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THE LOW-ENERGY EFFECTIVE FIELD THEORY FROM FOUR-DIMENSIONAL SUPERSTRINGS

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ABSTRACT

We derive the low-energy effective supergravity field theory obtained from a fermionic formulation of four-dimensional superstrings. It contains a single dilaton supermultiplet parametrizing an $SU(1,1)/U(1)$ manifold, three self-conjugate sets of matter supermultiplets parametrizing $SO(m,2)/SO(m) \times SO(2)$ manifolds, and chiral matter supermultiplets parametrizing a product of $SU(M,1)/SU(M) \times U(1)$ manifolds. We derive the Yukawa couplings of the theory and identify some of the self-conjugate matter fields as Higgses. We also discuss the possible pattern of supersymmetry breaking.

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String theory provides an appropriate framework for quantum gravity and the unification of all the particle interactions [1]. The rich variety of suitable conformally-invariant and modular-invariant string theories is now being explored, as are their possible phenomenological consequences at energies below the Planck scale. Earlier efforts were concentrated on superstrings in ten dimensions [2], six of which were compactified in some model-dependent way. For some of these models, the low-energy particle content [3] and their effective no-scale supergravity theory [4] are well-documented. More recently, conformal- and modular-invariant superstring theories have been formulated directly in four dimensions, using either a suitable spectrum of world-sheet fermions [5, 6] or bosonic coordinates [7]. The low-energy effective field theory given by these four-dimensional superstring theories has not yet been discussed, and is the subject of this paper.

We show that the theory contains a single dilaton supermultiplet whose interactions are obtained by a simple truncation of $N = 4$ supergravity [8] and have the familiar $SU(1,1)/U(1)$ structure [9]. The theory also contains three sets of self-conjugate matter supermultiplets Φ_i , whose interactions are obtained by first truncating $N = 4$ supergravity to $N = 2$, and then again to $N = 1$. These fields parametrize $O(m,2)/O(m) \times O(2)$ Kähler manifolds [10]. Finally, the twisted sector yields $N = 1$ chiral matter supermultiplets C parametrizing a product of $SU(M,1)/SU(M) \times U(1)$ manifolds [11]. We derive the Yukawa interactions of the effective field theory, which include Φ^3 trilinear couplings among the self-conjugate fields and ΦC^2 couplings to the matter fields. Therefore we identify Higgs fields as belonging to the self-conjugate Φ sector. Finally, we discuss the possibility that supersymmetry is spontaneously broken by some variant of gaugino condensation [12, 13], and study the appearance of soft supersymmetry breaking for the self-conjugate scalars Φ (e.g. the Higgses) and for the matter fields C (e.g. the squarks and sleptons).

We start with a brief review of the fermionic construction of four-dimensional superstring theories [5, 6]. They have the following world-sheet field content in the light-cone gauge—left-movers: 2 transverse bosonic coordinates $X^i(\tau)$, their fermionic partners $\psi^i(\tau)$, and another 18 Majorana-Weyl fermions x^m, y^m, ω^m ($m = 1, 2, \dots, 6$); right-movers: 2 transverse bosonic coordinates $X^i(\bar{\tau})$ and 44 real fermions $\bar{\psi}^A$ ($A = 1, 2, \dots, 44$). With these particle contents, the left- and right-movers are each conformal anomaly-free, and supersymmetry is realized non-linearly on the left-moving sector alone with, as the supercurrent,

$$G = \psi^i \partial_\tau X_i + \sum_{m=1}^6 x^m y^m \omega^m. \quad (1)$$

A consistent string theory is specified by fixing the boundary conditions on the fermions, which we take to be either periodic (Ramond) or antiperiodic (Neveu-Schwarz), in some modular-invariant combination. The general solution consistent with multi-loop modular invariance is constructed as follows. Let F be the full set of 64 left- and right-moving fermions, choose a ‘basis’ $B \equiv \{B_0 = F, B_1, \dots, B_P\}$ of $P + 1$ subsets of F , and impose the following conditions [6]:

- a) None of the subsets B_i is a product of any two $B_j \circ B_k$ ($\equiv B_j \cup B_k - B_j \cap B_k$) or more of the others.

- b) In order to maintain world-sheet supersymmetry, the fermion parity operators $(-1)^{B_i}$ either commute or anticommute with the supersymmetry charge G [eq. (1)].
- c) In order to respect multi-loop modular invariance, for any subsets B_i in B

$$n(B_i) = 2n(B_i \cap B_j) = 4n(B_i \cap B_j \cap B_k \cap B_l) = 0 \pmod{8} \quad (2)$$

where n is the net number of the left-movers minus right-movers in any subset of F .

- d) Let Ξ be the set of all subsets of F which are products of elements of the basis B . To any element $X \in \Xi$ there corresponds a sector denoted $R_X NS_{F \circ X}$ with the fermions in X periodic and the rest antiperiodic, whose physical states are further restricted by the projections $(-1)^{B_i} = \delta_X C(X|B_i)$, where: i) $\delta_X = -1 (+1)$ if the fermionic coordinates ψ^i belong (do not belong) to the set X , and ii) the coefficients $C(X|B_i) = \pm 1$ are computed starting with an arbitrary choice of the $C(B_i|B_j)$ for $i > j$ and using the rules

$$\left. \begin{aligned} C(X|Y) &= \epsilon_{X \cap Y}^2 C(Y|X) = -\epsilon_X C(X|F \circ X \circ Y) : \epsilon_X = e^{i\frac{\pi}{8}n(X)} \\ C(X|Y_1 \circ Y_2 \circ \dots \circ Y_k) &= (\delta_X)^{k-1} C(X|Y_1) C(X|Y_2) \dots C(X|Y_k) \end{aligned} \right\} \quad (3)$$

The fermionic contribution to the integrand of the one-loop partition-function can then be written as

$$Z_F = \sum_{\alpha \in \Xi} \delta_\alpha \pi \frac{[1 + \delta_\alpha C(\alpha|\beta)(-1)^\beta]}{2} R_\alpha NS_{F \circ \alpha} \quad (4)$$

It was shown in ref. [6] that the string theory will have at least one supersymmetry if and only if the basis B contains the set $S \equiv \{\psi^i, x^m (m = 1, 2, \dots, 6)\}$, and $C(S|Y) = -1$ for any set Y whose intersection with S is empty: $Y \cap S = \emptyset$. A generic string theory with $N = 4$ supersymmetries^{*)} has $B_{N=4} = \{F, S, \zeta_i\} : \zeta_i \cap S = \emptyset$, whilst a generic $N = 2$ superstring theory has $B_{N=2} = \{F, S, b_1, \zeta_i\}$, where $b_1 \cap S = \{\psi^i, x^1, x^2\}$. Finally, the generic $N = 1$ superstring theory is obtained from $B_{N=1} = \{F, S, b_1, b_2, \zeta_i\}$, with $b_2 \cap S = \{\psi^i, x^3, x^4\}$ [6].

Our construction of the effective $N = 1$ supergravity field theory at the tree level of the string topological expansion is based on the observations that the part of the theory containing the graviton, gauge, and some of the matter supermultiplets is a truncation of an $N = 4$ supergravity theory [8], whilst the remaining matter supermultiplets can be obtained by truncating an $N = 2$ supergravity theory [10]. Specifically, the sectors of the $N = 1$ string theory formed by the products of the elements F , S , and ζ_i of the set $B_{N=1}$ above are just those states of the $N = 4$ theory defined by

*) It is possible to construct other $N = 4$ models with more than one set of the form of S , as we will see later.

$B_{N=4}$ above which are retained after the projections $(-1)^{b_1}$ and $(-1)^{b_2}$. Similarly, the sectors of the $N = 1$ theory which involve the set b_1 (or b_2) (or $S \circ b_1 \circ b_2$) can be obtained by a truncation of the $N = 2$ theory defined by the corresponding $B_{N=2}$ above, using the projections $(-1)^{b_2}$ etc.

We first consider in detail the truncation of the bosonic part of the $N = 4$ supergravity theory, concentrating on the case when the massless bosons come from the $R_\phi NS_F$ sector which contains the graviton multiplet. (The generalization to the case when massless gauge multiplets exist in $R_{\mathfrak{g}_1} NS_F \circ \mathfrak{g}_1$ is straightforward.) In the left- (right-) moving part, the tachyon mass level is -1 (-2). Each boson (antiperiodic fermion) oscillator raises the mass level by $2(1)$ units. Therefore the massless bosonic states consistent with the $(-1)^S = -1$ and $(-1)^{S_1} = +1$ projections of this theory are:

- (a₄) $|\psi^i\rangle_L \otimes |X^j\rangle_R$, which contains the graviton (symmetric and traceless), dilaton (trace), and an antisymmetric part which is equivalent in four dimensions to a pseudoscalar;
- (b₄) $|x^m\rangle_L \otimes |X^j\rangle_R$ ($m = 1, 2, \dots, 6$), which are the six gauge bosons that together with (a₄) constitute the bosons of the $N = 4$ gravity supermultiplet and gauge a non-compact $U(1)^6$ group;
- (c₄) $|\psi^i\rangle_L \otimes |\bar{\psi}^A \bar{\psi}^B\rangle_R$, which are the massless gauge bosons of a gauge group G ; and
- (d₄) $|x^m\rangle \otimes |\bar{\psi}^A \bar{\psi}^B\rangle_R$, which are the six real scalar superpartners of the gauge bosons (c₄).

The bosonic part of the Lagrangian for $N = 4$ supergravity coupled to Yang-Mills is [8]:

$$\begin{aligned}
\mathcal{L} = & -\frac{1}{2}R + D_\mu \phi_\alpha D^\mu \phi^\alpha - \frac{1}{2} \text{Tr} \partial_\mu \phi_{ij} \partial^\mu \phi^{ij} \\
& - \frac{1}{4} \text{Re} \left(\frac{\phi^1 - \phi^2}{\phi^1 + \phi^2} \right) \text{Tr} F_{\mu\nu}^2 - \frac{1}{4} \text{Im} \left(\frac{\phi^1 - \phi^2}{\phi^1 + \phi^2} \right) \text{Tr} F_{\mu\nu} \tilde{F}^{\mu\nu} \\
& - \frac{\text{Tr}(F_{\mu\nu} \phi_{ij}) \text{Tr}(F^{\mu\nu} \phi^{ij})}{2|\phi^1 + \phi^2|^2} \\
& + |\phi^1 + \phi^2|^2 \left(\frac{1}{2} \text{Tr} |[\phi_{ik}, \phi_{kj}]|^2 - \frac{4}{9} |\text{Tr} \phi^{kl} [\phi_{ik}, \phi_{kj}]|^2 \right)
\end{aligned} \tag{5}$$

where

$$\phi_\alpha = \eta_{\alpha\beta} (\phi^\beta)^*, \quad \phi^\alpha \phi_\alpha = \phi_1 \phi_1^* - \phi_2 \phi_2^* = 1 \tag{6a}$$

$$\left. \begin{aligned}
\phi_{ij}^I &= -\phi_{ji}^I, \quad (\phi_{ij}^I)^* = \phi^{ijI} = -\frac{1}{2} \epsilon^{ijkl} \phi_{kl}^I, \\
\text{Tr}(\phi_{ij} \phi^{kl}) &= -\frac{1}{2} \delta_i^k \delta_j^l
\end{aligned} \right\} \tag{6b}$$

After elimination of the constraints (6a), the remaining two real components ϕ_α correspond to the dilaton and the two-index antisymmetric tensor among the states (a₄) above. The fields ϕ_{ij}^I correspond to the 6 dim G physical states (d₄) above, and include the 36 negative metric auxiliary scalars of the

vector multiplets related to the vectors (b₄) above. The latter are eliminated using the 36 constraints of local SU(4) and equation (6b). The derivative $\mathbb{D}_\mu$ covariantizes with respect to local SU(4) as well as G, while D_μ covariantizes a local U(1) only. Finally, in equation (5), R is the curvature scalar and the $F_{\mu\nu}$ are field strengths for G and U(1)⁶.

Now we make the truncation to $N = 2$ of the $N = 4$ spectrum (a₄) to (d₄) above, using the projection $(-1)^{b_1} = -1$ and introducing the notation $\bar{\psi}^a$ for the subset of $\bar{\psi}^A$ fermions contained in b₁, $\bar{\psi}^{a'}$ for the rest. The remaining states are:

- (a₂) $|\psi^i\rangle_L \otimes |X^j\rangle_R$, which includes the graviton and dilaton;
- (b₂) $|X^{1,2}\rangle_L \otimes |X^j\rangle_R$, which yields one gauge field in the dilaton multiplet and one in the graviton multiplet;
- (c₂) $|\psi^i\rangle_L \otimes |\bar{\psi}^a \bar{\psi}^b, \bar{\psi}^{a'} \bar{\psi}^{b'}\rangle_R$, which gauge a subgroup G' of G, with their scalar partners $|X^{1,2}\rangle_L \otimes |\bar{\psi}^a \bar{\psi}^b, \bar{\psi}^{a'} \bar{\psi}^{b'}\rangle_R$; and
- (d₂) $|X^{3,4,5,6}\rangle_L \otimes |\bar{\psi}^a \bar{\psi}^{b'}\rangle_R$, which belong to scalar $N = 2$ multiplets.

Thus, from the $N = 4$ gauge multiplets (c₄, d₄) above, those corresponding to unbroken generators of G reduce to $N = 2$ vector multiplets, whilst the broken generators of G reduce to $N = 2$ scalar multiplets.

At the level of the Lagrangian (5), this $N = 2$ truncation means setting the gauge fields of G/G' : $W_\mu^{[ab']} = 0$. Also, the scalars $\phi^{[AB]}$ split into $\phi^{[ab'], [a'b']}$ in the adjoint of G' and $\phi^{[ab']}$ in some non-trivial representation R' of G' , and the rest are set to zero. Similarly, out of the 36 scalars ϕ_{ij}^I we keep only the $\phi_{12}^{1,2}$ which combine with the two vectors (b₂) above to form the $N = 2$ vector multiplet, and the 16 scalars $\phi_{13,14}^{3,4,5,6}$ which are in auxiliary $N = 2$ scalar multiplets, and set the rest to zero. The above auxiliary scalar fields are eliminated using the reduced form of the constraints (6b), as follows. We denote the reduced $\phi_{12} = \Phi_0$, $\phi_{13} = \Phi_1$, and $\phi_{14} = \Phi_2$. Then (6b) tells us that $\phi_{34} = -\Phi_0^*$, $\phi_{24} = \Phi_1^*$, $\phi_{23} = -\Phi_2^*$, and

$$\text{Tr } \Phi_0 \Phi_0^* = -\frac{1}{2}, \quad \text{Tr } \Phi_0^2 = 0, \quad (7a)$$

$$\text{Tr } \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} (\Phi_1^*, \Phi_2^*) = -\frac{1}{2} \mathbb{1}, \quad \text{Tr } \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} (\Phi_1, \Phi_2) = 0. \quad (7b)$$

Use of these 13 constraints and the remaining local $SU(2) \times SU(2) \times U(1)$ subgroup of SU(4) eliminates all the negative metric auxiliary fields. From the constraints (6a) we see that the scalars of the $N = 2$ vector multiplets parametrize the Kähler manifold $O(m,2)/O(m) \times O(2)$, where $m \equiv \dim G'$. From the constraints (6b) we see that the scalar fields of the $N = 2$ multiplets coming from the truncation of $N = 4$ parametrize the quaternionic space $O(p,4)/O(p) \times O(4)$, where $p \equiv \dim G - \dim G'$. The constraints (6a) show that the scalars of the dilaton multiplet parametrize the Kähler manifold $SU(1,1)/U(1)$. Note that the scalar potential derived from equation (5) is positive semidefinite with flat directions, because the six negative signature components of the scalars correspond to U(1)⁶ and hence do not enter in the potential. Finally, in the notation of ref. [10], the

homogeneous second-order function F which specifies the interactions of the $N = 2$ gauge multiplets and gravity is

$$F = \frac{i X_1 X_a \bar{X}^a}{X_0} \quad (8)$$

where X_0 , X_1 , and X_a belong to the $N = 2$ auxiliary, dilaton, and gauge multiplets, respectively.

Finally we truncate the $N = 2$ theory above to $N = 1$ using the second constraint $(-1)^{b_2} = -1$. Similarly to the previous truncation, the right-moving fermions $\bar{\psi}^a$ are split into the $\bar{\psi}^{\tilde{a}}$ ($\bar{\psi}^{\tilde{a}}$) which belong (do not belong) to b_2 , and the $\bar{\psi}^{a'}$ are split into the $\bar{\psi}^{\tilde{a}'}$ and $\bar{\psi}^{\tilde{a}'}$. The massless physical states with $(-1)^{b_2} = -1$ are:

- (a₁) $|\psi^i\rangle_L \otimes |X^j\rangle_R$, the graviton and dilaton as before;
- (b₁) $|\psi^i\rangle_L \otimes |\bar{\psi}^{\tilde{a}} \bar{\psi}^{\tilde{b}}, \bar{\psi}^{\tilde{a}'} \bar{\psi}^{\tilde{b}'}, \bar{\psi}^{\tilde{a}} \bar{\psi}^{\tilde{b}'}, \bar{\psi}^{\tilde{a}'} \bar{\psi}^{\tilde{b}'}\rangle_R$, which are gauge vectors of some subgroup G'' of G' ,
- (c₁) $|x^{1,2}\rangle_L \otimes |\bar{\psi}^{\tilde{a}} \bar{\psi}^{\tilde{b}}, \bar{\psi}^{\tilde{a}'} \bar{\psi}^{\tilde{b}'}\rangle_R$, which are two scalars in a representation R_0 of G'' ,
- (d₁) $|x^{3,4}\rangle_L \otimes |\bar{\psi}^{\tilde{a}} \bar{\psi}^{\tilde{b}'}, \bar{\psi}^{\tilde{a}} \bar{\psi}^{\tilde{b}'}\rangle_R$, which are two scalars in a representation R_1 of G'' ,
- (e₁) $|x^{5,6}\rangle_L \otimes |\bar{\psi}^{\tilde{a}} \bar{\psi}^{\tilde{b}'}, \bar{\psi}^{\tilde{a}} \bar{\psi}^{\tilde{b}'}\rangle_R$, which are two scalars in a representation R_2 of G'' .

Thus the $N = 2$ graviton multiplet just gives us the $N = 1$ graviton multiplet, whilst the $N = 2$ gauge multiplets of G' give $N = 1$ vector multiplets of G'' , and the rest are $N = 1$ matter multiplets.

At the field level, we set to zero the gauge fields $W_{\mu}^{[ab], [a'b']}$ of G'/G'' . The only components of Φ_0 which survive are those corresponding to the generators of G'/G'' as well as the auxiliary scalars which completed the $N = 2$ multiplets of the vectors $|x^{1,2}\rangle_L \otimes |X^j\rangle_R$. The constraints on Φ_0 are still given by the relations (7a), but the trace is taken over the non-vanishing components of Φ_0 only. Thus Φ_0 parametrizes the manifold $O(m_0, 2)/O(m_0) \times O(2)$, where $m_0 = \dim R_0 = \dim G' - \dim G''$. The surviving components of Φ_1 and Φ_2 are (d₁) and (e₁) above, together with the auxiliary negative metric fields that accompanied the $N = 2$ gauge field $|x^{3,4,5,6}\rangle_L \otimes |X^i\rangle_R$. Since these components of Φ_1 and Φ_2 correspond to different generators of G' , $\text{Tr } \Phi_1 \Phi_2 = 0 = \text{Tr } \Phi_1 \Phi_2^*$. Therefore the constraints (7b) reduce to

$$\text{Tr } \Phi_i \Phi_i^* = -\frac{1}{2}, \quad \text{Tr } \Phi_i \Phi_j = 0 \quad (i, j = 1, 2). \quad (9)$$

Since these constraints factorize, Φ_1 and Φ_2 parametrize $[O(m_1, 2)/O(m_1) \times O(2)] \times [O(m_2, 2)/O(m_2) \times O(2)]$, where $m_i = \dim R_i$ ($i = 1, 2$). Of course, the dilaton field is not affected by this second truncation and still parametrizes $SU(1, 1)/U(1)$.

To specify the complete $N = 1$ supergravity Lagrangian we need only determine the Kähler potential $\mathcal{G}$ and the chiral gauge kinetic function f_{ab} [14]. The factorized form of the scalar manifold means that

$$\mathcal{G} = \mathcal{G}_D + \mathcal{G}_0 + \mathcal{G}_1 + \mathcal{G}_2 + \ln |w|^2 + \mathcal{G}_c \quad (10)$$

where W is an analytic function of the self-conjugate matter fields, and G_c is the Kähler potential for the chiral multiplets which we discuss later. It is well known that

$$G_0 = \ln(z + z^*) \quad (11)$$

with $z \equiv (\phi_1 - \phi_2)/(\phi_1 + \phi_2)$ in terms of the fields ϕ_α ($\alpha = 1, 2$) of equations (5) and (6a). The $G_{0,1,2}$ pieces of G are calculated as follows. Writing the components of Φ_0 as $(\eta_0^0, \eta_0^1, \eta_0^i)$, the constraints (7a) [cf. eq. (9)] take the forms

$$-|\eta_0^0|^2 - |\eta_0^1|^2 + |\eta_0^i|^2 = \frac{1}{2}, \quad -\eta_0^0{}^2 - \eta_0^1{}^2 + \eta_0^i{}^2 = 0. \quad (12)$$

The phase of η_0^0 can be rotated away using the local $U(1)$, and the constraints (12) take the forms

$$\eta_0^0 = \frac{1}{\sqrt{2}} \left(-1 - |\hat{\eta}_0^1|^2 + |\hat{\eta}_0^i|^2 \right)^{-\frac{1}{2}}, \quad \hat{\eta}_0^1{}^2 = -1 + \hat{\eta}_0^i{}^2, \quad (13)$$

in terms of the scaled fields $\hat{\eta}_0^{1,i} \equiv \eta_0^{1,i}/\eta_0^0$. Then

$$G_0 = -2 \ln \eta_0^0 = \ln \left(1 + \sqrt{(1 - \hat{\eta}_0^1{}^2)(1 - \hat{\eta}_0^i{}^2)} - \hat{\eta}_0^1 \hat{\eta}_0^{i*} \right) \quad (14)$$

and $G_{1,2} = -2 \ln \eta_{1,2}^0$ are similar. Note that all the fields appearing in $G_{0,1,2}$ must belong to self-conjugate representations of the residual gauge group G'' . The superpotential W can be determined by inspection of the $N = 1$ gravitino mass term coming from the corresponding term in $N = 4$, which is [8]

$$\begin{aligned} \mathcal{L}_m &= -\frac{2}{3} (\phi_1 + \phi_2) \text{Tr} \phi^{kl} [\phi_{1k}, \phi_{1l}] \bar{\chi}_\mu \sigma^{\mu\nu} \chi_\nu + \text{h.c.} \\ &\rightarrow -4 (\phi_1 + \phi_2) \text{Tr} \Phi_0 [\Phi_1, \Phi_2] \bar{\chi}_\mu \sigma^{\mu\nu} \chi_\nu + \text{h.c.} \end{aligned} \quad (15)$$

After absorbing a phase in the gravitino fields, the coefficient in equation (15) can be equated to the standard $m_{3/2} = e^{-G/2}$ of $N = 1$ supergravity, yielding

$$e^{-G/2} = 4 |\phi_1 + \phi_2| |\text{Tr} \Phi_0 [\Phi_1, \Phi_2]| = 4 \left(\frac{2}{z + z^*} \right)^{\frac{1}{2}} \eta_0^0 \eta_1^0 \eta_2^0 |\text{Tr} \hat{\eta}_0 [\hat{\eta}_1, \hat{\eta}_2]| \quad (16)$$

where the $\hat{\eta}_{1,2}^i$ have been rescaled by the $\eta_{1,2}^0$ as in equation (13). Formula (16) confirms the factorization (10) and yields

$$W = 2\text{Tr} \hat{\eta}_0 [\hat{\eta}_1, \hat{\eta}_2]. \quad (17)$$

To complete the specification of the $N = 1$ supergravity theory, we note that the chiral kinetic function

$$f_{ab} = z \delta_{ab} \quad (18)$$

as can be read directly from equation (5).

Two comments are in order. One is that the explicit analysis above was done in the $R_\phi \text{NS}_F$ sector of the generic four-dimensional string theory. The effect of massless states in the $R_{\bar{\eta}_1} \text{NS}_{F \circ \bar{\eta}_1}$ sectors would be to enlarge the gauge groups involved, but their analysis would follow the same pattern. Secondly, so far we have restricted ourselves to the truncation of $N = 4$, which does not describe the interactions of additional chiral $N = 1$ multiplets which can exist in the generic string spectrum and are needed to describe quarks and leptons. They can be included in the theory at the $N = 2$ level, as we now illustrate briefly with a simple example. If we choose $b_1 = \{\psi^i, x^1, x^2, y^3, \dots, y^6, \bar{\psi}^1, \dots, \bar{\psi}^{16}\}$ in the generic $N = 2$ theory $B_{N=2}$ above, then the projections $(-1)^S$ and $(-1)^{b_1}$ applied to the $R_{b_1} \text{NS}_{F \circ b_1}$ ($R_{S \circ b_1} R_{F \circ S \circ b_1}$) sector yield prospective chiral fermions (and their chiral scalar partners). These chiral multiplets parametrize an $SU(M,2)/SU(M) \times SU(2) \times U(1)$ manifold. This can be seen by a reduction of an $N = 4$ theory specified by $B'_{N=4} = \{F, S, S', \bar{b}, \zeta_i\}$, where $S'(\bar{b})$ contains the left-(right-) movers of the set b_1 above, with $C(S'|X) = -1$ if $X \cap S' = \emptyset$, and we keep only the sectors in which S' and $\bar{b}$ appear in the combination $S' \circ \bar{b} = b_1$. In order to obtain chiral asymmetry in the $N = 1$ theory, the set b_2 in $B_{N=1}$ above may take the form $b_2 = \{\psi^i, x^3, x^4, \omega^1, \omega^2, \omega^5, \omega^6, \bar{\psi}^1 \dots \bar{\psi}^{10}, \bar{\psi}^{17} \dots \bar{\psi}^k\}$, with $k = 22, 30$, or 38 . The projection $(-1)^{b_2}$ truncates the theory to $N = 1$, yielding two sets of chiral $O(16)$ generations, each of which parametrizes a pair of $SU(M,1)/SU(M) \times U(1)$ manifolds with $M = 64$ in this simple example. The gauge symmetry may be reduced by the projections $(-1)^{f_i}$, whilst the Yukawa couplings couple the self-conjugate matter multiplet Φ_0 to pairs of chiral matter fields. Thus we abstract the following form of Kähler potential for the chiral matter fields $C \equiv \Sigma_\alpha C^{(\alpha)}$:

$$\mathcal{G}_c = \sum_\alpha \mathcal{G}^{(\alpha)} + \ln |W_c|^2 \quad (19)$$

where the $\mathcal{G}^{(\alpha)}$ are typical $SU(M,1)/SU(M) \times U(1)$ [11] terms

$$\mathcal{G}^{(\alpha)} = \ln (1 - C^{(\alpha)} \cdot C^{(\alpha)}) \quad (20)$$

and

$$W_c = 2\text{Tr}' \Phi_0 [C, C] \quad (21)$$

where the trace Tr' is quite model-dependent. In the simple model introduced above, the Yukawa couplings must be consistent with an E_8 invariance broken by the $(-1)^{F_1}$ projections, and couple different $C^{(\alpha)}$. We see from the form of equation (21) that it is natural to identify the Higgses as coming from the self-conjugate matter multiplets Φ .

The effective supergravity theory [eqs. (10), (11), (14), and (17) to (21)] is significantly different from that found [4] by a truncation of ten-dimensional supergravity designed to mimic the heterotic string compactified on a Calabi-Yau manifold [3]. In particular, that theory had two gauge-singlet dilaton multiplets (S, T) corresponding to the dilaton scalar of the $D = 10$ theory and the overall size of the compactification manifold. The v.e.v.'s of these two dilaton fields determined between them the gauge coupling constant (S) and the string coupling constant (T). Here, the gauge coupling is determined by the string coupling, which is in turn determined by the v.e.v. of z [eqs. (11) and (18)]. This could be fixed by a mechanism analogous to the gaugino condensation [12, 13] used in the previous Calabi-Yau truncation [4]. The condensation of gauginos in some subgroup G''' of G'' would give an effective superpotential

$$W_0(z) = h \exp(-\lambda z) : \lambda = \frac{8\pi^2}{C_2(G''')} . \quad (22)$$

The superpotential (22) alone cannot fix z , but should be combined with some other term $W_1(z)$. In the case of Calabi-Yau truncation, it was thought [13, 15] that this could be generated by a v.e.v. for a three-index antisymmetric tensor field with indices in the six compactified dimensions, but this mechanism is not directly applicable to four-dimensional strings. Alternatively, the condensation of gauginos in a second subgroup of G'' might provide a $W_1(z)$ suitable for fixing z [16]. Let us assume that z is determined in some such way whose details are not important. In the case of Calabi-Yau truncation [3, 4], the value of T and hence of the string coupling was not determined by a tree-level analysis of the effective low-energy theory, nor was the scale $m_{3/2} = e^{-g/2}$ of supersymmetry breaking^{*)}. The hope has been expressed that both T and $m_{3/2}$ might be determined by radiative corrections in the effective low-energy theory [18]. There is an analogous problem here: to determine $m_{3/2} = [1/(z + z^*)]^{1/2} \eta_0^0 \eta_1^0 \eta_2^0 |W|$ we must specify the values of $\eta_{0,1,2}^0$. Non-perturbative or higher-order terms in the string topological expansion might play an important role in their determination, but we concentrate here on radiative corrections in the low-energy effective theory. It is convenient to discuss these using an alternative representation of the fields in $O(m,2)/O(m) \times O(2)$ manifolds:

$$\mathcal{G}_0 = \ln \sum_{a=1}^{m_0} (\bar{z}_{a_0} + \bar{z}_{a_0}^*)^2 \Rightarrow \eta_0^0 = \frac{1}{\sum_{a=1}^{m_0} (\bar{z}_{a_0} + \bar{z}_{a_0}^*)^2} . \quad (23)$$

*) It may also be possible to construct realistic models with supersymmetry broken by D-terms [17], a possibility we will study elsewhere.

The one-loop radiative corrections to the effective potential have the form

$$V_1 = \frac{1}{64\pi^2} \Lambda^4 f(m_{3/2}^2/\Lambda^2) \quad (24)$$

where Λ is a cut-off of order m_P provided by the string excitations. When we insert the representation (23) for η^0 into (24) we see that V_1 has an $O(m)$ symmetry. If there is a non-trivial minimum, it breaks $O(m) \rightarrow O(m-1)$ and yields $(m-1)$ Goldstone bosons corresponding to real parts of the ξ_a . In addition, there are n Goldstones corresponding to the imaginary parts of the ξ_a . We expect all these fields to be pseudo-Goldstone bosons with masses given at the two-loop level by gauge and/or Yukawa interactions. Turning to the chiral matter scalars: if $\mathcal{G}_c$ [eq. (19)] had appeared independently of $\mathcal{G}_0$ in the one-loop effective potential, their soft supersymmetry-breaking scalar masses could have been non-zero. However, the forms (10) and (24) of $\mathcal{G}$ and V_1 tell us that

$$\left. \frac{\partial^2 V_1}{\partial c^i \partial c_j^\dagger} \right|_{c=0} \propto \frac{\partial V_1}{\partial \mathcal{G}} \propto \frac{\partial V_1}{\partial \mathcal{G}_0} \quad (25)$$

which vanishes at the one-loop level in any non-trivial minimum of the type discussed above in connection with the ξ_a . Thus the squared scalar masses do not appear before the two-loop level, whereas we expect non-zero gaugino masses at the one-loop level [19]. Similar results were obtained previously [20] in the case of the Calabi-Yau truncation, but it was subsequently pointed out [21] that Kaluza-Klein modes could in principle modify that effective low-energy theory in such a way that scalar masses would be induced at the one-loop level. Kaluza-Klein modes are of course absent in the four-dimensional approach to superstrings used here, so the vanishing of the one-loop scalar masses is a hard result here.

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