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GLUON CONDENSATE AND EFFECTIVE LINEAR POTENTIAL

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A B S T R A C T

It is shown that a model in which the QCD vacuum is simulated by a stochastic background field leads to an asymptotically linear potential. If the fluctuations are assumed to be Gaussian, a correlation length of 0.4 to 0.8 fm is obtained.

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There are many investigations¹⁻¹⁰, in which the complex structure of the QCD vacuum is approximated by a fluctuating external color field. In ref. 10 the application of this approach to Green's functions in Euclidean space time of heavy quarks was motivated and investigated. It could be shown that some contradictory statements about non-existence^{1,2} and existence of local potentials¹¹ could be resolved, if a finite correlation time of the fluctuating field was assumed and compared with the correlation time of the quark system. The short distance expansion of this model corresponds¹² to a non-relativistic version of the Borel-improved SVZ sum rules¹³. In this paper we show that a finite correlation time and length leads to an asymptotically linear potential. Hence the model seems to be quite realistic since it describes several essential aspects of non-perturbative QCD in quite different domains.

We consider the Schwinger function (i.e. Green's function in Euclidean space time) of a heavy non-relativistic quark-antiquark pair, interacting through a Coulomb-potential, in an external fluctuating color potential¹⁰ A_μ^a . In the C. M. system of the pair it has the form:

$$G(\vec{x}_f, T; \vec{x}_i, -T) = \left\langle \int_{\substack{\vec{x}(-T) = \vec{x}_i \\ \vec{x}(+T) = \vec{x}_f}} [d\vec{x}] \exp \left\{ - \int_{-T}^T (H_{q\bar{q}}(\vec{x}, t) + H_{int}(\vec{x}, A, t)) dt \right\} \right\rangle_A \quad (1)$$

Here $\vec{x}$ is the distance between the quark and antiquark, $[d\vec{x}]$ denotes the quantum mechanical path integration and $\langle \rangle_A$ the averaging over the color potential;

$$H_{q\bar{q}} = \frac{\mu}{2} \dot{\vec{x}}^2 + \frac{(\lambda_q + \lambda_{\bar{q}})^2 - \lambda_q^2 - \lambda_{\bar{q}}^2}{8} \frac{g_s^2}{4\pi} \frac{1}{|\vec{x}|} \quad (2a)$$

$$H_{int} = \frac{1}{2} g_s \lambda_q^a A_4^a(\vec{x}/2, t) + \frac{1}{2} g_s \lambda_{\bar{q}}^a A_4^a(-\vec{x}/2, t) \\ + \frac{1}{4} g_s \lambda_q^a \dot{\vec{x}} \cdot \vec{A}^a(\vec{x}/2, t) - \frac{1}{4} g_s \lambda_{\bar{q}}^a \dot{\vec{x}} \cdot \vec{A}^a(-\vec{x}/2, t); \quad (2b)$$

The Gell-Mann matrices λ_q and $\lambda_{\bar{q}}$ act on the quark and antiquark color spinor respectively; μ is the reduced mass, g_s the strong coupling constant.

We may interchange the averaging over the paths $[x]$ and the external field A .

We then expand the field average of H_{int} into linked clusters¹⁴:

$$\langle \exp - \int_{-T}^T H_{int} dt \rangle = \exp \sum_{n=1}^{\infty} (-1)^n C_n / n! \quad (3)$$

$$C_n \equiv \langle\langle \left(\int_{-T}^T H_{int} dt \right)^n \rangle\rangle_A$$

is a linked cluster (i.e. the connected part of the expectation value).

We first discuss the general case and show that for finite correlation time and length each cluster gives rise to an asymptotically linear potential.

We fix the gauge by choosing the modified coordinate gauge⁷:

$$A_4^a(\vec{0}, t) = 0; \quad \vec{x} \cdot \vec{A}^a(\vec{x}, t) = 0. \quad (4)$$

In that gauge a gauge-invariant quarkonium state with a stretched string

$$q_A(\vec{x}/2, t) \left[P \exp -i \int_{-\vec{x}/2}^{\vec{x}/2} \vec{A}(\vec{y}, t) d\vec{y} \right]_{AB} \bar{q}_B(-\vec{x}/2, t) \quad (5a)$$

becomes a global singlet:

$$q_A(\vec{x}/2, t) q_A(-\vec{x}/2, t). \quad (5b)$$

This implies that the effective Hamiltonian to be derived in the following should be only applied to a two-quark state in a global singlet.

The gauge (4) yields the following connection between the potential and the field tensor:

$$A_4^a(\vec{x}, t) = \int_0^1 x_k F_{4k}^a(\alpha \vec{x}, t) d\alpha \quad (6a)$$

$$A_e^a(\vec{x}, t) = \int_0^1 x_k F_{ke}(\alpha \vec{x}, t) \alpha d\alpha \quad (6b)$$

A typical term in the cluster expansion (3) is thus:

$$C_n = (-g_s/2)^n x_{i_1}^{(1)} \dots x_{i_n}^{(n)} \lambda_q^{a_1} \dots \lambda_q^{a_n} \int_{-T}^T \int_{-T}^T dt_1 \dots dt_n \int_0^1 \dots \int_0^1 d\alpha_1 \dots d\alpha_n \cdot \ll F_{4i_1}^{a_1}(\alpha_1 \vec{x}^{(w)/2}, t_1) \dots F_{4i_n}^{a_n}(\alpha_n \vec{x}^{(w)/2}, t_n) \gg_A \quad (7)$$

with $x^{(i)} \equiv x(t_i)$.

Since

$$\vec{x}^{(i)} = \vec{x}^{(1)} + \dot{\vec{x}}(t_1)(t_i - t_1) + \dots \quad (8a)$$

and the time difference $(t_i - t_1)$ is controlled by the correlation time Λ_t , we have in static approximation

$$\vec{x}^{(i)} = \vec{x}^{(1)} + O(\dot{\vec{x}}) \quad (8b)$$

We introduce the new variables

$$\begin{aligned} \tau_i &= (t_i - t_1); \quad \delta_i = |\vec{x}|(\alpha_i - \alpha_1); \quad i=2 \dots n \\ \beta_1 &= |\vec{x}| \alpha_1 \end{aligned} \quad (9)$$

From time-translation, space-rotation and parity-invariance follows that only clusters with even n contribute and we obtain the following form for the cluster:

$$\begin{aligned} C_n &= |\vec{x}|^n (g_s/2)^n \lambda_q^{a_1} \dots \lambda_q^{a_n} \cdot \\ &\int_{-T}^T dt_1 \int \dots \int d\tau_2 \dots d\tau_n \frac{1}{|\vec{x}|^n} \int_0^{|\vec{x}|} d\beta_1 \int \dots \int d\delta_2 \dots d\delta_n \cdot \\ &\cdot R^{a_1 \dots a_n}(\beta_1, \delta_2 \dots \delta_n, \tau_2, \dots \tau_n) \end{aligned} \quad (10)$$

If we are interested in bound states, we may choose T very large as compared to the correlation time Λ_t , hence we extend the limits of integration of

τ_i from $-\infty$ to $+\infty$. If furthermore the correlation time of the quark-anti-quark system is large as compared to the correlation time Λ_t of the fields (in a Coulomb potential this implies that $|x|$ has to be large against Λ_t and that the static approximation is justified, see ref. 10), we may decouple the τ_i integration and obtain as an additive contribution of the n'th cluster to the effective Hamiltonian:

$$H_{int}^{eff, n}(\vec{x}(t)) = \frac{(-1)^n}{n!} (g_s/2)^n \lambda_q^{a_1} \dots \lambda_q^{a_n} \cdot \int_0^{|\vec{x}|} d\beta_1 \int \dots \int d\delta_2 \dots d\delta_n \int_{-\infty}^{+\infty} d\tau_2 \dots d\tau_n \cdot R^{a_1 \dots a_n}(\beta_1, \delta_i, \tau_i) \quad (11)$$

We assume that R is a bounded function and that it falls off with a correlation length Λ_x if the difference variables δ_i increase. Then for $|\vec{x}| \gg \Lambda_x$ the integration limits for δ_i become independent of $|\vec{x}|$ and we are only left with a linear $|\vec{x}|$ -dependence due to the integration over β_1 . So we can construct a total effective Hamiltonian describing the interaction of the $q\bar{q}$ -pair with the fluctuating (vacuum) color field as a sum of local potential, each of which is asymptotically linear.

Little more can be said about the general case and we therefore discuss now the special case, where the fluctuations are assumed to be Gaussian. In that case the dynamics of the background field is determined by the two-point cluster

$$\langle F_{4k}^a(\vec{x}, t) F_{4l}^b(\vec{x}', t') \rangle = \delta_{ij} R^{ab}(\vec{x}, \vec{x}', t-t') \quad (12a)$$

with

$$R^{ab}(\vec{0}, \vec{0}, 0) = -\delta^{ab} \frac{\pi^2}{24} \left\langle \frac{\alpha_s}{\pi} F_{\mu\nu}^a F^{a\mu\nu}(0) \right\rangle \quad (12b)$$

We compare it with the gauge-invariant quantity

$$G_2((x-x')^2) = \left\langle \frac{\alpha_s}{\pi} F_{\mu\nu}^a(\vec{x}, x_0) \cdot \left[P \exp -i \int A_\mu(z) dz^\mu \right]_{ab} F^{b\mu\nu}(\vec{x}', x'_0) \right\rangle \quad (13)$$

$$z = x' + (x - x')\tau \quad 0 \leq \tau \leq 1$$

Due to the fixed gauge (4) we have

$$R^{aa}(\vec{x}, x_0; \vec{x}', x'_0) = \frac{\pi^2}{3} G_2(|\vec{x} - \vec{x}'|^2) \quad (14)$$

and

$$R^{aa}(\vec{0}, x_0; \vec{0}, x'_0) = \frac{\pi^2}{3} G_2(|x_0 - x'_0|^2)$$

We hence make for R^{ab} the ansatz, compatible with (12) to (14):

$$R^{ab}(\vec{x}, x_0; \vec{x}', x'_0) = \delta^{ab} \frac{\pi^2}{24} G_2(0) \cdot R\left(\frac{|\vec{x} - \vec{x}'|^2}{\Lambda^2}, \frac{(x_0 - x'_0)^2}{\Lambda^2}\right) \quad (15)$$

This yields after some elementary manipulations with the integration variables the effective Hamiltonian:

$$H_{int}^{eff}(\vec{x}(t)) = |\vec{x}(t)| \frac{\pi^2}{18} G_2(0) \cdot \Lambda^2 \cdot \int_{-\infty}^{+\infty} d\tau \int_0^{|\vec{x}|/\Lambda} d\rho (1 - \Lambda \rho / |\vec{x}|) R(\rho^2, \tau^2) \quad (16)$$

where we have used that the quarks are triplets and that the quark-antiquark state is a singlet. Asymptotically ($|\vec{x}| \gg \Lambda$) we obtain thus:

$$H_{int}^{eff}(\vec{x}) = \sigma |\vec{x}| + \gamma \quad (17a)$$

with

$$\sigma = \frac{\pi^2}{18} G_2(0) \Lambda^2 \int_{-\infty}^{+\infty} d\tau \int_0^{\infty} d\rho R(\rho^2, \tau^2) \quad (17b)$$

$$\gamma = -\frac{\pi^2}{18} G_2(0) \Lambda^3 \int_{-\infty}^{+\infty} d\tau \int_0^{\infty} d\rho \rho \cdot R(\rho^2, \tau^2) .$$

We specify R further to the functions

$$A) \quad R(\rho^2, \tau^2) = e^{-(\rho^2 + \tau^2)^{\alpha/2}} ; \quad B) \quad R(\rho^2, \tau^2) = e^{-|\rho|^{\alpha}} e^{-|\tau|^{\alpha}}$$

which yields

$$A): \quad \Lambda^2 = \frac{18 \alpha}{\pi^3 \Gamma(2/\alpha) G_2(0)} \sigma ; \quad \gamma = \frac{6 \sqrt{2\alpha} \Gamma(3/\alpha)}{\pi^{5/2} [\Gamma(2/\alpha)]^{3/2}} \frac{\sigma^{3/2}}{[G_2(0)]^{1/2}}$$

$$B): \quad \Lambda^2 = \frac{9 \alpha^2}{\pi^2 [\Gamma(1/\alpha)]^2 G_2(0)} \sigma ; \quad \gamma = \frac{3 \alpha \Gamma(2/\alpha)}{\pi [\Gamma(1/\alpha)]^2} \frac{\sigma^{3/2}}{[G_2(0)]^{1/2}}$$

Using $\sigma = 1/(2\pi) \text{ GeV}^2$ (corresponding to the Regge slope $\alpha' = 1 \text{ GeV}^2$ and $G_2(0) = (0.012 \dots 0.02) \text{ GeV}^4$ ¹³ we find values (for $\alpha = 1$ and 2) in the range $\Lambda \approx (0.4 \dots 0.8) \text{ fm}$, $\gamma \approx (0.3 \dots 0.5) \text{ GeV}$. The latter value is quite compatible with the intercept of the linear part of the successful semi-phenomenological potential of ref. 15.

The correlation length of the gauge-invariant expression $G_2(x)$ (eq. (13)) has been investigated in an $SU(2)$ lattice calculation¹⁶. $G_2(x)$ is found to be of the form $G_2(x) = G_2(0)e^{-x/\Lambda}$ with $\sigma\Lambda^2 = 0.064$ ($G_2(0)\Lambda^4 = 0.0019$). This corresponds to $\lambda \approx 0.13 \text{ fm}$. Such a short correlation length would be very dangerous for sum rule analyses, which take only into account the 4-dimensional condensate $G_2(0)$. It would also (consistently) imply that the stochastic field which simulates the vacuum has large non-Gaussian contributions.

Finally we shall shortly discuss the case of two other derivations of a linear potential from gluon condensates. In ref. 17 a constant condensate is assumed, but the dipole interaction is iterated and summed up in a specific way. The resulting slope $\sigma = \sqrt{8/5} \frac{\pi}{3} \sqrt{G_2}$ agrees very well with the phenomenological value, but see ref. 18 for a critical discussion. In ref. 19 a condensate with finite correlation length is assumed, as in the present paper, but there its interaction to lowest order with the exchanged gluons is investigated. This yields an asymptotically linear potential with the slope $\sigma = 3\pi/8 \alpha_s(\infty) G_2 \lambda^2$ where $\alpha_s(\infty)$ is the effective color charge at large distances.

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