Optimizing heavy ion experiments to probe dense nuclear matter

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Methods of extracting the azimuth of the reaction plane are compared using theoretically generated test data. We show that the required resolution can be achieved by experiments that are conceptually simpler than fully exclusive 4π measurements.

One of the primary goals of relativistic heavy ion research is to learn about the thermodynamic and transport properties of hot, dense nuclear matter.¹ Recent measurements of collective flow characteristics with heavy nuclei^{2,3} and the development of novel numerical techniques⁴ to solve the nonequilibrium transport equations [referred to as the Vlasov-Uehling-Uhlenbeck or Boltzmann-Uehling-Uhlenbeck equations (VUU)], including for the first time compression potential effects, have brought that goal much closer to realization. The three main observables used up to now¹ to extract information on the nuclear equation of state were (1) the pion excitation function, (2) composite fragment yields, and (3) in-plane transverse momenta. All three observables are related to associated *single particle* inclusive differential cross sections. The integrated pion and composite inclusive cross sections give, of course, the corresponding average multiplicity times the reaction cross section. The moments of the baryon inclusive cross section provide the basis for collective flow analysis. Even though these observables correspond to *single particle* quantities, most data on them have been obtained using semiexclusive 4π multiparticle detection systems—the streamer chamber⁵ and the Plastic Ball Wall.⁶ Obviously, 4π exclusive data contain much more information than is actually used or needed in connection with the above observables. The purpose of this paper is to suggest a conceptually simpler experiment that may be able to provide much more detailed information on collective flow than has been possible up to now. The proposed experiment⁷ would involve a high-resolution single-particle spectrometer together with a multiplicity array to constrain the magnitude of the impact parameter and a time-of-flight wall to determine the azimuthal orientation of the reaction plane.

Theoretically, the ideal inclusive experiment would measure the invariant *triple* differential single particle distribution as a function of the *vector* impact parameter $\mathbf{b} = (b \cos \Phi_R, b \sin \Phi_R)$, where Φ_R is the azimuth of the reaction plane. The total angular momentum, which can be written as the vector product of $\mathbf{b}$ and the incident momentum, is of course normal to the reaction plane. The calculated distribution for fragments of type i ,

$$f_i(\mathbf{p}, \mathbf{b}) = \lim_{t \rightarrow \infty} \int d^3r f_i(\mathbf{r}, \mathbf{p}, t; \mathbf{b}), \quad (1)$$

is the output of a detailed theoretical computation. We have in mind several models based on the Boltzmann equation: cascade,⁸ fluid dynamics,⁹ and more general nonequilibrium transport equations such as VUU with momentum-independent⁴ or momentum-dependent¹⁰ interactions. As emphasized many times,^{1,11,12} the flow characteristics are expected to depend strongly not only on the magnitude of the impact parameter but also on the azimuthal angle of the particles relative to the azimuthal angle Φ_R of the reaction plane. The problem experimentally, of course, is that there is no direct measure of the vector impact parameter. The strategy in 4π experiments is to estimate these two geometrical parameters (b and Φ_R) by measuring the momenta $\mathbf{p}(\nu)$ of as many charged fragments ν as possible. The multiplicity is used to constrain the magnitude of $\mathbf{b}$, and a global variable such as the kinetic flow tensor,¹³

$$F_{ij} = \sum_{\nu} \frac{p_i(\nu) p_j(\nu)}{2m(\nu)}, \quad (2)$$

is used to estimate Φ_R . While in principle such detailed measurements provide the most complete experimental characterization of the event possible, it is by no means clear that for the specific purpose of estimating the above two geometrical parameters such exclusive measurements are really necessary. Furthermore, in practice there is always a compromise between the number of particles identified and momentum analyzed and the resolution and acceptance of such multiparticle detectors. In the case of visual detectors such as the streamer chamber, the price paid for multiparticle measurements is relatively low statistics and limited particle identification, while in highly segmented detector systems such as the Plastic Ball Wall, the price paid is a limited resolution and restricted momentum space coverage. Single particle detectors, on the other hand, can be tailored for high resolution and wide acceptance,¹⁴ but at the price of losing the absolutely crucial impact parameter information. Clearly, there must exist an optimized experiment exploiting the advan-

tages of both approaches while minimizing their relative disadvantages.

In this paper we study the optimum degree of exclusivity that is needed to estimate the vector impact parameter (b, Φ_R) with sufficient accuracy to allow precise measurements of triple differential distributions. How well the vector impact parameter needs to be determined and how difficult it is to achieve the required accuracy depends on the strength of collective flow effects. If flow angles and aspect ratios¹³ of typical events were large, then it would be easy to design a sufficiently accurate Φ_R meter. However, current data indicate^{2,3} that the collective transverse momenta are modest, $\langle p_{\text{perp}} \rangle \sim 100$ MeV/c. Such transverse momenta are in fact about by a factor of 2 smaller than expected from nonviscous fluid dynamics, although they appear to be about a factor of 2 larger than the prediction attributed to cascade models.¹ Furthermore, the sensitivity of the collective transverse momenta to the compression potential was found within the framework of VUU to be only on the order of 50 MeV/c for a factor of 2 variation in the effective compressibility modulus.¹ The smallness of the transverse momenta and their relative insensitivity to the interesting physics implies that a determination of Φ_R to sufficient accuracy is not trivial. To see how much accuracy is needed, consider the average transverse momentum $\langle p_x(y) \rangle$ in the reaction plane, as a function of rapidity y . This observable, proposed in Ref. 15, is the most sensitive probe of collective flow up to now. The accuracy with which this in-plane momentum can be estimated depends on how well Φ_R can be determined. Let

$$\overline{p_x(y)} = \langle p_{\text{perp}} \cos(\Phi - \Phi_R) \rangle$$

denote the exact average in-plane transverse momentum. Estimates of Φ_R based on a finite number of final fragments of course have a finite dispersion $\Delta\Phi$. The effect of that dispersion is to reduce the measured transverse momentum by a factor

$$\langle \cos\Delta\phi \rangle \approx 1 - \frac{1}{2}\Delta\Phi^2.$$

The measured transverse momentum is thus biased downward by

$$\Delta p_x(y) \approx \frac{1}{2}\Delta\Phi^2 \overline{p_x(y)}. \quad (3)$$

We want a measurement with an accuracy better than the amount given by (3). Since we need an accuracy of 20–30 MeV/c on transverse momenta ~ 100 MeV/c in order to achieve sufficient sensitivity to the compression potential,

Φ_R should be determined to an accuracy

$$\Delta\Phi \leq \left[2 \frac{|\Delta p_x|}{\overline{p_x}} \right]^{1/2} \sim 35\text{--}45 \text{ deg}. \quad (4)$$

It is important to emphasize that the above estimate of the required resolution of Φ_R is based on what we now know about the magnitude of $\overline{p_x}$ from experiments^{2,3} and about the required sensitivity from VUU calculations.⁴

Next we compare four different methods to estimate Φ_R to test if the required sensitivity can be achieved with less exclusive measurements. We consider the following four alternatives: (1) kinetic flow analysis,¹³ (2) transverse velocity analysis,¹⁵ (3) transverse projectile fragmentation analysis,¹⁶ and (4) pseudorapidity (angular) analysis.¹⁷

The first method requires the most exclusivity since the center-of-mass momenta $\mathbf{p}(\nu)$ of all the fragments within the acceptance of the detector must be determined. This, of course, requires mass identification as well as lab momentum measurements. With these c.m. momenta the following restricted kinetic flow matrix can be constructed:

$$F_{ij}(y_c) = \sum_{\nu} \theta[y(\nu) - y_c] \frac{p_i(\nu)p_j(\nu)}{2m(\nu)}, \quad (5)$$

where $-y_{\text{c.m.}} \leq y_c \leq y_{\text{c.m.}}$ is a center-of-mass rapidity cut and $\theta(x) = 1$ if $x \geq 0$ and 0 if $x < 0$. By diagonalizing this flow matrix, the length and orientation of the three principal flow axes can be estimated. The azimuthal angle of the longest forward principal axis provides an estimate of the orientation Φ_R of the reaction plane. The rapidity cut y_c is introduced above to study the effect of less than 4π acceptance.

The transverse velocity method is motivated by the difficulty of particle identification. If the mass of the fragments is not determined, then it is not possible to transform to the center-of-mass frame, and the flow matrix (5) cannot be constructed. On the other hand, transverse velocities are boost-invariant along the beam axis and are relatively easy to measure with a time-of-flight wall. Obviously, such a wall needs to be sufficiently segmented so that the polar and azimuthal angles of the velocity of a particle in addition to the time of flight can be determined with good accuracy. With the time of flight, path length, and polar angle, the rapidity $y(\nu)$ and transverse velocity $\mathbf{v}_{\text{perp}}(\nu)$ of each charged particle hitting the wall can be determined. Motivated by the results of Ref. 15, we construct a restricted collective transverse velocity variable (calculated in the laboratory frame) as a function of the rapidity cut y_c as follows:

$$\mathbf{V}_{\text{perp}}(y_c) = \sum_{\nu} \epsilon[y(\nu) - y_{\text{c.m.}}] \theta[y(\nu) - y_{\text{c.m.}} - y_c] w(v_{\text{perp}}) \mathbf{v}_{\text{perp}}(\nu), \quad (6)$$

where $\epsilon(x) = +1$ if $x \leq 0$ and -1 if $x > 0$, and $w(v_{\text{perp}})$ is a weight factor. We shall consider the following two choices for the weight factor w : (1) $w(v_{\text{perp}}) = 1$ and (2) $w(v_{\text{perp}}) = 1/v_{\text{perp}}$. In this case the azimuth of $\mathbf{V}_{\text{perp}}$ provides the required estimate of Φ_R . A recent experimental study of collective azimuthal alignment also used trans-

verse velocities.¹⁸

In the third method we simply restrict the transverse velocity variable in (6) to y_c near the projectile rapidity and restrict the sum in (6) to include only fragments with $A > A_c (\geq 1)$ and $Z > Z_c (\geq 1)$ as suggested in Ref. 16.

The fourth method that we consider involves the mea-

surement of only the polar and azimuthal angles ($\theta(\nu), \phi(\nu)$) of the fragments. This is by far the easiest experiment since neither particle identification nor velocity measurement is involved. With these angles we can define only a pseudotransverse velocity along the lines of Ref. 17:

$$\eta_{\text{perp}}(\theta_c) = \sum_{\nu} \theta[\theta_c - \theta(\nu)] w(\theta(\nu)) \times (\cos\phi(\nu), \sin\phi(\nu)). \quad (7)$$

We have considered different possible weight factors, but shall report only on results with the simplest one [$w(\theta)=1$], since we did not find dramatic differences with others.

To test these different estimators of ϕ_R we employed the exclusive event generator FREESCO.¹⁹ The parameters of the calculation have been updated to best reproduce the measured flow characteristics of the Plastic Ball Wall experiment.² FREESCO provides complete events as a function of the impact parameter including heavy projectile and target fragments. Baryon number, charge, momentum, and energy are conserved in each event. Since the parameters have been tuned to incorporate available information on the transverse flow momenta, the generated events provide an ideal “data” set for our problem. For illustration, we have studied Nb + Nb at 400

MeV/nucleon at an intermediate impact parameter $b=2.7$ fm. The rms fluctuations of ϕ_R around the true reaction plane were computed using the different estimators for various rapidity and fragment mass and charge cuts from a data set consisting of 2500 FREESCO events. Results are shown in Figs. 1 and 2.

In Fig. 1 the rms dispersions of the reaction plane azimuthal angle Φ_R are compared for methods (1) and (2) as a function of the normalized c.m. rapidity cut $-1 \leq \alpha = y_c / y_{c.m.} \leq 1$. The flow matrix estimator is indicated by the crosses. Solid dots display the transverse velocity estimator with weight factor $w(v_{\text{perp}})=1$; open symbols represent the transverse velocity estimator with weight factor $w(v_{\text{perp}})=1/v_{\text{perp}}$. Recall from (4) that we need a dispersion of less than about 35–45 deg. The statistical error associated with the transverse velocity estimator is $\sim 4\%$ in the region of the minimum ($\alpha \approx 0.2$). Also shown in Fig. 1 is the multiplicity of fragments used in the analysis for a given rapidity cut (solid line). The value of the multiplicity of used fragments is to be read off the right scale. The total average multiplicity of $A \geq 1$ fragments for this reaction was 105.1. The results in Fig. 1 include all fragments with $A \geq 1$. Results with more restrictive cuts on the fragment mass and charge are shown in Fig. 2. We see that while the more exclusive flow matrix analysis leads to significantly smaller dispersions than the transverse velocity method, both methods lead to adequate resolution of ϕ_R at least in certain rapidity cut regions. As α increases and the average multiplicity

FIG. 1. Root mean squared fluctuation of the estimated reaction plane as a function of the rapidity cut using different methods and different weights. All fragments (except pions) are taken into account in the analysis. Crosses: flow tensor analysis based on Eq. (5); full dots and open symbols: transverse velocity analysis (6) with weight factors as shown in the figure. The solid line displays the average multiplicity of fragments used in the analysis as a function of the rapidity cut. The multiplicity values are to be read off the scale on the right. The figure is based on a sample of 2500 FREESCO events. Statistical uncertainties are $\sim 4\%$ in the region of interest ($\alpha \approx 0.2$).

FIG. 2. Root mean squared fluctuation of the estimated reaction plane as a function of the rapidity cut with cuts on the fragment mass and charge. Crosses, full dots, open symbols, and the solid line have the same meaning as in Fig. 1, except that unbound neutrons are now excluded from the analysis. Squares represent the flow analysis with $Z \geq 2$, and diamonds and triangles display the results of the transverse velocity analysis for $Z \geq 2$ and $Z > 2$, respectively, in the projectile fragmentation region.

ty of the analyzed fragments decreases, we see in both methods the expected $1/\sqrt{M}$ increase of the dispersion. However, as first noted in Ref. 15, for rapidity cuts approaching and going past zero in the center-of-mass frame, the dispersion via the transverse velocity method decreases significantly because the random fluctuations from the central fireball component (participant region) are reduced.

In Fig. 2 we show how the dispersion in ϕ_R is affected by fragment mass and charge cuts. The minimum dispersion in Fig. 1 was $\Delta\phi \approx 26$ deg for $w=1$ and $\Delta\phi \approx 22$ deg for $w=1/v_{\text{perp}}$ and utilized the transverse velocities of ≈ 39 fragments including neutrons. Excluding free neutrons with a $Z_c=1$ cut left only ≈ 22 charged fragments in the analysis on the average at that rapidity cut. This loss, of course, increases the rms dispersion. We found that with weight $w=1$ the minimum in the rms dispersion was increased to $\Delta\phi \approx 44$ deg. However, we see from the open symbols in Fig. 2 that with the weight factor $w=1/v_{\text{perp}}$ the dispersion is still below ≈ 35 deg at $\alpha \approx 0.2$. We conclude that the desired resolution on ϕ_R can be achieved with the transverse velocity method restricted to charged particles by judicious choice of the weight factor. It is clear that the resolution would further improve by considering heavier systems such as Au + Au.

In Fig. 2 we also show the effect of higher mass and charge cuts in the projectile fragmentation region. Excluding neutrons with a rapidity cut $\alpha=0.6$ decreases the average number of fragments from ≈ 20 to ≈ 10 and increases the dispersion from 38 to 56 deg with the transverse velocity method involving $w=1$. However, if we further demand that only fragments with charge $Z \geq 2$ be included in the analysis, then the dispersion with the transverse velocity method (diamonds) decreases back to 29 deg even though we found on the average only approximately two fragments in that kinematic range. Restricting further to heavy fragments with $A > 4$ and $Z > 2$ (triangles), the dispersion decreased to the amazing value of 11 deg even though there was only one such fragment on the average for the reaction considered. From this we conclude that the projectile fragment transverse velocity analysis¹⁶ can provide the required resolution of ϕ_R for intermediate impact parameters in collisions involving $A > 100$ nuclei. Of course, for more central collisions, where no projectile remnant survives, this method will fail. In that case the transverse velocity method with the weight $w=1/v_{\text{perp}}$ and a rapidity cut $\alpha \approx 0.2$ can provide the required resolution.

Finally we quote our results for the pseudorapidity analysis. Including all charged fragments (excluding neutrons and pions) we found that $\Delta\phi \approx 68, 62$, and 55 deg

for $\theta_c = 45, 30$, and 10 deg, respectively. This resolution is not sufficiently good. However, by restricting to $Z > 1$ fragments with angles less than 10 deg, the four fragments left in the analysis on the average led to a dispersion of $\Delta\phi \approx 37$ deg. Including only the one fragment with $Z > 2$ led to $\Delta\phi \approx 16$ deg. It thus appears that for intermediate impact parameters when at least one heavy projectile fragment survives, a rather good estimate of the reaction plane is possible even with the pseudorapidity method restricted to the forward angles and $Z > 2$. Unfortunately, without rapidity information this method fails for more central collisions.

In conclusion, we found that the azimuthal orientation of the reaction plane can be determined to approximately 30 deg accuracy in sufficiently noncentral collisions by measuring a few heavy projectile fragments as proposed in Ref. 16. For more central collisions, where the method of extracting the reaction plane from projectile fragments breaks down, a time-of-flight wall system could achieve the desired resolution. The importance of having such high resolution on ϕ_R is that it would allow for the first time detailed measurements of the triple differential cross section for different fragments. Thus far only the measurement of the average transverse momenta of those fragments is possible. Furthermore, by measuring and identifying a single fragment in a single arm spectrometer rather than with the system designed to estimate (b, ϕ_R) , the autocorrelation bias emphasized in Ref. 15 is automatically eliminated. In this way the cleanest triple differential distributions can be extracted. The importance of studying the detailed triple differential distributions rather than only the transverse momenta has already been demonstrated in early experiments,² where at least two distinct flow features were uncovered. Only when detailed comparisons of such distributions with the results of VUU calculations are made can we gain further confidence in the extracted nuclear equation of state. Finally, we emphasize that while we have demonstrated that there are conceptually straightforward experiments that could significantly advance the program of extracting the equation of state from nuclear collisions, any specific proposal could be tested using the FREESCO event generator¹⁹ in the above manner by first filtering the computer-generated "data" according to the anticipated acceptance.

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