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CP VIOLATION IN THE STANDARD MODEL WITH FOUR GENERATIONS

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ABSTRACT

CP violation in the standard model with four generations of quarks is carefully examined using rephasing invariants of the mixing matrix. A condition for CP invariance is presented and proved. CP violation in different sections is connected by orthogonal conditions. Some interesting examples of mixing matrices are discussed.

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The problem of generation and flavour mixing is one of the outstanding problems to be solved in modern particle physics. Since Kobayashi and Maskawa¹⁾, this problem has naturally included the problem of CP violation. Recently there have been many discussions on the properties of this type of CP violation, characterized by rephasing and convention-independent languages²⁾⁻⁷⁾. Basically, there are two approaches. One works directly with the mixing matrices²⁾⁻⁴⁾ and the other, believing that mass matrices are the origins of mixing matrices, works on the mass matrices⁵⁾⁻⁷⁾. Though the second approach may help to provide a new line in the attempt to connect mixing matrices and quark masses, it is not so straightforward as the first where practical use is concerned. In the example of the standard model with three generations (SM3), the necessary and sufficient condition for CP conservation is, in the first approach, the vanishing of the quantity

$$t \equiv \frac{V_{ub}V_{td}}{V_{ub}V_{td}} = \frac{V_{ub}V_{td}}{V_{ub}V_{td}} \quad (1)$$

while in the second approach, either (i) $t = 0$, and/or (ii) two charge $2/3$ (charge $-1/3$) quarks are degenerate. Obviously, when condition (ii) is satisfied, $t = 0$, so the statement of $t = 0$ is enough. Also, it is really difficult to find it directly relevant to any CP violation processes of the quantity C :

$$C = \{i\prod_{i < j} (m_{u_i}^2 - m_{u_j}^2) \prod_{i < j} (m_{d_i}^2 - m_{d_j}^2)\} t, \quad (2)$$

while the imaginary and real parts of the quartets introduced in Ref. 3) as

$$Q_{jk;\beta\gamma} = V_{j\beta}V_{k\gamma}V_{j\gamma}^*V_{k\beta}^* \quad (3)$$

appear everywhere when discussing CP violations. Note that the element appearing in one quartet is in a 2×2 submatrix of the mixing matrix. This kind of relevance and simplification of the first approach appears in the discussion of CP violation in the standard model with four generations (SM4) as well. As SM3 is better and better understood, the problem of the fourth generation in general becomes more and more attractive. As discussed by Gronau et al.⁶⁾ using the second approach, the condition for CP conservation in SM4 has six equations. We shall show that it has three equations if we use the first approach. We shall then suggest a feasible method to judge the amount of CP violation in SM4. As is well known, t is universal for nine different quartets in SM3, so it is the measure of CP violation in SM3 which is 10^{-4} experimentally. However, there is no universal imaginary part for SM4. We shall also connect CP violation from different

sections and use the relation we get to examine some interesting matrices appearing in the literature.

There are $(N-1)^2 N^2 / 4$ quartets for SMN, but not all of them are independent in the sense that the reality of two quartets may be the sufficient condition for the third to be real. Now the sufficient condition for CP invariance of SM4 is that three independent non-zero quartets are real (three equalities). We find that the three submatrices, which are not in one line (or column), may have independent non-zero quartets. We can always find three independent non-zero quartets in a sensible mixing matrix by choosing submatrices without zero elements.

The method of the proof is simple, using the physical irrelevance of rephasing of the quark operators and renumbering of the generations. Also, the weak charged current interaction does not change if we exchange all up for all down quarks. Suppose we find three independent non-zero quartets which are located as follows (plaquettes marked by 1,2,3 represent corresponding submatrices):

$$\left(\begin{array}{ccc} \boxed{1} & \boxed{2} & \boxed{4} \\ \boxed{3} & & \\ \boxed{5} & x & x \\ & x & x \end{array} \right) \quad (4)$$

and are real. We can immediately say that the whole matrix can be rephased to real, if the 2×2 submatrix 4 or 5 is not degenerate, e.g.,

$$V_{13} V_{24} - V_{14} V_{23} \neq 0 \quad (5)$$

It is not difficult to prove that when inequality (5) is not satisfied, the quartets 1 and 2 are not independent. The argument goes as follows. We first make 7 elements in the first line and first column to be real by using all possible rephasing of the eight quarks (note that the one common phase of the quarks does not affect the shape of the matrix). Since submatrix 1 has a real quartet, element V_{22} has to be real. Consequently, elements V_{23} and V_{32} have to be real since quartets 2 and 3 in Eq. (4) are real. Then V_{24} and V_{42} have to be real due to the orthogonality of the 4×4 matrix. We also have the following equations on the same basis:

$$V_{13} \text{Im} V_{33} + V_{14} \text{Im} V_{34} = 0$$

$$V_{23} \text{Im} V_{33} + V_{24} \text{Im} V_{34} = 0$$

which have only a vanishing solution if Eq. (5) is satisfied. So V_{33} and V_{34} are real, as are V_{43} and V_{44} . The arguments are exactly the same if the three relevant quartets are located in the top right or other corners. It would at first seem that the mixing matrices which have the minimal number of zero elements necessary to invalidate the above proof are those with $V_{23} = V_{32} = 0$ or $V_{22} = V_{33} = 0$. However, one can always move both zeros to the sides of the mixing matrix by renumbering generations and then the above proof goes through. Of course, if all four submatrices in the four corners are degenerate, we shall have difficulties. However, in this case we can choose then the three quartets as the following:

$$\begin{pmatrix} x & x & x & x \\ & \boxed{1} & \boxed{2} & x \\ & \boxed{3} & & x \\ & & x & x \end{pmatrix} \quad (6)$$

If there are four zeros in the mixing matrix, e.g., $V_{13} = V_{24} = V_{31} = V_{42} = 0$, we can check the three quartets on the diagonal line,

$$\begin{pmatrix} & & 0 & x \\ \boxed{1} & & & 0 \\ 0 & \boxed{2} & & \\ x & 0 & \boxed{3} & \end{pmatrix} \quad (7)$$

We can always make V_{11} , V_{22} , V_{33} , V_{44} , V_{12} , V_{14} and V_{23} real by rephasing quark operators. It is easy to prove that all elements become real after this rephasing operation if the three quartets are real. When the number of zeros increases, the proof becomes more and more trivial. A comment follows the proof. The conditions of CP invariance for SM3 in both approaches dealing with mixing and mass matrices respectively are given by only one equation, but for SM4 they are given by different numbers of equations [for the mass matrix, six equations are required⁶⁾]. The reason for this is still under investigation.

Having zero matrix elements or degenerate submatrices is a miracle accident

for a realistic mixing matrix^{*)}. However, a very special case is worth studying, which is that few of the 2×2 submatrices in the mixing matrix are orthogonal in the sense of a complex product. As proved before³⁾, their corresponding quartets must be real. This situation happens if there is some underlying substructure⁹⁾. For example, one finds that the following four submatrices are separately orthogonal:

$$\begin{pmatrix} \boxed{1} & \boxed{2} \\ \boxed{3} & \boxed{4} \end{pmatrix} \quad (8)$$

In this case, the reality of quartet 2 (or 3) is the consequence of the horizontal (or vertical) orthogonality of submatrix 1 (which causes quartet 1 to be real). Essentially we do not have enough independent real quartets. Therefore matrix (8) may still have a non-trivial CP phase.

So far we have proved the condition for a 4×4 mixing matrix to be CP invariant. On the other hand, if we find that any one quartet of the matrix is not real, then it tells us that there are non-trivial CP phases in the matrix, i.e., CP is not conserved. The quartet which gives the largest imaginary part may contribute the largest CP violation in some sense. Specifically, in Ref. 10) we suggested a formula to estimate the amount of CP violation in SM3, which is

$$R_A = t/\gamma(KM) \quad , \quad (9)$$

where R_A is the optimum event rate of CP asymmetry events in terms of mixing parameters. We do not use asymmetry parameters as the measure of CP violation because we have examples of having very large CP asymmetry parameters, but with corresponding R_A 's being small¹¹⁾. $\gamma(KM)$ is the absolute value squared of the KM elements which appear in the decay width of the relevant particle, and is $|V_{ii}|^2$ for d, c, t quarks and $|V_{i-1,i}|^2$ for s and b quarks. When the branching ratio of the discussed channel is not greatly suppressed (e.g., for the K complex) due to the existence of too many competitive channels, R_A is at the same order of the practical event rate, e.g.,

*) For an example, see Eq. (16) in Chau and Botella, Ref. 8).

$$\delta = \{\Gamma(K_L \rightarrow \pi^- 1^+ \nu) - \Gamma(K_L \rightarrow \pi^+ 1^- \bar{\nu})\} / \{\Gamma(K_L \rightarrow \pi^- 1^+ \nu) + \Gamma(K_L \rightarrow \pi^+ 1^- \bar{\nu})\}$$

$$\approx 3.3 \times 10^{-3} \approx t/S_1^2 \quad . \quad (10)$$

The immediate generalization of Eq. (9) to SM4 is

$$R_A = |\text{Im} Q_{jk}; \beta \gamma| / \gamma(\text{KM}) \quad . \quad (11)$$

For heavy quarks, the actual event rates are smaller than R_A due to the enrichment of decay channels. A wrong impression is that since we have more (i.e., three versus one) non-trivial phases in the mixing matrix, CP violation in the four-generation case becomes larger than that in the three-generation case.

We will conclude by examining some possible patterns of SM4 mixing matrices proposed in the literature¹²⁾⁻¹⁴⁾. Before doing so, let us establish a relation between quartets in one line, which is the consequence of the orthogonality of the mixing matrix. The point is that the imaginary parts of any three quartets with two elements in common, shown as follows

$$(12)$$

sum to zero. For example,

$$\text{Im} V_{34}^* V_{44} V_{33} V_{43}^* + \text{Im} V_{34}^* V_{44} V_{31} V_{41}^* + \text{Im} V_{34}^* V_{44} V_{32} V_{42}^* = 0 \quad , \quad (13)$$

etc. The consequence of this is that if the products $V_{31} V_{41}^*$ and $V_{32} V_{42}^*$ are small or $V_{13} V_{14}^*$ and $V_{23} V_{24}^*$ are, the bottom right quartets cannot have a large imaginary part. In other words, CP violation is controlled by small (though not the smallest) matrix elements.

The following is one possible naive extension of the SM3 mixing matrix¹²⁾

$$\begin{pmatrix} 1 & \lambda & \lambda^3 & \lambda^5 \\ \lambda & 1 & \lambda^2 & \lambda^4 \\ \lambda^3 & \lambda^2 & 1 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^3 & 1 \end{pmatrix}, \quad (14)$$

where $\lambda \approx 0.23$ is a parameter to show the orders of magnitude of matrix elements. The K physics is controlled by elements in the first two columns, that is to say, the ε depends on the imaginary part of a quadratic function of quartets made of elements in the two columns, and ε' on that of a linear function of quartets including V_{ud} and V_{us} ³⁾. From Eq. (11) we know the largest imaginary part of the relevant quartet for K physics is not larger than λ^6 , which is the same as in the three-generation case. Therefore, except for the case of V_{ub} being too small, the introduction of the new generation does not affect the K physics significantly, especially when taking the following factor into account: the dependence of M_{12} on m_t^2 and m_c^2 , (the mass of the fourth 2/3-charged quark) slows down when these masses approach and pass over the mass $m_W^{2(14)}$. Another possibility suggested by Chen and Li is the following pattern of the 4x4 mixing matrix¹³⁾

$$\begin{pmatrix} 1 & \lambda & \lambda^3 & \lambda^5 \\ \lambda & 1 & \lambda^2 & \lambda^4 \\ \lambda^3 & \lambda^2 & 1 & \lambda^2 \\ \lambda^4 & \lambda^3 & \lambda^2 & 1 \end{pmatrix}. \quad (15)$$

As far as K physics is concerned, this is similar to (12), i.e., no relevant quartet whose imaginary part is larger than λ^6 . The CP violation in b particles is also still limited to λ^6 in both cases (12) and (13), i.e., the best R_A may not be larger than λ^2 as in the SM3 case¹⁵⁾. Finally, the following pattern has not yet been ruled out according to Botella and Chau⁸⁾:

$$\begin{pmatrix} 1 & \lambda & \lambda^3 & \lambda^2 \\ \lambda & 1 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 & \lambda^3 \\ \lambda^2 & \lambda^2 & \lambda^3 & 1 \end{pmatrix}. \quad (16)$$

The character of this mixing matrix is that the b' (t') quarks mainly decay into the u and c (d and s) quarks. Here the K physics is strongly affected by the t' quark with an $\text{Im}Q$ no larger than λ^5 and we may get a large R_A for the b' particles (of order λ). If we exchange diagonal and non-diagonal elements in the low-down 2×2 submatrix, the big R_A will transfer from the b' to the t' systems.

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