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GAUGE COVARIANCE OF QCD COHERENT STATES

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ABSTRACT

By use of BRST invariance in a Hamiltonian formalism, the gauge-dependence of QCD coherent states is studied. The eikonal form of the leading coherent state operator is proved to hold in covariant gauges for matrix elements between physical states. At subleading level, the existence of infra-red singular parton correlations of Bloch-Nordsieck-violating type is shown to be a consequence of BRST covariance. It is moreover argued that the BRST invariance of the asymptotic states requires, besides the physicality conditions, also that incoming partons be in an overall colour singlet.

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1. - INTRODUCTION

During the past few years, a considerable effort has been made¹⁾⁻⁷⁾ to analyze the perturbative infra-red (IR) behaviour of QCD by means of coherent states¹⁾, i.e., of operator functionals of soft gluon fields which describe the gluonic cloud surrounding a hard scattering process²⁾⁻⁷⁾.

Although the original definition of coherent states in QED¹⁾ as states of minimal uncertainty does not go through to QCD, the approach based on asymptotic dynamics⁸⁾ is applicable instead³⁾⁻⁵⁾, and leads to a variety of interesting features, both theoretical and phenomenological.

- (1) Due to the intricate IR behaviour of QCD, arising from gluon interactions, there is no closed expression for the QCD coherent state operators^{3),4)}. However, a Hamiltonian formalism has been set up^{5),6)} which allows their explicit construction to any desired accuracy (leading, first subleading, and so on).
- (2) There is no precise relation of the operators so constructed to classical fields, mainly because of the difficulty of defining a non-trivial classical limit of QCD with external sources⁹⁾. However, the leading-order coherent state is defined in terms of dressed fields, whose equations of motion are very similar to the classical ones, except that they are non-local because of a built-in energy ordering⁵⁾. This relation is due to the validity, in QCD also, of the eikonal approximation, with the difference that there is an independent emission of dressed gluons (instead of free ones).
- (3) Due to the relation of coherent states with the soft tail of hadronic jets¹⁰⁾, there is a variety of coherent effects¹¹⁾ which were predicted^{12),13)}, and are now being checked experimentally in two-jet^{10),13)} and three-jet^{13),14)} events.
- (4) The construction of many-parton coherent states is especially interesting for initial-state interactions in Drell-Yan processes and parton-parton scattering at colliders. It has been shown^{6),7)} that the violation¹⁵⁾ of the Bloch-Nordsieck theorem is due to the occurrence, at subleading level, of IR singular parton-parton correlations. On the other hand, the analysis of coherence effects in initial jets^{11),16)} and many-parton scattering¹⁷⁾ is now under way.

The approach of asymptotic dynamics⁸⁾, on which these results are based, consists of the study of the asymptotic Hamiltonian

$$H_{as} = H_0 + H_s^E, \quad H = H_{as} + H_h^E, \quad (1.1)$$

where H_s^E is the soft part of the interaction Hamiltonian, whose vertices, by definition, transfer energy less than E , a scale that separates soft and hard partons.

In this decomposition, the S-matrix is rewritten as

$$S = \Omega_-^{E\dagger} S_h^E \Omega_+^E, \quad (1.2)$$

where

$$\Omega_{\pm}^E = T \exp \left[\int_{\mp\infty}^0 dt H_s^E(t) \right] \quad (1.3)$$

are soft evolution operators, and S_h^E is IR finite¹⁸⁾. As a consequence, the $\pm$ hard states

$$|h, \pm\rangle = \Omega_{\pm}^{E\dagger} |h\rangle \quad (1.4)$$

have S-matrix elements $\langle -|S|+ \rangle$ free of IR singularities, and we are led to define⁶⁾

$$U_{\pm}^E(A) = \langle h | \Omega_{\pm}^E | h \rangle (\langle 0 | \Omega_{\pm}^E | 0 \rangle_h)^{-1} \quad (1.5)$$

as coherent state operators, embodying the IR behaviour of the theory, when made explicit in terms of free gluon fields.

Despite the nice properties listed above, the approach described in Eqs. (1.1)-(1.5) has some conceptual difficulties. First of all, the decompositions (1.1) and (1.2) are frame-dependent, even if the meaningful results listed above can be checked to be Lorentz-covariant. This difficulty is shared to some extent by QED in the charged sector¹⁹⁾. On the other hand, in QCD we are in the funny situation that the candidate asymptotic states (1.4) are not expected to be acceptable for partonic configurations which are inconsistent with confinement: that is, the formalism may have problems for good reasons. It has already been

argued^{7),20)} that only local colour singlets are consistent with finiteness of short-distance observables.

Furthermore, both states and Hamiltonian are gauge-dependent. The results (1)-(4) have mostly been obtained by using physical-gauge Ward identities; one may wonder whether the same picture of IR behaviour goes through at operator level when quantizing (1.1) in covariant gauges. One may also ask which states of type (1.4) are gauge-invariant, and therefore have a better chance of being acceptable.

The purpose of this paper is to clarify the latter points by studying the gauge-dependence of the soft Hamiltonian (1.1) and the states (1.4) in a class of covariant gauges in which BRST symmetry^{21),22)} holds. Our results are the following:

- (i) The hierarchy of coherent state operators in (1) and (2), and in particular the eikonal form of the leading operator are valid in the physical sector. In other words, one should eliminate unphysical degrees of freedom (like ghosts and scalar gluons) by taking matrix elements between physical states.
- (ii) The existence of parton correlations of Bloch-Nordsieck-violating type [cf. point (4)] is a consequence of BRST invariance at subleading level.
- (iii) The requirement that the states (1.4) (properly modified) be BRST-invariant is most naturally satisfied by states $|h\rangle$ which are both physical and colour-singlet.

The contents of the paper are as follows. In Section 2 we set up the Hamiltonian BRST formalism and we define the hierarchy of IR approximations to $\Omega_{\pm}^E$. In Section 3 we prove the eikonal form of the leading-order coherent state in the physical sector, and we study its gauge-dependence. In Section 4 we set up the condition of BRST covariance at subleading level, we exhibit properties (ii) and (iii), and we discuss our results. A few details are left to the Appendices.

2. - ASYMPTOTIC HAMILTONIAN DYNAMICS IN COVARIANT GAUGES

The method of asymptotic dynamics in QCD³⁾⁻⁶⁾ involves several steps. Firstly, the QCD Hamiltonian and the partonic Fock space are decomposed into a hard and a soft part^{3),4)}. This is done by introducing an upper energy transfer E and an IR cut-off λ .

Secondly, the perturbative time-ordered expansion of the soft evolution operator is rearranged into a v -ordered form, where $v < E$ is the energy transfer^{5),6)}. This allows us to define successive approximations to the soft evolution operator which are better related to the hierarchy of IR singularities.

Finally, the coherent state operator is explicitly constructed as a functional of soft gluon fields, order by order in this new expansion (leading, first subleading, and so on).

In this section we shall describe the first two steps, in the Hamiltonian formalism for covariant gauges, with particular emphasis on the treatment of ghosts and of unphysical degrees of freedom. An essential tool will be the study of BRST symmetry²¹⁾, and of the corresponding charge in the soft sector of the Fock space.

Let us start with the QCD Lagrangian

$$\mathcal{L} = \mathcal{L}_{cl} + \mathcal{L}_{GF} + \mathcal{L}_{gh} \quad , \quad (2.1)$$

where

$$\begin{aligned} \mathcal{L}_{cl} &= -\frac{1}{2} \text{Tr} F_{\mu\nu} F^{\mu\nu} + \sum_f \bar{\psi}_f (i\not{D} - m_f) \psi_f \quad , \\ D_\mu &= \partial_\mu + ig A_\mu \quad , \quad A_\mu = t_a A_\mu^a \quad , \quad ig F_{\mu\nu} = [D_\mu, D_\nu] \quad , \end{aligned} \quad (2.2)$$

is the classical Lagrangian,

$$\mathcal{L}_{GF} = -\partial^\mu \tilde{B} \cdot \tilde{A}_\mu + \frac{1}{2} \alpha \tilde{B}^2 \quad (2.3)$$

is the gauge-fixing term as a function of the auxiliary scalar octet field $\tilde{B}$ (given by $-\partial_\mu \tilde{A}^\mu / \alpha$ by the equations of motion) and

$$\mathcal{L}_{gh} = 2i \text{Tr} (\partial_\mu \bar{c} D^\mu c) \quad (2.4)$$

is the ghost term as a function of anticommuting octet fields $c_a = c_a^+$, $\bar{c}_a = \bar{c}_a^+$.

Following Kugo and Ojima²³⁾, we shall choose the set $\{\underline{A}_i (i=1,2,3), \underline{B}, \underline{c}, \bar{\underline{c}}\}$ of independent field variables, with conjugate momenta

$$\underline{\Pi}_i = -\underline{F}_{0i}, \quad \underline{\Pi}_B = -\underline{A}_0, \quad \underline{\Pi}_c = i \dot{\bar{\underline{c}}}, \quad \underline{\Pi}_{\bar{\underline{c}}} = -i D_0 \underline{c}, \quad (2.5)$$

so that $\underline{A}_0$ will be viewed as momentum conjugate to $\underline{B}$, rather than as an independent variable.

By a standard Legendre transform on (2.1), we get the Hamiltonian

$$H = H_0 + H_I, \quad (2.6)$$

where H_0 is the quadratic part, and the interaction density

$$\mathcal{H}_I = g (\underline{\Pi}_i \cdot \underline{A}^i - \underline{\Pi}_0 \cdot \underline{\Pi}_B) \quad (a)$$

$$-g \partial_i \underline{A}_j \cdot (\underline{A}^i \times \underline{A}^j) - g \underline{\Pi}_i \cdot (\underline{\Pi}_B \times \underline{A}_i) \quad (b)$$

(2.7)

$$+ \frac{1}{4} g^2 (\underline{A}_i \times \underline{A}_j) \cdot (\underline{A}^i \times \underline{A}^j) \quad (c)$$

$$-g (\underline{\Pi}_B \times \underline{c}) \cdot \underline{\Pi}_c - i g (\underline{A}_i \times \underline{c}) \cdot \partial^i \bar{\underline{c}}, \quad (d)$$

shows (a) quark-gluon, (b) three-gluon, (c) four-gluon and (d) gluon-ghost interaction vertices.

The Lagrangian (2.1) is invariant under global BRST transformations²¹⁾

$$\delta \underline{A}_\mu = \lambda D_\mu \underline{c} , \quad \delta \psi = -ig \lambda \underline{c} \cdot \underline{t} \psi , \quad (a) \quad (2.8)$$

$$\delta \underline{c} = \frac{1}{2} g \lambda \underline{c} \times \underline{c} , \quad \delta \underline{\bar{c}} = i \lambda \underline{B} , \quad \delta \underline{B} = 0 , \quad (b)$$

part of which [Eq. (2.8a)] is a gauge change with parameter $\lambda \underline{c}(x)$, where λ is a constant anticommuting variable. Correspondingly, the Nöther charge

$$Q_B = \int d^3x \left[i \underline{B} \cdot \underline{\pi}_c - \underline{c} \cdot (D^i \underline{\pi}_i - g \underline{j}_0) + \frac{1}{2} g (\underline{c} \times \underline{c}) \cdot \underline{\pi}_c \right] \quad (2.9)$$

is conserved, and commutes with H in (2.6).

In the following, we shall rely on the general observation²²⁾ that Q_B allows a clear definition of physical and gauge degrees of freedom. As expected, Q_B in (2.9) starts with a linear combination of the QCD constraints (i.e., transversality, $\underline{B} \sim \partial_\mu \underline{A}^\mu = 0$, and Gauss's law, $D^i \underline{\pi}_i - g \underline{j}_0 = 0$), and is furthermore Hermitian and nilpotent ($Q_B^2 = 0$). Therefore, one can define physical states in the Heisenberg representation by

$$Q_B |phys\rangle_{Heis} = 0 , \quad (2.10)$$

while gauge states are of the form $Q_B | \rangle$ and gauge operators are of the type $[Q_B, 0]_\pm$.

Let us now quantize (2.6) in the interaction representation at $t = 0$. We can rewrite (2.7), (2.9) and (2.10) in terms of the (free) gluon fields

$$\underline{A}_\mu = \sum_{\sigma=\pm} \int d[q] \underline{A}_\mu^\sigma(q) e^{-i\sigma q \cdot x} , \quad (2.11)$$

where

$$\begin{aligned} \underline{A}_\mu^+ &= \underline{A}_\mu(q) , \quad \underline{A}_\mu^- = \underline{A}_\mu^\dagger(q) , \quad d[q] = d^3q [2\omega_q (2\pi)^3]^{-1} , \\ [A_{\mu a}(q) , A_{\nu b}^\dagger(q')] &= -g_{\mu\nu} \delta_{ab} 2\omega_q (2\pi)^3 \delta^3(q - q') , \end{aligned} \quad (2.12)$$

and of similar expansions for quark and ghost fields.

By using free field equations of motion in the Feynman gauge ($\alpha = 1$)

$$\partial_\mu \tilde{A}^\mu = -\tilde{B} \quad , \quad \square \tilde{A}_\mu = 0 \quad , \quad (2.13)$$

the BRST charge (2.9) becomes

$$Q_B = Q_B(t=0) = Q_0 + Q_1 \quad , \quad (2.14)$$

where

$$Q_0 = \int d^3x \left(\partial_\mu \tilde{A}^\mu \cdot \tilde{c} - \partial_\mu \tilde{A}^\mu \cdot \dot{\tilde{c}} \right) = - \sum_\sigma \int d[p] \, p \cdot A_\alpha^\sigma(p) \tilde{c}_\alpha^{-\sigma}(p) \quad , \quad (a) \quad (2.15)$$

$$Q_1 = \int d^3x \, \tilde{c} \cdot \left[\dot{\tilde{c}}_0 + (\dot{\tilde{A}}_i - \partial_i \tilde{A}_0) \times \tilde{A}^i + \frac{i}{2} \tilde{c} \times \dot{\tilde{c}} \right]_{t=0} \quad (b)$$

are the quadratic and cubic parts respectively.

It is straightforward to check that the evolution operator $U_I(0, -\infty)$ in the interaction representation transforms the free charge Q_0 into the complete one, i.e.,

$$U_I(0, -\infty) Q_0 = Q_B \quad U_I(0, -\infty) = (Q_0 + Q_1) \quad U_I(0, -\infty) \quad , \quad (2.16)$$

so that the physicality condition becomes

$$Q_0 |phys\rangle_I = 0 \quad . \quad (2.17)$$

Due to (2.15a), Eq. (2.17) defines transverse states

$$\partial^\mu \tilde{A}_\mu^{(+)} |phys\rangle_I = 0 \quad (2.17')$$

in the zero-ghost vector, but is in general consistent with more complicated ghost and scalar gluon configurations.

Let us now define the soft part of the interaction Hamiltonian H_S^E by replacing the expansion (2.11) in (2.7) and by introducing the constraint

$$\lambda < \nu = \left| \sum_i \sigma_i \omega_i \right| < E, \quad \sigma_i = \pm, \quad (2.18)$$

on the energy transfers at the various vertices. We obtain

$$H_I(t) = H_h^E + H_s^E, \quad H_s^E(t) = H_f + H_g + H_G + H_4, \quad (2.19)$$

where

$$\begin{aligned} H_f(t) &= g \sum_{\sigma} \int d[p] d[q] p_f^{\sigma}(\underline{p}) \hat{p}_{\mu} A_{\sigma}^{\mu}(\underline{q}) e^{-i\sigma \hat{p} q t} \Theta(E_p - \omega_q) \Theta(E - \hat{p} q), \quad (a) \\ H_g(t) &= \frac{-ig}{3!} f_{a_1 a_2 a_3} \sum_{\sigma_i} \int d[q_1] d[q_2] d[q_3] (2\pi)^3 \delta^3(\sum \sigma_i q_i) e^{-i \sum \sigma_j \omega_j t} \\ &\quad \cdot \Theta(E - |\sum_j \sigma_j \omega_j|) \Gamma_{\mu_1 \mu_2 \mu_3}(\sigma_i q_i) A_{a_1}^{\mu_1 \sigma_1}(\underline{q}_1) A_{a_2}^{\mu_2 \sigma_2} A_{a_3}^{\mu_3 \sigma_3}, \quad (b) \\ H_G(t) &= g f_{a_1 a_2 a_3} \sum_{\sigma_i} \int d[q_1] d[q_2] d[q_3] (2\pi)^3 \delta^3(\sum \sigma_i q_i) \exp(-i \sum \sigma_j \omega_j t) \cdot \\ &\quad \cdot \Theta(E - |\sum_j \sigma_j \omega_j|) \bar{c}_{a_1}^{\sigma_1} c_{a_2}^{\sigma_2} \sigma_1 q_1^{\mu} A_{\mu a_3}^{\sigma_3}(\underline{q}_3), \quad (c) \\ &\quad \left(\hat{p}_r = p_r / E_p, \quad \underline{p}_f = \sum_f d_f^{\dagger} \underline{t}_f d_f \right), \end{aligned} \quad (2.20)$$

and Γ_{μ} is the 3g vertex.

Note that in (2.20) we have dropped H_4 (which does not contribute up to the first subleading level discussed here) and we have used the eikonal form for H_f , because the vertex $g \rightarrow q\bar{q}$ has no soft fermion singularity.

Analogously, the Fock space $\mathcal{H}$ of quarks, gluons and ghosts is decomposed as $\mathcal{H} = \mathcal{H}_h \times \mathcal{H}_s$, where $\mathcal{H}_h$ includes partonic states $|p\rangle$ with $E_p > E$, and their direct products. Because of the decomposition (2.19) the S-matrix can be written in the form

$$S = \Omega_{-}^{E\dagger} S_h \Omega_{+}^E, \quad (2.21)$$

where $\Omega_{\pm}^E = U_s(0, \mp\infty)$ are Möller operators for the soft Hamiltonian, while T_h , which is screened by H_h , only allows transitions with $\nu > E$, i.e., acts essentially on $\mathcal{H}_h$. Therefore all information about IR singularities is embodied in $\Omega_{\pm}^E$.

The next step is to define a hierarchy of IR approximations for the soft evolution operators

$$\Omega_{\pm}^E = U_s(0, \mp\infty) = T \exp \left[-i \int_{\mp\infty}^0 H_s^E(t) dt \right], \quad (2.22)$$

by exploiting the fact that time and energy transfer ν are conjugated variables.

By defining the ν -transforms $h^{\pm}(\nu)$

$$H_s^E(t) = \sum_{\sigma=\pm} \int_{\lambda}^E d\nu h^{\sigma}(\nu) e^{-i\sigma\nu t} \quad (2.23)$$

and the ν -Hamiltonian operators

$$\frac{d}{d\nu} \Omega_{\pm}^{\nu} = \Delta_{\pm}(\nu) \Omega_{\pm}^{\nu}, \quad (a)$$

(2.24)

$$\Omega_{\pm}^E = P_{\nu} \exp \left[\int_{\lambda}^E d\nu \Delta_{\pm}(\nu) \right], \quad (b)$$

the following expansion has been derived⁶⁾

$$\Delta_{\pm}(\nu) = \sum_{n=1}^{\infty} \Delta_{\pm}^{(n)}(\nu),$$

$$\Delta_{\pm}^{(n+1)}(\nu) = \sum_{\{\sigma_i\}} \int_{\lambda}^{\nu} d\nu_1 \dots d\nu_n \frac{[h^{\sigma_n}(\nu_n), [\dots [h^{\sigma_1}(\nu_1), h^{\sigma}(\nu)]]]}{(\sigma_n \nu_n + \dots \sigma \nu \pm i\varepsilon) \dots (\sigma_1 \nu_1 + \sigma \nu \pm i\varepsilon)(\sigma \nu \pm i\varepsilon)} \quad (2.25)$$

where $\nu_i < \nu$ and multiple commutators of h^{σ} 's occur. In particular, the first term $\Delta_{\pm}^{(1)}$ is given by

$$\Delta_{\pm}^{(1)}(\nu) = \sum_{\sigma} h^{\sigma}(\nu) / \sigma \nu = \frac{1}{\nu} (h^{+}(\nu) - h^{-}(\nu)), \quad (a)$$

(2.26)

$$\Omega_{\pm}^{E(1)} = P_{\nu} \exp \left[\int_{\lambda}^E \frac{d\nu}{\nu} (h^{+}(\nu) - h^{-}(\nu)) \right] \quad (b)$$

and corresponds to replacing (2.23) into the time-ordered expansion (2.22) with

the v -ordering requirement $\lambda \ll v_1 \dots \ll v_n \ll E$.

Further terms in the expansion (2.25) have the same basic v^{-1} behaviour due to the constraint $v_i < v$ in the denominators, but are depressed at operator level with respect to (2.26) by further powers of g . Therefore, Eq. (2.25) defines the hierarchy of IR approximations we were looking for.

3. - EIKONAL FORM OF LEADING-ORDER COHERENT STATE

The expansion defined by (2.24b) and (2.25) allows an explicit construction of the coherent state operators $U_{\pm}(A)$ as functionals of the free gluon field. They are defined by taking matrix elements of $\Omega_{\pm}^E$ between the states of $\mathcal{H}_h$ occurring in a given hard scattering process, i.e., [cf. Eq. (1.4)]

$$\begin{aligned} (U_{\pm}^E)_{\underline{\beta}, \underline{\alpha}} \langle 0 | \Omega_{\pm}^E | 0 \rangle_h &= \langle p_1, \beta_1 \dots p_n, \beta_n | \Omega_{\pm}^E | p_1, \alpha_1 \dots p_n, \alpha_n \rangle_h \\ &= (U_{\pm}^E)_{\underline{\beta}, \underline{\alpha}} U_{\pm}^S, \end{aligned} \quad (3.1)$$

where we have explicitly factorized the soft operator $U_{\pm}^S$, giving the action of $\Omega_{\pm}^E$ on the hard vacuum state.

The leading order approximation $U^{E(1)}$ comes from $\Omega^{(1)}$ and $\Delta^{(1)}$ in (2.26), which correspond to strongly-ordered energy transfers. It has been stated⁶⁾, on the basis of physical gauge Ward identities, that the soft Hamiltonian can be replaced by its eikonal form in the energy-ordered region. Here we shall show that this is indeed the case for quantization in covariant gauges, provided matrix elements between physical states are taken.

Let us start by noticing that the constraint $v = |\sum_i \sigma_i \omega_i| < E$ in (2.18) does not imply that all energies occurring at the vertex are soft. In other words, H_s^E connects states of $\mathcal{H}_h$ and $\mathcal{H}_s$, besides those of $\mathcal{H}_s$ amongst themselves. If two partons at the vertex are faster than the third ($\omega_1 \approx \omega_2 \gg \omega_3$) then, by momentum conservation, necessarily $\sigma_2 = -\sigma_1$, and hence

$$v = |\omega_1 - \omega_2 (q_1 + \sigma_1 \epsilon_3 q_3) + \sigma_1 \epsilon_3 \omega_3| \simeq \hat{q}_1 \cdot q_3 < E \quad (3.2)$$

implies that q_3 is soft ($\hat{q}_{1\mu} = q_{1\mu}/\omega_1$), while the others may be fast. Alternatively, all energies ω_i are soft and of the same order and σ_i is undetermined.

In the strongly-ordered situation, the eikonal form of the 3g vertex should apply. By setting $q_1 = p$, $q_3 = -q$, one has

$$\Gamma_{\lambda\mu\nu}(p, -p+q, -q) = 2p_\nu g_{\lambda\mu} - p_\lambda g_{\mu\nu} - p_\mu g_{\nu\lambda} + O(|q|/|p|), \quad (3.3)$$

where the first term is the eikonal one, while the latter two are non-eikonal and couple soft gluons to fast scalar gluon fields $p \cdot A^\sigma(p)$ which are non-trivial in covariant gauges.

By distinguishing ordered and non-ordered energy regions in (2.20), we obtain the decomposition

$$H_s^E(t) = H_{eik}(t) + K(t) + H'_s(t), \quad (3.4)$$

where the eikonal part is defined as usual by

$$H_{eik}(t) = g \sum_{\sigma} \int d[p] d[q] p^a(p) \hat{p}_\mu A_\mu^\sigma(q) \exp(-i\sigma \hat{p} q t) \cdot \Theta(E - \hat{p} \cdot q) \Theta(E_p - \omega_q), \quad (3.5)$$

with

$$p_\mu(p) = p_\mu^f + p_\mu^g + p_\mu^c, \\ p_{ga} = i f_{abc} A_\mu^{b\dagger}(p) A_c^\mu(p), \quad p_{ca} = f_{abc} \sum_{\sigma} \bar{c}_b^\sigma(p) c_c^\sigma(p), \quad (3.6)$$

$H'_s(t)$ involves only soft parton energies of the same order and is therefore sub-leading, and $K(t)$ includes non-eikonal terms due to ghost- and scalar gluon-interactions in the strongly-ordered region, i.e.,

$$K(t) = \frac{1}{2} g \sum_{\sigma} \int d[p] d[q] \Theta(E_p - \omega_q) \Theta(E - \hat{p} \cdot q) \exp(-i\sigma \hat{p} q t) \cdot \left\{ A_\mu^a(q) \left[\hat{p} \cdot A_b^\dagger(T^a)_{bc} A_c^\mu - \frac{1}{2} p_c^a \hat{p}^\mu \right] + i c_a^\sigma(q) \bar{c}^\dagger T^a p \cdot A \right\} + h.c. \quad (3.7)$$

In order to show that the simple eikonal term gives rise to the leading IR singularities, we have to prove that $K(t)$ does not give IR leading contributions between physical states. The main point is that $K(t)$ is a gauge operator, in the sense that

$$K(t) = \{Q_0, O(t)\} + \dots, \quad (3.8)$$

where the dots stand for subleading terms $O(\omega_q/E_p)$, and

$$O(t) = -\frac{1}{2}g \sum_{\sigma} \int d[p] d[q] \frac{1}{E_p} \Theta(E_p - \omega_q) \Theta(E - \hat{p}q) \exp(-i\sigma \hat{p}qt) \cdot \\ \cdot f_{abc} [A_{\mu}^{a\dagger} \bar{c}_b A_c^{\mu\sigma}(q) - i \bar{c}_a^{\dagger} \bar{c}_b c_c^{\sigma}(q)] + h.c. \quad (3.9)$$

This can be easily checked from the (anti)commutation relations

$$[Q_0, A_{\mu}^{\sigma a}(p)] = -\sigma p_{\mu} c^{\sigma a}(p), \quad [Q_0, d_a^{\sigma}(p)] = 0, \\ \{Q_0, \bar{c}_a^{\sigma}(p)\} = i\sigma p \cdot A_a^{\sigma}(p), \quad \{Q_0, c_a^{\sigma}(p)\} = 0. \quad (3.10)$$

Due to (3.8), the v -Hamiltonian operator has the form

$$\Delta^{(1)}(v) = \Delta_{eik}(v) + \{Q_0, \Delta_0(v)\} + \dots, \quad (3.11)$$

with the commutation relation

$$[Q_0, \Delta^{(1)}] = [Q_0, \Delta_{eik}] = -g Q(v) \int d[q] \sum_{\sigma} c_{\sigma}^{\sigma}(q) \delta(\omega_q - v), \quad (3.11')$$

where $Q(v)$ is the colour charge above energy v . Therefore, the eikonal evolution operator

$$\Omega_{eik}^E = P_v \exp \left[\int_{\lambda}^E dv \Delta_{eik}(v) \right] \quad (3.12)$$

satisfies the commutation relation

$$\Omega_{eik}^E Q_0 = (Q_0 + Q_{1,eik}) \Omega_{eik}^E, \quad (3.13)$$

which is similar to Eq. (2.16), with Q_1 replaced by its strong-ordering form

$$Q_{1,eik} = g \int d[q] Q_a(\omega_q) \sum_{\sigma} c_{\sigma a}(q), \quad (a)$$

(3.14)

$$Q(\omega) = \int d[q] g(q) \Theta(\omega_q - \omega). \quad (b)$$

Therefore, the leading evolution operator takes the form

$$\Omega^{E(1)} = \Omega_{eik}^E \left[1 + \int_{\lambda}^E d\nu \Omega_{eik}^{\nu \dagger} \{ Q_0, \Delta_0(\nu) \} \Omega_{eik}^{\nu} + \dots \right], \quad (3.15)$$

where the additional terms will be shown not to contribute between physical states. In fact, by (3.13), the second term in (3.15) can be written as

$$\begin{aligned} & \{ Q_0, \Omega_{eik}^E \Omega_{eik}^{\nu \dagger} \Delta_0(\nu) \Omega_{eik}^{\nu} \} + \\ & + \Omega_{eik}^E \Omega_{eik}^{\nu \dagger} \left[q \int d[q] Q(\omega_q), \Delta_0(\nu) \right] \sum_{\sigma} c_{\sigma}^{\sigma}(q) \Omega_{eik}^{\nu}, \end{aligned} \quad (3.16)$$

and both terms have vanishing matrix elements between physical states, the first one by the physicality condition (2.17), and the second one because $\Delta_0(\nu)$, involving colour-singlet operators harder than $\nu > \omega_q$, commutes with $Q(\omega_q)$.

We thus conclude that the leading evolution operator takes the naive eikonal form (3.12) when evaluated between physical states in $\mathcal{H}$.

The range of validity of the eikonal approximation is further clarified by two results, proved in Appendix A. Firstly, one can show that partons in $\mathcal{H}_h$ interact with those in $\mathcal{H}_s$ always with eikonal vertices, i.e.,

$$\begin{aligned} H_s^E &= H_{eik}(E_i > E) + H_s(\omega_i < E) \\ &\equiv H_{eik}^> + H_s^<. \end{aligned} \quad (3.17)$$

This means that hard active partons can be treated as external currents. While the reduction of $H_s^<$ to the eikonal form in the strong-ordering region is an IR leading result, the decomposition (3.17) is valid to all IR orders.

Secondly, one can prove that the gauge term $K(t)$ in (3.7) does not contribute, neither to $\Delta^{(1)}$ in (2.25), nor to Δ_C , i.e., the Coulomb interaction term defined previously⁵⁾. In other words, one has

$$\Delta_C(\nu) = \frac{i\pi}{\nu} [h^+(\nu), h^-(\nu)] =$$

$$= \frac{i\pi}{v} \left[h_{eik}^+(v), h_{eik}^-(v) \right] + \dots, \quad (3.18)$$

where matrix elements between physical states and subleading terms are understood.

A final remark concerns the explicit expansion of the leading-order coherent state operator $U_{eik}(A)$ and its gauge-transformation properties. By replacing (3.12) into the definition (3.1), one finds^{5),6)} in this case the factorized expression

$$\left[U_{eik}(\underline{p}_1, \dots, \underline{p}_n) \right]_{\underline{\beta}, \underline{\alpha}} = \prod_{i=1}^n U_{\underline{\beta}_i \underline{\alpha}_i}^E(\underline{p}_i), \quad (3.19)$$

where the single parton coherent state is defined by

$$d_{\underline{\alpha}}^E(\underline{p}) = \Omega_{eik}^{E\dagger} d_{\underline{\alpha}}(\underline{p}) \Omega_{eik}^E = U_{\underline{\alpha}\underline{\beta}}^E(\underline{p}) d_{\underline{\beta}}(\underline{p}), \quad (3.20)$$

and is given by

$$U^E(\underline{p}) = P_{\omega} \exp \left[ig \int_{\lambda}^E d[q] \frac{p^{\mu}}{p \cdot q} \pi_{\mu}^{\omega q}(q) \right] \quad (3.21)$$

in terms of the "dressed" gluon field

$$i \pi_{\mu a}^{\omega}(q) = \left[U^{\omega}(q) \right]_{ab} \left(A_{\mu b}(q) - A_{\mu b}^{\dagger}(q) \right). \quad (3.22)$$

Without analyzing here Eq. (3.21) [whose phenomenological and theoretical content is discussed elsewhere^{6),10)}], let us note the BRST transformation properties of the dressed fields (3.20). From Eq. (3.13), we obtain

$$\left[Q_0, d_{\underline{\alpha}}^E \right] = \Omega_{eik}^{E\dagger} \left[Q_0 + Q_{1,eik}, d_{\underline{\alpha}} \right] \Omega_{eik}^E, \quad (3.23)$$

where on the left-hand side Q_0 generates the free BRST transformation, and $Q_{1,eik}$ the non-Abelian rotation. Therefore, by (3.20), we obtain

$$[Q_0, U_{eik}^E(\underline{p})] = -g \int d[q] \sum_{\sigma} \underline{c}_{\sigma}^{\omega_q} \cdot \underline{t} \quad U_{eik}^E(\underline{p}), \quad (3.24)$$

where $\underline{c}^{\omega}(\underline{q})$ is the dressed ghost field: in other words, $U_{eik}^E(\underline{p})$ changes by a $\underline{p}$ -independent colour rotation.

It is interesting to check, by means of (3.12) and (3.24), that the S-matrix (2.21) is invariant, while the coherent state operator is only invariant when the hard partons are in an overall colour singlet. This follows from (3.24) because of the factorization property (3.19), or directly from

$$Q_0 \Omega_{eik}^{E\dagger} |p_1 \dots p_n\rangle = \Omega_{eik}^{E\dagger} (Q_0 + Q_{1,eik}) |p_1 \dots p_n\rangle = 0 \quad (3.25)$$

by the physicality and the colour-singlet conditions. We shall see in Section 4 how this result generalizes to subleading terms.

4. - GAUGE COVARIANCE AND SUBLEADING TERMS

Let us now come to the discussion of the BRST transformation properties of the coherent states in general. For this purpose, we need the reduction of the commutation relation (2.16) to the soft Möller operators $\Omega_{\pm}^E$, and the general form of the latter.

The simplest way to find the BRST charge for the asymptotic Hamiltonian

$$H_{as} = H_0 + H_s^E \quad (4.1)$$

is to start from the decomposition (3.17)

$$H_s^E(t) = H_{eik}^>(t) + H_s^<(t), \quad (4.1')$$

where hard partons act as external sources, coupled with soft ones with eikonal vertices. We then find

$$Q_{B,as} = Q_0 + g \, Q(E) \cdot \int d[q] \sum_{\sigma} \underline{c}_{\sigma}^{\omega}(\underline{q}) + Q_{1,s}, \quad (4.2)$$

where the first term is the free charge (2.15a), the second one is the contribution to Q_1 of the eikonal interaction of hard partons [proportional to their colour charge $Q(E)$], and the third is a purely soft term, which by (2.15b) is defined by

$$Q_{1s} = ig f_{a_1 a_2 a_3} \sum_{\sigma_i}^E \int d[q_1] d[q_2] d[q_3] (2\pi)^3 \delta^3(\sum \sigma_i q_i) \cdot \\ \cdot \sigma_1 c^{a_3 \sigma_3}(q_3) \left[A_0^{a_1 \sigma_1}(q_1) q_1 \cdot A^{a_2 \sigma_2}(q_2) - \omega_1 A_\mu^{a_1 \sigma_1} A_{a_2 \sigma_2}^\mu - \frac{1}{2} \omega_1 \bar{c}^{a_1 \sigma_1} c^{a_2 \sigma_2} \right]. \quad (4.3)$$

We can rewrite (4.2) in a slightly different form, by isolating the eikonal contribution to Q_{1s} , i.e.,

$$Q_{B,as} = Q_0 + Q_{1,eik} + Q'_{1s}, \\ Q_{1,eik} = \int d[q] \sum_{\sigma}^E Q(\omega_q) \sum_{\sigma} c^\sigma(q), \quad (4.4)$$

where $Q_{1,eik}$ is now the same as in Eq. (3.13) and Q'_{1s} is defined as in (4.3), but with subtraction of the eikonal part, coming from the strong-ordering region $\omega_1 \approx \omega_2 \gg \omega_3$.

In conclusion, the soft Möller operators satisfy the commutation relation

$$Q_0 \Omega_\pm^{E\dagger} = \Omega_\pm^{E\dagger} (Q_0 + Q_{1s} + g Q(E) \int d[q] \sum_{\sigma} c^\sigma(q)) \\ = \Omega_\pm^{E\dagger} (Q_0 + Q'_{1s} + Q_{1,eik}), \quad (4.5)$$

which replaces Eq. (2.16) and is the starting point of our discussion. To leading order, by setting $\Omega_\pm^E = \Omega_{eik}$, $Q'_{1s} = 0$, Eq. (4.5) reduces to Eq. (3.13).

Let us first use Eq. (4.5) as a Ward identity constraint on IR subleading terms in $\Omega_\pm^E$. We shall see that its validity implies the existence of partonic correlations of Bloch-Nordsieck-violating type.

We can write in general⁶⁾

$$\Omega_{\pm}^E = \Omega_{\text{eff}}^E V, \quad (a)$$

$$V^\dagger d_\alpha V = d_\alpha + \int_{\lambda} d\nu \left[d_\alpha, \Delta_{\text{eff}}^{(2)}(\nu) + \Delta_{\text{eff}}^{(3)}(\nu) + \dots \right], \quad (b) \quad (4.6)$$

where the interesting term (giving IR singular parton-parton correlations) is given by

$$\int d\nu \Delta_{\text{eff}}^{(3)}(\nu) \Big|_{\text{ppA}} = \frac{1}{2} i \{a_1 a_2 a\} \int d[p_1] d[p_2] d[q] \cdot \{ \rho^{a_1}(p_1), \rho^{a_2}(p_2) \} \sum_{\sigma} A_{\rho}^{\sigma a}(q) G_{\sigma}^{\rho}(p_1, p_2, q), \quad (4.7)$$

and the current $G_{\sigma}^{\mu} = G_{f\sigma}^{\mu} + G_{g\sigma}^{\mu}$ has a fermionic part

$$G_f^{\mu\rho} = \frac{1}{2} g^3 \int \frac{d^4 q_1}{(2\pi)^3} \frac{\delta(p, q_1) (p_1 p_2)}{q_1^2 [p_2(q_1+q) + i\epsilon\sigma]} \frac{p_2^{\mu}}{p_2 \cdot q} - (1 \leftrightarrow 2) \quad (4.8a)$$

and a gluonic part

$$G_g^{\mu\rho} = -\frac{1}{2} g^3 \int \frac{d^4 q_1}{(2\pi)^3} \frac{d^4 q_2}{(2\pi)^3} \frac{\delta(p, q_1) \delta(p_2, q_2)}{q_1^2 q_2^2} (2\pi)^3 \delta^3(q_1 + q_2 + q) \cdot p_1 p_2 p_2 p_2 \Gamma^{\mu\mu_2\mu} (q_1, q_2, q) (\omega + q_1^0 + q_2^0 + i\epsilon\sigma)^{-1}. \quad (4.8b)$$

It is straightforward to check that G_g^{μ} is not conserved, being

$$G^{\sigma} \cdot q = G_g^{\sigma} \cdot q = -\frac{1}{2} g^3 \int \frac{d^4 q_1 d^4 q_2}{(2\pi)^3 q_1^2 q_2^2} \delta(p, q_1) \delta(p_2, q_2) \cdot \delta^3(q_1 + q_2 + q) [p_2 \cdot q_1 p_{10} - q_{10} p_1 \cdot p_2 - (1 \leftrightarrow 2)]. \quad (4.9)$$

Therefore, the gauge transformation property of V in (4.6) is non-trivial and should be explained on the basis of Eq. (4.5).

In fact, by replacing (4.6a) into (4.5) we get, by (3.13),

$$V^\dagger Q_0 V + \Omega_\pm^{E\dagger} Q'_1 \Omega_\pm^E = Q_0 \quad (4.10)$$

and therefore, by (4.6b)

$$\int dv \left[Q_0, \Delta_{\text{eff}}^{(2)}(v) \right] = - \int dv \left[Q'_{1s}, \Delta_{\text{eik}}(v) \right], \quad (a)$$

(4.11)

$$\int dv \left[Q_0, \Delta_{\text{eff}}^{(3)}(v) \right] = - \frac{1}{2} \int dv_1 dv_2 \left[\left[Q'_{1s}, \Delta_{\text{eik}}(v_1) \right], \Delta_{\text{eik}}(v_2) \right]. \quad (b)$$

In other words, the non-conserved parts of the currents occurring in $\Delta_{\text{eff}}^{(i)}$ (generated by Q_0) are related to multiple commutators of Q'_{1s} with Δ_{eik} .

In particular, by (3.10), (4.3) and (4.7), the ppA part of the right-hand side of Eq. (4.11b) is given by

$$i g^3 \{ a_1 a_2 a_3 \} \sum_{\sigma_i} \int d[p_1] d[p_2] \{ p^{a_1}, p^{a_2} \} \int^E d[q_1] \dots d[q_3] \cdot$$

$$(2\pi)^3 \delta^3(\sum \sigma_i q_i) \cdot \frac{\sigma_1 (\hat{p}_2 q_1 - \omega, \hat{q}_1 \cdot \hat{q}_2)}{(\hat{p}_1 q_1)(\hat{p}_2 q_2)} c^{a_3 \sigma_3}(q_3), \quad (4.12)$$

and coincides with the left-hand side, after doing the q_{10} and q_{20} integrations in Eq. (4.9).

The meaning of this check of Eq. (4.11b) is better seen from the explicit form of the coherent state operator defined in (3.1). By separating $G_\mu^\pm$ into the antisymmetric (Coulomb) part G_μ^A and the symmetric one G_μ^S , we obtain⁷⁾

$$U_\pm^E(\underline{p}_1 \dots \underline{p}_n) = \prod_i U_{\text{eik}}(\underline{p}_i) \cdot U_\pm^C(\underline{p}_1 \dots \underline{p}_n) \cdot V(\underline{p}_1 \dots \underline{p}_n), \quad (4.13a)$$

where

$$V(\underline{p}_1 \dots \underline{p}_n) = 1 + i \sum_{i,j} g^3 f_{abc} t_{ia} t_{jb} \cdot$$

$$\cdot \int d[q] G_S^\mu(\underline{p}_i, \underline{p}_j, \underline{q}) (A_{\mu c}(\underline{q}) + A_{\mu c}^\dagger(\underline{q})) \quad (4.13b)$$

depends on $A+A^+$ (and G_S^μ), and is therefore the Bloch-Nordsieck-violating term⁶⁾. Equations (4.11b) and (4.12) then show that the non-conserved part of G_S^μ in (4.9) is non-vanishing by BRST invariance.

It should be remarked that, although this argument proves that $G_S^\mu \neq 0$, it does not predict the explicit magnitude of Bloch-Nordsieck violation in Drell-Yan processes¹⁵⁾. An explicit evaluation (Appendix B) shows that

$$G_S^\mu = \frac{1}{8} g^3 \left(\frac{1}{v_{12}} - 1 \right) j_{12}^\mu + G'^\mu + G''^\mu, \quad (4.14)$$

where $v_{12} = [1 - p_1^2 p_2^2 / (p_1 p_2)^2]^{\frac{1}{2}}$, $j_{12}^\mu = p_1^\mu / p_2 \cdot q - p_2^\mu / p_1 \cdot q$ and

$$\begin{aligned} G'^\mu &= \frac{1}{8} g^3 \sqrt{p_1^2} n_2^\mu \left[\left(\frac{p_1 p_2 + \sqrt{p_1^2 p_2^2}}{2(p_1 q)(p_2 q)} \right)^{1/2} - \frac{\sqrt{p_1^2}}{p_1 q} \right] - (1 \leftrightarrow 2), \\ G''^\mu &= \frac{1}{8} g^3 \frac{p_1 p_2}{(p_1 q)(p_2 q)} \left(\frac{p_1 q}{p_1 p_2} \right)^2 \frac{p_2^2 q_1^\mu}{q_1^2 v_{12}^2} - (1 \leftrightarrow 2), \\ n_2^\mu &= v_{12}^{-2} \left(\frac{p_2^\mu}{p_1 p_2} - \frac{p_2^2 p_1^\mu}{(p_1 p_2)^2} \right), \end{aligned} \quad (4.15)$$

are components of G_{gS}^μ respectively in the plane (p_1, p_2) and out of it, which vanish in the massless limit.

Since Bloch-Nordsieck violation involves the interference $G_S^\mu \cdot j_{12}$ with the leading current j_{12} , neither G'_μ nor G''_μ contribute to it (after g -integration), the full result coming from the first term in (4.14). Both instead contribute to the non-conserved part

$$G_S \cdot q = G_{gS} \cdot q = \frac{1}{8} g^3 \frac{\sqrt{p_1^2} p_2 q - \sqrt{p_2^2} p_1 q}{(p_1 p_2) - \sqrt{p_1^2 p_2^2}} \left(\frac{(p_1 p_2) + \sqrt{p_1^2 p_2^2}}{2(p_1 q)(p_2 q)} \right)^{1/2}, \quad (4.16)$$

which agrees with (4.9). It is conceivable that (4.16) will instead contribute to higher orders, and to more complicated processes.

Let us now come back to Eq. (4.5), in order to discuss the gauge-dependence of the asymptotic coherent-state basis. We shall slightly change the definition (1.4), which does not leave the vacuum state invariant. Following (3.1) we shall rather define

$$\begin{aligned} |p_1 \alpha_1, \dots, p_n \alpha_n; \pm\rangle &= \Omega_{\pm}^{E\dagger} U_{\pm}^s |p_1 \alpha_1, \dots, p_n \alpha_n\rangle_h \\ &= \sum_{\underline{\beta}} [U^{E\dagger}(p_1, \dots, p_n)]_{\underline{\beta}, \underline{\alpha}} |p_1 \beta_1, \dots, p_n \beta_n\rangle_h, \end{aligned} \quad (4.17)$$

by factorizing the soft operator

$$U_{\pm}^s = \langle 0 | \Omega_{\pm}^E | 0 \rangle_h = T \exp \left[-i \int_{\mp\infty}^0 H_s^E(t) dt \right], \quad (4.18)$$

which gives the action of $\Omega_{\pm}^E$ on the hard vacuum state. This redefinition is acceptable, because it changes the hard S-matrix elements by multiplication with the vacuum fluctuation $\langle 0 | U_{\pm}^{s\dagger} U_{\pm}^s | 0 \rangle$, which is IR finite.

The states (4.17) have a simple gauge transformation property. In fact, by the definition (4.18) we have, as in (2.16),

$$(\mathcal{Q}_0 + \mathcal{Q}_{1s}) U_{\pm}^s = U_{\pm}^s \mathcal{Q}_0, \quad (4.19)$$

and therefore, by replacing (4.5) into (4.17), we get the final result

$$\begin{aligned} \mathcal{Q}_0 |h, \pm\rangle &= \Omega_{\pm}^{E\dagger} U_{\pm}^s \cdot \\ &\cdot \left(\mathcal{Q}_0 + \mathcal{Q}(E) \cdot \int d[q] \sum_{\underline{\beta}} \underline{c}^{\beta}(\underline{q}) \right) |h\rangle, \end{aligned} \quad (4.20)$$

where $|h, \pm\rangle$ belongs to the Fock space spanned by (4.17).

Equation (4.20) shows that, for the asymptotic state $|h, \pm\rangle$ to be BRST invariant, it is not sufficient that the base state $|h\rangle$ be physical. We should naturally require that $|h\rangle$ be colour-singlet too.

Note that the result (4.20) is a direct generalization of the one in QED¹⁹⁾. One may then wonder whether, similarly to QED, it would be possible to modify further the definition of the states (4.17) in the charged sector, so as to have both a vanishing charge (4.20) and a finite S-matrix. This is certainly an open question; but we are inclined to think that it is impossible, just because of the non-factorizable IR singularities of type (4.13) which are present in the states (4.17).

To sum up, we have shown in this paper that the Hamiltonian formulation in covariant gauges is a perfectly viable approach to coherent states, which leaves unchanged the nice features (1)-(4) in the Introduction.

Furthermore, we have exhibited [Eq. (4.11)] the interesting relation between BRST invariance at subleading level and the existence of singular parton correlations of Bloch-Nordsieck-violating type [which only disappears between colourless clusters⁷⁾]. We have also argued that asymptotic coherent states which are BRST-invariant must be in an overall colour singlet [Eq. (4.20)].

Both questions have to do with the existence of an asymptotic partonic theory. Within the limitations of perturbation theory, the results above are consistent with the confinement assumption. However, we cannot rigorously exclude, at the moment, the fact that a legal redefinition of the states (4.17) could lead to a vanishing (4.20) in the charged sector too.

APPENDIX A

EIKONAL APPROXIMATION FOR HARD PARTONS AND COULOMB TERM

Here we want to prove two results quoted in Section 3: that hard partons are described by eikonal vertices to any IR order, and that the Coulomb term (3.18) takes its eikonal form to leading order. In both cases we have to show that gauge operators do not contribute.

In dealing with hard partons, let us rewrite the decomposition (3.4) by including in H_{eik} and K the interaction between hard ($E_p > E$) and soft ($\omega_i < E$) partons only, i.e.,

$$H_s^E(t) = H_{\text{eik}}^>(t) + K^>(t) + H_s^<(t), \quad (\text{A.1})$$

where the first two terms are defined as in Eqs. (3.5) and (3.7), but with the constraint $E_p > E > \omega_q$, while $H_s^<(t)$ involves soft-parton self-couplings only.

It follows that $K^>$ is a gauge operator in the sense that

$$K^>(t) = \{ Q_{\text{oh}}, O^>(t) \}, \quad (\text{A.2})$$

where $O^>$ is defined as in Eq. (3.9) with $E_p > E > \omega_q$ and Q_{oh} is the restriction of Q_0 to $\mathcal{H}_h$, i.e.,

$$Q_{\text{oh}} = - \sum_{\sigma} \int d[p] p \cdot A_{\alpha}^{\sigma}(p) c_{\alpha}^{-\sigma}(p) \Theta(E_p - E). \quad (\text{A.3})$$

Note that, since Q_{oh} acts only on hard operators in $O^>$, Eq. (A.2) is identically true, that is does not contain the IR subleading terms of Eq. (3.8). By then using the transformation properties (3.10) and $Q_{\text{oh}}^2 = 0$, we get from (A.1) and (A.2) the Ward identities

$$[Q_{\text{oh}}, H_s^E(t)] = 0 = [Q_{\text{oh}}, \Omega_{\pm}^E] \quad (\text{A.4})$$

which mean that the soft Möller operators map the physical sector of $\mathcal{H}_h$ into itself.

We can now show that $K^>$ does not contribute to the IR singularities of the S-matrix in the physical sector. In fact, a typical contribution to the time integrals would be

$$\begin{aligned} & \Omega_-^{E\dagger} S_h^E U_s(0,t) \{Q_{oh}, O^>(t)\} U_s(t,0) \Omega_+^E \\ & = \left\{ Q_{oh}, \Omega_-^{E\dagger} S_h^E U_s(0,t) O^>(t) U_s(t,0) \Omega_+^E \right\}, \end{aligned} \quad (A.5)$$

where we have used (A.4) and the property $[Q_{oh}, S_h^E] = 0$. It follows that (A.5) is a pure gauge term and drops out from matrix elements between hard physical states.

The argument in (A.5) is easily generalized to terms which are non-linear in $K^>$, due to Eq. (A.4). We can conclude that $K^>$ can be incorporated in H_h^E so that the soft Hamiltonian takes the form (3.17).

Let us now discuss the structure of Coulomb-like terms. From Section 3 we know that the IR leading dynamics is given by the soft Hamiltonian

$$H_s^E(t) = H_{eik}(t) + K(t) + \dots \quad (A.6)$$

The operator $\Omega^{(1)} \approx Q_{eik}$ contributes to emission probabilities one IR and one collinear logarithm for each power of α_s . Other IR leading contributions without collinear logarithms come^{3),5)} from the $\delta(\sigma_1 v_1 + \sigma v)$ term in $\Delta^{(2)}$ of Eq. (2.25) that we call the Coulomb term, i.e.,

$$\Delta_{\pm}^{(2)}(v) \rightarrow \pm \Delta^c(v) = \pm \frac{i\pi}{v} [h^+(v), h^-(v)]. \quad (A.7)$$

We want to show that $h^{\pm}(v)$ in (A.7) can be replaced by its eikonal form, apart from non-leading terms. In fact, by replacing (A.6) in (A.7), we get

$$\begin{aligned} \Delta^c(v) &= -\frac{i\pi}{v} g^2 \int d[p_1] d[p_2] \delta(v - \hat{p}_1 \cdot q) \delta((\hat{p}_1 - \hat{p}_2) \cdot q) \\ &\cdot \left(\rho^a(\underline{p}_1) \hat{p}_1^\mu + J_K^{a\mu}(\underline{p}_1) \right) \left(\rho^a(\underline{p}_2) \hat{p}_2^\mu + J_{K\mu}^a(\underline{p}_2) \right), \end{aligned} \quad (A.8)$$

where

$$J_k^{a\mu} = -\frac{1}{2E_p} f_{abc} \left\{ Q_0, A_b^{\mu\dagger}(\underline{p}) \bar{c}_c(\underline{p}) \right\} + h.c. \quad (A.9)$$

is a gauge operator. We can then decompose (A.8) in the form

$$\Delta^C(\nu) = \Delta_{eik}^C(\nu) + \Delta_K^C(\nu), \quad (A.10)$$

where

$$\begin{aligned} \Delta_{eik}^C(\nu) &= \frac{i\pi}{\nu} \left[h_{eik}^+(\nu), h_{eik}^-(\nu) \right] \\ &= -\frac{i\alpha_s}{2\nu} \int d[p_1] d[p_2] \frac{(p_1 \cdot p_2) \underline{p}(p_1) \cdot \underline{p}(p_2)}{[(p_1 \cdot p_2)^2 - p_1^2 p_2^2]^{1/2}}, \end{aligned} \quad (A.11)$$

and Δ_K^C is the remaining part, which by (A.9) contains at least a gauge operator factor.

We can now repeat the argument of Section 3. Since $[Q_0, \Delta^C] = 0$, (because Q_0 commutes with $\underline{p}$ and J_k^μ), the commutator $[Q_0, \Delta^{(1)\pm\Delta^C}] = [Q_0, \Delta_{eik}]$ is still given by (3.11') and therefore the evolution operator

$$\Omega_\pm^0 = P_\nu \exp \left[\int_\lambda^\nu d\nu \left(\Delta_{eik}(\nu) \pm \Delta_{eik}^C(\nu) \right) \right] \quad (A.12)$$

satisfies the same commutator relation (3.13) as Ω_{eik} . The elimination of the gauge terms follows as in Eqs. (3.15) and (3.16).

APPENDIX B

INTEGRALS FOR THE BLOCH-NORDSIECK-VIOLATING CURRENT

We want to prove here the explicit expressions of $G_S^\mu = G^{\mu+} + G^{\mu-}$ in Eqs. (4.8) and (4.9).

The fermionic integral in (4.8a) is easily evaluated in the collinear frame $p_1^\mu = (1, 0, v_1)$, $p_2^\mu = (1, 0, -v_2)$ to yield

$$G_{fs}^\mu = -g^3 \frac{p_2^\mu}{p_2 q} \int \frac{d^3 q_\perp}{(2\pi)^3} \frac{\hat{p}_1 \hat{p}_2}{q_{1\perp}^2 + \hat{p}_1^2 q_{13}^2} P(\hat{p}_2 q + (v_1 + v_2) q_{13})^{-1} -$$

$$- (p_1 \leftrightarrow p_2) = g^3 j_{12}^\mu / 8 v_{12} , \quad (B.1)$$

by complex integration in the q_{13} variable.

The gluonic integral in (4.8b) (which is not apparently Lorentz-covariant) is more complicated and will eventually be evaluated in the symmetric frame $v_1 = v_2$. By doing the δ -function integrations in the collinear frame, and the $q_{1\perp}$ integration à la Feynman, we obtain

$$G_{gs}^\mu = \frac{g^3}{8\pi} \int_0^1 \frac{dx}{\sqrt{x(1-x)}} \left[q_\perp^2 (x \hat{p}_2^2 + (1-x) \hat{p}_1^2) + q_\perp^2 \hat{p}_1^2 \hat{p}_2^2 \right]^{-1/2} \cdot$$

$$\cdot \left[b^\mu + \frac{(a^\mu + (2x-1) \hat{p}_1 \hat{p}_2 q_\perp^\mu) ((1-x) \hat{p}_1^2 \hat{p}_2 q + x \hat{p}_2^2 \hat{p}_1 q)}{x(1-x) q_\perp^2 (v_1 + v_2)^2 + x \hat{p}_2^2 (\hat{p}_1 q)^2 + (1-x) \hat{p}_1^2 (\hat{p}_2 q)^2} \right] , \quad (B.2)$$

where we have introduced the notation

$$a^\mu = 2 \hat{p}_1 q \hat{p}_2^\mu + \hat{p}_1 \hat{p}_2 \hat{p}_2 q n_1^\mu - (1 \leftrightarrow 2) ,$$

$$b^\mu = \hat{p}_1^2 n_2^\mu - \hat{p}_2^2 n_1^\mu , \quad (B.3)$$

and the vectors n_i^μ , with

$$n_{1\mu} = (v_1 + v_2)^{-1} (v_1, 0, 1) = \frac{1}{v_{12}} \left(\frac{\hat{p}_{1\mu}}{\hat{p}_1 \cdot \hat{p}_2} - \frac{\hat{p}_1^2 \hat{p}_{2\mu}}{(\hat{p}_1 \cdot \hat{p}_2)^2} \right) ,$$

$$\hat{p}_1 n_1 = \hat{p}_2 n_2 = 0, \quad \hat{p}_1 n_2 = \hat{p}_2 n_1 = 1. \quad (\text{B.4})$$

We are able to do the integration (B.2) for $v_1 = v_2$ only, when some x -dependence disappears, and it becomes of the form

$$\begin{aligned} & (q_1^2 + \hat{p}^2 q_3^2)^{-1/2} \cdot \int_0^1 dx (x(1-x))^{-1/2} \left[b^\mu / \sqrt{\hat{p}^2} + \right. \\ & \left. + \frac{\alpha^\mu + \hat{p}_1 \hat{p}_2 (2x-1) q_1^\mu}{|q_1| (v_1 + v_2)} \cdot \frac{x \sqrt{\alpha_2} + (1-x) \sqrt{\alpha_1}}{(x_1 - x)(x - x_2)} \right], \end{aligned} \quad (\text{B.5})$$

where

$$\begin{aligned} \alpha_1 &= \hat{p}_1^2 (\hat{p}_2 q)^2 (4 q_1^2 v^2)^{-1}, \quad \alpha_2 = \hat{p}_2^2 (\hat{p}_1 q)^2 (4 q_1^2 v^2)^{-1}, \\ x_{1,2} &= \frac{1}{2} (1 + \alpha_2 - \alpha_1) \pm \frac{1}{2} [1 + 2(\alpha_1 + \alpha_2) + (\alpha_1 - \alpha_2)^2]^{1/2}. \end{aligned} \quad (\text{B.6})$$

The integral (B.5) can be performed by complex integration, because the poles $x_{1,2}$ lie outside the interval $0 < x < 1$. After some algebra, based on the definitions (B.3)-(B.6), we obtain the expression

$$G_{qs}^\mu = -\frac{1}{8} q^3 j_{12}^\mu + G'^\mu + G''^\mu, \quad (\text{B.7})$$

which, by adding (B.1), reduces to Eq. (4.14).

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