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NUCLEAR EFFECTS IN THE PRODUCTION OF HIGH-MASS DRELL-YAN PAIRS

BY π^-W AND π^-D INTERACTIONS

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A B S T R A C T

We calculate the cross-section $\pi^-W \rightarrow \mu^+\mu^-+X$ taking into account the EMC effect and parton-multiple scattering in the nucleus. In the $d\sigma/dQ^2dx_F$ distributions the nuclear effects are small. For the $d\sigma/dq_t$ distribution, however, the multiple scattering of the incoming parton is important. By comparing the mean $\langle q_t^2 \rangle$ of the lepton pair from π^-W and π^-D interactions we obtain a quark-nucleon cross-section σ_{qN} as a function of the virtuality Q^2 of the quark. We show that the nuclear effects bring perturbative QCD in satisfactory agreement with the data.

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1. - INTRODUCTION

Deep inelastic scattering and lepton pair production have been extensively studied during the last decade since they should test in an accurate way perturbative QCD¹⁾. The first generation of experiments has confirmed the presence of logarithmic scaling violations as predicted by the Altarelli-Parisi equations²⁾. However, a big surprise came in 1983 with the so-called EMC effect^{3),4)}, i.e., the fact that structure functions from free nucleons are not identical to that from bound nucleons. This has generated a lot of theoretical work. In the region $0.1 < x < 0.8$ two explanations exist. The first one^{5),6)} which is classical in nuclear physics, proposes an enhancement of the pion field leading to an increase of the sea and a decrease of the momentum carried by the valence quarks. The second one⁷⁾ called Q^2 rescaling, consists in a modification of the confinement size for a free nucleon in nuclear matter. For $x > 0.8$, clusters of six quarks⁸⁾ and Fermi motion play an important role. These nuclear effects should be present in other reactions. Indeed, an anomalous A dependence of jets⁹⁾ and produced hadrons¹⁰⁾ in large transverse momentum reactions on nuclei has been observed. Since for statistical reasons most experiments on lepton pair production have been performed on heavy nuclei, they should be sensitive to nuclear effects¹¹⁾⁻¹³⁾. Until now they have in fact been mainly used to test perturbative QCD. Experiments with several beams and targets¹⁴⁾ observed large corrections to the naive Drell-Yan cross-section which are in qualitative agreement with first-order predictions¹⁵⁾. Moreover the correct W and Z production¹⁶⁾ rate predicted from first-order QCD¹⁷⁾ constitutes a test of the Drell-Yan mechanism in the high energy regime. A lot of activity has been devoted to the transverse momentum properties of lepton pairs. If first-¹⁸⁾ and second-order¹⁹⁾ corrections are adequate in the large q_t range ($q_t \sim Q$), it is necessary to treat soft-gluon effects²⁰⁾⁻²²⁾ carefully to understand the low and intermediate q_t range. The recent unified description²³⁾ of the whole q_t range describes correctly the W and Z transverse momentum spectrum and ISR data in the low lepton pair mass range²⁴⁾.

Although the Drell-Yan mechanism seems to be well understood in the high energy regime, the recent data of the high statistics NA10 experiment on π^-W have shown that the same formalism fails to fully describe the K factor in the whole Q range²⁵⁾, the transverse momentum distributions²⁶⁾ for large masses and the angular distributions²⁷⁾. Moreover, the NA10 experiment has established the presence of two nuclear effects²⁸⁾: a decrease of the cross-section $d\sigma/dQ^2 dx_F$ for negative and small x_F values at large Q^2 values, a difference of the mean square

transverse momentum between W- and D-targets and an increase with q_t of the ratio of the W and D cross-sections. The A dependence of the integrated cross-section, however, is equal to one.

It has recently been shown that by combining the soft-gluon exponentiation scheme^{29),30)} with EMC structure functions, the predicted K factor³¹⁾ is in close agreement with data. This challenge motivated us to reanalyze lepton pair production with a careful incorporation of nuclear effects which are of two types: EMC effect and rescattering of partons inside the nuclear matter.

The paper is organized as follows. We discuss in Section 2 the EMC effect and we compare our predictions to the NA10 data for the cross-section $d\sigma/dQ^2 dx_F$. We give in Section 3 the QCD formalism needed to compute the transverse momentum distributions for free nucleons. Section 4 is devoted to multiple scattering effects. Our numerical results for the triple differential cross-section $1/q_t \cdot d\sigma/dq_t dQ dy$ and our conclusions are displayed in Section 5.

2. - THE EMC EFFECT IN DRELL-YAN PRODUCTION OF LEPTON PAIRS

Deep inelastic scattering of muons and electrons has shown that the structure functions of nuclei are not a simple sum of nucleon structure functions modified by nucleon motion and binding. The measurement of $F_2(x)$ in nuclei gives a depletion of the valence part of the structure function in the region of $0.2 < x < 0.7$ and a still disputed increase at small x for the sea. Among the numerous explanations of this behaviour three models have received considerable attention: (a) the classical nuclear physics picture^{5),6)}; (b) the Q^2 -rescaling model⁷⁾; (c) the cluster model⁸⁾. The classical picture allows for an increase of pions in nuclei and balances momentum by depleting the nucleon large- x distribution. The necessary change of the Bjorken variable $x = Q^2/2p \cdot q$ in the nucleus is sometimes also called nuclear binding correction. This prescription³²⁾ gives, for a nucleon in the nucleus, modified kinematical variables $p \cdot q = (m+\epsilon)v - p_z q_z \approx (m+\epsilon-p_z)v$ where $\epsilon < 0$ is the separation energy and p_z the momentum of the nucleon. The nuclear structure function becomes

$$F_2^A(x) = F_2^{\text{Nucleon}}(x) + \frac{1}{A} \sum_{i=1}^A \int \rho_i(\vec{p}) F_2^N\left(\frac{x}{1 + \frac{\epsilon_i - p_z}{m}}\right) d^3\vec{p} \quad (1)$$

In principle, the F_2^{meson} includes all mesons which contribute to the nucleon-nucleon interaction and therefore are present in the nucleus. In reality this classical nuclear physics model picks only the pion field as important.

The needed size of the separation energy³³⁾ $|\epsilon| \approx 50$ MeV for heavy nuclei is much larger than the Coulomb-corrected measured separation energy. The model relies entirely on the folding prescription which assumes that deep inelastic scattering off the nucleus is obtained by an incoherent sum of scatterings with partons in nucleonic and mesonic clusters. It has been shown, however, that interference diagrams are not negligible for small x . The hadronic tensor $W_{\mu\nu} \propto \langle A | [j_{\mu}^{(x)}, j_{\nu}] | A \rangle$ contains a piece where a quark is exchanged between two nucleons (Fig. 1). This correction is due to the Pauli principle between quarks in two overlapping nucleons. It amounts to $\approx 5\%$ and has been estimated for small x in Ref. 34) and calculated for $x > 0.3$, e.g., in He^3 in Ref. 35).

For the region of x -values of the data we are going to discuss ($0.15 < x < 0.7$), the classical nuclear physics picture amounts to a x rescaling form of $F_2^A(x) \approx F_2^N(x/(1+(\bar{\epsilon}_1/m)))$. A choice of $|\bar{\epsilon}| \approx 50$ MeV reproduces the EMC effect approximately (see Fig. 2). We use this model to see possible differences to the Q^2 rescaling model in the Drell-Yan production when soft-gluon effects are important.

The Q^2 rescaling model is based on soft-gluon radiation, a concept fundamental to the QCD corrections calculated in this paper. It states that at a given Q^2 the nuclear $F_2^A(x, Q^2)$ can be obtained from the nucleon structure $F_2^N(x, Q^2)$ by adding the radiation of soft gluons with transverse momenta in the interval $1/R_A^2 < p_T^2 < 1/R_N^2$, where R_A and R_N are the confinement scales characteristic of the nucleus and the nucleon. This leads to a nuclear structure function

$$F_2^A(x, Q^2) = F_2^N(x, Q^2 \xi^2) \quad (2)$$

with

$$\xi = \left(\frac{R_A}{R_N} \right)^{\alpha_s(Q_0^2)/\alpha_s(Q^2)} \quad (3)$$

where the rescaling factor ξ also contains the ratio of strong coupling constants $\alpha_s(Q_0^2)/\alpha_s(Q^2)$ as exponent. $Q_0^2 = 0.75 \text{ GeV}^2$ was chosen as the starting point of the evolution in the model of Ref. 36). Experiments in a certain region of Q^2 allow ξ to be estimated. A description of the SLAC data⁴⁾ with $R_A/R_N = 1.23$ was

proposed³⁶⁾ for gold (A=197). Because of the small A dependence we used this value for W (A=184). It has been speculated that with increasing Q^2 a picture of the nucleus becomes true where all partons communicate freely with each other. Thus the rescaling factor would have an additional Q^2 dependence³⁷⁾ beyond Eq. (2). We do not consider this effect here.

The above choices of nuclear corrections were applied to the following quark distribution functions in the proton

$$x u_v(x) = N_u x^{\eta_1} (1-x)^{\eta_2} \quad (4)$$

$$x d_v(x) = N_d x^{\eta_3} (1-x)^{\eta_4} \quad (5)$$

with $N_u = 2/B(\eta_1, \eta_2+1)$, $N_d = 1/B(\eta_3, \eta_4+1)$ and for the sea quarks

$$x S(x) = x \bar{u} = x \bar{d} = x \bar{s} = A_s (1-x)^{\eta_5} \quad (6)$$

The constants at the starting point for the evolution ($Q_0^2 = 25 \text{ GeV}^2$) are given by:

$$\eta_1 = 0.632, \quad \eta_2 = 0.327$$

$$\eta_3 = \eta_1, \quad \eta_4 = \eta_2 + 1$$

$$\eta_5 = 8.0$$

A_s is determined from the energy momentum sum rule assuming that the gluons carry 47% of the proton momentum. For the pion we use a valence distribution

$$x V_\pi(x) = N_v x^{\eta_5} (1-x)^{\eta_6} \quad (7)$$

with $\eta_5 = 0.527$ and $\eta_6 = 1.16$, $N_v = 1/B(\eta_5, \eta_6+1)$ and a sea distribution

$$x S_\pi(x) = A_s (1-x)^{\eta_7}, \quad \eta_7 = 8.4 \quad (8)$$

We remark that the above structure functions agree with the F_2^p measurement in the interval $0.15 < x < 0.7$ ³⁸⁾.

The projectile pion distribution can be considered as an approximate parametrization of NA10 data³⁹⁾. The nuclear pion distribution is obtained with a pion distribution function $f_{\pi/A}$ in the nucleus, for which we took the model of Ref. 13). We approximated $f_{\pi/A}$ as

$$f_{\pi/A}(y) = \frac{N_0}{(2\pi)^{1/2} \sigma} \exp \left\{ - (y - y_0)^2 / 2\sigma^2 \right\} \quad (9)$$

with $\sigma = 0.12$ and $N_0 = 0.235$. This gives a momentum carried by the pion of 0.06 roughly compensating the x-loss of the nucleon which is $|\epsilon/m| = 0.053$. In Fig. 2 we show the calculation of the EMC ratio $\rho = F_2^W(x)/F_2^d(x)$ for the isoscalar target

$$F_2^A = \frac{5}{18} \cdot x (u_V^A + d_V^A) + \frac{4}{3} (xS^A) \quad (10)$$

in the two models. The x-rescaling plus pion model has a larger increase for small x-values and gives a larger depletion at large x, which would be stopped by the Fermi motion correction. The Q^2 rescaling model gives a smaller sea-quark enhancement and less large x-depletion. If we compare to the SLAC data σ_A/σ_D ⁴⁾ for Au ($A=197$) we find that the two models give an upper and a lower limit to the nuclear correction in the region $0.2 < x < 0.7$ we are concerned with. We will see how much accuracy the Drell-Yan data need to improve the deep inelastic scattering data. Of course, there remains the natural advantage of the Drell-Yan experiment that one can directly access the $\bar{q}$ distribution at small x by using a proton⁴⁰⁾ beam. We refer to the literature¹³⁾ for a simulation of this effect. Also beyond the scope of the present investigation of the π^-W and π^-D data is a discussion of cluster effects at $x > 1$ ¹²⁾. Probably the use of nuclear beams at CERN may allow for an investigation of Drell-Yan production in this kinematical region.

Using the pion and nuclear structure functions, we have calculated the double differential cross-sections $Q^2 d\sigma/dx_F dQ^2$ for $\pi A \rightarrow \mu^+ \mu^- + X$. The Born⁴¹⁾ cross-section is given as

$$\frac{Q^2 d\sigma^{(0)}}{dx_F dQ^2} = \frac{4\pi\alpha^2}{9s} \frac{1}{x_1 + x_2} \sum_f e_f^2 \left\{ \bar{q}_\pi(x_1) q_A(x_2) + q_\pi(x_1) \bar{q}_A(x_2) \right\} \quad (11)$$

with

$$Q^2 = (p(\mu_1) + p(\mu_2))^2, \quad x_F = \{p_z(\mu_1) + p_z(\mu_2)\} / \{\sqrt{S}/2\}$$

The invariant hadron centre-of-mass energy is $\sqrt{S} = 19.2$ GeV. The lepton momenta determine the momentum of the virtual photon and of the partons for $q + \bar{q} \rightarrow \gamma \rightarrow \mu^+ + \mu^-$

$$\frac{x_1 \sqrt{S}}{2} - \frac{x_2 \sqrt{S}}{2} = \frac{x_F \sqrt{S}}{2}$$

The invariant mass Q^2 of the photon is

$$x_1 \cdot x_2 \cdot S = Q^2$$

A solution of these equations gives the points x_1 and x_2 where the parton densities have to be evaluated

$$\begin{aligned} x_1 &= \frac{1}{2} (x_F + \sqrt{x_F^2 + 4\tau}) \\ x_2 &= \frac{1}{2} (-x_F + \sqrt{x_F^2 + 4\tau}) \\ \tau &= Q^2/S \end{aligned} \quad (12)$$

The QCD corrections have been evaluated to first order¹⁵⁾. The expression is given in the Appendix. The main part of the correction comes from the annihilation diagram for which a resummation of infra-red parts has been performed³⁰⁾. We shall briefly recall it now. The large terms in the first-order calculation are (see the Appendix):

- (i) the $(1 + (\alpha_s/2\pi)C_F\pi^2)$ which comes from the analytic continuation of the quark form factor from space-like to time-like Q^2 ;
- (ii) the infra-red singular term $[1 + \alpha_s((Q^2)C_F/\pi)\ln(1-x_1)\ln(1-x_2)]$.

These terms, which give 90% of the first-order correction, can be exponentiated. The soft-gluon formula is therefore obtained by replacing in (A.1)

$$\left(1 + \frac{\alpha_s(Q^2)}{2\pi} C_F \pi^2\right) \quad \text{by} \quad \exp\left(\frac{\alpha_s(Q^2)}{2\pi} C_F \pi^2\right) \quad (13)$$

and

$$\left(1 + \frac{\alpha_s(Q^2)}{\pi} C_F \ln(1-x_1) \ln(1-x_2)\right)$$

by

$$\exp \left\{ -\frac{12\pi C_F}{(33-2N_f)} \left[\ln \frac{Q^2}{\Lambda^2} (1-x_1) - \ln \frac{\alpha_s(Q^2)}{\alpha_s(Q^2/(1-x_1))} + (x_1 \rightarrow x_2) \right. \right. \\ \left. \left. - \ln \frac{Q^2}{\Lambda^2} (1-x_1)(1-x_2) \ln \frac{\alpha_s(Q^2)}{\alpha_s(Q^2/(1-x_1)(1-x_2))} \right] \right\} \quad (14)$$

$$\text{with } \alpha_s(Q^2) = \frac{12\pi}{(33-2N_f)} \frac{1}{\ln(Q^2/\Lambda^2)},$$

N_f being the number of flavours. This is our basic QCD formula which we will compare to NA10 data. The Compton piece is evaluated to first order. The resulting K factor is rather independent of the nuclear corrections and of the kinematical choice of x_F ($K \approx 1.8$). Let us remark that the exponentiation of the π^2 terms gives $K = 1.63$. In Figs. 3a-3g, we show a comparison of our calculation with the NA10 data for $d^2\sigma/d\sqrt{\tau}dx_F$ for various bins of $\sqrt{\tau}$ as a function of x_F . These data have been corrected for Fermi-motion²¹⁾. The correction $R(x_1, x_2)$ is $<4\%$ and very close to unity for most of the kinematical region. Larger corrections only occur for large $\sqrt{\tau}$ above the T region. It would be advantageous to have raw data without the ill-defined Fermi correction procedure.

One sees that, in general, the shape of the cross-sections is well reproduced by the structure functions and the magnitude is correctly calculated with the soft-gluon summation technique. The theoretical results without QCD corrections are shown in Figs. 3a and 3g. For small $\sqrt{\tau} \approx 0.30$, the large x_F values test the nuclear parton densities near $x_2 = 0.16$, where the pion model shows an enhancement of the theoretical cross-section. The opposite is true for the small negative x_F (i.e., large x_2) values. The two nuclear models bracket the experimental results for most cases, besides the large lepton masses $\sqrt{\tau} \geq 0.6$, where there is a 1σ deviation of the data from theory. It looks as if the effective x_F would need to be shifted by $\Delta x_F = +0.075$. As will be discussed in Section 4, multiple scattering effects of the parton in the nucleus cannot explain such a shift. Apparently, it is also not visible in the $\sqrt{S} = 23.7$ GeV data.

3. - TRANSVERSE MOMENTUM DISTRIBUTIONS FOR FREE NUCLEONS

The transverse momentum distribution of lepton pairs is certainly one of the more subtle quantities to compute in the framework of perturbative QCD. It arises mostly from gluon radiation since at the Born level little transverse momentum from the hadron wave function is expected. The first-order prediction¹⁸⁾ (see Fig. 4) is only reliable when the transverse momentum q_t is of the order of magnitude of the lepton pair mass Q . Second-order corrections have been computed for non-singlet contributions¹⁹⁾. For $Q \sim 5$ GeV, they are important in the range $3 < q_t < 5$ GeV. In terms of the K_T factor defined as:

$$K_T = \left[\frac{d\sigma}{dQ^2 dy dq_t^2} \Big|_{O(\alpha_s) + O(\alpha_s^2)} \right] / \left[\frac{d\sigma}{dQ^2 dy dq_t^2} \Big|_{O(\alpha_s)} \right] \quad (15)$$

one gets $K_T \sim 2.3$ at $q_t \sim 3$ GeV and $K_T \sim 1.5$ at $q_t \sim 5$ GeV.

Let us consider the first-order correction to $q\bar{q}$ annihilation. The distribution reads:

$$\frac{d\sigma^{(1)}}{dQ^2 dy dq_t^2} = \frac{d\sigma^{(0)}}{dQ^2 dy} \frac{1}{q_t^2} \frac{\alpha_s C_F}{2\pi} \cdot 2 \ln \frac{Q^2}{q_t^2} = \frac{d\sigma^{(0)}}{dQ^2 dy} \cdot \frac{dP^{(1)}}{dq_t^2} \quad (16)$$

with $C_F = 4/3$ and $d\sigma^{(0)}/dQ^2 dy$ refers to the Born cross-section. Since $q_t^2 \ll Q^2$ the logarithmic term is sizeable and one has therefore to include higher-order corrections (see Fig. 5). This can be done by taking into account exactly transverse momentum conservation²¹⁾. The sum of cross-sections for the production of n gluons reads:

$$\frac{dP}{dq_t^2} = \sum_{n=1}^{\infty} \frac{1}{n!} \prod_{i=1}^n \frac{dP^{(1)}}{dk_{ti}^2} \delta(q_t - \sum_{i=1}^n k_{ti}) \quad (17)$$

Going into the impact parameter space, we get:

$$\frac{dP}{dq_t^2} = \frac{1}{2} \int_0^\infty b db J_0(bq_t) \exp \left\{ \frac{16}{3\pi} \int_0^{q_{\max}} \frac{dq}{q} \ln \frac{Q}{q} \alpha_s(q) \cdot [J_0(bq) - 1] \right\} \quad (18)$$

The transverse momentum cross-section reads

$$\frac{d\sigma}{dQ^2 dy dq_t^2} = \frac{d\sigma^{(0)}}{dQ^2 dy} \cdot \frac{dP}{dq_t^2} \quad (19)$$

An explicit formalism valid both in the domain $q_t \ll Q$ and $q_t \sim Q$ has been recently derived²³⁾. The first-order perturbative q_t distribution has the exact form [without assuming factorization as in Eq. (16)]:

$$\begin{aligned} \frac{d\sigma}{dQ^2 dq_t^2 dy} &= \frac{d\sigma^{(0)}}{dQ^2 dy} \cdot (1 + F + S) \cdot \delta(q_t^2) + B \left(\frac{\ln Q^2/q_t^2}{q_t^2} \right)_+ \\ &+ \frac{C}{(q_t^2)_+} + Y(q_t^2). \end{aligned} \quad (20)$$

where $Y(q_t^2)$ is the piece regular at $q_t = 0$, σ_0 is the zeroth-order Drell-Yan cross-section, F , B and C are functions of Q^2 and y .

The distribution S in Eq. (20) comes from the quark-quark gluon-vertex with x and k_T being the momenta of the quark after gluon radiation. The differential probability for this process is $dP/dq_t^2 dx$

$$\frac{dP}{dq_t^2 dx} = \frac{\alpha_s(q_t^2)}{2\pi} C_F \frac{1}{q_t^2} \cdot \left(\frac{1+x^2}{1-x} \right) \quad (21)$$

Integrating over x one gets S :

$$S(q_t^2, Q^2, y) = \frac{\alpha_s}{2\pi} C_F \cdot 2 \cdot \left\{ \left(\frac{\ln(Q^2/q_t^2)}{q_t^2} \right)_+ - \frac{3}{(2q_t^2)_+} \right\} \quad (22)$$

In the soft-gluon approximation ($1+x^2 \sim 2$) used in Eq. (16), only the first term is obtained. One performs a Fourier transform of the singular piece into impact parameter space b and factors the collinear singularities into the structure functions. After resummation of $S(b, Q^2, y)$ [i.e., the replacement of $(1+S)$ into $\exp(S)$ and inverse Fourier transform] one gets:

$$\frac{d\sigma}{dQ^2 dq_t^2 dy} = \frac{d\sigma^{(0)}}{dQ^2 dy} \int_0^\infty \frac{d^2 \vec{b}}{4\pi} e^{i\vec{b}\vec{q}_t} R(b^2, Q^2, y) \exp S(b^2, Q^2, y) + Y(q_t^2, Q^2, y) \quad (23)$$

where $S(b^2, Q^2, y)$ is the Sudakov form factor in the double leading logarithmic approximation

$$S(b^2, Q^2, y) = \int_0^{q_{t, \max}^2} \frac{dq^2}{q^2} [J_0(bq) - 1] \frac{\alpha_s(q^2)}{\pi} C_F \left\{ \ln \frac{Q^2}{q^2} - \frac{3}{2} \right\} \quad (24)$$

We choose $(q_t^2)_{\max} = Q^2$ as the upper limit of the integration. The result remains almost unchanged by any other reasonable choice [see Ref. 23) for details].

Let us recall that upon integration over q_t we recover the first-order perturbative cross-section $d\sigma/dQ^2 dy$. This formalism is well adapted to W and Z production since the lepton pair mass being large we are in a perturbative regime where first order gives the correct production rate. This is not the case for lepton pair production at fixed target energies since it has been shown experimentally that an extra K factor is needed to get agreement with data for $d\sigma/dQ^2$. This extra K factor comes from the resummation of soft gluons as we have seen in the previous section. Therefore, we will extend the previous formalism in order to include these soft-gluon effects by replacing in the function R :

$$\left(1 + \frac{\alpha_s}{2\pi} C_F \pi^2 \right) \quad \text{by} \quad \exp \left(\frac{\alpha_s C_F \pi^2}{2\pi} \right)$$

and $1 + \alpha_s/\pi C_F \ln(1-x_1) \ln(1-x_2)$ by Eq. (14).

4. - MULTIPLE SCATTERING EFFECTS IN DRELL-YAN q_t DISTRIBUTIONS

For a long time, the Drell-Yan process $hN \rightarrow \ell^+ \ell^- X$ has been considered unchanged by nuclear targets. Indeed in the NA10 experiment²⁸⁾ the ratio of total πA cross-sections between W ($A=184$) and deuteron gives a dependence $\sigma_W/\sigma_D = A^\alpha$ with $\alpha = 0.994 \pm 0.01$ in various τ intervals. The same linear A dependence, however, does not hold for q_t distributions. In the above experiment ratios up to two were obtained for $1/A_W (d\sigma^W/dq_t)/1/A_D (d\sigma^D/dq_t)$ for transverse momenta up to 4 GeV. In the literature^{42),43)}, the angle straggling off the incoming parton has been pointed out as a possible source for such an A -dependence. In Ref. 42), general estimates were given for $\langle q_t^2 \rangle$ as a function of A ; in Ref. 43), the first hadronic scattering before lepton-production has been calculated. We will calculate all rescatterings and consider the initial state interaction in the same theoretical framework as the soft-gluon corrections used for q_t distributions.

The Drell-Yan process has the remarkable advantage that the incoming parton has a well-defined virtuality which is of the order of the produced dilepton mass Q^2 . Such a parton has a differential cross-section for scattering with other partons which is due to multiple soft-gluon exchange. Following the model of Ref. 44) for double-jet production, we have for the transverse momentum distribution of parton-parton scattering

$$\frac{1}{\sigma_0} \frac{d\sigma_{qq}}{dq_t^2} = \int d^2\vec{b} e^{i\vec{q}_t \cdot \vec{b}} e^{S(b)}$$

with $S(b)$ for quark-quark scattering given as

$$S(b) = \int_0^{k_{t,max}} \frac{dk_t^2}{k_t^2} \frac{\alpha_s(k_t^2)}{\pi} \cdot \frac{4}{3} \left\{ \ln \frac{\hat{s}}{k_t^2} - \frac{3}{2} \right\} \left\{ J_0(k_t b) - 1 \right\} \quad (29)$$

where the scale $\hat{s} = Q^2$.

This function $S(b)$ is the same Sudakov factor which enters into the calculation of the q_t distribution, and sums multiple gluon bremsstrahlung and vertex correction graphs [cf. Eq. (24)]. Of course, the total normalization of the parton-parton cross-section σ_0 is not fixed. We assume that σ_0 can be determined empirically from the difference of mean $\langle q_t^2 \rangle$ in two different nuclei. Using the Glauber theory we relate the total parton-nucleon cross-section to the shape of the transverse distribution $1/\sigma_0 (d\sigma^{qq}/dk_t^2)$. The qq-differential cross-section is given in Glauber theory as

$$\frac{d\sigma_{qq}}{dq_t^2} = \left| \int e^{i\vec{q}_t \cdot \vec{b}} (1 - e^{-\Omega_{qq}(b)}) d^2\vec{b} \right|^2 \approx \left| \int e^{i\vec{q}_t \cdot \vec{b}} \Omega_{qq}(b) d^2\vec{b} \right|^2 \quad (26)$$

Parametrizing the Sudakov shape function $e^{S(b)}$ as

$$e^{S(b)} = e^{-b^2/2\sigma^2} \quad (27)$$

we assume there exists a constant c independent of σ , such that

$$\Omega_{qq}(b) = c e^{-b^2/\sigma^2} \quad (28)$$

This choice of the profile function is consistent with Eq. (27). The quark-nucleon cross-section is obtained from a folding with the quark density in the nucleon. In general there would be also the possibility of having gluon constituents in the nucleon. But since we determine the total interaction cross-section σ_0 empirically, we do not think that this will matter

$$\Omega_{qN}(b) = B \int \Omega_{qq}(\vec{b}-\vec{s}) g_{q/N}(\vec{s}) d^2\vec{s} \quad (29)$$

The distribution of quarks in the nucleon $\rho(s)$ is normalized to 1. B is the number of quarks in the nucleon. Using Eq. (28) we get for the total quark-nucleon cross-section

$$\sigma_{qN} = B \cdot c \cdot \pi \cdot \sigma^2 \quad (30)$$

This is the main result, which guarantees that the mean $\langle q_t^2 \rangle$ will be independent of Q^2 whereas σ may and does indeed vary with Q^2 . In Fig. 6 we plot the Sudakov shape function $e^{S(b)}$ as a function of b for various Q^2 . One clearly can recognize how the effective size of the virtual quark decreases with increasing Q^2 . In this picture with higher Q^2 more gluon structure of the quark in the hadron is explicitly considered and the effective gluon cloud of the quark becomes smaller. A typical "size" of the quark at $Q = 5$ GeV is 0.3 fm and at $Q = 8$ GeV we find 0.23 fm. Using the scaling relation of Eq. (26), we deduce cross-sections σ_{qN} varying between 2.5 mb at $Q = 5$ GeV and 0.8 mb at $Q = 12$ GeV. The proportionality constant $B \cdot c$ is fixed by the following Glauber derivation of the parton-multiple scattering in the nucleus. Let the nuclear density $\rho(r)$ for $A > 16$ be parametrized as a Fermi distribution⁴⁵⁾

$$\rho(r) = \rho_0 / (1 + \exp[(r-r_0)/c]) \quad (31)$$

where $r_0 = (1.15 A^{1/3} - 0.87 A^{-1/3})$ fm, $c = 0.55$ fm and ρ_0 is normalized so that $\int d\vec{r} \rho(r) = A$. For the deuteron we use an analytical hard-sphere density of radius $R_c = 2.8$ fm. In general multiple scattering corrections for the deuteron are negligible. For analytical calculations, the above Fermi distribution can be approximated by a hard-sphere density of radius $R_c = 1.2$ fm $A^{1/3}$.

Let us call P_i the probabilities normalized to A that the incoming parton has undergone $i-1$ hadronic scatterings before it annihilates into a lepton pair. With σ_{DY} as elementary hadron-nucleon Drell-Yan cross-section, $\sigma_{DY} P_1$ is the Drell-Yan cross-section for no rescattering in the nucleus:

$$\sigma_{DY} P_1 = \left(\frac{\sigma_{DY}}{\sigma_{qN}} \right) \cdot \int d^2b \int_{-\infty}^{+\infty} dz e^{-\sigma_{qN} \int_{-\infty}^z \rho(b,z') dz'} \sigma_{qN} \rho(b,z) \quad (32)$$

The individual factors give the probabilities that a Drell-Yan interaction takes place at (b,z) and that no quark-nucleon interaction has preceded it between $-\infty$ and z . These probabilities have to be summed over all the impact parameters to yield the total Drell-Yan cross-section. The above expression can be integrated by the usual trick that the derivative of the integral of the thickness function is again the thickness function,

$$\frac{d}{dz} F(z) = \frac{d}{dz} \int_{-\infty}^z \sigma_{qN} \rho(b,z') dz' = \sigma_{qN} \rho(b,z) \quad (33)$$

$$\sigma_{DY} P_1 = \left(\frac{\sigma_{DY}}{\sigma_{qN}} \right) \cdot \int d^2b \left(1 - e^{-\sigma_{qN} \int_{-\infty}^{+\infty} \rho(b,z) dz} \right) = \frac{\sigma_{DY}}{\sigma_{qN}} \cdot \int d^2b \left(1 - e^{-T(b)} \right) \quad (34)$$

where we call $T(b) = \int_{-\infty}^{+\infty} dz \rho(b,z)$ the thickness function³⁴⁾.

In a similar way we can derive $\sigma_{DY} P_2$ as

$$\begin{aligned} \sigma_{DY} \cdot P_2 = \left(\frac{\sigma_{DY}}{\sigma_{qN}} \right) \int d^2 \vec{b} \int_{-\infty}^{+\infty} dz e^{-\sigma_{qN} \int_{-\infty}^z g(b, z') dz'} \cdot \sigma_{qN} \cdot g(b, z) \cdot \\ \cdot \int_{-\infty}^{+\infty} d\tilde{z} e^{-\sigma_{qN} \int_{\tilde{z}}^{+\infty} g(b, z') dz'} \cdot \sigma_{qN} \cdot g(b, \tilde{z}) \end{aligned} \quad (35)$$

It reads

$$\sigma_{DY} P_2 = \left(\frac{\sigma_{DY}}{\sigma_{qN}} \right) \cdot \int d^2 \vec{b} \left\{ (1 - e^{-T(b)}) - T(b) e^{-T(b)} \right\} \quad (36)$$

In general we have

$$\sigma_{DY} \cdot P_n = \left(\frac{\sigma_{DY}}{\sigma_{qN}} \right) \cdot \int d^2 \vec{b} \left\{ 1 - \left(\sum_{k=0}^{n-1} \frac{1}{k!} T^k \right) e^{-T} \right\} \quad (37)$$

The total nuclear Drell-Yan cross-section sums up to A times the elementary cross-section σ_{DY}

$$\sigma_{DY}^A = \sum_{i=1}^n P_i \cdot \sigma_{DY} = A \cdot \sigma_{DY} \quad (38)$$

One can easily show this as follows. Let us define X as the sum of the arguments of the b integrals [Eqs. (34), (36), (37)]

$$X = (1 - e^{-T}) + [1 - (1+T)e^{-T}] + [1 - (1+T+\frac{1}{2}T^2)e^{-T}] + \dots \quad (39)$$

then $dX/dT = 1$ and $X(T=0) = 0$ which gives $X = T$ and

$$\sum_{i=1}^n P_i = \frac{1}{\sigma_{qN}} \int d^2 \vec{b} T(b) = A \quad (40)$$

This justifies to speak of P_i as probabilities normalized to A nucleons. In the low density and/or low cross-section limit $\sigma_{qN} \rightarrow 0$ one finds that $P_1 \rightarrow A$

$$P_1 \Rightarrow \frac{1}{\sigma_{qN}} \int d^2 \vec{b} (1 - (1 - \sigma_{qN} g(b)) \dots = \int d^2 \vec{b} g(b) = A \quad (41)$$

Note, however, that even for large rescattering cross-section, this formula never gives the limit $\sigma_{DY} \propto A^{2/3}$, which is measured in Drell-Yan production on nuclei with $Q^2 < (4 \text{ GeV})^2$. Probably this cross-section comes from quark-antiquark annihilations between partons which have been produced in the hadronic

interactions of the projectile with the target⁴⁶⁾. This hadronization is not taken into account in our formalism. The mean number of rescatterings is given as

$$\langle n \rangle = \frac{1}{A} \sum_n P_n \cdot n = \frac{1}{A \sigma_{qN}} \int d^2b \left(\frac{1}{2} T^2(b) + T(b) \right) \quad (42)$$

The derivation of this equation uses the same method as before. We define X_n as the argument of P_n

$$P_n = \frac{1}{\sigma_{qN}} \int d^2b X_n(b) = \frac{1}{\sigma_{qN}} \int d^2b \sum_{k=n}^{\infty} \frac{T^k}{k!} e^{-T} \quad (43)$$

Then

$$\frac{dX_n}{dT} = \frac{T^{n-1}}{(n-1)!} e^{-T} \quad (44)$$

$$\sum_{n=1}^{\infty} n \cdot \frac{dX_n}{dT} = \sum_{n=1}^{\infty} \left\{ \frac{(n-1) T^{n-1}}{(n-1)!} + \frac{T^{n-1}}{(n-1)!} \right\} e^{-T} = T+1 \quad (45)$$

and

$$\sum_{n=1}^{\infty} n \cdot X_n \Big|_{T=0} = 0 \quad (46)$$

which leads to $\sum_{n=1}^{\infty} T_n X_n = \frac{1}{2} T^2(b) + T(b)$ proving Eq. (42). The differential cross-section to make a Drell-Yan pair after 1, ..., n rescatterings is given by a multiple folding of the differential distribution of Eq. (24) and Eq. (29)

$$\frac{1}{\sigma_{DY}} \frac{d\sigma^{DY}}{dq_t^2} = \int d^2b e^{i\vec{q}_t \cdot \vec{b}} e^{S(b)} = \frac{1}{\sigma_0} \frac{d\sigma_{qq}}{dq_t^2} \approx e^{-\frac{q_t^2 \sigma^2}{2}} \cdot \left(\frac{\sigma^2}{2\pi} \right) \quad (47)$$

Neglecting the x-loss of the parton in the multiple scattering we obtain for the transverse Drell-Yan cross-section with a nuclear target

$$\frac{1}{\sigma_{DY}} \frac{d\sigma^{\pi+A \rightarrow \mu\mu+X}}{dq_t^2} = \sum_{n=1}^{\infty} P_n^A e^{-\frac{q_t^2 \sigma^2}{2n}} \cdot \left(\frac{\sigma^2}{2\pi n} \right) \quad (48)$$

The mean $\langle q_t^2 \rangle$ is given by averaging with the above cross-section as

$$\langle q_t^2 \rangle_A = \frac{1}{A} \frac{2}{\sigma^2} \sum_{n=1}^{\infty} n \cdot P_n^A \quad (49)$$

The total nuclear cross-section equals A-times the Drell-Yan nucleon cross-section as derived before. Only the relative weight in transverse momentum space is shifted by multiple scattering. No parton is lost for the Drell-Yan process.

Since we have chosen the quark nucleon cross-section $\sigma_{qN} = B \cdot c \pi \sigma^2$ related to the shape of the transverse momentum distribution $\exp(-q_t^2 \sigma^2 / 2)$, the difference of mean transverse momentum $\langle q_t^2 \rangle$ between different nuclei is independent of σ^2 , i.e., independent of Q^2 and can be calculated with Eq. (42)

$$\begin{aligned} \Delta q_t^2 &= \langle q_t^2 \rangle_{A_1} - \langle q_t^2 \rangle_{A_2} = \\ &= \frac{1}{A_1} \frac{2}{\sigma^2} \sum n \cdot P_n^{A_1} - \frac{1}{A_2} \frac{2}{\sigma^2} \sum n \cdot P_n^{A_2} = \\ &= \frac{\sigma_{qN}}{\sigma^2} \left\{ \frac{1}{A_1} \int d^2b \rho_{A_1}^2(b) - \frac{1}{A_2} \int d^2b \rho_{A_2}^2(b) \right\} \end{aligned} \quad (50)$$

$$\Delta q_t^2 = \pi \cdot B \cdot c \left\{ \frac{1}{A_1} \int d^2b \rho_{A_1}^2(b) - \frac{1}{A_2} \int d^2b \rho_{A_2}^2(b) \right\} \quad (51)$$

So we can fix the normalization of the quark-nucleon cross-section from the measured difference of $\langle q_t^2 \rangle$. In the NAL0 experiment²⁸⁾ this difference has been measured for tungsten ($A = 183$) and the deuteron to

$$\Delta q_t^2 = \langle q_t^2 \rangle_W - \langle q_t^2 \rangle_D = (0.17 \pm 0.03 \pm 0.02) \frac{\text{GeV}^2}{c^2}$$

and is independent⁴⁷⁾ of Q^2 in agreement with our physical reasoning. Using hard sphere distributions

$$\rho_A = \frac{A}{\frac{4\pi}{3} R_A^3} 2\sqrt{R_A^2 - b^2} \quad (52)$$

for an analytical calculation we get for the difference between a nucleus A and the deuteron

$$\Delta q_t^2 = \langle q_t^2 \rangle_A - \langle q_t^2 \rangle_D = \frac{9 \cdot B \cdot c}{8\pi} \left\{ \frac{A^{1/3}}{(1.2 \text{ fm})^2} - \frac{2}{(2.8 \text{ fm})^2} \right\} \quad (53)$$

The numerical calculations confirm this result. Using the experimental value of $\langle q_t^2 \rangle_W - \langle q_t^2 \rangle_D$, we obtain $B \cdot c \cdot \pi = 3.24$ and for the quark-nucleon cross-sections the values shown in Fig. 7 as a function of Q^2/Λ^2 with $\Lambda = 0.2 \text{ GeV}$. In the region $5 \text{ GeV} < Q < 12 \text{ GeV}$, these cross-sections can be parametrized as

$$\sigma_{qN}(Q^2) = (4.57 - 0.45 \ln Q^2/\Lambda^2) \text{ mb}$$

If one extends this calculation to a larger interval, one obtains

$$\sigma_{qN}(Q^2) \approx 4300 \text{ mb} / (\ln Q^2/\Lambda^2)^4$$

for $Q > 1 \text{ GeV}$. This empirical determination is consistent with the bound $\sigma_{qN} < 10 \text{ mb}$ for $Q^2 > 10 \text{ GeV}^2$ which has been obtained in deep inelastic muon scattering off nuclear targets⁴⁸⁾.

The Drell-Yan process provides the unique chance to measure this cross-section of a quark with given Q^2 on a nucleon. We remark that it is this value which determines which partons in nuclear relativistic ion collisions contribute to the thermalization to eventually form a plasma. Clearly a measurement of the q_t distribution of lepton-pairs with $Q^2 > (4 \text{ GeV})^2$ in nucleus-nucleus collision is very important. We note that the previous calculation⁴²⁾ of one rescattering only does not obtain the characteristic $A^{1/3}$ dependence of $\langle q_t^2 \rangle_A - \langle q_t^2 \rangle_D$. This is due to the neglect of the higher-order terms in the rescattering series. In spite of this the calculation of Ref. 42) gives too high a $\Delta q_t^2 \approx 0.4 \text{ GeV}^2$ compared to the NAL0 data. This large value probably comes from a too large cross-section σ_{qN} . Having fixed the quark-nucleon cross-section in our calculation we have evaluated the differential transverse cross-section for tungsten and the deuteron given by Eq. (47) and compared to the data. The theoretical results are shown in Fig. 8 for $Q = 5 \text{ GeV}$ and $Q = 8 \text{ GeV}$. The data show a slight energy dependence, but have not been analyzed yet as a function of Q^2 . We would favour such an analysis.

Using our formalism we can give a lower limit of Q^2 related to the target size such that the effect of rescattering is smaller than 10%

$$1 - \frac{1}{\sigma_{qN} \cdot A} \int d^2b \cdot \frac{1}{2} (\sigma_{qN} g(b))^2 = 0.9$$

which gives

$$\ln Q^2/\Lambda^2 > 4.8 \cdot A^{1/12} \quad (54)$$

At asymptotic $Q^2 \rightarrow \infty$ the rescattering effect should vanish since a quark with very high virtuality has a vanishing cross-section in our model. This formula should replace estimates of Ref. 44) based on qualitative arguments.

To conclude this chapter, let us estimate the mean x -loss of a parton after one collision. Knowing the transferred transverse momentum, it is easy to calculate the energy-loss of the scattered parton E_ℓ which is equal to the energy acquired by the struck parton

$$E_\ell = (k_z^2 + q_t^2)^{1/2} - k_z = \left[\left(\frac{1}{2} x\sqrt{s} \right)^2 + q_t^2 \right]^{1/2} - \frac{1}{2} x\sqrt{s} \quad (55)$$

$$\Delta x = \frac{E_\ell}{\frac{1}{2} x\sqrt{s}} \approx \frac{2q_t^2}{x^2 s'} \quad (56)$$

Choosing for $\langle x^2 \rangle \approx 1/9$ we get at $p_L = 194$ GeV an x loss $\Delta x \approx 0.075$, which by accident equals the shift necessary to fit the large $\sqrt{\tau}$ data. But it must be remembered that only a very small fraction $\leq 6\%$ of the partons undergo a rescattering at large Q^2 in our calculation. We can therefore exclude this shift due to rescattering in the same nucleus. In principle along the lines of Ref. 49), one can derive the double differential distribution $dN/dx dq_t$ of the parton after traversing the nucleus. For the experimental data considered here the x -degradation of the parton seems to play not a very important rôle. In the ratio of x_1 distributions of the pion deduced from W and D only the last x -bin seems to be affected.

5. - CALCULATIONS OF TRIPLE DIFFERENTIAL CROSS-SECTIONS IN TUNGSTEN AND THE DEUTERON

In a second attempt we have incorporated the multiple scattering correction in the soft-gluon programme used in Ref. 23) for W and Z productions. We have evaluated at incoming π energies of 194 GeV/c and 284 GeV/c the triple differential cross-sections $1/q_t (d^3\sigma/dq_t dQ dY)$ for π on tungsten including the nuclear corrections to the structure functions

$$\begin{aligned} \frac{d\sigma^{\pi W}}{dQ^2 dq_t^2 dy} &= \frac{1}{2} \sum_{n=1}^{\infty} P_n \int db \cdot b \cdot J_0(bq_t) \frac{4\pi\alpha^2}{9Q^2 S} \sum_f e_f^2 \cdot \\ &\cdot \{ [\bar{q}_\pi(x_1, 1/b^2) q_A(x_2, 1/b^2) + (1 \leftrightarrow 2)] + \dots \} \cdot \\ &\cdot \{ e^{n S(b^2, Q^2, y)} \cdot R(b^2, Q^2, y) \} + Y(q_t^2, Q^2, y) \quad (57) \end{aligned}$$

As one sees from Eq. (57) due to multiple scattering the product of n-Sudakov form factors appears in impact parameter space with the appropriate weights P_n . The quark-distributions for the nucleus are the modified nucleon distributions due to the EMC effect. Since we need evolution starting at small Q^2 we have used the Duke-Owens parametrization of proton and pion structure functions⁵⁰⁾ with $\Lambda = 200$ MeV. (The q_t distribution is not too sensitive to a specific choice.) We have not modified the regular part Y of the gluon corrections, since it is contributing in a region of q_t where the accuracy of the above expression is no longer guaranteed. Second-order corrections would have to be fully calculated to be reliable in this region. We have indicated a theoretical error by adding or subtracting the $O(\alpha_s^2)$ part and compared to the hard-gluon $O(\alpha_s)$ calculation which must be correct for asymptotically large q_t^2 . Note the EMC effect in the rescaling model enters the above expression in a non-trivial manner

$$q_A(x_2, 1/b^2) = q_N(x_2, \frac{1}{b^2} \cdot \left(\frac{R_A}{R_N} \right)^{2\bar{\alpha}_s/\alpha_s(1/b^2)}) \quad (58)$$

The impact parameter integral contains different scale sizes in the structure functions when we discuss the nucleus. In nucleus-nucleus collisions this would manifest itself in a rescaled effective q_t . Define $\tilde{\xi}$ as the rescaling factor

$$\tilde{\xi}(q_t) = \left(\frac{R_A}{R_N} \right)^{2\bar{\alpha}_s/\alpha_s(q_t)}$$

Then,

$$\frac{1}{6} \frac{d\sigma^{AA}}{dq_t^2}(q_t^2) = \tilde{\xi}^2 \cdot \frac{1}{6} \frac{d\sigma^{NN}}{dq_t^2}((\tilde{\xi}q_t)^2) \quad (59)$$

if we neglect multiple scattering corrections. The above equation is not totally true for π -nucleus collisions since the incoming parton in the pion is not affected by the scale change $\tilde{\xi}q_t$. In Fig. 9, we show the comparison of our theoretical results with experiment. Let us recall that previously²⁶⁾ there was no set of parameters Λ and freezing constant Λ_c of α_s , which could fit simultaneously the low and high mass experiments. Especially the theoretical high mass q_t distribution was always too flat. With the new calculation including Q^2 rescaling and multiple scattering effects we get a good description of these data. For the large Q^2 bin, where rescattering is very small ($\leq 5\%$ for $Q=12$ GeV), we are in good agreement with data using Q^2 rescaling (see Figs. 9b and 9e). This model is slightly preferred over x rescaling. At small masses the soft-gluon framework given in Eq. (57) is valid up to $q_t \sim 3$ GeV. As can be seen in Figs. 9a, 9c and 9d, the theoretical curve describes correctly the data. The Q^2 rescaling model is indistinguishable from the x rescaling model. In the kinematical region of $3 \text{ GeV} < q_t < 4.5 \text{ GeV}$ where the soft-gluon resummation formula is unreliable, we have indicated by a shaded area the uncertainty of the calculation. In order to improve the theoretical picture in this q_t range, inclusion of second-order corrections [Ref. 19)] in the formalism of Ref. 23) is needed. Only at very large q_t the hard $O(\alpha_s)$ correction seems to be approached experimentally.

We have also calculated π deuteron cross-sections which have not been analyzed experimentally (see Fig. 10). Ideally the best procedure would first be to examine the Λ parameter and the QCD machinery with these data and then go to the heavier nucleus to see the nuclear effects clearly. This still has to be done. At the moment we are predicting deuteron data from fixing the π -W results.

Using the full formula of Eq. (57) we see that the ratio of transverse cross-sections is sensitive to both nuclear corrections. It increases slower than the calculations of the simplified model mostly because of the EMC effect. If there was a rescaling of q_t due to the EMC effect in the nuclear case like in Eq. (59), the effective quark-nucleon cross-section would have to be modified as $\tilde{\sigma}_{qN} = \tilde{\xi}^2 \sigma_{qN}$, i.e., it would have to be increased by 50% in order to be in agreement with the data. Since the data are integrated over a limited region of Y and Q^2 , we did not try an iterative procedure to fix σ_{qN} exactly. Equation (57) is not useful in the region of intermediate q_t . At large q_t^2 we have

calculated the multiple scattering corrected hard-gluon calculation. We expect that with asymptotic q_t^2 there is a return of the ratio to unity which comes from one hard single scattering event.

The QCD calculation at the discussed energies is much more difficult than at the energy regime of the collider. Reversely we can hope to extract from Drell-Yan experiments stricter controls of our abilities to do higher-order QCD calculations. These must include the nuclear effects of multiple scattering and modified structure functions as we have shown. In the $d\sigma/dQ^2 dx_F$ distributions the required accuracy on Drell-Yan experiments is very high in order to discriminate between different models of nuclear structure functions. Dedicated experiments with protons on nuclei may investigate the nuclear sea-quarks better. In the q_t distributions there also remains the sensitivity of the cross-section to the behaviour of α_s for small q_t^2 . The summation of soft gluons has allowed us to reduce the dependence on yet uncalculable non-perturbative effects but this infra-red sensitivity may give the possibility to investigate the modification of the effective QCD coupling constant in the nuclear medium due to colour dielectric effects⁵¹⁾. In the future we believe it is this interplay of hadron physics with the well-shaped tool of perturbative QCD which will keep Drell-Yan physics interesting.

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APPENDIX

We give in this Appendix the general expression for the cross-section $d\sigma/dQ^2 dx_F$ to first-QCD order. The annihilation piece reads:

$$\begin{aligned}
 \frac{d\sigma^A}{dQ^2 dx_F} = & \frac{4\pi\alpha^2}{9Q^2 S} \int_{x_2}^1 dt_1 \int_{x_2}^1 dt_2 \sum_f e_f^2 \left[\bar{q}_f(t_1, Q^2) q_f(t_2, Q^2) + (1 \leftrightarrow 2) \right] \\
 & * \left\{ \frac{\delta(t_1 - x_1) \delta(t_2 - x_2)}{(x_1 + x_2)} \left[1 + \frac{\alpha_s(Q^2) C_F}{2\pi} \left(1 + \pi^2 + \frac{2\pi^2}{3} \right. \right. \right. \\
 & \left. \left. - \frac{3}{2} \ln \frac{x_1 x_2}{(1-x_1)(1-x_2)} + 2 \ln \frac{x_1}{(1-x_1)} \ln \frac{x_2}{(1-x_2)} \right) \right] \\
 & + \frac{\alpha_s(Q^2) C_F}{2\pi} \frac{\delta(t_2 - x_2)}{(x_1 + x_2)} \left[\frac{t_1^2 + x_1^2}{t_1^2 (t_1 - x_1)_+} \ln \frac{(x_1 + x_2)(1-x_2)}{x_2(t_1 + x_2)} \right. \\
 & \left. + \frac{3}{2(t_1 - x_1)_+} - \frac{2}{t_1} - \frac{3x_1}{t_1^2} \right] + (1 \leftrightarrow 2) \tag{A.1} \\
 & + \frac{\alpha_s(Q^2) C_F}{2\pi} \left[\frac{(t_1 + t_2)(\tau^2 + t_1^2 t_2^2)}{t_1^2 t_2^2 (t_1 + x_2)(t_2 + x_1)} \frac{1}{(t_1 - x_1)(t_2 - x_2)_{++}} \right. \\
 & \left. \left. - \frac{2}{t_1 t_2 (t_1 + t_2)} \right] \right\}
 \end{aligned}$$

$$\frac{d\sigma^c}{dQ^2 dx_F} = \frac{4\pi\alpha^2}{9Q^2 S} \frac{\alpha_s(Q^2)}{2\pi} \int dt_1 \int dt_2 \cdot$$

$$\left\{ \sum_q e_q^2 g_\pi(t_1, Q^2) [q_A(t_2, Q^2) + \bar{q}_A(t_2, Q^2)] \cdot \right.$$

$$\left[\frac{\delta(t_2 - x_2)}{(x_1 + x_2) t_1^3} \left((x_1^2 + (t_1 - x_1)^2) \cdot \ln \frac{(x_1 + x_2)(1 - x_2)}{x_2(t_1 + x_2)} - 6x_2(t_1 - x_1) + t_1^2 \right) \right.$$

$$\left. + \frac{\tau^2 + (t_1 t_2 - \tau)^2}{t_1^3 t_2^2 (t_2 + x_1)} \frac{1}{(t_2 - x_2)_+} + \frac{t_1(t_2 + x_1)(t_2 - x_2) + 2\tau(t_1 + t_2)}{t_1^2 t_2^2 (t_1 + t_2)^2} \right]$$

$$+ (1 \leftrightarrow 2) \}$$

(A.2)

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FIGURE CAPTIONS

- Fig. 1 Quark exchange between nucleons not included in the folding model of Eq. (1).
- Fig. 2 Ratio of F_2^{Au}/F_2^D as a function of x compared to σ^A/σ^D from Ref. 4). Full curve: Q^2 rescaling. Dashed curve: x rescaling.
- Fig. 3a-g Double differential cross-sections $d\sigma/d\tau dx_F$ per nucleon for π^-W as a function of x_F for different $\sqrt{\tau}$ intervals at $p_{lab}^\pi = 200$ GeV/c. Full curve: Q^2 rescaling. Dashed curve: no EMC effect. Dotted-dashed curve: x rescaling. In Figs. 3a and 3g, the dotted curves correspond to the zero-order prediction.
- Fig. 4 First-order graphs. 4a: annihilation diagrams; 4b: Compton diagrams.
- Fig. 5 Soft-gluon emission in $q\bar{q}$ collision.
- Fig. 6 The Sudakov form factor as a function of b for various Q . Full curve: $Q = 5$ GeV. Dashed curve: $Q = 8$ GeV. Dotted-dashed curve: $Q = 12$ GeV.
- Fig. 7 The quark-nucleon cross-section as a function of Q^2/Λ^2 .
- Fig. 8 The ratio σ^W/σ^D as a function of q_t . Triangle points: $p_{lab}^\pi = 286$ GeV/c. Circle points: $p_{lab}^\pi = 140$ GeV (The NA10 data are integrated over the Q range). Full curve: $Q = 5$ GeV. Dashed curve: $Q = 8$ GeV.
- Fig. 9a-f The differential cross-sections $1/q_t d\sigma/dq_t dQdy$ as a function of q_t for several mass bins and energies for π^-W . Full curve: Q^2 rescaling. Dashed curve: x rescaling. The shaded area gives the theoretical uncertainty.
- Fig. 10 The differential cross-section $1/q_t d\sigma/dq_t dQdy|_{y=0}$ as a function of q_t for π^-D . 10a: $Q = 5$ GeV. 10b: $Q = 8$ GeV.

- Figure 1 -

- Figure 2 -

- Figure 3 -

- Figure 4 -

- Figure 5 -

- Figure 6 -

- Figure 7 -

- Figure 8 -

- Figure 9 -

- Figure 10 -