CERN-TH.4618/86

SUPERSYMMETRY AND THE LATTICE: A RECONCILIATION?

G. Curci

INFN and Dipartimento di Fisica dell'Università, Pisa

and

G. Veneziano

CERN, Geneva

ABSTRACT

Contrary to common prejudice, we claim that supersymmetric gauge theories can be studied non-perturbatively on the lattice (by using known ideas and techniques). We discuss in detail super-Yang-Mills theory and propose some explicit measurements which are both physically interesting and within present or near-future computer capabilities.

CERN-TH.4618/86

December 1986

1. INTRODUCTION

The non-perturbative properties of supersymmetric (SUSY) gauge theories have been the subject of many investigations in recent years [1–4]. A good grasp of them is obviously important if one believes that, in a fundamental theory of all interactions, strongly coupled SUSY gauge theories will play a role somewhere between the Planck (or GUT) scale and the low-energy (100 GeV) scale of present accelerator physics.

One currently popular example of such a scenario is offered by superstring theories in which SUSY breaking may involve gluino condensation [5] due to a strongly interacting super-Yang–Mills interaction. But, of course, other examples come to mind since these theories contain, at the Planck scale, huge SUSY gauge groups, one of which could evolve into a strongly coupled theory before the SUSY-breaking scale is reached.

The methods so far employed for the non-perturbative study of SUSY gauge theories have been based on general arguments (Witten index [1], Ward identities [2]), on effective Lagrangians [3], and on explicit analytic calculations [4]. These methods have led to a considerable understanding of non-perturbative properties (especially those concerning the vacuum), beyond what could have been expected on the basis of the QCD precedent.

Yet, many questions have remained unanswered. Numerical Monte Carlo calculations could answer them as well as provide a check of the existing claims. Questions of particular relevance would include those of possible SUSY anomalies, of confinement and chiral symmetry breaking, of the low-lying spectrum and, finally, of the effects of explicit soft SUSY-breaking terms.

Our claim here is that, contrary to widespread beliefs (briefly reviewed in Section 2), SUSY and the lattice can happily coexist.

In Section 2 we shall give the general argument for our claim based on lattice SUSY and chiral Ward identities (a more rigorous justification in the spirit of Symanzik’s approach [6] is given in the Appendix).

In Section 3 we shall list some explicit ‘measurements’, apparently feasible with present or near-future computer capabilities, which could provide valuable answers to the physically relevant questions mentioned above.

2. GENERAL IDEAS: CHIRAL AND SUSY WARD IDENTITIES

Many problems prevent a direct lattice formulation of SUSY field theories suitable for Monte Carlo calculations. The first obvious one is that, lacking lattice generators of the Poincaré group, it is impossible to fulfil the (continuum) algebra of SUSY transformations. Previous attempts [7] include, as a possible way out, the proposal to use lattice formulations with a ‘SLAC’ derivative operator [8], which contains long-range correlations. But it is known [9] that such formulations do not reproduce a Lorentz-invariant theory in the continuum limit.

Another problem is represented by the balancing between bosonic and fermionic modes required by SUSY: a naïve lattice fermion formulation produces too many fermions [10].

The obstacle is that a lattice formulation without species doubling and with explicit continuous chiral symmetry has been shown not to exist [11, 12]. The conclusion that these observations suggest is that an explicit breaking of SUSY (and chirality) is unavoidable.

These arguments are the lattice version of the general problem of defining a manifestly SUSY-invariant regularization.

The simple idea behind our proposal is that, rather than try to have some SUSY on the lattice, one should let the lattice spoil SUSY if it so wishes. Our requirement is that SUSY, like chiral symmetry, should only be recovered in the continuum limit (this being well defined for an asymptotically free theory as it is the limit of vanishing bare coupling).

In this paper we shall consider the simple case of SUSY Yang-Mills (SYM). Such a theory is a particular case of an ordinary gauge theory with a Majorana fermion λ belonging to the adjoint representation. The only two parameters that define the continuum theory are the gauge coupling (equivalently a scale Λ) and the mass of the gluino m_λ . The SUSY limit corresponds to $m_\lambda = 0$ and coincides with the chiral limit in which the axial symmetry, $\lambda \rightarrow e^{i\alpha\gamma_5}\lambda$, is not explicitly broken to any order in perturbation theory. At the non-perturbative level, $U(1)_A$ breaks down to a discrete subgroup $[Z_{2N} \text{ for } SU(N)_{\text{gauge}}]$, but again the Z_{2N} symmetric point (in parameter space) should coincide with the SUSY limit.

We conclude that if the continuum limit of the lattice theory exists and its massless limit can be defined on the basis of chiral symmetry, then this limit *must* represent the SYM theory in which we are interested.

Fortunately some nice progress has recently been made [12, 13] on the way to define such a chiral limit. Let us illustrate the point in terms of lattice Ward identities (WIs).

In the Wilson formulation, the lattice action is

$$S_{\text{lattice}} = S_U + S_F \quad (1)$$

with S_U being the standard gauge field action and

$$\begin{aligned} S_F &= a^4 \sum_{\mathbf{x}} \left\{ \frac{1}{2a} \text{Tr} \sum_{\mu} \left[\bar{\lambda}(\mathbf{x}) (\gamma_{\mu} - r) U_{\mu}(\mathbf{x}) \lambda(\mathbf{x} + \hat{\mu}) U_{\mu}^{\dagger}(\mathbf{x}) \right. \right. \\ &\quad \left. \left. - \bar{\lambda}(\mathbf{x} + \hat{\mu}) (\gamma_{\mu} + r) U_{\mu}^{\dagger}(\mathbf{x}) \lambda(\mathbf{x}) U_{\mu}(\mathbf{x}) \right] + \right. \\ &\quad \left. + (M_0 + \frac{4r}{a}) \text{Tr} \bar{\lambda}(\mathbf{x}) \lambda(\mathbf{x}) \right\} \equiv \\ &\equiv \sum_{\mathbf{x}, \mathbf{x}'} \frac{1}{2} \bar{\lambda}(\mathbf{x}) K(\mathbf{x}, \mathbf{x}') \lambda(\mathbf{x}') \end{aligned} \quad (2)$$

The partition function average of a gauge-invariant operator Ω is defined by

$$\begin{aligned}\langle \Omega \rangle &= \int dU d\lambda \exp(-S_u - S_F) \Omega(U, \lambda) \\ &\equiv \int dU \exp(-S_u + \frac{1}{2} \ln \det K) \tilde{\Omega}(U, G) \quad (3)\end{aligned}$$

where the Majorana nature of the gluino field is taken into account via the factor 1/2 in front of $\ln \det K$, and $\tilde{\Omega}(U, G)$ is obtained from Ω by replacing each $\lambda \bar{\lambda}$ pair by the fermionic propagators $G = (K)^{-1}$.

The axial WIs follow the change of variables: $\lambda \rightarrow e^{i\alpha\gamma_5}\lambda$ and $U \rightarrow U$. They read

$$\langle \Delta_\mu J_\mu^5(x) \Omega \rangle = \langle [D_A(x) + X_A(x)] \Omega \rangle - \langle \delta_A(x) \Omega \rangle \quad (4)$$

where J_μ^5 is the current, generating the axial transformation, which we need not specify [14] (except to remark that it goes to the usual continuum current for $a \rightarrow 0$); D_A is the symmetry-breaking term coming from the explicit mass term; and X_A is the breaking term due to the lattice regularization.

With the choice

$$J_\mu^5(x) = \text{Tr} \bar{\lambda}(x) \gamma_\mu \gamma_5 U_\mu(x) \lambda(x+\hat{\mu}) U_\mu^\dagger(x) \quad (5)$$

the symmetry-breaking terms are

$$D_A(x) = 2M_0 J(x)^5 \equiv 2M_0 \text{Tr} \bar{\lambda}(x) \gamma_5 \lambda(x) \quad (6)$$

$$\begin{aligned}X_A(x) &= -r \sum_\mu \text{Tr} [\bar{\lambda}(x) \gamma_5 U_\mu(x) \lambda(x+\hat{\mu}) U_\mu^\dagger(x) - \\ &\quad - 4 \bar{\lambda}(x) \gamma_5 \lambda(x)] \quad (7)\end{aligned}$$

and the axial transformations are

$$\delta_A(x) = \gamma_5 \lambda(x) \frac{\delta}{\delta \lambda(x)} + \bar{\lambda}(x) \gamma_5 \frac{\delta}{\delta \bar{\lambda}(x)} \quad (8)$$

Note that, for $a \rightarrow 0$, $X_A \approx \sum_\mu [-r (a^2/8) \text{Tr} \bar{\lambda}(x) \gamma_5 \overleftrightarrow{D}_\mu^2 \lambda(x)]$, so X_A is an ‘irrelevant’ operator that formally vanishes in the continuum limit. However, the insertion of X_A in a Green function can induce, in the loop expansion, divergences of $O(1/a)$ that can compensate the explicit factor a of the X_A vertices. By means of this mechanism X_A can mix with ‘relevant’ operators of dimension ≤ 4 . In Refs. [12, 13] it is shown that, as $a \rightarrow 0$, the effect of X_A is threefold:

- i) a rescaling of J_μ^5 and of δ_A (if it acts on composite operators with products of fields at the same point);

ii) a redefinition of the mass-breaking term (6):

$$D_A \rightarrow D_A - 2 \bar{M}_A J^5$$

iii) a new dimension-four breaking term proportional to the Adler-Bell-Jackiw (ABJ) anomaly.

In summary it is possible to show that for $a \rightarrow 0$,

$$\begin{aligned} \langle \partial_\mu (Z_A J_\mu^5)(x) \Omega \rangle &\approx 2(M_0 - \bar{M}_A) \langle J^5(x) \Omega \rangle \\ &+ \langle \frac{\alpha N}{\pi} G \tilde{G}(x) \Omega \rangle - \langle \delta_A' \Omega \rangle \end{aligned} \quad (9)$$

where we have ignored additional contact terms.

We stress that the result, Eq. (9), does not depend on the specific form of the lattice axial current [14]. Other (infinitely many) possible choices are related by finite renormalizations, i.e. different values of Z_A in Eq. (9).

For the subsequent discussion it is useful to report the standard argument [15] that relates the combination $M_0 - \bar{M}$ to the renormalized mass.

Let us consider the WI [$\Omega = \lambda(y)\bar{\lambda}(z)$ in Eq. (9)]:

$$\begin{aligned} i(p-p')_\mu \Gamma_\mu^5(p, p') &= 2(M_0 - \bar{M}_A) \Gamma^5(p, p') + \\ &+ \Gamma_{G\tilde{G}}^5(p, p') - \bar{S}'(p) \gamma_5 - \gamma_5 \bar{S}'(p') \end{aligned} \quad (10)$$

where $S(p)$ is the gluino propagator with the pole at the mass m .

With the definitions

$$\begin{aligned} \Gamma^5(p, p') &= Z_{DA}^{-1} \tilde{\Gamma}^5(p, p'), \\ \bar{S}'(p) &= Z_2^{-1} \tilde{\bar{S}}'(p), \quad \tilde{\bar{S}}'(p) = -i \not{p} \tilde{S}_1(p^2) + \tilde{S}_0(p^2) \end{aligned}$$

where $\tilde{\Gamma}^5(p, p')$ and $\tilde{\bar{S}}^{-1}(p)$ are normalized at the tree value for on-shell momenta, the WI for $p = p'$ implies that

$$M_0 - \bar{M}_A = m Z_{DA} Z_2^{-1} \tilde{S}_1(-m^2) \quad (11)$$

from which it follows that in the chiral limit $M_0 - \bar{M}_A \rightarrow 0$.

In practical Monte Carlo calculations the chiral limit corresponds to the tuning of M_0 to a critical value M_{cr} such that

$$M_{cr} - \bar{M}_A(M_{cr}, r, g_0) = 0$$

This tuning can be done by measuring some quantity [13] which is sensitive to the chiral limit (e.g. in QCD the Goldstone boson mass in a flavour non-singlet channel; see below for our case).

A similar scheme can be used for deriving (broken) SUSY WIs.

Define now a SUSY change of variables as

$$\begin{aligned} \delta_S U_\mu(x) &= i g \gamma_\mu \lambda(x) U_\mu(x) \\ \delta_S \lambda(x) &= \frac{1}{2} \sigma_{\alpha\beta} \cdot G_{\alpha\beta}^{\text{lattice}}(x) \end{aligned} \quad (12)$$

where $G_{\alpha\beta}^{\text{latt}}$ is a suitable combination of links which, in the $a \rightarrow 0$ limit, gives the continuum gauge field strength (e.g. the open plaquette).

Note that the SUSY transformations, Eq. (12), commute with the lattice gauge transformations

$$\begin{aligned} U_\mu(x) &\rightarrow U_\mu^\omega(x) = e^{i\omega(x)} U_\mu(x) e^{-i\omega(x)} \\ \lambda(x) &\rightarrow \lambda^\omega(x) = e^{i\omega(x)} \lambda(x) e^{-i\omega(x)} \end{aligned} \quad (13)$$

The SUSY change of variables will induce WIs of the general form

$$\langle \Delta_\mu S_\mu(x) \Omega \rangle = \langle [D_S(x) + X_S(x)] \Omega \rangle - \langle \delta_S(x) \Omega \rangle \quad (14)$$

with explanations of the symbols analogous to those for Eq. (4).

A possible choice for S_μ is

$$S_\mu(x) = \text{Tr} \sigma_{\alpha\beta} G_{\alpha\beta}^{\text{lattice}}(x) \gamma_\mu U_\mu(x) \lambda(x+\hat{\mu}) U_\mu^\dagger(x) \quad (15)$$

In this case we obtain

$$D_S(x) = -M_0 \text{Tr} \sigma_{\alpha\beta} G_{\alpha\beta}^{\text{lattice}}(x) \lambda(x) \quad (16)$$

We do not report the (ugly) expression for X_S , as its precise form is not necessary for our subsequent discussion. The only information we need is that X_S is an ‘irrelevant’ operator that can mix with operators of dimension $\leq 9/2$, and that the SUSY transformations commute with generalized Becchi–Rouet–Stora (BRS) transformations.

On the basis of power-counting analysis and Slavnov invariance we can argue that, in the $a \rightarrow 0$ limit, X_S can mix with two classes of operators:

I) gauge-invariant operators; a basis is given by $\partial_\mu S_\mu$, $\partial_\mu T_\mu$, $\gamma_\mu D_\nu G_{\mu\nu} \lambda$, and $\sigma_{\alpha\beta} G_{\alpha\beta} \lambda$, where

$$T_\mu(x) = G_{\mu\nu}(x) \delta_\nu \lambda(x) \quad (17)$$

II) operators that are the BRS transformation of operators of dimension 7/2 and 5/2.

The class II operators can be ignored in gauge-invariant correlations or S-matrix elements (or more simply if we work in the background gauge).

The possible mixings show the crucial difference between axial and SUSY WIs as we know it from the continuum theory [16]. Whilst X_A has the effect of inducing an anomaly which could not be reabsorbed in a gauge-invariant redefinition of $J_\mu^{(5)}$, the ‘physical’ dimension-9/2 operators that can originate from X_S (as $a \rightarrow 0$) appear to consist only of divergences of two gauge-invariant operators S_μ and T_μ , and operators that redefine the explicit mass SUSY-breaking term and the SUSY transformations of fields.

Thus for SUSY we obtain WIs which in the continuum limit become

$$\begin{aligned} & \langle (\alpha \partial_\mu S_\mu + \beta \partial_\mu T_\mu)(x) \Omega \rangle = \\ & = - (M_0 - \bar{M}_S) \langle \sigma G \lambda(x) \Omega \rangle - \langle \tilde{\delta}_S \Omega \rangle \end{aligned} \quad (18)$$

where $\tilde{\delta}_S$ is the renormalized transformation of the fields (we ignore possible contact terms). The correctly normalized SUSY current becomes

$$\hat{S}_\mu = \alpha S_\mu + \beta T_\mu \quad (19)$$

which is a combination of the allowed dimension-7/2 operators. By contrast, in the chiral case, the axial current mixes with itself as it is the only allowed dimension-3 operator. We remark that

$$\alpha = 1 + O(g_0^2), \quad \beta = O(g_0^2), \quad \tilde{\delta}_S = \delta_S + O(g_0^2)$$

and in the limit $a \rightarrow 0$ the continuum WIs are recovered.

In order to show that the chiral limit defines the SUSY point, let us consider the WI in the case $\Omega = A\rho(y)\bar{\lambda}(z)$.

We obtain

$$i(p-p')_\mu \hat{S}_\mu p(p, p') = - (M_0 - \bar{M}_S) \Gamma_{p(\sigma G \lambda)}(p, p') + \sigma_{p\tau} p'_\tau S^{-1}(p) + \dots \quad (20)$$

The dots in Eq. (20) represent the contributions of the other terms of the WI.

For $p = p'$ a simple analysis of the relevant form factors shows that

$$(M_0 - \bar{M}_S) = m Z_{DS} Z_2^{-1} \tilde{S}_1(-m^2) \quad (21)$$

where

$$\left. \Gamma_{p(\sigma G \lambda)}(p, p) \right|_{p^2 = -m^2} = Z_{DS}^{-1} \sigma_{p\mu} p_\mu + \dots$$

Taking into account the analogous result obtained from the axial WI, Eq. (11), we conclude that

$$(M_0 - \bar{M}_S) = Z_{SA} (M_0 - \bar{M}_A) \quad (22)$$

where

$$Z_{SA} = 1 + O(g_0^2)$$

This result, Eq. (22), is intuitively simple as the axial WI and SUSY WI are related by a SUSY transformation.

Moreover, Eq. (22) shows that in the continuum limit the chiral limit defines the SUSY point, and vice versa.

The following observations are in order:

- 1) The presence of a breaking term X_S coming from the lattice regularization is as necessary as the presence of X_A in Eq. (4). Indeed, the two terms are related by supersymmetry in the continuum limit, so that a regularization giving $X_S = 0$ would imply $X_A = 0$ in contrast with general theorems [15].
- 2) In the continuum limit X_S induces two effects: it renormalizes D_S , and it introduces new operators that can be reabsorbed in a gauge-invariant redefinition of the SUSY current and of the SUSY transformations. These redefinitions represent lattice artefacts with deviations, proportional to the bare coupling constant, which are harmless in the continuum limit, thanks to asymptotic freedom.
- 3) If we set ourselves at $m_\lambda = 0$ we turn off both the soft breaking of chirality and of SUSY.

In summary, by proceeding as explained above, we recover in the continuum limit SUSY WIs for Green's functions involving elementary fields. We expect no difficulty in extending these arguments to the case of composite (gauge-invariant) operators which we propose to measure on the lattice.

We note that lattice regularization is completely equivalent to other continuum regularization. We meet the same problems when we try to interpret the regularization artefacts. This equivalence is more transparent in the Symanzik approach.

In the Appendix we rederive the results of this section by this latter approach, outlining the equivalence of the lattice regularization with the familiar dimensional regularization [17].

3. SUGGESTED MEASUREMENTS

Although our considerations can easily be extended to an $SU(N)$ gauge group, here we shall discuss only the case of $SU(2)$ for which numerical calculations are the easiest possible. These naturally split into two classes.

3.1 Quenched approximation calculations

This is the (non-SUSY) approximation in which dynamical gluino loops are neglected, i.e. where in Eq. (13) the $\frac{1}{2} \ln \det K$ term is set to zero. It is similar to quenched QCD except that a Majorana fermion in the adjoint representation replaces a set of Dirac fermions in the fundamental + antifundamental.

An interesting question is whether, in this limit, confinement and spontaneous chiral symmetry breaking occur, as they seem to do in quenched QCD. Arguments based on large N expansions and anomalous WIs [18] suggest that dynamical chiral symmetry breaking should also occur here. On the other hand, the fermion colour can be now screened by gluons, and this could lead to surprises.

In order to check these points, one should measure the correlations

$$G_S = \langle \bar{\lambda} \lambda(0) \bar{\lambda} \lambda(\tau) \rangle \quad (23a)$$

$$G_P = \langle \bar{\lambda} \gamma_5 \lambda(0) \bar{\lambda} \gamma_5 \lambda(\tau) \rangle \quad (23b)$$

A first measurement could be done in the quenched OZI conserving limit, corresponding to Fig. 1a. If confinement and spontaneous chiral symmetry breaking hold in this approximation, one should observe that $G_{S,P}$ falls off exponentially at large T separations but with different rates. Indeed, the mass gap should go to zero for G_P in the chiral limit (corresponding to a massless pseudoscalar Goldstone boson), whilst the behaviour of G_S is still controlled by a non-vanishing mass gap. Both gaps would instead go to zero together, irrespective of whether there is confinement or not, if chiral symmetry is unbroken in the quenched OZI approximation—in the first case because of colour-singlet massless fermion pairs, and in the second case because of massless gluino–gluon intermediate states.

As a second step one could consider, still within the quenched approximation, the OZI violating diagrams of Fig. 1b. Thus one can check whether the ABJ anomaly provides a way of avoiding Goldstone's theorem and of giving mass to the pseudoscalar boson as well.

3.2 Dynamical gluinos

Adding dynamical gluinos amounts to resolving the exact SUSY theory. The fermionic determinant corresponding to a Majorana spinor in the adjoint representation has to be added. After this inclusion we expect new signals, i.e.

- i) in the OZI approximation (Fig. 2a: closed fermion loops added to Fig. 1a) the anomaly is still turned off. The situation should be compared with that of the quenched OZI approximation after a retuning of the Wilson parameter needed in order to reach the chiral limit of the complete SUSY theory. Probably no dramatic change will occur, but surprises cannot be ruled out. Adding the OZI-violating diagrams (Fig. 2b), one should observe that
- ii) the behaviour of the correlations G_S and G_P is now *completely* changed, i.e.,
 - a) there is no longer any massless state, neither in G_S nor in G_P , because of $U(1)_A$ anomaly effects;
 - b) intermediate-state-effect contributions should cancel out instead in the correlation

$$G \equiv \langle \bar{\lambda}(1+\gamma_5)\lambda(x) \bar{\lambda}(1+\gamma_5)\lambda(y) \rangle \quad (24)$$

which should become x, y independent in the SUSY limit because of the WI [3],

$$\sigma_\mu \partial_\mu^x G(x, y) = \langle (\delta_S \bar{\delta}_S \bar{\lambda}(1+\gamma_5)\lambda(x)) \bar{\lambda}(1+\gamma_5)\lambda(y) \rangle =$$

$$= -\langle (\bar{\delta}_S \bar{\lambda}(1+\gamma_5)\lambda) (\delta_S \bar{\lambda}(1+\gamma_5)\lambda) \rangle = 0$$

- c) intermediate states add up instead in the correlation

$$\bar{G} \equiv \langle \bar{\lambda}(1+\gamma_5)\lambda(x) \bar{\lambda}(1-\gamma_5)\lambda(y) \rangle \quad (25)$$

which should fall to its large-distance limit as $\exp(-mT)$, where m is the mass of a lowest-lying Wess-Zumino supermultiplet. Note that no further tuning of the parameters of the theory is needed when going from the OZI limit to the full theory.

- iii) The asymptotic values of G and $\bar{G}$ should correspond to the square of the gluino-gluino condensate $\langle \bar{\lambda}\lambda \rangle$, which is expected to be finite^{*)} and is given by

$$\frac{g^2}{32\pi^2} \langle \bar{\lambda}\lambda \rangle = \pm c \Lambda^3 \quad (26)$$

in the two Z_4/Z_2 related vacua of the theory (corresponding to a Witten index $\Delta_W = 2$). Instanton calculations give a value for c :

^{*)} The actual physical value can only be obtained, however, after proper subtraction and normalization of the operator $\bar{\lambda}\lambda$ is performed as in Ref. [13].

$$c = 4/\sqrt{5} \quad (27)$$

and it would be interesting to check this prediction.

- iv) A way to provide evidence of the presence of two vacua could be to study the theory close to the SUSY limit ($-\epsilon\Lambda \leq m_\lambda \leq \epsilon\Lambda$). For $m_\lambda \neq 0$, one of the two states is the true vacuum (which one depends on the sign of m_λ). Starting from a sign of m_λ and letting the system relax to one vacuum, one can gradually change that sign and observe the system leave the metastable state, showing a situation of spontaneously broken symmetry.

Another important issue that would be worth exploring in SYM theory is the question of confinement versus Higgs phase for such theory. Arguments have been put forward [19] according to which SYM theory would be realized in the Higgs phase, i.e. with massive gluons and short-range forces among static coloured sources. Study of Wilson loops (with the trace taken in the fundamental representation) would certainly pin down this question.

4. OUTLOOK

In this paper we have argued that (asymptotically free) supersymmetric gauge theories can be studied non-perturbatively on the lattice.

The absence of a supersymmetric lattice regularization does not prevent us from defining a SUSY continuum limit more than the lack of a chiral regularization prevents us from considering a chiral limit. In both cases, of course, bare parameters have to be tuned in order to obtain the desired continuum theory. For SYM theory it is fortunate that the chiral limit and the SUSY limit are one and the same.

The appeal of our observation clearly depends on the possibility of extending the arguments to more general SUSY gauge theories. We foresee no problems of principle, just extra technical complications.

Consider, for instance, the addition of a chiral matter superfield in the $N \oplus \bar{N}$ representation of $SU(N)_c$, the so-called one-flavour SQCD case. Seen as a gauge theory, this model can contain many parameters besides the gauge coupling g and the gluino mass m_λ . These are given by the following terms of dimension ≤ 4 in the Lagrangian:

$$\begin{aligned} & -m \bar{\Psi} \Psi - \mu^2/2 (\phi^* \phi + \tilde{\phi}^* \tilde{\phi}) - \hat{\mu}^2 (\phi \tilde{\phi} + h.c.) \\ & + h \bar{\Psi} \lambda \phi + \tilde{h} \bar{\tilde{\Psi}} \lambda \tilde{\phi} + h.c. \\ & + \eta (\phi^* \tau \phi)^2 + \tilde{\eta} (\tilde{\phi}^* \tau \tilde{\phi})^2 + \hat{\eta} (\phi^* \tau \phi) (\tilde{\phi}^* \tau \tilde{\phi}) \\ & + \beta (\phi \tau \tilde{\phi}) (\phi \tau \tilde{\phi}) \end{aligned}$$

where ϕ , $\tilde{\phi}$, ψ , and $\tilde{\psi}$ are the fields occurring in the chiral matter superfields.

The supersymmetric continuum theory corresponds to $m = \mu, \hat{\mu} = 0, h = -\tilde{h} = g = \text{gauge coupling}$ and to a precise colour structure of the quartic terms giving:

$$g^2 \sum_a (\phi^* T^a \phi - \tilde{\phi}^* T^a \tilde{\phi})^2$$

Thus, compared with SYM, this continuum theory possesses one extra free parameter m .

The condition $m = \mu$ can be imposed by tuning the matter fermion Wilson parameter. As for the other parameters, more tuning will be necessary since lattice operators that go naïvely to the desired ones of the continuum theory actually mix with unwanted ones under renormalization. At finite a these unwanted effects should be tuned out by measuring correlations which are sensitive to explicit SUSY breaking; e.g.

$$G_1 = \langle \bar{\lambda}(1+\gamma_5)\lambda(x) \phi\tilde{\phi}(0) \rangle, \\ G_2 = \langle \phi\tilde{\phi}(x) \phi\tilde{\phi}(0) \rangle, \text{ etc}$$

should become x -independent in the SUSY limit. Thus, in principle, a finite set of measurements will allow the tuning of all the free parameters so as to define a supersymmetric theory. Once this is done, all further measurements will provide information and/or check existing claims regarding the non-perturbative properties of SQCD, such as the breaking of chiral symmetries, the existence of N [for $SU(N)$] well-behaved supersymmetric vacua as long as $m \neq 0$, and the runaway of such vacua in the massless limit.

Acknowledgements

We are grateful to many colleagues at Pisa and CERN for discussions. We would also like to thank M. Karliner, H.B. Nielsen and G.C. Rossi for their interest in our work and for useful comments. One of us (G.C.) wishes to thank the Theory Division of CERN for the warm hospitality enjoyed while most of this work was done.

REFERENCES

- [1] M. Dine, R. Rohm, N. Seiberg and E. Witten, Phys. Lett. **156B** (1985) 55.
- [2] E. Witten, Nucl. Phys. **B202** (1982) 253.
S. Cecotti and L. Girardello, Phys. Lett. **110B** (1982) 39.
- [3] G. Veneziano, Phys. Lett. **124B** (1983) 357.
V.A. Novikov, M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. **B229** (1983) 407.
K. Konishi, Phys. Lett. **135B** (1984) 439.
G.M. Shore and G. Veneziano, Int. J. Mod. Phys. **1** (1986) 499.
- [4] G. Veneziano and S. Yankielowicz, Phys. Lett. **113B** (1982) 321.
T.R. Taylor, G. Veneziano and S. Yankielowicz, Nucl. Phys. **B218** (1983) 493.
See also G.M. Shore and G. Veneziano, Ref. [3], and references therein.
- [5] V.A. Novikov, M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. **B260** (1985) 157, and references therein.
D. Amati, Y. Meurice, G.C. Rossi and G. Veneziano, Nucl. Phys. **B263** (1986) 591, and references therein.
I. Affleck, M. Dine and N. Seiberg, Nucl. Phys. **B256** (1985) 557, and references therein.
- [6] K. Symanzik, Nucl. Phys. **B226** (1983) 187 and 205.
- [7] H. Nicolai and P. Dondi, Nuovo Cimento **41A** (1977) 1.
S. Elitzur, E. Rabinovici and A. Schwimmer, Phys. Lett. **119B** (1982) 165.
T. Banks and P. Windey, Nucl. Phys. **B198** (1982) 226.
J. Bartels and G. Kramer, Z. Phys. **C20** (1983) 159.
J. Bartels and J.B. Bronzan, Phys. Rev. **D28** (1983) 818.
- [8] S.D. Drell, M. Weinstein and S. Yankielowicz, Phys. Rev. **D14** (1976) 478 and 1627.
- [9] L.H. Karsten and J. Smit, Nucl. Phys. **B144** (1978) 536.
- [10] K.G. Wilson, Phys. Rev. **D10** (1974) 2445.
K. Wilson, Proc. 13th Int. School of Subnuclear Physics: New phenomena in subnuclear physics, Erice, 1975, ed. A. Zichichi (Plenum Press, Inc., New York, 1977), p. 13.
- [11] H.B. Nielsen and N. Ninomiya, Nucl. Phys. **B185** (1981) 20.
- [12] L.H. Karsten and J. Smit, Nucl. Phys. **B183** (1981) 103.
- [13] M. Bochicchio, L. Maiani, G. Martinelli, G.C. Rossi and M. Testa, Nucl. Phys. **B262** (1985) 331.
- [14] G. Curci, Phys. Lett. **167B** (1986) 425.
- [15] S.L. Adler, in 1970 Brandeis University Summer Institute in Theoretical Physics, eds. S. Deser, M. Grisaru and H. Pendleton (MIT Press, Cambridge, Mass., 1970), Vol. 1.
- [16] For a review see, for example,
P. Breitenlohner and D. Maison, in Proc. Nuffield Workshop on Supersymmetry and its Applications: Superstrings, anomalies, and supergravity, Cambridge, 1985, eds. G. Gibbons, S. Hawking and P. Townsend (Cambridge Univ. Press, Cambridge, UK, 1986).

- [17] J.C. Collins, Nucl. Phys. **B80** (1974) 341.
P. Breitenlohner and D. Maison, Commun. Math. Phys. **52** (1977) 11, 39, and 55.
G. Bonneau, Nucl. Phys. **B167** (1980) 261 and **B171** (1980) 477.
- [18] G. Veneziano and S. Yankielowicz, Ref. [4].
G. Veneziano, Phys. Lett. **95B** (1980) 90.
- [19] M.A. Shifman, private communication.

APPENDIX

The formulation of gauge theories on a lattice with finite size L and spacing a has the advantage of defining a theory without infrared and ultraviolet divergences with exact local gauge invariance. In the limit $L \rightarrow \infty$, $a \rightarrow 0$, the lattice theory is equivalent, in the sense of Symanzik [6], to the continuum theory with a cut-off of order $1/a \rightarrow \infty$.

Explicitly, the lattice theory is equivalent, order by order in g and a , to a local effective Lagrangian (LEL) on the continuum given by

$$LEL = \mathcal{L}_{\text{continuum}} + \sum_{k=1}^{\infty} a^k \mathcal{L}_k \quad (\text{A.1})$$

where $\mathcal{L}_k$ are local operators of dimensions $4+k$. Equivalence à la Symanzik means that a Green function with N -external lines computed on the lattice possesses to all orders in perturbation theory the asymptotic small- a expansion,

$$G_{\text{latt}}^{(N)}(p_1 \dots p_N, g, a) = \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} a^j (ln a)^k G_{LEL}^{jk}(p_1 \dots p_N, g) \quad (\text{A.2})$$

where g is the set of bare parameters of the theory, and the G^{jk} coefficients can be computed in the continuum theory with the rules of the LEL. An effective action defines a set of Feynman rules and a subtraction procedure of the ultraviolet divergences à la Zimmermann.

We are interested in the leading-order term of the expansion in a in Eq. (A.2) as we neglect terms that vanish in the continuum limit. We thus have

$$G_{\text{latt.}}^{(N)} \approx G_{LEL}^{(N)} \quad (\text{A.3})$$

and the $\approx$ sign signifies equality of the Green functions computed in the two schemes up to quantities that vanish as $a \rightarrow 0$.

It is useful to consider the effective action in the dimensional renormalization scheme (DRS), where the finite part of a Feynman diagram is obtained by the minimal subtraction of the divergent parts. The comparison of the formulations of the theory is more transparent as the lattice Green functions are equivalent, modulo a finite renormalization, to the minimally subtracted functions of the DRS where $1/a$ is the mass scale.

The LEL is obtained by a rescaling of the fields and of the coupling constant (preserving the BRS symmetry) of the minimally subtracted theory. After the renormalization of the gluino self-mass, we impose the condition $S^{-1}(p=0, m=0) = 0$ on the gluino self-energy. The LEL is then given by

$$S_w \approx \int dx \mathcal{L}_{\text{eff.}} = \int dx \left[\frac{1}{4} Z_3 G_{\mu\nu} G_{\mu\nu} + \frac{1}{2\xi} (\partial_\mu A_\mu)^2 + \tilde{Z}_3 \partial_\mu \bar{c} D_\mu c + \frac{1}{2} Z_2 \bar{\lambda} \gamma_\mu D_\mu \lambda + \frac{1}{2} Z_m m \bar{\lambda} \lambda \right] \quad (\text{A.4})$$

where, for SU(2),

$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - g \frac{\tilde{Z}_1}{\tilde{Z}_3} A_\mu \times A_\nu$$

and

$$D_\mu \phi = \partial_\mu \phi - g \tilde{Z}_1 / \tilde{Z}_3 A_\mu \times \phi$$

(c is the ghost field).

The finite constants $Z_i = Z_i(g, \xi, r)$ relate the regularizations: they can be computed following the scheme of Symanzik.

The action is invariant under the BRS transformations:

$$\begin{aligned} \delta_G A_\mu &= \tilde{Z}_3 \partial_\mu c - g \tilde{Z}_1 A_\mu \times c \\ \delta_G \bar{c} &= -\frac{1}{\xi} \partial_\mu A_\mu \\ \delta_G c &= \frac{1}{2} g \tilde{Z}_1 c \times c \\ \delta_G \lambda &= \tilde{Z}_1 c \times \lambda \end{aligned} \quad (\text{A.5})$$

In this framework the definition of external currents is equivalent to the study of the divergences induced by the insertion of composite operators in well-defined Green functions.

Let us first consider the definition of the lattice axial current. On the lattice, several definitions are possible, which in the limit $a \rightarrow 0$ reproduce the continuum current $j_\mu^5(x) = N[1/2 \bar{\lambda}(x) \gamma_\mu \gamma_5 \lambda(x)]$ ($N[\text{Op}]$ defines the 'finite part' of the operator Op according to the rules of the LEL). We choose one of the possible forms, for example that given by Eq. (5):

$$J_\mu^5(x) = \text{Tr} \bar{\lambda}(x) \gamma_\mu \gamma_5 U_\mu(x) \lambda(x+\hat{\mu}) U_\mu^\dagger(x) \quad (\text{A.6})$$

According to an analysis of lattice symmetries in the limit $a \rightarrow 0$, $J_\mu^5(x)$ can only mix with $j_\mu^5(x)$ and we can write

$$J_\mu^5(x) \approx (1 + h_A) j_\mu^5(x) \quad (\text{A.7})$$

in the sense that $\langle J_\mu^5(x) \Pi \rangle \approx (1 + h_A) \langle j_\mu^5(x) \Pi \rangle$, where Π stands for a product of elementary fields at different points. Equivalently, $J^5 \approx (1 + h_p) j^5$. As we have mentioned in Section 2, a finite rescaling Z_A of J_μ^5 is necessary in order to recover the generator of the axial transformations defined in Eq. (8).

In the LEL formulation this rescaling is obtained by considering the axial WI in the DRS:

$$Z_2 \partial_\mu \langle j_\mu^5(x) \Pi \rangle = Z_m m \langle j^5(x) \Pi \rangle + \langle X_A^{\text{DRS}} \Pi \rangle - \langle \delta_A \Pi \rangle \quad (\text{A.8})$$

where

$$X_A^{\text{DRS}}(x) = Z_2 N \left[\frac{1}{4} \bar{\lambda}(x) (\gamma_\mu \gamma_5 + \gamma_5 \gamma_\mu) \overleftrightarrow{D}_\mu \lambda(x) \right] \quad (\text{A.9})$$

In DRS in $n = 4 + \epsilon$ dimensions, $\gamma_\mu \gamma_5 + \gamma_5 \gamma_\mu \neq 0$ and the symmetry-breaking term $X_A^{\text{DRS}}(x)$ looks like an ‘irrelevant operator’. It formally vanishes in the $\epsilon \rightarrow 0$ limit.

However, in the loop-expansion, terms of the form ϵ/ϵ can develop and X_A^{DRS} can mix with operators of dimension ≤ 4 that survive in the $\epsilon \rightarrow 0$ limit.

As shown in Ref. [17], as $\epsilon \rightarrow 0$, X_A^{DRS} can be rewritten in the form

$$X_A^{\text{DRS}}(x) = Z_2 p \partial_\mu j_\mu^5 + Z_m q m j^5 + r N [G_{\mu\nu} \tilde{G}_{\mu\nu}] \quad (\text{A.10})$$

where p , q , and r are finite constants.

The normalized continuum current is given by

$$\hat{j}_\mu^5(x) = Z_2 (1 - p) j_\mu^5 \approx \frac{Z_2 (1 - p)}{1 + h_A} J_\mu^5 \quad (\text{A.11})$$

with normalized axial WI,

$$\partial_\mu \hat{j}_\mu^5(x) = Z_m m (1 + q) j^5 + r N [G \tilde{G}] + \delta_A \quad (\text{A.12})$$

The normalized lattice axial current is given by

$$\hat{J}_\mu^5 = Z_A J_\mu^5$$

where

$$Z_A = \frac{Z_2}{1+h_A} (1-\phi) = 1 + f_A \quad (\text{A.13})$$

and from the lattice axial WI [Eq. (9)] it follows that

$$(M_0 - \bar{M}_A) = m \frac{Z_m(1+g)}{1+h_p} \quad (\text{A.14})$$

So we reobtain the result that $M_0 - \bar{M}_A$ vanishes in the chiral limit.

The definition of the SUSY current is more involved as there are two operators of the continuum theory,

$$J_\mu = N \left[\frac{1}{2} \sigma_{\alpha\beta} G_{\alpha\beta} \gamma_\mu \lambda \right], \quad t_\mu = N [G_{\mu\nu} \gamma_\nu \lambda] \quad (\text{A.15})$$

with which the mixing of the naïve lattice SUSY current is possible. The mixing can be taken into account by defining two lattice operators S_μ and T_μ , which in the naïve $a \rightarrow 0$ limit give s_μ and t_μ . Let us consider the combinations

$$\langle (S_\mu + \alpha_{s\tau} T_\mu) \pi \rangle \simeq (1+h_s) \langle J_\mu \pi \rangle \quad (\text{A.16})$$

$$\langle (T_\mu + \alpha_{\tau s} S_\mu) \pi \rangle \simeq (1+h_\tau) \langle t_\mu \pi \rangle \quad (\text{A.17})$$

Now we write the continuum SUSY WI in the DRS:

$$\begin{aligned} (Z_2 Z_3)^{1/2} \langle \partial_\mu J_\mu(x) \pi \rangle &= \\ &= \langle N [\Delta_m(x) + \Delta_{g.f.}(x) + X_s(x)] \pi \rangle + \langle \delta_s \pi \rangle \end{aligned} \quad (\text{A.18})$$

where

$$\begin{aligned} \Delta_m &= -\frac{m}{2} Z_3^{1/2} Z_2^{-1/2} \sigma_{\alpha\beta} G_{\alpha\beta} \lambda, \\ \Delta_{g.f.} &= Z_2^{1/2} Z_3^{-1/2} (\xi^{-1} \partial_\nu \partial_\mu A_\mu - g \tilde{Z}_1 \partial_\nu \bar{c} \times c) \gamma_\nu \lambda, \\ X_s &= -\frac{1}{2} g Z_2^{3/2} Z_3^{1/2} \tilde{Z}_1 \tilde{Z}_3^{-1} (\bar{\lambda} \gamma_\nu \times \lambda) \gamma_\nu \lambda \end{aligned} \quad (\text{A.19})$$

and

$$\delta_S = \delta_S^A + \delta_S^\lambda = \sqrt{\frac{Z_2}{Z_3}} \gamma_\mu \lambda \frac{\delta}{\delta A_\mu} + \sqrt{\frac{Z_3}{Z_2}} \frac{1}{2} \sigma_{\alpha\beta} G_{\alpha\beta} \frac{\delta}{\delta \lambda} \quad (\text{A.20})$$

The gauge-fixing term induces a different normalization for gluons and gluinos ($Z_2 \neq Z_3$) and gives the SUSY-breaking term $\Delta_{g.f.}$. However, $\Delta_{g.f.}$ is the Slavnov variation of an operator,

$$\Delta_{g.f.} = -\sqrt{\frac{Z_2}{Z_3}} \delta_G \partial_\nu \bar{c} \gamma_\nu \lambda, \quad (\text{A.21})$$

so it is harmless on gauge-invariant correlations.

Once again the regularization induces a breaking term that formally vanishes as $\epsilon \rightarrow 0$.

By taking into account the relevant operators with dimension 9/2 and 7/2, we can write

$$\begin{aligned} \langle N[X_S(x)] \Pi \rangle &= \langle N[c_1 \partial_\mu \delta_\mu + c_2 \partial_\mu t_\mu + \\ &+ c_3 m \sigma_{\alpha\beta} G_{\alpha\beta} \lambda + c_4 \delta_G \partial_\nu \bar{c} \gamma_\nu \lambda + \\ &+ c_5 m \delta_G \bar{c} \lambda](x) \cdot \Pi \rangle + \\ &+ c_6 \langle \delta_S^A(x) \Pi \rangle, \end{aligned} \quad (\text{A.22})$$

showing the possible mixings compatible with power counting and BRS symmetry. The SUSY WI can be rewritten in the form

$$\begin{aligned} \langle \partial_\mu \hat{J}_\mu(x) \Pi \rangle &= \langle N[(1 + \ell_m) \Delta_m + (1 + \ell_{gf}) \Delta_{gf} \\ &+ m \ell_{gf}^{(2)} \delta_G (\bar{c} \lambda)](x) \Pi \rangle + \langle (\delta_S^{1A}(x) + \delta_S^\lambda(x)) \Pi \rangle, \end{aligned} \quad (\text{A.23})$$

where the normalized SUSY current is given by

$$\hat{J}_\mu = (\bar{z}_2 \bar{z}_3)^{1/2} \left((1 + c_1') j_\mu + c_2' t_\mu \right) \quad (\text{A.24})$$

The conclusion is that no anomalies appear in the SUSY WIs, for Green functions of gluons and gluinos at different points, as the breaking term due to the regularization can be reabsorbed (in the chiral limit) in a redefinition of the current of the SUSY transformations. However, the problem of anomalous WIs can reappear at the level of WIs involving composite operators. To our knowledge a complete analysis of such WIs is lacking.

If we assume that no problems arise when we consider general gauge-invariant correlations, the lattice SUSY current that is conserved as $a \rightarrow 0$ is given by the expression

$$\begin{aligned} \hat{S}_\mu = (\bar{z}_2 \bar{z}_3)^{1/2} & \left(\frac{1 + c_1'}{1 + h_s} (S_\mu + \alpha_{s\tau} T_\mu) + \right. \\ & \left. + \frac{c_2'}{1 + h_\tau} (T_\mu + \alpha_{\tau s} S_\mu) \right) \equiv (1 + k_1) S_\mu + k_2 T_\mu \end{aligned} \quad (\text{A.25})$$

where k_1 and k_2 are functions of the bare coupling and vanish in the continuum limit.

By comparing the lattice SUSY WI [Eq. (18)] and the LEL SUSY WI, we conclude that

$$M_0 - \bar{M}_S \propto m \quad (\text{A.26})$$

and

$$M_0 - \bar{M}_S = Z_{SA} (M_0 - \bar{M}_A) \quad (\text{A.27})$$

where Z_{SA} is a combination of finite renormalization constants.

The above involved construction shows that it is possible to find a conserved SUSY current with a suitable combination of dimension-7/2 lattice operators.

It is important to note that the mixing coefficients vanish in the continuum limit. (The above construction does not introduce infrared divergences as the mixing coefficients are determined for non-exceptional values of the momenta.) The use of the LEL, as an intermediate step, shows that the lattice regularization is not different from a standard continuum regularization, and the occurrence of possible anomalies is due to the general structure of regularization artefacts.

a) Quenched OZI approximation

b) quenched OZI-violating diagrams

Fig. 1

a) OZI-conserving diagrams

b) OZI-violating diagrams

Fig. 2 (a) + (b) give the full SYM theory