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GEOMETRICAL HIERARCHY AND SPONTANEOUS SYMMETRY BREAKING

K. Farakos
Physics Dept., Nat. Tech. Un., Athens

G. Koutsoumbas*
CERN - Geneva

M. Surridge
Physics Dept., Amsterdam Un., Amsterdam

and G. Zoupanos
Physics Dept., Nat. Tech. Un., Athens

A B S T R A C T

A four-dimensional gauge theory, where Higgs fields and the corresponding potential appear naturally, is obtained by dimensionally reducing a pure gauge theory over a compact coset space S/R . We show, using an explicit example, that a hierarchy of the scales in the coset-space can change the spontaneous symmetry-breaking of the four-dimensional gauge theory.

*) On leave from Physics Dept., Nat. Tech. Un., Athens, Greece

The present popular version of the old idea of unification of all interactions, is that such a unification is realized in higher dimensions. The most ambitious program in this direction is based on generalizations of the old Kaluza-Klein idea⁽¹⁾ while many new ideas have been introduced with the development of the superstring theory⁽²⁾. An important step, from the particle physics point of view, is to obtain a realistic theory in four dimensions and eventually to obtain in low energies the standard model. In fact, there exists an interesting program, the Coset-Space Dimensional Reduction (C.S.D.R.), which permits a detailed examination of the various features of the four dimensional theory such as its spontaneous breaking, fermionic spectrum etc. The C.S.D.R. scheme is based on the observation that by imposing symmetry conditions on a Yang Mills theory one can obtain a Yang-Mills-Higgs theory in lower dimensions⁽³⁾. There exists already a considerable amount of work⁽⁴⁻¹⁶⁾ on the C.S.D.R. scheme that starts from the fundamental work of Forgacs and Manton⁽⁴⁾ and includes attempts to embed the C.S.D.R. scheme in the superstring program⁽¹⁵⁾.

Here we shall consider the Higgs potential of the four dimensional effective gauge theory, which is obtained from a pure Yang - Mills theory through C.S.D.R. In particular we shall demonstrate a novel feature, namely that a change of the scales in the coset space, when possible, can change the

pattern of the spontaneous symmetry breaking of the four dimensional gauge theory.

Let us briefly recall the dimensional reduction procedure using symmetric gauge fields⁽⁴⁾. One starts with a pure field theory with gauge group G defined on an extended space-time manifold M which is assumed to be a direct product:

$$M = M_4 \times S/R \tag{1}$$

where M_4 is a four dimensional Minkowski space-time and S/R is a compact coset space. In general S is assumed to be a compact semi-simple Lie group and R is a Lie subgroup of S . The gauge group is also assumed to be compact and simple. One assumes that the metric of M is block diagonal.

One defines symmetric gauge fields by demanding that the action of S on the gauge field A_μ is compensated by a gauge transformation. Given a coset space S/R , it is possible to determine explicitly both the compensating gauge transformation and the dependence of A_μ on the coset space coordinates. When A_μ is inserted into the Yang-Mills Lagrangian on M , one can integrate trivially over the coset space coordinates and obtain a Yang-Mills-Higgs theory in four dimensions. The four-dimensional theory has the following characteristic features: (i) The gauge group, which we denote by H , is the centralizer in G of a subgroup R_G (isomorphic to R) of G . (ii) The Higgs fields can be determined, as well as the Higgs potential.

Formally, the Higgs potential takes the form:

$$V(\Phi) \propto \kappa^{rr'} \kappa^{ss'} \text{Tr}((f_{rst} \Phi_t - (\Phi_s, \Phi_r))(f_{r's't'} \Phi_{t'} - (\Phi_{s'}, \Phi_{r'}))) \quad (2)$$

where $\Phi_s(x)$ is a scalar field taking values in the Lie algebra of the initial gauge group G , and κ is a matrix related to the metric tensor of the coset space. In component form, $\Phi_s(x) = \Phi_s^a(x) t^a$, where t^a are generators of G . The indices s, s', r, r' correspond to generators of S and f 's are the structure constants of S .

The above expression for $V(\Phi)$ is only formal because Φ_s must satisfy the constraints⁽⁴⁾

$$(\Phi_m, \Phi_s) - f_{rst} \Phi_t = 0 \quad (2')$$

where s takes all values, but the index m is restricted to values corresponding to generators of R . These constraints imply that some components Φ_s^a are zero, some are constants, and the rest can be identified with components of the genuine Higgs fields. One can easily find the genuine Higgs content of the dimensionally reduced theory. The gauge fields, which become scalars in four dimensions, transform under R as a vector as specified by the embedding:

$$\text{ad } S = \text{ad } R + v \quad (3)$$

and $v = \sum s_k$, where each s_k is an irreducible representation of R . Then the multiplets under H of independent Higgs fields can be found by decomposing the adjoint representation of G under $R_G \times H$:

$$\text{ad } G = \Sigma(r_k, h_k) \quad (4)$$

where r_k is some irreducible representation of R_G and h_k is some irreducible representation of H . For each pair of (r_k, s_j) , where r_k and s_j are identical irreducible representations, there is a Higgs multiplet h_k in the four dimensional theory. The rule given above for finding which Higgs multiplets occur in four dimensions is a consequence of solving eq. (2'). When the $V(\Phi)$ is expressed in terms of the unconstrained independent Higgs fields, it remains a quartic polynomial which is invariant under gauge transformations of the final gauge group H , and its minimum determines the vacuum expectation value of the Higgs fields.

The minimization of this potential remains in general a difficult problem even though the work which has been done and the methods that have been developed considerably facilitate the problem⁽¹⁶⁾. However, if S has an isomorphic image S_G within G which contains R_G in a consistent way, then it is possible to allow the Φ_s to become generators of S_G . In this case the potential $V(\Phi)$ is zero, which is clearly the absolute minimum value that it can take. These non-zero vacuum expectation values of the Higgs fields break the symmetry from H to K , where all elements of K commute with the generators $\langle \Phi_s \rangle$ of S_G . Thus one deduces that the final unbroken symmetry group K , is just the centralizer of S_G in G ⁽¹⁴⁾. Recently it was shown⁽¹²⁾ that the fermions of such

a theory become all very massive after spontaneous symmetry breaking. Therefore this attractive possibility is ruled out in searches to obtain the standard model from C.S.D.R. of a gauge group in higher dimensions. Thus we return to the original problem to derive the scalar potential of the four dimensional theory in terms of the genuine Higgs fields and to minimize it.

Our observation is that in the general case (when $S \neq G$) by changing the scales in the coset space, when it is possible, one can change the pattern of symmetry breaking of the gauge theory (including the case that the theory remains unbroken). On the other hand when SCG one cannot change the pattern of symmetry breaking. In the following we shall demonstrate our points in two examples. In the first example let us consider a pure Yang-Mills with $G = SU_5$ in ten dimensions, where the extra dimensions belong to the non-symmetric six-dimensional coset space $Sp_4/(SU_2 \times U_1)_{\text{non-max}}$. We are going to identify R_G as the $SU_2 \times U_1$ subgroup appearing in the decomposition

$$SU_5 \supset SU_3 \times SU_2 \times U_1$$

$$24 = (8,1)(0) + (1,3)(0) + (\bar{3},2)(1) + (3,2)(-1) + (1,1)(0) \quad (6)$$

From eq(6) it is obvious that the centralizer of $SU_2 \times U_1$ is $SU_3 \times U_1$, which according to the previously given rules is the gauge group of the effective theory in four dimensions. The decomposition of the adjoint of Sp_4 under $(SU_2 \times U_1)_{\text{non-max}}$ is:

$$Sp_4 \supset (SU_2 \times U_1)_{\text{non-max}} \quad (7)$$

$$10 = 3_0 + 1_{2,0,-2} + 2_{1,-1}$$

Therefore from eqs (6) and (7) and according to the rules we find that the genuine Higgs fields β appearing in the four-dimensional theory transform as a complex 3_{-1} of $H = SU_3 \times U_1$. Note that Sp_4 has no isomorphic image in SU_5 . The decomposition (6) suggests the introduction of the following generators of SU_5 :

$$Q_{SU_5} = \{Q^c(0), Q^i(0), Q_{ac}(1), Q^{ac}(-1), Q(0)\} \quad (8)$$

where $\rho = 1, \dots, 8$; $i = 1, 2, 3$; $a = 1, 2, 3$; $c = 1, 2$. The generators Q_{SU_5} are in full correspondence with the decomposition (6). Their commutation relations are given in the appendix 1 of ref. (16). For the construction of the Higgs potential we shall also need to introduce the following generators of Sp_4 according to the decomposition (7):

$$T_{Sp_4} \equiv \{T^i(0), T_+(2), T_0(0), T_-(-2), T_-^c(-1), T_{c+}(1)\} \quad (9)$$

where $i = 1, 2, 3$; $c = 1, 2$. Their commutation relations can be found in appendix 6 of ref. (16).

In order to express the formal Higgs potential of eq. (2) in terms of the genuine Higgs fields, we need explicit expressions for the scalar fields Φ_s . Again the decomposition (7) suggests the following change in the notation of the scalar fields Φ_s :

$$\{\Phi_s: s = 1, \dots, 10\} \rightarrow \{\Phi, \Phi_+, \Phi_0, \Phi_-, \Phi_-^c, \Phi_{c+}\} \quad (10)$$

Then the solution of the constraints (3) in terms of the genuine Higgs field is:

$$\Phi^i = Q^i; \quad \Phi_{c+} = \beta^a(-1)Q_{ac}(1); \quad \Phi_+ = 0 \quad (11)$$

$$\Phi_0 = \frac{3}{5} Q; \quad \Phi_-^c = \beta_a(1)Q^{ac}(-1); \quad \Phi_- = 0$$

Finally let us consider the matrix κ in eq (2). Since the constraints (3) have to be satisfied the indices r, r', s, s' in eq. (2) correspond to generators of S which are not generators of R . In the new notation introduced in eq. (10) the constraints (3) exclude four values (corresponding to $T_i(0)$ and $T_0(0)$). Therefore the matrix κ is 6×6 and it is the metric tensor of the coset space $Sp_4/SU_2 \times U_1$. $SU_2 \times U_1$ invariance requires that κ in the notation of eq. (9) take the form:

$$\kappa = \begin{array}{ccccc|cc} & T_{1+} & T_{2+} & T_-^1 & T_-^2 & T_- & T_+ & \\ \left. \begin{array}{c} 0 \\ 0 \\ \bar{R}_1^2 \\ 0 \\ 0 \\ 0 \end{array} \right\} & \begin{array}{c} 0 \\ 0 \\ 0 \\ \bar{R}_1^2 \\ 0 \\ 0 \end{array} & \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} & \begin{array}{c} \bar{R}_1^2 \\ \bar{R}_1^2 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} & \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \bar{R}_2^2 \end{array} & \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \bar{R}_2^2 \end{array} & \begin{array}{c} T_{1+} \\ T_{2+} \\ T_-^1 \\ T_-^2 \\ T_- \\ T_+ \end{array} & (12) \end{array}$$

where R_1, R_2 are in principle arbitrary radius parameters.

In order for the kinetic term of the genuine Higgs fields β to have ordinary form we have to make the redefinition

$$\beta' = \frac{\sqrt{2}}{gR_1} \beta \quad (13)$$

where g is the coupling constant of the resulting four dimensional theory.

Now we can rewrite the potential (2), taking into account the solution of the constraints (11), as

$$\begin{aligned} \frac{1}{g^2} V(\Phi) = & \frac{2}{(R_1)^4} \text{Tr}(\Phi_i^\dagger \Phi_i + \Phi_0^2) + \frac{2}{R_2^4} \text{Tr} \Phi_0^2 + \\ & + \frac{4}{R_1^4} \left(\frac{1}{R_2^2} - \frac{1}{R_1^2} \right) \text{Tr}(\Phi_-^c \Phi_{c+}) \\ & + \frac{2}{R_1^4} \text{Tr} \{ (\Phi_{c+}, \Phi_{d+}) (\Phi_-^d, \Phi_-^c) + (\Phi_{c+}, \Phi_-^d) (\Phi_{d+}, \Phi_-^c) \} \end{aligned} \quad (14)$$

Expressing the Φ 's in eq. (14) in terms of the genuine Higgs β according to eq. (11) and using the redefinition (13) we obtain:

$$\begin{aligned} \frac{1}{g^2} V(\beta') = & \frac{1}{g^2} \left(\frac{3}{R_1^4} + \frac{2}{R_2^4} \right) \frac{6}{5} - 4 \left(\frac{2}{R_1^2} - \frac{1}{R_2^2} \right) (\beta'^+ \beta') \\ & + 3g^2 (\beta'^+ \beta')^2 \end{aligned} \quad (15)$$

It is clear from the mass term in the potential that the theory

will remain unbroken in the vacuum, if $R_2 < R_1/\sqrt{2}$ and will break to $SU_2 \times U_1$ if $R_2 > R_1/\sqrt{2}$. It is also clear that such choices of R_1 and R_2 , since they hold for a huge range of the parameters, are natural.

In contrast to the previous case, when S has an isomorphic image S_G within G , as we have already mentioned it is expected^(14,5) that the four dimensional gauge group G breaks into $K = C_G(S_G)$. This result should hold even when one introduces different radii. The reason is that the potential (2) being positive definite takes its minimum value zero when

$$\langle \phi_S \rangle \equiv \langle \phi_S^a \rangle \cdot Q_a = Q_S = Q_G \quad (16)$$

The latter is due to the fact that each term vanishes independently from its proportionality constant which contains the radii.

Let us demonstrate the above result in an explicit example. Consider the same $G = SU_5$ and the same coset-space S/R $S/R = Sp_4/(SU_2 \times U_1)_{\text{non-max}}$. However the Sp_4 is now embedded in SU_5 as follows

$$SU_5 \supset Sp_4 \quad (17)$$

Then the $R = SU_2 \times U_1$ can be obtained from the following decomposition

$$SU_5 \supset SU_4 \times U_1 \supset (SU_2 \times SU_2) \times U_1 \supset (SU_2 \times U_1) \times U_1 \quad (18)$$

and the decomposition of the adjoint of SU_5 under $(SU_2 \times U_1) \times U_1$ is:

$$24 = 1(0,0) + 2(1/2,-1) + 2(-1/2,-1) + 2(-1/2,1) + 2(1/2,1) + \\ + 1(1,0) + 1(0,0) + 1(-1,0) + 3(0,0) + 3(1,0) + 3(0,0) + 3(-1,0) \quad (19)$$

Therefore in this example $H \equiv C_{SU_5}(SU_2 \times U_1) = U_1 \times U_1$ and $K = C_{SU_5}(Sp_4) = \emptyset$. Let us demonstrate that $K = \emptyset$ by considering the potential and the breakdown of H .

We introduce the following generators of SU_5 according to the decomposition (19):

$$Q_{SU_5} \equiv \{Q, F^a, G^a, F_a, G_a, R_+, R, R_-, R^i, Q^{i+}, Q^i, Q^{i-}\} \quad (20)$$

where $a = 1, 2$ and $i = 1, 2, 3$. The commutation relations of Q_{SU_5} are given in table I. From eqs (7) and (19) we find that the genuine Higgs fields appearing in the four dimensional theory transform as $\beta = (-1, 0)$, $\nu_1 = (-\frac{1}{2}, -1)$, $\nu_2 = (-\frac{1}{2}, 1)$ under $H = U_1 \times U_1$.

The solution of the constraints (3) in terms of the genuine Higgs fields is now:

$$\Phi^i = R^i; \quad \Phi^+ = \beta R^+; \quad \Phi^{c-} = \frac{1}{\sqrt{2}}(\nu_1^+ G^c - \nu_2^+ \epsilon^{cd} F_d) \quad (21)$$

$$\Phi_0 = R; \quad \Phi^- = \beta^+ R^-; \quad \Phi_{c+} = \frac{1}{\sqrt{2}}(\nu_1 G_c - \nu_2 \epsilon_{cd} F^d) \quad (22)$$

At this point it is worth mentioning that by choosing β, v_1, v_2 equal to one, the embedding of Sp_4 in SU_5 is determined.

The matrix κ in eq. (2) is given by eq. (12) as before. The potential (2) using the solution of the constraints is

$$\begin{aligned} V(\varphi) = & \frac{4}{R_1^4} (\beta - v_1 v_2)^+ (\beta - v_1 v_2) + \frac{2}{R_1^4} (2 - v_1^+ v_1 - v_2^+ v_2)^2 + \\ & + \frac{4}{R_1^4} (v_1^+ v_1 - v_2^+ v_2)^2 + \frac{4}{R_1^4} (\beta^+ \beta - 1)^2 + \frac{2}{R_1^2 R_2^2} (v_1 - \beta v_2^+)^+ (v_1 - \beta v_2^+) + \\ & + \frac{2}{R_1^2 R_2^2} (v_2 - \beta v_1^+)^+ (v_2 - \beta v_1^+) \end{aligned}$$

We observe that for $\beta = v_1 = v_2 = 1$ this potential takes its minimum value zero independently of the values of the radii R_1, R_2 .

In conclusion, in the framework of the C.S.D.R. scheme, we have pointed out that the introduction of a suitable but natural hierarchy of scales in the coset space, when possible, can change the symmetry breaking pattern of the resulting four-dimensional gauge theory. In addition we have shown that in the case that S has an isomorphic image in G the introduction of radii does not change the breaking pattern. In the light of this new result many of the negative results that have been obtained previously in the attempts to obtain

a realistic four-dimensional theory from C.S.D.R. of a gauge theory in higher dimensions should be reexamined.

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TABLE 1

$$(Q, F^a) = -\frac{5}{2} F^a$$

$$(Q, G^a) = -\frac{5}{2} G^a$$

$$(R, F^a) = \frac{1}{\sqrt{2}} F^a$$

$$(R, G^a) = -\frac{1}{\sqrt{2}} G^a$$

$$(R, R_+) = \sqrt{2} R_+$$

$$(R, Q^{m+}) = \sqrt{2} Q^{m+}$$

$$(R^m, F^a) = -(R^m)^{ab} F^b$$

$$(R^m, G^a) = -(R^m)^{ab} G^b$$

$$(R^m, Q^{n+}) = f^{mnp} Q^{p+}$$

$$(R_m, Q^{n-}) = f^{mnp} Q^{p-}$$

$$(R^m, Q^n) = f^{mnp} Q^p$$

$$(Q^m, F^a) = -(Q^m)^{ab} F^b$$

$$(Q^m, G^a) = (Q^m)^{ab} G^b$$

$$(Q^m, Q^{n+}) = -\sqrt{2} \delta^{mn} R_+$$

$$(R_-, F^a) = -G^a$$

$$(R_+, G^a) = -F^a$$

$$(Q^{m-}, F^a) = -\sqrt{2} (Q^m)^{ab} G^b$$

$$(Q^{m+}, G^a) = -\sqrt{2} (Q^m)^{ab} F^b$$

$$(R_+, Q^m) = \sqrt{-1} Q^{m+}$$

$$(R_+, Q^{m-}) = -\sqrt{2} Q^m$$

$$(R_+, R_-) = \sqrt{2} R$$

$$(Q^{m+}, Q^{n-}) = \sqrt{2} m n R + f^{mnp} R^p$$

$$(F^a, F_b) = -\frac{5}{2} \delta^{ab} Q + \frac{1}{\sqrt{2}} \delta^{ab} R - (R^m)^{ab} R^m - (Q^m)^{ab} Q^m$$

$$(G^a, G_b) = -\frac{5}{2} \delta^{ab} Q - \frac{1}{\sqrt{2}} \delta^{ab} R - (R^m)^{ab} R^m + (Q^m)^{ab} Q^m$$

$$(F^a, G_b) = -\delta^{ab} R_+ - \sqrt{2} (Q^m)^{ab} Q^{m+}$$

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