CERN-TH.4650/87

HIGHER DERIVATIVE SUPERGRAVITY IS EQUIVALENT
TO STANDARD SUPERGRAVITY COUPLED TO MATTER. I.

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A B S T R A C T

We generalize a recent theorem by Whitt (the equivalence of $R+\alpha R^2$ gravity with the Einstein theory coupled to a scalar field with specified non-linear self-couplings) to $N = 1$ supergravity in four dimensions. Pure $R+\alpha R^2$ supergravity turns out to be equivalent to standard supergravity coupled to two chiral multiplets. The corresponding Kähler metric is the same as in no-scale models, but the superpotential is different. We give also four generalizations of the above theorem: i) we couple matter and allow the coefficient of R^2 to be field-dependent; ii)-iii) we consider arbitrary powers of the scalar curvature R and iv) we consider arbitrary functions of matter fields, R , $\square R$, $\square^2 R$ etc. Peculiar phenomena occur when no-scale models are coupled to higher powers of R . We present some applications of the above results, in particular in connection with superstring phenomenology.

CERN-TH.4650/87
February 1987

Recently, higher-derivative supergravity theories¹⁾⁻⁴⁾ have become more and more popular, principally because of their applications to superstring theories⁵⁾ in the field limit^{6),7)} [in particular to understand the supersymmetric completion of the terms needed to cancel the anomalies^{8),9)}].

Unfortunately, the corresponding Lagrangians are very hard to construct. Even in four dimensions where we have a powerful tensor calculus¹⁰⁾⁻¹²⁾, life is not so easy since computing the Lagrangian by brute-force tensor calculus leads to some 10^4 terms, which are however expected to simplify dramatically in the final answer.

It is natural to look for better methods giving directly the simple final answer, bypassing the intermediate steps. Obviously, the simple final answer would be easier to guess, having some physical intuition of its meaning and of the reasons why it must be simple, after all. Some major insights into these issues can be obtained from the analysis of the corresponding problems in ordinary gravity.

The particle spectrum (at the linearized level) for the more general Lagrangian quadratic in the gravitational curvatures^{*)}

$$S = \int d^4x \sqrt{-g} [R - 2\Lambda + \alpha R^2 + \beta R_{\mu\nu} R^{\mu\nu}] \quad (1)$$

was studied in Refs. 13)-14). The spectrum contains (besides the usual massless graviton) a massive ($m^2 = -1/\beta$) spin-2 poltergeist (negative-metric) and a positive-metric scalar field with mass

$$m_0^2 = \frac{1}{2(3\alpha + \beta)} \quad (2)$$

In the limit $\beta \rightarrow 0$ the poltergeist decouples (and renormalizability is lost) and we remain with a totally acceptable spectrum, i.e. the graviton plus a physical scalar (provided $\alpha > 0$, otherwise it is a tachyon^{**)} .

*) Recall that a linear combination of R^2 , $R_{\mu\nu} R^{\mu\nu}$ and $R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta}$ is a total derivative, by the Gauss-Bonnet theorem.

**) In Ref. 15) it was argued that in the superstring effective Lagrangian the gravitational curvature must enter through the 4D Gauss-Bonnet combination, in order for the poltergeists to decouple. This is correct if one demands that only massless states are described by the effective Lagrangian. But the absence of negative-norm states, by itself, allows an R^2 term to be added. It is tempting to identify the corresponding particle with mass Eq. (2), with a physical string state.

That Eq. (1) with $\beta = 0$ is equivalent to a massive scalar coupled to ordinary gravity was then proved at the full non-linear level by Whitt¹⁶⁾. Indeed, Eq. (1) ($\beta = 0$) is equivalent to

$$S = \int d^4x \sqrt{-g} [(1 + 2\alpha\phi)R - \alpha\phi^2 - 2\Lambda] \quad (3)$$

as can be seen by integrating out the auxiliary Gaussian field ϕ . The Weyl rescaling $g_{\mu\nu}|_{\text{new}} = (1 + 2\alpha\phi)g_{\mu\nu}$ puts Eq. (3) in the form

$$S = \int d^4x \sqrt{-g} \left[R - \frac{6\alpha^2 \partial^\mu \phi \partial_\mu \phi}{(1 + 2\alpha\phi)^2} - \frac{\alpha\phi^2 + 2\Lambda}{(1 + 2\alpha\phi)^2} \right] \quad (4)$$

which corresponds to the coupling of gravity to a (positive-metric) scalar with specific self-couplings [notice that the kinetic term in Eq. (4) may be put in the canonical form by a field redefinition].

The main purpose of the present letter is to give the supersymmetric counterpart of the above result^{*}), as well as some generalizations and applications.

The most naive idea to extend Whitt's argument would be to try the Gaussian trick of Eq. (3). This cannot work since the counting of the physical degree of freedom is different in supergravity (vs. ordinary gravity) since also the auxiliary fields propagate physical particles¹⁷⁾.

However, the trick that does work is even more elementary, and this is the first reason why we are actually able to prove a much stronger result (see Generalization D below) than in the ordinary case.

We shall work in the framework of superconformal tensor calculus^{11),12)} which has two main advantages:

- i) By definition, we do not need to perform any Weyl rescaling. We have only to use the Kugo-Uehara gauge¹¹⁾ in the final result. This is the second reason why the proof is actually more elementary than in the ordinary case.

^{*}) At the linearized level, this was shown some time ago by Ferrara, Grisaru and Van Nieuwenhuizen, see Ref. 17).

ii) It is more flexible for the generalizations.

In this formalism, the Lagrangian of (pure) standard supergravity is^{18),11)}

$$-\frac{3}{2} [S_0 \bar{S}_0]_D \quad (5)$$

where S_0 (Weyl weight $w = 1$, chiral weight $n = 1$) is the compensator chiral multiplet^{18),11)}

$$S_0 = (z_0, -i\chi_R, h_0) \quad (6)$$

whose first two components can be fixed at will, by a choice of dilation, chiral, and S-supersymmetry gauge conditions. In Eq. (5) $[...]_D$ is the superconformally invariant action density for a real vector multiplet with $w = 2$ [see Refs. 18) and 11)]. The R^2 -term is contained in the following superconformal action density

$$[R \bar{R}]_D \quad (7)$$

where the superconformal chiral multiplet R ($w = n = 1$), called the scalar curvature multiplet^{*}, is defined to be^{10),12),19)}

$$R = S_0^{-1} \Sigma(\bar{S}_0) \quad (8)$$

where $\Sigma(\dots)$ is the superconformal chiral projector^{11)**)}, which maps a conformal multiplet with weights $w-n = 2$ into a chiral multiplet with weights $w' = n' = w+2$.

Then the supersymmetric version of Eq. (1) ($\beta = 0$) is

$$e \mathcal{L} = -\frac{3}{2} [S_0 \bar{S}_0]_D + \frac{3}{2} \lambda [R \bar{R}]_D \quad (9)$$

(with a suitable superconformal gauge condition). [For the moment, we ignore the cosmological constant; $\lambda \neq 0$ will be a special case of our generalization below].

*) Our R differs from that of Refs. 10) and 19) in normalization and in the fact that here R is a superconformal multiplet, whereas in Refs. 10) and 19) it is a Poincaré multiplet.

***) Our $\int$ differs in sign with respect to that of Ref. 11).

Equation (9) may be rewritten as

$$-\frac{3}{2} [S_0 \bar{S}_0]_D + \frac{3}{2} \lambda [A \bar{A}]_D + \left(\frac{3}{2} [\Lambda (A - R)]_F + h.c. \right) \quad (10)$$

where A ($w = n = 1$) and Λ ($w = n = 2$) are chiral multiplets and $[\dots]_F$ is the superconformal action density^{18),11)} for a chiral multiplet with $w = n = 3$.

In fact, the super-Lagrange multiplier Λ enforces the constraint $A = R$, so that Eq. (10) reduces to Eq. (9). Now we need the following

LEMMA

$$\begin{aligned} [\Lambda R]_F + h.c. &= \\ &= [(S_0^{-1} \Lambda) \bar{S}_0 + S_0 (\bar{S}_0^{-1} \bar{\Lambda})]_D + \text{surface term.} \end{aligned} \quad (11)$$

Indeed,

$$\begin{aligned} [\Lambda R]_F + h.c. &\stackrel{\text{def.}}{=} [(S_0^{-1} \Lambda) \Sigma(\bar{S}_0)]_F + h.c. = \\ &= \frac{1}{2} [(S_0^{-1} \Lambda) \Sigma(\bar{S}_0) + S_0 \Sigma(\bar{S}_0^{-1} \bar{\Lambda})]_F + \\ &+ \frac{1}{2} [(S_0^{-1} \Lambda) \Sigma(\bar{S}_0) - S_0 \Sigma(\bar{S}_0^{-1} \bar{\Lambda})]_F + h.c. \end{aligned} \quad (12a)$$

By the fundamental Lemma of the tensor calculus^{*)}

$$[(S_0^{-1} \Lambda) \Sigma(\bar{S}_0) - S_0 \Sigma(\bar{S}_0^{-1} \bar{\Lambda})]_F + h.c. = \text{Total Divergence.} \quad (12b)$$

From the definition of $[\dots]_D$ ^{18),11)}, we have

$$\begin{aligned} [(S_0^{-1} \Lambda) \bar{S}_0 + S_0 (\bar{S}_0^{-1} \bar{\Lambda})]_D &= \\ &= \frac{1}{2} [(S_0^{-1} \Lambda) \Sigma(\bar{S}_0) + S_0 \Sigma(\bar{\Lambda} \bar{S}_0^{-1})]_F + h.c. \end{aligned} \quad (12c)$$

*) For a proof, see the first paper of Ref. 18), in particular Section 4, Eq. (33).

Equation (11) then follows, comparing Eqs. (12a) - (12c).

Using the Lemma, Eq. (10) may be rewritten as

$$-\frac{3}{2} [S_0 \bar{S}_0]_D + \frac{3}{2} \lambda [A \bar{A}]_D - \frac{3}{2} [\Lambda \bar{S}_0 S_0^{-1}]_D - \frac{3}{2} [\bar{\Lambda} S_0 \bar{S}_0^{-1}]_D + \left(\frac{3}{2} [\Lambda A]_F + h.c. \right) \quad (13)$$

It is convenient to define the chiral multiplets with zero weights T and C as

$$C = \sqrt{\lambda} S_0^{-1} A \quad (\lambda > 0) \quad (14)$$

$$T = S_0^{-2} \Lambda \quad (15)$$

In terms of these fields, Eq. (13) reads

$$-3 [\Phi(T, C) S_0 \bar{S}_0]_D + ([g(T, C) S_0^3]_F + h.c.) \quad (16)$$

where

$$2\Phi = 1 + T + \bar{T} - \bar{C}C \quad (17)$$

$$g = \frac{3}{2\sqrt{\lambda}} C T \quad (18)$$

Now the proof of the supersymmetric Whitt theorem is complete, since Eq. (16) is just the superconformal expression of the general $N = 1$ Lagrangian of Ref. 18); the corresponding Kähler-invariant function $G^{18)}$ is $(T \rightarrow T + \frac{1}{2})$,

$$G = -3 \ln(T + \bar{T} - \bar{C}C) + \ln \left[\frac{g}{2\lambda} |C|^2 |T - \frac{1}{2}|^2 \right] \quad (19)$$

Note that the Kähler potential is the same as in the no-scale models²⁰⁾, so $(\lambda > 0)$ the kinetic terms are positive in the usual domain (biholomorphic to B_2).

Note that since we disregarded a surface term in Eq. (11), the equivalence between Eq. (9) and Eqs. (16) - (18) is, strictly speaking, restricted to the classical equation of motion.

FOUR GENERALIZATIONS

For didactical reasons we have preferred to discuss the theorem in the simplest situation, corresponding to the one discussed by Whitt. However, for the application we have in mind, we need a more general statement. For the same didactical reasons as before we prefer to give step-by-step generalizations, instead of giving directly the most general possible result, even if no new idea or technique is involved in each step.

A. Coupling to matter

Instead of Eq. (9), we can consider the more general case

$$e \mathcal{L} = -\frac{3}{2} [H(z^\alpha, \bar{z}^\beta) s_0 \bar{s}_0]_D + \frac{3}{2} \lambda [K(z^\alpha, \bar{z}^\beta) R \bar{R}]_D + ([h(z^\alpha) s_0^3]_F + h.c.) \quad (20)$$

where $H(z^\alpha, \bar{z}^\beta)$ and $K(z^\alpha, \bar{z}^\beta)$ are arbitrary functions of the zero-weight chiral multiplets Z^α ($\alpha = 1, \dots, N$) and $h(z^\alpha)$ is holomorphic. Gauge vectors can also be added, without changing the result.

Using the same argument as before, we can rewrite Eq. (20) in the standard form¹⁸⁾ with

$$2 \Phi = T + \bar{T} - K(z^\alpha, \bar{z}^\beta) \bar{c} c + H(z^\alpha, \bar{z}^\beta) \quad (21a)$$

$$g = h(z^\alpha) + \frac{3}{2f\lambda} c T. \quad (21b)$$

B. More general couplings of the scalar curvature multiplet

We can add non-linear couplings of the chiral multiplet R , as of any other (matter) chiral multiplet. Then, we consider

$$e\mathcal{L} = -\frac{3}{2} [\Psi(z^\alpha, z^\beta, s_0^{-1}R, \bar{s}_0^{-1}\bar{R}) s_0 \bar{s}_0]_D + \quad (22)$$

$$+ ([h(z^\alpha) s_0^3]_F + h.c.)$$

where Ψ is an arbitrary real function and h is holomorphic. (By the Lemma, we do not lose any generality by assuming that h does not depend on $s_0^{-1}R$). Obviously, Eq. (20) is a special case of Eq. (22). Again we find

$$2\Phi = T + \bar{T} + \Psi(z^\alpha, \bar{z}^\beta, C, \bar{C}) \quad (23a)$$

$$g = h(z^\alpha) + \frac{3}{2} C T \quad (23b)$$

(here $C = s_0^{-1}A$).

C. Arbitrary powers of the scalar curvature

The problem with Eq. (22) is that, despite its containing arbitrary powers of the scalar curvature multiplet R , its component form is only quadratic in the scalar curvature R (at least if SUSY is not broken). This is explained, for instance, in Ref. 21). In order to have an arbitrary function of the scalar curvature R , one should consider the following superconformal density [see e.g. Ref. 21)]

$$e\mathcal{L} = \quad (24)$$

$$= -\frac{3}{2} [\Omega\{z^\alpha, z^\beta, s_0^{-1}R, \bar{s}_0^{-1}\bar{R}, s_0^{-2}\bar{z}(\bar{R}), \bar{s}_0^{-2}\bar{z}(R)\} s_0 \bar{s}_0]_D$$

$$+ ([h(z^\alpha) s_0^3]_F + h.c.)$$

where $\Omega\{\dots\}$ is an arbitrary real function and h is holomorphic (again, we are losing no generality by assuming h to be a function of Z^α only).

Now Eq. (24) is equivalent to

$$\begin{aligned}
 & -\frac{3}{2} [\Omega \{z, \bar{z}, s_0^{-1} A, \bar{s}_0^{-1} \bar{A}, s_0^{-2} B, \bar{s}_0^{-2} \bar{B}\} s_0 \bar{s}_0]_D + \\
 & + \left(\frac{3}{2} [\Lambda (A - \mathcal{R})]_F + \frac{3}{2} [M (B - \Sigma(\bar{\mathcal{R}}))]_F + \right. \\
 & \quad \left. + [h s_0^3]_F + h.c. \right) \quad (25)
 \end{aligned}$$

where B and M are chiral multiplets with weights $w = n = 2$ and $w = n = 1$, respectively. As before,

$$[\Lambda \mathcal{R}]_F = [\Lambda \bar{s}_0 s_0^{-1}]_D \quad (26a)$$

and

$$\begin{aligned}
 [\mathcal{M} \Sigma(\bar{\mathcal{R}})]_F &= [\Sigma(M \bar{\mathcal{R}})]_F = \\
 &= [M \bar{\mathcal{R}}]_D \equiv [M \bar{A}]_D \quad (26b)
 \end{aligned}$$

by the constraint $\bar{A} = \bar{\mathcal{R}}$, Eq. (25).

Using Eqs. (26) we see that Eq. (25) is again equivalent to a standard supergravity with

$$2\Phi = T + \bar{T} + Q\bar{c} + c\bar{Q} + \Omega(z, \bar{z}, c, \bar{c}, F, \bar{F}) \quad (27a)$$

$$g = h(z) + \frac{3}{2} (Tc + QF) \quad (27b)$$

where

$$\begin{aligned}
 c &= s_0^{-1} A & T &= s_0^{-2} \Lambda \\
 F &= s_0^{-2} B & Q &= s_0^{-1} M.
 \end{aligned} \quad (28)$$

D. Addition of terms containing $\square^n R$

There is still one generalization of our theorem, i.e., we can consider terms which contain $\square^n R$. So we shall consider Lagrangians which are arbitrary functions of matter fields, $R, \square R, \square^2 R, \dots$. These last fields are contained in the components of the multiplets ^{*}) $\bar{\Sigma}[S_0^{-1}\Sigma(\bar{R})], \Sigma[\bar{S}_0^{-1}\bar{\Sigma}(S_0^{-1}\Sigma(\bar{R}))]$, etc.

In this general case, as is already clear from Generalization C, the theorem is proved recursively. That is, we define $\bar{A}$ to be equal to $\bar{R}$ by the Lagrange multiplier term $[\bar{\Lambda}(\bar{A}-\bar{R})]_F$, then B_1 to be $\Sigma(\bar{R})$ adding a term $[M_1(B_1-\Sigma(\bar{R}))]_F$, $\bar{B}_2$ to be $\bar{\Sigma}[S_0^{-1}\Sigma(\bar{R})]$ by the constraint $[\bar{M}_2(\bar{B}_2-\bar{\Sigma}[S_0^{-1}\Sigma(\bar{R}))]]$ and so on. Then, starting from the highest n , we use the identities

$$\begin{aligned} & [H_m(B_m - \Sigma[\bar{S}_0^{-1}\bar{\Sigma} \dots \bar{S}_0^{-1}\Sigma(\bar{R})])]_F = \\ & = [H_m(B_m - \Sigma(\bar{S}_0^{-1}\bar{B}_{m-1}))]_F = \\ & = [H_m B_m]_F - [M_m \bar{S}_0^{-1}\bar{B}_{m-1}]_D \end{aligned} \quad (29)$$

The first term on the right-hand side of Eq. (29) is reabsorbed in the superpotential g and the second one in the function Φ of the standard supergravity form¹⁸⁾.

Going on with this procedure, we end up with a Lagrangian of the usual form¹⁸⁾ specified by the two functions

$$\begin{aligned} 2\Phi = & T + \bar{T} + \sum_{i=1} (c_{i-1} \bar{Q}_i + \bar{c}_{i-1} Q_i) + \\ & + \Omega(z^\alpha, \bar{z}^\beta, c_i, \bar{c}_j) \end{aligned} \quad (30)$$

$$g = h(z^\alpha) + \frac{3}{2} \sum_{i=0} c_i Q_i \quad (31)$$

^{*}) All other cases can be reduced to the one considered here.

where

$$Q_0 = T, \quad C_0 = S_0^{-1} A \quad (32)$$

and, for $i \neq 0$

$$C_i = S_0^{-2} B_i, \quad Q_i = S_0^{-1} M_i. \quad (33)$$

Obviously, the number of fields increases as we add higher and higher derivatives, since the number of degrees of freedom propagated by the gravitational multiplet increases too.

To appreciate the power of the above methods, the reader should compare the simplicity of our exact results with the complexity of the corresponding analysis at the linearized level by standard techniques.

Other possible extensions will be published elsewhere.

E. Two remarks on negative-norm states

1. From Eq. (30) we see that Φ is linear in T as in no-scale models. However, Φ is not plurisubharmonic, because of the fields Q . Thus we have generically as many poltergeists as Q fields. This may look odd, since in many cases the linear approximation does not predict such states. For instance, $R^n (n > 2)$ does not contribute to the (flat space) linearized equations of motion, so no new state appears. In these cases we can show that the above poltergeists decouple in the equivalent theory too. We illustrate the mechanism in a global model. Consider the free Wess-Zumino multiplet $\mathcal{L}_0 = [A\bar{A}]_D + (m[A^2]_F + \text{h.c.})$. The vacuum is $\langle A \rangle = 0$. Let us add a higher derivative term

$$\mathcal{L} = \mathcal{L}_0 + \alpha^2 [\Xi(\bar{A}) \bar{\Xi}(A) A \bar{A}]_D \quad (34)$$

which does not contribute to the quadratic term. By the same argument as before, Eq. (34) is shown to be equivalent to

$$\begin{aligned} \mathcal{L} = & [X\bar{X}]_D + [S\bar{S}]_D - [Z\bar{Z}]_D + \\ & + \left(S^{-3/4} m[(Z-X)]_F + \frac{1}{2\alpha} [S(1+\sqrt{5} \frac{Z+X}{Z-X})]_F + h.c. \right) \end{aligned} \quad (35)$$

where now the kinetic terms are "canonical". Z seems to be a poltergeist, but actually it decouples (in the linearized case). Indeed, Eq. (35) is ill-defined around the putative supersymmetric state $\langle A \rangle = 0$, which means $\langle S \rangle = \langle Z-X \rangle = 0$.

The model can be defined by a limit procedure^{*)}: we add a compact perturbation $\epsilon(X-Z)$ to the superpotential and then take $\epsilon \rightarrow 0$. SUSY is unbroken (see footnote) and the linearized spectrum consists of a positive- norm multiplet of mass $O(m^2)$ and two multiplets of mass $O(\alpha^{-2}\epsilon^{-2})$. Thus the unwanted states decouple when $\epsilon \rightarrow 0$ (or $\alpha \rightarrow 0$), as physically expected.

2. Anyhow, linear-level ghost decoupling may not be enough for a consistent theory. Indeed, the scalar potential of the equivalent standard theory is still unbounded below. It is natural to ask for non-linear decoupling. This puts severe constraints on the geometry of the underlying σ -model. The most spectacular example is a standard no-scale model coupled to higher curvatures as above. In this case all unwanted states are automatically decoupled at the full non-linear level. In fact, the function Ω has the form²⁰⁾

$$\Omega = Z^1 + \bar{Z}^1 + \text{terms which do not contain } Z^1. \quad (36)$$

Letting $Z^1 \rightarrow Z^1 + T$ [see Eq. (30)], we eliminate T from the kinetic terms. Then T is a Lagrange multiplier enforcing $C_0 = 0$ [see Eq. (31)]. But then [again by Eq. (30)], it is Q_1 which has no kinetic term; by Eq. (31) it enforces $C_1 = 0$, so now it is Q_2 which has no kinetic term ... At the end all Q 's and C 's are eliminated and we remain with

$$2\Phi = \Omega(Z^\alpha, \bar{Z}^\beta, C_i=0, \bar{C}_j=0) \quad (37)$$

*) Notice that, even if the potential is not bounded from below, we can still give a meaning to the Witten index [Ref. 22)], which turns out to be + 1.

i.e. with the original no-scale model in which we just cancel all higher curvature terms!^{*)} [Similar remarks apply to low-energy superstrings²³⁾.]

APPLICATIONS AND DISCUSSION

Basically there are two large classes of potential applications. The first class corresponds to the same applications of the original Whitt paper¹⁶⁾. The idea is that once you have proved the equivalence with the standard theory, all the classical theorems of General Relativity [such as positive energy and action, singularity theorem and geodesic (non-) completeness, black hole laws²⁵⁾.....] are extended automatically to the higher derivative phase, provided (of course) that the resulting standard theory satisfies the specific assumptions of each theorem²⁶⁾.

Of course, the above applies to the supersymmetric case too, where there is also some additional general theorem [e.g., for SUSY-breaking²⁷⁾].

However, the applications we have in mind are of a different nature; we want to simplify the computations of the component field Lagrangian for higher derivative supergravity.

To be concrete, let us consider the particular case studied in Refs. 1) and 2), which is related to superstring theory.

The Lagrangian is of the form
(some standard supergravity) +

$$+ [f_1(z) \mathcal{R} \bar{\mathcal{R}}]_D + [f_2(z) E_a E^a]_D + \quad (38)$$

$$+ ([f_3(z) W_{\alpha\beta\gamma} W^{\alpha\beta\gamma}]_F + h.c.)$$

where $f_i(z)$ $i = 1, 2, 3$ are arbitrary (but f_3 must be holomorphic) functions of the chiral multiplets Z^α , and E_a is the Ricci multiplet^{19), 28)} (here E_a is defined to be a superconformal multiplet with $w = 1$, $n = 0$). $W_{\alpha\beta\gamma}$ is the chiral Weyl multiplet^{19), 28)}.

*) Of course this is not true for more general higher derivative terms than those considered in this paper. See Ref. 24).

The last term in Eq. (38) is easy enough to be handled directly by standard techniques. The second term, which by itself would not be so simple, is completely trivialized by the result we have proved.

Unfortunately, there still remains a term, i.e., $[f_2 E_a E^a]_D$ which is very difficult to compute. The problem with this term is that, if $f_2 \neq 0$, the theory is definitely not equivalent to one of the standard supergravities of Cremmer et al.¹⁸⁾. This can be seen in two different ways. First of all, from the linearized analysis [see Refs. 13) and 14)] we know that the equivalent Lagrangian must contain (at least) one massive spin-2 supermultiplet (of negative norm). Negative norm is not a problem, from a formal point of view. But no one has written down explicitly the full non-linear coupling of a massive spin-2 multiplet to supergravity. Then, a preliminary step towards an equivalence theorem for (Ricci)²-supergravity is the construction of the putative equivalent standard theory, that is the most general coupling of massive higher-spin multiplets to $N = 1$ supergravity.

The same conclusion is obtained from the formal structure of the super-Legendre transformation needed in the proof of equivalence. Work along these lines is in progress.

Anyhow, one could also try a different strategy; i.e., we can limit ourselves to $f_2 = f_3 + \bar{f}_3$ in Eq. (38), to reduce the combination of supercurvatures in Eq. (33) to the Gauss-Bonnet one^{19),*)}, up to terms quadratic in R , which can be handled using the above results. We know [see Ref. 15)] that the Gauss-Bonnet combination does not propagate the spin-2 poltergeist, so we can hope that this particular theory may be equivalent to a simpler one without exotic matter multiplets. But I am a bit sceptical.

We stress that our results give some relevant information for the Ricci multiplet too. Indeed, comparing the recursive construction of general higher curvature invariants given by Ref. (21), with the recursive procedure we adopted to generalize the theorem, we see that once we have understood the equivalence for the simplest case, i.e., just $E_a E^a$, we have also the equivalence for arbitrary functions of the invariant $E_a E^a$, as well as of other related invariants.

*) As was the case in Refs. 1) and 2).

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