian probability distribution of multiple Coulomb scattering. The fitted mean angle is $\theta_0^F = 0.0257$ rad.
5. The angular acceptance of the measurement at 272 MeV/c.
6. The geometry of the integral over θ' and ϕ' , which sums the in-scattering and out-scattering effects to and from angle θ .
7. The $\bar{p}p$ differential elastic cross section $d\sigma(\theta)/dt$ at $p=272$ MeV/c together with the separate contributions $d\sigma^C(\theta)/dt$, $d\sigma^{CN}(\theta)/dt$ and $d\sigma^N(\theta)/dt$. For the parameters of the nuclear amplitude $p=0$, $b=40$ (GeV/c) $^{-2}$, $\sigma_{tot}=263$ mb and $\eta=0$ are taken.
8. The measured differential elastic cross section at (a) 272 MeV/c and (b) 233 MeV/c together with the results of the fits. The error bars represent statistical errors only.
9. The correlation between p and b for fits with different input-values for σ_{tot} at 272 MeV/c. The contours correspond to a χ^2 increase of 1 above the minimum.
10. The measured values for p below 0.75 GeV/c assuming spin-dependence $\eta=0$ from the experiments mentioned in Table 6. The curve shows the result of the dispersion calculation from Iwasaki et al.

The error bars represent statistical errors only. The systematic error on our results amounts to ± 0.05 at both momenta.

11. The measured values for p below $0.75 \text{ GeV}/c$ assuming a spin-dependence η as predicted by the Paris model. The curves show the predictions from the Paris and Nijmegen $\bar{p}p$ potential models. The error bars represent statistical errors only. The systematic error on our measurement amounts to ± 0.05 at both momenta.
12. The measured values for b below $0.75 \text{ GeV}/c$ assuming spin-dependence $\eta=0$. The error bars represent statistical errors only. The systematic error on our measurement is $\pm 5 (\text{GeV}/c)^{-2}$ at $272 \text{ MeV}/c$ and $\pm 7 (\text{GeV}/c)^{-2}$ at $233 \text{ MeV}/c$.

fig. 1

fig. 2

fig. 3

fig. 4

fig. 5

fig. 6

fig. 7

fig. 8b

fig. 8a

fig. 9

fig. 10

fig. 11

fig. 12