CERN-TH.4622/86

A PHENOMENOLOGICAL QUARK MATTER EQUATION OF STATE

I: THE ZERO TEMPERATURE CASE

Frédérique Grassi^{*)}

CERN - Geneva
and
Observatoire de Paris, 92195 Meudon, France

A B S T R A C T

It is necessary to know the equation of state of quark-gluon matter to discuss properties of the primordial quark-hadron transition or of condensed stellar objects. In this paper, the methods of relativistic quantum many-body theory are applied to the study of quarks interacting through some phenomenological potential, at zero temperature. In the case of colour-dependent potentials, the Hartree field is known to be (approximately) zero. In the case of colour-independent confining potentials, it is shown that the Hartree approximation for the quark plasma yields results close to those obtained by Kalman for the scalar plasma, and also has some common features with the model derived by Walecka and Chin for nuclear matter: quarks undergo a first-order phase transition, they are in a massive collective bound state at low density and have a massless ideal gas behaviour at high density. An interesting feature of this calculation is that it is independent of the actual shape of the interquark potential provided that this potential is screened by pair creation - as is normally expected. These results for quark matter will be generalized to non-zero temperatures and the non-virtual gluon exchange contribution to the equation of state computed, in a following paper.

^{*)} Address after 1 January 1987: Fermilab, Batavia, IL 60510, U.S.A.

1. - INTRODUCTION

Quark matter is expected to play an important rôle in cosmology and in astrophysics. In cosmology, if a quark-hadron transition actually occurred, it could have a number of interesting consequences such as the production of density fluctuations or formation of primordial objects^{1),2)} - while the spectrum of density fluctuations must usually be added by hand in scenarios of galaxy formation, appearance of small black holes³⁾ or strange nuggets⁴⁾⁻⁸⁾ - which might provide an explanation for the missing problem, gravitational wave emission - whose detection could provide information on details concerning the transition^{4),9),10)}, etc. In astrophysics, quark matter might be present at the centre of neutron stars and quark stars might even exist. They are expected to have maximum masses, maximum moments of inertia and maximum redshifts close to those of neutron stars, but might perhaps be distinguished from them, by causing glitches in the pulse period of pulsars - as proposed by Fechner and Joss¹¹⁾ - or by cooling particularly fast - as first suggested by Iwamoto¹²⁾. In addition, quark matter droplets, either emitted by these objects or produced in hadronic collisions in the upper atmosphere, might be the clue to understanding anomalies in cosmic rays such as the centauro events or the - still controversial - Cygnus X-3 muon excess [see 13), 4) and their followers].

However, possible quark matter applications to cosmology and astrophysics are plagued with the fact that we do not know the exact equation of state of quark matter. For instance, the possibility that strange matter might be more stable than nuclear matter depends on what its free energy is at a given density. In the same way, the existence of stable quark stars is still an unresolved issue, because it depends (in particular) on whether or not the density at which the quark phase becomes energetically favourable - i.e., the transition takes place - is above the maximum density allowed for a stable neutron star.

In the absence of an exact equation of state, one must resort to using either a perturbation-based equation of state or a phenomenological equation of state. Perturbative equations of state were used for instance to study quark core stars and quark stars by Kislinger and Morley¹⁴⁾, Freedman and McLerran¹⁵⁾, Baluni¹⁶⁾, ... They have been computed to increasing orders in the coupling constant, starting from the simple ideal Fermi gas treatment of Collins and Perry¹⁷⁾ or the first-order calculations - close to those carried out in QED - by Baym and Chin¹⁸⁾ at zero temperature, or Chin¹⁹⁾ at non-zero temperature, up to the more complex calculations at order α_s^2 and zero temperature of Freedman

and McLerran²⁰⁾ and Baluni²¹⁾, or at order $\alpha_s^{3/2}$ and non-zero temperature by Shuryak²²⁾ and Kapusta²³⁾; the most general result the author is aware of being that of Toimela²⁴⁾ at non-zero chemical potential, quark masses and temperature, at order $\alpha_s^2 \ln \alpha_s$.

But these perturbative calculations are marred by uncertainties. First, one does not know precisely their domain of validity: the running coupling constant for strong interactions, α_s , at a given spacelike momentum q^2 is, in the one-loop approximation, of the order of $1/\ln(-q^2/\Lambda^2)$, where Λ is a quantity to be obtained from experimental data. If Λ turns out to be small, α_s would be small at reasonable densities, while if Λ is large, α_s would be small only at high densities, possibly excluding neutron stars or heavy ion collisions (though a more refined analysis taking into account two-loop effects or temperature dependence could decrease the value of α_s for a given density). Second, as mentioned for instance by Kapusta²³⁾, the choice of a renormalization point to implement formulae for the grand potential is not obvious when one non-massless quark flavour, or several different quark flavours are present. Finally, while at zero temperature, one may think that the perturbative calculations will converge, the situation is not the same at non-zero temperature. At low order, there is the so-called plasmon puzzle: the $\alpha_s^{3/2}$ contribution is larger than the α_s contribution for a range of reasonable values of α_s . While this might still be cured - perhaps for instance by a self-consistent calculation of the electric gluon mass including the plasmon effect²⁵⁾, it is believed [see, for instance the review papers by Linde²⁶⁾, Shuryak²⁷⁾ or Gross, Pisarski and Jaffe²⁸⁾], that the grand potential cannot be computed beyond the third order, because the next terms would then be finite but of the same order of magnitude.

Due partly to the fact that perturbative calculations rest on weak ground, but mostly to the possibility that non-perturbative effect - not yet included in perturbative calculations as is the plasmon effect - might be at work, phenomenological quark matter equations of state have been developed and used in astrophysics and cosmology in many papers. Because of its simplicity, the MIT bag equation of state has been widely applied to investigate the quark-hadron phase transition [see, for example, 11),29),30) for stars, 1),31)-37) for cosmology]. The main problem raised by the use of the MIT bag equation of state for astrophysical and cosmological purposes is that it has been designed to describe matter at ordinary temperatures and densities. Hence it is not clear how one should extrapolate it to the higher temperatures or densities expected to prevail in stars or in the early Universe. A few attempts have been made though to

compute the temperature and density dependence of the MIT vacuum constant [e.g., 38) and 39)]. In addition, other equations of state have been investigated, many of which are based [e.g., 40) and 41)] on the idea developed by Vinciarelli⁴²⁾, Bardeen, Chanowitz, Drell, Weinstein and Yan⁴³⁾, Friedberg and Lee⁴⁴⁾, ..., that confinement arises inside hadrons because some phenomenological field would increase the quark masses outside hadrons. While the origin of this phenomenological field is a little mysterious, it was suggested by Wagoner and Steigman⁴⁵⁾ to describe the interactions between quarks by using phenomenological potentials such as those hypothesized in potential models to fit experimental data on quarkonium. This idea was taken up by Olive⁴⁶⁾ to describe the quark-gluon plasma in the Thomas-Fermi approximation and the resulting equation of state was applied in studies of the primordial quark-hadron phase transition by Källman⁴⁷⁾ and Schramm and Olive⁴⁸⁾.

In the model presented below, at zero temperature, and in a forthcoming paper at non-zero temperature, an approach somehow similar to that of Olive is followed. The main technical differences for the treatment of quarks is that a Dirac equation with an interquark potential is used and a Hartree approximation is made. This has the interesting consequence that results will be independent of the actual shape of the potential provided that it is screened by pair creation. In particular, it will be shown that the quarks are subject to a first-order phase transition, that they are in a massive collective bound state at low density and behave like a massless ideal gas at high density. The treatment of gluons will be conceptually different from that of Olive: they are not considered as particles interacting through some - not yet put in evidence - phenomenological potential, but as suggested by Hakim and Collin⁴⁹⁾, as modes propagating in the quark plasma, i.e., they are emitted when the plasma is disturbed from equilibrium, in much the same way that a solid disturbed from equilibrium would emit phonons. (This has the advantage that, provided that quarks are confined, gluons are confined as well.)

The outline of this paper is as follows. In Section 2, the part played by colour in the Hartree approximation is examined and (finite) Hartree self-consistent equations are obtained. The numerical solution of these equations for potentials with scalar effects, or potentials with scalar and vector effects, is discussed in Sections 3 and 4, respectively. Finally, concluding remarks, reservations and future problems are discussed in Section 5.

2. - HARTREE APPROXIMATION

The model presented below is relativistic, as for astrophysical and cosmological purposes one is primarily interested in light quarks. The equation of motion chosen is Dirac's, which has the advantage of being a little simpler than Bethe-Salpeter's and, contrary to Klein-Gordon's, allows us to keep track of the spins. [One may also want to add that, according to quarkonia spectroscopy, all these three equations yield sensible results - compare for instance 50) and 51).]

In the Dirac equation, the following structure of the interaction potential is assumed:

$$V(r) = V_v(r) \gamma_\mu^{(1)} \gamma^{\mu(2)} - V_s(r) 1^{(1)} 1^{(2)} \quad (2.1)$$

V must have a vector component to account for one-gluon exchange, which should dominate the interaction at short distances. A scalar component is necessary, phenomenologically, because in the non-relativistic expansion of the Bethe-Salpeter equation, the spin-orbit term has opposite sign for scalar or vector potentials, the right sign to get the observed ordering of the 3P_j levels of charmonium being that of scalar potentials⁵²⁾. [This conclusion was also reached in a more rigorous approach in 53).] The confining potential is thus usually assumed to be this scalar component. It is, however, possible that the confining potential also enters the vector component [e.g., 54)]. As a matter of fact, one does not know whether the interaction potential does not have a more complicated structure (with tensor, axial or pseudo-tensor terms). However, potentials of the form (2.1) are the simplest ones leading to good theoretical predictions for quarkonia. Also their structure may be considered as just an extrapolation to quark matter of the structure of the potentials used to describe nuclear matter (and account for its properties) in mean field theories.

Finally, colour must be taken into account. While it is normal to multiply the one-gluon exchange term by a factor of $\lambda^{(1)} \cdot \lambda^{(2)} / 4$, it is questionable to do the same thing for the confining part. First there is no theoretical reason to do this: it is enough to multiply the one-gluon exchange term by $\lambda^{(1)} \cdot \lambda^{(2)} / 4$ to render the colour-singlet energetically favourable or to get the correct sign and relative sizes of meson and baryon spin-spin splittings. Second, it has been said that a colour-independent confining potential could lead to forces between the quarks in different hadrons, either confining or strongly repulsive, if a description in terms of potential models was to be used. But the use of these potential models in this case is rather controversial⁵⁵⁾. On the other hand,

however, if potential models developed to describe quark-antiquark states are extrapolated to describe baryons, a factor of 1/2 between the anti-triplet quark-quark potential and the singlet quark-antiquark potential is phenomenologically favoured [see, for instance, 56)], and so this could be a reason for multiplying the confining potential also, by $\lambda^{(1)} \cdot \lambda^{(2)} / 4$.

Because they lead to quite different equations of state, the following four possibilities for V_s and V_v of Eq. (2.1) are considered:

$$\begin{cases} V_s(r) = \alpha V_c(r) \\ V_v(r) = V_G(r) \tilde{\lambda}^{(1)} \tilde{\lambda}^{(2)} / 4 - \beta V_c(r) \end{cases}$$

with either (I): $\alpha = 1$ and $\beta = 0$ or (II): α and $\beta \neq 0$ (2.2a)

$$\begin{cases} V_s(r) = \alpha V_c(r) \tilde{\lambda}^{(1)} \tilde{\lambda}^{(2)} / 4 \\ V_v(r) = [V_G(r) - \beta V_c(r)] \tilde{\lambda}^{(1)} \tilde{\lambda}^{(2)} / 4 \end{cases}$$

with either (III): $\alpha = 1$ and $\beta = 0$ or (IV): α and $\beta \neq 0$ (2.2b)

As will be seen, it is not necessary to specify the actual shape of V_G and V_c or the values of α and β in our approximation.

Starting from the Dirac equation for one particle, it is straightforward to derive the following expression, relating the two-point and four-point Green functions:

$$(i\partial - m)G(x, y) = \delta^4(x - y) - i \int d^3z G(x, z; y, z^+) V(|x - z|) \Big|_{z_0 = x_0} \quad (2.3)$$

where the notation z^+ means that z^{0+} is infinitesimally greater than z_0 .

Insertion of the Hartree approximation

$$G(x, y; z, t) = G(x, z) G(y, t) \quad (2.4)$$

into (2.3) then leads to an equation in terms of two-point Green functions only:

$$[i\partial - m + i \int d^3z G(z, z^+) V(|x - z|) \Big|_{z_0 = x_0}] G(x, y) = \delta^4(x - y) \quad (2.5)$$

The Hartree potential

$$U_H = -i \int d^3z G(z, z^+) V(|x-z|) \Big|_{z_0=x_0} \quad (2.6)$$

has been said to be identically zero for scalar or vector potentials of type III and IV, i.e., when (2.1) is multiplied by an overall factor of $\lambda^{(1)} \cdot \lambda^{(2)} / 4$ [see, for example, 57) and 58)]. This is somehow surprising as, in the spirit of the Hartree approximation, a quark whose colour is, say, red, interacting with other quarks whose colours are such as to ensure colour neutrality of the whole plasma, is replaced by a free red quark in an exterior field whose colour must be antired [see Eqs. (2.3) and (2.5)]. Hence intuitively U_H should not be null. Let us see this more precisely.

The computational argument leading to $U_H \equiv 0$ is usually presented as follows. At equilibrium, the two-point Green function is proportional to the unity matrix in colour space (because in the Green functions, the operators ϕ and $\bar{\phi}$ - where all three colours participate equally - are applied to colour neutral states). Components of the Hartree field may then be written in the form:

$$U_{H\mu\sigma}^{pn} = -i \int d^3z G(z, z^+)_{mm}^{\lambda\tau} V(|x-z|)_{\mu\sigma, \tau\lambda} \frac{\lambda_a^{(2)p\tau} \lambda^{(3)a, mm}}{4} \quad (2.7)$$

[in this formula - and in what follows - Greek letters denote Lorentz indices, Latin letters from the beginning of the alphabet stand for gluon colour indices while Latin letters from the middle of the alphabet are used for the quark colour indices. Also interaction potentials of type III and IV have been written in the form $V(r) = \mathcal{V}(r) \lambda^{(1)} \cdot \lambda^{(2)} / 4$.]

The λ matrices have no diagonal element, except for $\lambda^{(3)}$ and $\lambda^{(8)}$. But these two matrices are traceless, hence U_H should vanish identically.

However, when reasoning this way, one ignores that the Hartree field felt by a quark with indices μ and n is due to all the other quarks except itself, hence the summation in (2.7) should be carried out only for $m \neq n$. For fermions, this is due to the fact that, before the Hartree approximation is performed, (2.3) contains a four-point Green function $G_2(x, z; y, z^+) \Big|_{z_0=x_0}$. In this function, the operators $\phi(x)$ and $\phi(z)$ must create or absorb, in the < 1 , particles whose quantum numbers (colour, spin, energy level, etc.) must be different.

In order to compute U_H exactly, one may add and subtract the term corresponding to "m = n, spin z = spin x, etc.":

$$\begin{aligned}
 U_{H\mu\sigma}^{pn} = & -i \sum_{\substack{\text{all } m's \\ \text{all spin } z \\ \text{etc}}} \int d^3z G(z, z^+)^{\lambda\tau}_{mm} \mathcal{V}(|x-z|)_{\mu\sigma, \tau\lambda} \frac{\lambda_a^{(z)pn} \lambda^{(z)a, mm}}{4} \\
 & + i \int d^3z G(z, z^+)^{\lambda\tau}_{nn} \Big|_{\substack{\text{spin } z = \text{spin } x \\ \text{etc}}} \mathcal{V}(|x-z|)_{\mu\sigma, \tau\lambda} \frac{\lambda_a^{(z)pn} \lambda^{(z)a, nn}}{4}
 \end{aligned} \quad (2.8)$$

The first term is zero for reasons mentioned previously and the second term may be computed by noting that:

$$G(z, z^+)^{\lambda\tau}_{nn} \Big|_{\substack{\text{spin } z = \text{spin } x \\ \text{etc}}} \lambda^{(z)a, nn} \lambda_a^{(z)pn} = \frac{1}{3} G(z, z^+)^{\lambda\tau}_{nn} \Big|_{\substack{\text{spin } z = \text{spin } x \\ \text{etc}}} \quad (2.9)$$

where n is fixed.

Hence for type III and IV potentials:

$$U_{H\mu\sigma}^{pn} = -\frac{1}{3} \int d^3z \mathcal{V}(|x-z|)_{\mu\sigma, \tau\lambda} [-i G(z, z^+)^{\lambda\tau}_{nn} \Big|_{\substack{\text{spin } z = \text{spin } x \\ \text{etc}}}] 1^{pn} \quad (2.10)$$

where

$$\begin{aligned}
 \mathcal{V}(|x-z|) &= V_G \gamma_\mu \gamma^\mu - \alpha V_c 11 \quad \text{for type III potentials} \\
 \mathcal{V}(|x-z|) &= [V_G - \beta V_c] \gamma_\mu \gamma^\mu - \alpha V_c 11 \quad \text{for type IV potentials}
 \end{aligned}$$

While for type I and II potentials, a similar calculation, with unity matrices instead of λ -matrices, leads to:

$$\begin{aligned}
 U_{\mu\sigma}^{nm} = & - \int d^3z \mathcal{V}_c(|x-z|)_{\mu\sigma, \tau\lambda} [-iG(z, z^+)_{nn}^{\lambda\tau}] 1^{nm} \\
 & + \int d^3z \mathcal{V}_c(|x-z|)_{\mu\sigma, \tau\lambda} [-iG(z, z^+)_{nn}^{\lambda\tau} \Big|_{\substack{\text{spin } z = \text{spin } x \\ \text{etc}}}] 1^{nm} \\
 & - \frac{1}{3} \int d^3z \mathcal{V}_c(|x-z|)_{\mu\sigma, \tau\lambda} [-iG(z, z^+)_{nn}^{\lambda\tau} \Big|_{\substack{\text{spin } z = \text{spin } x \\ \text{etc}}}] 1^{nm}
 \end{aligned}
 \tag{2.11}$$

where

$$\mathcal{V}_G(|x-z|) = V_G \gamma_\mu \gamma^\mu$$

$$\mathcal{V}_c(|x-z|) = \alpha V_c 11 \quad \text{for type I potentials}$$

$$\mathcal{V}_c(|x-z|) = -\beta V_c \gamma_\mu \gamma^\mu + \alpha V_c 11 \quad \text{for type II potentials}$$

In some cases, the term on the right-hand side of (2.10) and the second and third terms on the right-hand side of (2.11) might actually be negligible. In order to see when this happens, let us develop the field operators in terms of particle operators

$$\psi(x) = \int d^4k \sum_{m=1}^3 \sum_{\alpha=1,2} [b_\alpha^m(k) u_\alpha^m(k) e^{-ikx} + d_\alpha^m(k)^\dagger v_\alpha^m(k) e^{ikx}] \tag{2.12}$$

Thus:

$$\begin{aligned}
 & 1_{\tau\lambda} [-iG(z, z^+)_{nn}^{\lambda\tau} \Big|_{\substack{\text{spin } z = \text{spin } x \\ \text{etc}}}] \\
 & = \int d^4k \sum_m \sum_\alpha [\langle b_\alpha^m(k)^\dagger b_\alpha^m(k) \rangle \bar{u}_\alpha^m(k) u_\alpha^m(k) + \langle d_\alpha^m(k) d_\alpha^m(k)^\dagger \rangle \bar{v}_\alpha^m(k) v_\alpha^m(k)] \Big|_{\substack{\text{quantum} \\ \text{numbers} = \\ \text{those of particle} \\ \text{at } x}}
 \end{aligned}$$

$$= \begin{cases} \langle b_\alpha^m(k)^\dagger b_\alpha^m(k) \rangle \bar{u}_\alpha^m(k) u_\alpha^m(k) & \text{if there is a quark at } x \\ \langle d_\alpha^m(k) d_\alpha^m(k)^\dagger \rangle \bar{v}_\alpha^m(k) v_\alpha^m(k) & \text{if there is an antiquark at } x \end{cases}$$

$$= \begin{cases} \frac{1}{e^{\beta(p_0 - \mu)} + 1} & \text{if there is a quark at } x \end{cases} \tag{2.13a}$$

$$= \begin{cases} \frac{1}{e^{\beta(-p_0 + \mu)} + 1} & \text{if there is an antiquark at } x \end{cases} \tag{2.13b}$$

In the same way, one gets:

$$(\gamma_0)_{\tau\lambda} [-i G(z, z^+)]_{nn}^{\lambda\tau} \Big|_{\text{etc}}^{\text{spin } z = \text{spin } x} = \begin{cases} \frac{p_0}{m} \frac{1}{e^{\beta(p_0 + \mu)} + 1} & \text{if there is a quark at } x \quad (2.13c) \\ \frac{-p_0}{m} \frac{1}{e^{\beta(-p_0 + \mu)} + 1} & \text{if there is an antiquark at } x \quad (2.13d) \end{cases}$$

and

$$(\gamma_i)_{\tau\lambda} [-i G(z, z^+)]_{nn}^{\lambda\tau} \Big|_{\text{etc}}^{\text{spin } z = \text{spin } x} = \begin{cases} \frac{p_i}{m} \frac{1}{e^{\beta(p_0 + \mu)} + 1} & \text{if there is a quark at } x \quad (2.13e) \\ \frac{-p_i}{m} \frac{1}{e^{\beta(-p_0 + \mu)} + 1} & \text{if there is an antiquark at } x \quad (2.13f) \end{cases}$$

= 0 in the rest frame of particle at x

So one sees that the first term in (2.11) will contain terms such as (2.13a), (2.13c), (2.13e) multiplied by the number of quarks and terms like (2.13b), (2.13d), (2.13f) multiplied by the number of antiquarks, so that the second term and probably the third term as well, should be small compared to the first one. In the same way, since (2.10) contains only the contribution of one particle, it should not be very big. Hence when a large number of particles is present, one may assume

$$U_H \sim 0 \quad \text{for type III-IV potentials} \quad (2.14a)$$

$$\sim - \int d^3z [-i G(z, z^+)] \mathcal{V}_c(1x-z1) \quad \text{for type I-II potentials} \quad (2.14b)$$

So type III and IV potentials are expected to have no effect in the Hartree approximation while type I potentials have scalar effects and type II potentials have both scalar and vector effects. [One should note, in addition, that when a Hartree-Fock approximation is made, terms appearing in the Hartree approximation, like the right-hand side of (2.10) or the second and third terms of (2.11), are exactly cancelled by similar ones produced by the Fock approximation⁵⁹⁾].

In order to see what these effects are precisely, one must solve Eq. (2.5) with the Hartree field defined in (2.6), now given by (2.14b). Equation (2.5) may be written in momentum space as below⁶⁰⁾

$$(\not{p} - m - U_H) G(p) = 1 \quad (2.15)$$

A glance at (2.1) and (2.5) suggests that U_H only has scalar and vector components. Assuming that the medium is uniform, the coefficient of the vector component can only be a factor of the four velocity. Hence in the rest frame, U_H may actually be decomposed as:

$$U_H = (U_0^H 1 + U_3^H \gamma_0) 1 \quad (2.16)$$

where 1 denotes the unity matrix in either the γ -matrix space or colour space according to the context. Equation (2.15) then reads:

$$\begin{aligned} & [\gamma_0 \not{p}_0^H - \vec{\gamma} \vec{p} - m_H] G(p) = 1 \\ & \text{where} \\ & p_0^H = p_0 - U_0^H \\ & m_H = m + U_3^H \quad (\text{this quantity is designated thereafter as the effective mass}) \end{aligned} \quad (2.17)$$

This leads to the spectral function:

$$A(p) = \frac{\gamma_0 \not{p}_0^H - \vec{\gamma} \vec{p} + m_H}{2 \sqrt{\vec{p}^2 + m_H^2}} [2\pi \delta(p_0^H - \sqrt{\vec{p}^2 + m_H^2}) - 2\pi \delta(p_0^H + \sqrt{\vec{p}^2 + m_H^2})] \cdot 1 \quad (2.18)$$

Hence, it follows that

$$\begin{aligned} -i G(z, z^+) &= \int \frac{d^4 p}{(2\pi)^4} G'(p) = \int \frac{d^4 p}{(2\pi)^4} \frac{1}{e^{\beta(p_0 - \mu)} + 1} A(p) \\ &= \int \frac{d^3 p}{(2\pi)^3} \frac{1}{e^{\beta(U_0^H + \sqrt{\vec{p}^2 + m_H^2} - \mu)} + 1} \frac{\gamma_0 \sqrt{\vec{p}^2 + m_H^2} - \vec{\gamma} \vec{p} + m_H}{2 \sqrt{\vec{p}^2 + m_H^2}} \\ &+ \int \frac{d^3 p}{(2\pi)^3} \frac{1}{e^{\beta(-U_0^H + \sqrt{\vec{p}^2 + m_H^2} + \mu)} + 1} \frac{-\gamma_0 \sqrt{\vec{p}^2 + m_H^2} - \vec{\gamma} \vec{p} + m_H}{2 \sqrt{\vec{p}^2 + m_H^2}} \\ &- \int \frac{d^3 p}{(2\pi)^3} \frac{-\gamma_0 \sqrt{\vec{p}^2 + m_H^2} - \vec{\gamma} \vec{p} + m_H}{2 \sqrt{\vec{p}^2 + m_H^2}} \cdot 1 \end{aligned} \quad (2.19)$$

The first two integrals represent the contribution of matter while the last one accounts for vacuum fluctuations. However, owing to the fact that our approach is essentially phenomenological, these fluctuations should not show up; the matter part only should be used throughout the calculations. Inserting the matter part

of (2.19) into (2.14b) then leads to the following set of self-consistent coupled equations:

$$U_0^H = \int d^3z V_v(|z-x|) \cdot n_{q-\bar{q}} \quad (2.20a)$$

$$U_s^H = - \int d^3z V_s(|z-x|) \cdot \bar{n}_{q+\bar{q}} \quad (2.20b)$$

where

$$n_{q-\bar{q}} \equiv 6 \int \frac{d^3p}{(2\pi)^3} \left[\frac{1}{e^{\beta(U_0^H + \sqrt{\vec{p}^2 + m_u^2} - \mu)} + 1} - \frac{1}{e^{\beta(-U_0^H + \sqrt{\vec{p}^2 + m_u^2} + \mu)} + 1} \right]$$

$$\bar{n}_{q+\bar{q}} \equiv 6 \int \frac{d^3p}{(2\pi)^3} \frac{m_u}{\sqrt{\vec{p}^2 + m_u^2}} \left[\frac{1}{e^{\beta(U_0^H + \sqrt{\vec{p}^2 + m_u^2} - \mu)} + 1} + \frac{1}{e^{\beta(-U_0^H + \sqrt{\vec{p}^2 + m_u^2} + \mu)} + 1} \right]$$

and it is implicit that only the confining part of V_v should be used in Eq. (2.20a), namely $V_v = -\beta V_c$. It may be noticed that (2.20a,b) are rather similar to the equations usually derived in the Hartree approximation, starting from a Dirac Lagrangian density with Yukawa coupling to scalar and vector particles^{40),61)-63)}:

$$U_0^H \equiv \frac{g_v^2}{m_v^2} n_{F-\bar{F}} \quad (2.21a)$$

$$U_s^H \equiv - \frac{g_s^2}{m_s^2} \bar{n}_{F+\bar{F}} \quad (2.21b)$$

[where $n_{F-\bar{F}}$ and $\bar{n}_{F+\bar{F}}$ are defined as in (2.20a,b) up to a factor of 1/3]. Also for purely scalar confining potentials (i.e., $\beta=0$), one only needs to solve an equation resembling (2.21b). Such an equation is usually encountered when dealing with a scalar plasma (i.e., fermions with a scalar Yukawa coupling)⁶⁴⁾⁻⁶⁶⁾.

However, expressions (2.20a,b) may still contain infinities, as $\int d^3z V_v(|x-z|)$ and $\int d^3z V_s(|x-z|)$ are infra-red divergent, for instance in the case of a linear confining potential. But, as advocated by Kogut and Susskind⁶⁷⁾, infinitely rising potentials are expected to be screened by the creation of quark-antiquark pairs. One may either suppose that there exist some fixed screening lengths c_s and c_v and use:

$$\int d^3r [-\beta V_c(r)] = \int_0^{c_v} d^3r [-\beta V_c(r)] \quad (2.22a)$$

$$\int d^3r [\alpha V_c(r)] = \int_0^{c_s} d^3r [\alpha V_c(r)] \quad (2.22b)$$

[in which case the solution of (3a,b) would depend on the shape of V_c] or assume that the maximum value of $|U_0^H|$ and $|U_s^H|$ felt by a quark is twice the Fermi energy ϵ_F (and not just twice the quark mass, because in dense matter, pairs can only be created above the Fermi sea). Hence one has

$$\int d^3r [-\beta V_c(r)] \Big|_{|U_0^H|=2\epsilon_F} = \left[\frac{U_0^H}{m_{q\bar{q}}(U_0^H, U_0^H; \mu)} \right] \Big|_{|U_0^H|=2\epsilon_F} \quad (2.23a)$$

$$\int d^3r [\alpha V_c(r)] \Big|_{|U_s^H|=2\epsilon_F} = \left[\frac{-U_s^H}{\bar{m}_{q\bar{q}}(U_s^H, U_s^H; \mu)} \right] \Big|_{|U_s^H|=2\epsilon_F} \quad (2.23b)$$

Now, if for instance $-\beta V_c$ (resp. αV_c) is switched off at some distance c , this distance is expected to be independent of U_0^H (resp. U_s^H). Though it may depend on the density through the chemical potential or on corrections to the particle mass due to the existence of the scalar confining potential (resp. on corrections to the chemical potential due to the existence of the vector confining potential). So (2.23a) [resp. (2.23b)] holds for all values of U_0^H (resp. U_s^H) and defining

$$\Delta \equiv \frac{3m^2}{\pi^2} \int d^3r [-\beta V_c(r)] \quad (2.24a)$$

$$\Gamma \equiv \frac{3m^2}{\pi^2} \int d^3r [\alpha V_c(r)] \quad (2.24b)$$

the set of equations which must be solved is

$$\frac{U_0^H}{m} = \Delta \left(1 + \frac{U_s^H}{m}\right)^3 \int d\tilde{z} \tilde{z}^1 \left\{ \frac{1}{e^{\beta \left[\frac{U_0^H}{m} + \left(1 + \frac{U_s^H}{m}\right) \sqrt{\tilde{z}^2 + 1} - \frac{H}{m} \right]} + 1} - \frac{1}{e^{\beta \left[\frac{U_0^H}{m} + \left(1 + \frac{U_s^H}{m}\right) \sqrt{\tilde{z}^2 + 1} + \frac{H}{m} \right]} + 1} \right\} \quad (2.25a)$$

$$\frac{U_s^H}{m} = -\Gamma \left(1 + \frac{U_s^H}{m}\right)^3 \int d\tilde{z} \frac{\tilde{z}^2}{\sqrt{\tilde{z}^2 + 1}} \left\{ \frac{1}{e^{\beta \left[\frac{U_0^H}{m} + \left(1 + \frac{U_s^H}{m}\right) \sqrt{\tilde{z}^2 + 1} - \frac{H}{m} \right]} + 1} + \frac{1}{e^{\beta \left[\frac{U_0^H}{m} + \left(1 + \frac{U_s^H}{m}\right) \sqrt{\tilde{z}^2 + 1} + \frac{H}{m} \right]} + 1} \right\} \quad (2.25b)$$

$$\Delta = \left\{ \frac{\frac{U_0^H}{m}}{\left(1 + \frac{U_s^H}{m}\right)^3 \int d\tilde{z} \tilde{z}^1 \left\{ \frac{1}{e^{\beta \left[\frac{U_0^H}{m} + \left(1 + \frac{U_s^H}{m}\right) \sqrt{\tilde{z}^2 + 1} - \frac{H}{m} \right]} + 1} - \frac{1}{e^{\beta \left[\frac{U_0^H}{m} + \left(1 + \frac{U_s^H}{m}\right) \sqrt{\tilde{z}^2 + 1} + \frac{H}{m} \right]} + 1} \right\}} \right\} \Big|_{\frac{U_0^H}{m} = 2\epsilon_F} \quad (2.25c)$$

$$\Gamma = \left\{ \frac{-\frac{U_s^H}{m}}{\left(1 + \frac{U_s^H}{m}\right)^3 \int d\tilde{z} \frac{\tilde{z}^2}{\sqrt{\tilde{z}^2 + 1}} \left\{ \frac{1}{e^{\beta \left[\frac{U_0^H}{m} + \left(1 + \frac{U_s^H}{m}\right) \sqrt{\tilde{z}^2 + 1} - \frac{H}{m} \right]} + 1} + \frac{1}{e^{\beta \left[\frac{U_0^H}{m} + \left(1 + \frac{U_s^H}{m}\right) \sqrt{\tilde{z}^2 + 1} + \frac{H}{m} \right]} + 1} \right\}} \right\} \Big|_{\frac{U_0^H}{m} = 2\epsilon_F} \quad (2.25d)$$

where dimensionless variables have been introduced (and it is implicit that T and ϵ_F are expressed in unit of m).

In the Hartree approximation, the potential never appears alone but always as an integrand [see (2.6)]. In addition, here, it is interesting to note that in the set of equations (2.24a-d), V_v and V_s have completely disappeared (because of the way screening has been introduced). Hence their solution will be independent of the particular shape of the potential or of the way screening occurred (namely, are V_v and V_s exponentially damped? Do they stop sharply at a certain distance? etc.). If, however, one wants to compute the screening lengths, the shape of the potential and the way it is screened should be specified, say, for instance

$$-\int d^3r [\alpha V_c(r)] = \int_0^c d^3r (kr - U) \quad (2.26)$$

Then once (2.24a-d) have been solved, the value of Γ which has been obtained may be used to solve (2.25) in terms of the screening length c [which will now depend on μ contrary to c_s and c_v of Eqs. (2.22a,b)]. Note that two different cut-offs have been introduced, one for U_0^H , the other one for U_s^H . This is normal as there is no reason for assuming that the scalar and vector components of the Hartree field have similar cut-offs. For instance, in the case of nuclear matter, these cut-offs are the masses of the exchanged scalar and vector particles. Finally, while at non-zero temperatures the integrals in (2.24a-d) must be evaluated numerically, at zero temperature, they take on the simpler form:

$$\left\{ d\xi \xi^2 \left[\frac{1}{e^{\beta \left[\frac{U_0^H}{m} + \left(1 + \frac{U_s^H}{m}\right) \sqrt{\xi^2 + 1} - \frac{\mu}{m} \right] + 1}} - \frac{1}{e^{\beta \left[-\frac{U_0^H}{m} + \left(1 + \frac{U_s^H}{m}\right) \sqrt{\xi^2 + 1} + \frac{\mu}{m} \right] + 1}} \right] \right\} \Big|_{T=0}$$

$$= \begin{cases} \frac{P_F^3}{3} & \text{if } \frac{\mu}{m} - \frac{U_0^H}{m} \geq 1 + \frac{U_s^H}{m} \\ -\frac{P_F^3}{3} & \text{if } \frac{U_0^H}{m} - \frac{\mu}{m} \geq 1 + \frac{U_s^H}{m} \\ 0 & \text{otherwise} \end{cases} \quad (2.27a)$$

$$\left\{ d\xi \frac{\xi^2}{\sqrt{\xi^2 + 1}} \left[\frac{1}{e^{\beta \left[\frac{U_0^H}{m} + \left(1 + \frac{U_s^H}{m}\right) \sqrt{\xi^2 + 1} - \frac{\mu}{m} \right] + 1}} + \frac{1}{e^{\beta \left[-\frac{U_0^H}{m} + \left(1 + \frac{U_s^H}{m}\right) \sqrt{\xi^2 + 1} + \frac{\mu}{m} \right] + 1}} \right] \right\} \Big|_{T=0}$$

$$= \begin{cases} \frac{1}{2} [P_F \sqrt{P_F^2 + 1} - \ln(P_F + \sqrt{P_F^2 + 1})] & \text{if } \left(\frac{U_0^H}{m} - \frac{\mu}{m} \right)^2 \geq \left(1 + \frac{U_s^H}{m} \right)^2 \\ 0 & \text{otherwise} \end{cases}$$

$$\text{where } P_F = \sqrt{\left(\frac{U_0^H}{m} - \frac{\mu}{m} \right)^2 / \left(1 + \frac{U_s^H}{m} \right)^2 - 1} \quad (2.27b)$$

Equations (2.24a-d) will be solved numerically in the next two sections. Once this is done, the equation of state may be computed easily. The (generic) phenomenological Lagrangian leading to the equation of motion (2.3) is

$$L = \int d^3x \bar{\Psi}(x) (i\partial - m) \Psi(x) - \frac{1}{2} \iint d^3x d^3z \bar{\Psi}(x) \bar{\Psi}(z) V(|x-z|) \Psi(z) \Psi(x) \quad (2.28)$$

From this, one can compute the energy momentum tensor. On the other hand, one may assume that the medium is uniform and isotropic, in which case

$$T_{\mu\nu} = (\epsilon + p) u_\mu u_\nu - p g_{\mu\nu} \quad (2.29)$$

where p is the pressure and ϵ is the energy density. In this way one can show that

$$\epsilon = \frac{\langle T^{00} \rangle}{V} = \frac{1}{V} \int d^3x \bar{\Psi}(x) \frac{i\gamma_0 \partial_0 + i\vec{\gamma} \vec{\partial} + m}{2} \Psi(x) = \int \frac{d^4p}{(2\pi)^4} \frac{p_0 \gamma_0 + \vec{p} \vec{\gamma} + m}{2} G^c(p) \quad (2.30a)$$

$$p = \frac{\langle T^{ii} \rangle}{3V} = \frac{1}{V} \int d^3x \bar{\Psi}(x) \frac{i\gamma_0 \partial_0 - i\vec{\gamma} \vec{\partial}/3 + m}{2} \Psi(x) = \int \frac{d^4p}{(2\pi)^4} \frac{p_0 \gamma_0 - \vec{p} \vec{\gamma}/3 + m}{2} G^c(p) \quad (2.30b)$$

These equations hold exactly and in the case of the Hartree approximation they may be implemented with (2.19) and yield:

$$\begin{aligned} \frac{\epsilon}{m^4} = & 3 \int \frac{d^3\vec{z}}{(2\pi)^3} \left(1 + \frac{U_3^H}{m}\right) \left\{ \frac{U_0^H}{e^{\left[\frac{U_0^H}{m} + \left(1 + \frac{U_3^H}{m}\right) \sqrt{\vec{z}^2 + 1} - \frac{\mu}{m}\right]_{+1}}} - \frac{1}{e^{\left[\frac{U_0^H}{m} + \left(1 + \frac{U_3^H}{m}\right) \sqrt{\vec{z}^2 + 1} + \frac{\mu}{m}\right]_{+1}}} \right\} \\ & + \left(1 + \frac{U_3^H}{m}\right) \left(\sqrt{\vec{z}^2 + 1} + \frac{\vec{z}^2}{\sqrt{\vec{z}^2 + 1} + \frac{1 + U_3^H/m}{\sqrt{\vec{z}^2 + 1}}} \right) \left\{ \frac{1}{e^{\left[\frac{U_0^H}{m} + \left(1 + \frac{U_3^H}{m}\right) \sqrt{\vec{z}^2 + 1} - \frac{\mu}{m}\right]_{+1}}} + \frac{1}{e^{\left[\frac{U_0^H}{m} + \left(1 + \frac{U_3^H}{m}\right) \sqrt{\vec{z}^2 + 1} + \frac{\mu}{m}\right]_{+1}}} \right\} \end{aligned} \quad (2.31a)$$

$$\begin{aligned} \frac{p}{m^4} = & 3 \int \frac{d^3\vec{z}}{(2\pi)^3} \left(1 + \frac{U_3^H}{m}\right) \left\{ \frac{U_0^H}{e^{\left[\frac{U_0^H}{m} + \left(1 + \frac{U_3^H}{m}\right) \sqrt{\vec{z}^2 + 1} - \frac{\mu}{m}\right]_{+1}}} - \frac{1}{e^{\left[\frac{U_0^H}{m} + \left(1 + \frac{U_3^H}{m}\right) \sqrt{\vec{z}^2 + 1} + \frac{\mu}{m}\right]_{+1}}} \right\} \\ & + \left(1 + \frac{U_3^H}{m}\right) \left(\sqrt{\vec{z}^2 + 1} - \frac{\vec{z}^2/3 + \frac{1 + U_3^H/m}{\sqrt{\vec{z}^2 + 1}}}{\sqrt{\vec{z}^2 + 1}} \right) \left\{ \frac{1}{e^{\left[\frac{U_0^H}{m} + \left(1 + \frac{U_3^H}{m}\right) \sqrt{\vec{z}^2 + 1} - \frac{\mu}{m}\right]_{+1}}} + \frac{1}{e^{\left[\frac{U_0^H}{m} + \left(1 + \frac{U_3^H}{m}\right) \sqrt{\vec{z}^2 + 1} + \frac{\mu}{m}\right]_{+1}}} \right\} \end{aligned} \quad (2.31b)$$

Again one may note that these equations have a particularly simple form at zero temperature. They read:

$$\frac{\varepsilon}{m^4} = \frac{3}{2\pi^2} \left(1 + \frac{U_s^H}{m}\right)^3 \left\{ \pm \frac{U_0^H}{m} \frac{P_F^3}{3} + \frac{P_F}{2} \sqrt{P_F^2 + 1} - \frac{1}{2} \ln(P_F + \sqrt{P_F^2 + 1}) \right. \\ \left. + \left(1 + \frac{U_s^H}{m}\right) \left[\frac{P_F}{4} \sqrt{P_F^2 + 1}^3 + \frac{P_F^3}{4} \sqrt{P_F^2 + 1} - \frac{P_F}{2} \sqrt{P_F^2 + 1} + \frac{1}{4} \ln(P_F + \sqrt{P_F^2 + 1}) \right] \right\} \quad (2.32a)$$

$$\frac{n}{m^4} = \frac{3}{2\pi^2} \left(1 + \frac{U_s^H}{m}\right)^3 \left\{ \pm \frac{U_0^H}{m} \frac{P_F^3}{3} - \frac{P_F}{2} \sqrt{P_F^2 + 1} + \frac{1}{2} \ln(P_F + \sqrt{P_F^2 + 1}) \right. \\ \left. + \left(1 + \frac{U_s^H}{m}\right) \left[\frac{P_F}{4} \sqrt{P_F^2 + 1}^3 - \frac{P_F^3}{12} \sqrt{P_F^2 + 1} - \frac{1}{4} \ln(P_F + \sqrt{P_F^2 + 1}) \right] \right\} \quad (2.32b)$$

As for the density, it may be obtained in a straightforward manner from (2.20a) and (2.23a):

$$n_{q\bar{q}} = \frac{3m^2}{\pi^2} \frac{U_0^H}{\Delta} \quad (2.33)$$

So we now have all the equations necessary to describe the thermodynamics of our quark plasma and we may turn to their numerical solution.

3. - POTENTIALS WITH PURELY SCALAR EFFECTS IN THE HARTREE APPROXIMATION

Type I potentials have purely scalar effects and one only needs to solve

$$\frac{U_s^H}{m} = \begin{cases} -\frac{\Gamma}{2} \left(1 + \frac{U_s^H}{m}\right)^3 \left[P_F \sqrt{P_F^2 + 1} - \ln(P_F + \sqrt{P_F^2 + 1}) \right] & \text{if } \frac{\mu}{m} \geq 1 + \frac{U_s^H}{m} \\ 0 & \text{otherwise} \end{cases} \quad (3.1a)$$

where

$$P_F = \sqrt{\left(\frac{\mu}{m}\right)^2 - \left(1 + \frac{U_s^H}{m}\right)^2} - 1$$

and

$$\Gamma = \left\{ \frac{-\frac{U_s^H}{m}}{\left(1 + \frac{U_s^H}{m}\right)^3 \frac{1}{2} \left[p_F^r \sqrt{p_F^{r^2} + 1} - \ln(p_F^r + \sqrt{p_F^{r^2} + 1}) \right]} \right\} \Big|_{\frac{U_s^H}{m} = \pm \frac{2\mu}{m}} \quad (3.1b)$$

where

$$p_F^r = \begin{cases} \sqrt{\left(\frac{\mu}{m}\right)^2 / \left(1 + \frac{U_s^H}{m}\right)^2 - 1} & \Big|_{\frac{U_s^H}{m} = \pm \frac{2\mu}{m}} \quad \text{if } \frac{\mu}{m} \geq \left(1 + \frac{U_s^H}{m}\right) \Big|_{\frac{U_s^H}{m} = \pm \frac{2\mu}{m}} \\ 0 & \text{otherwise} \end{cases}$$

In order to get a first insight into the solution, it is assumed that the screening length - and therefore Γ as well - is constant [this corresponds in fact to the ansatz (2.22a)]. For Γ constant and positive, (3.1a) is similar to the gap equation of the scalar plasma: it has been studied at zero temperature by Kalman⁶⁵⁾ and at non-zero temperature by Diaz-Alonso and Hakim^{64),66)}. As we are reminded in Fig. 1a, it leads to a possible mass transition for large Γ 's. With our definition (2.24b) for Γ , it is also possible that this quantity is negative. In this case, as can be inferred from (3.1a), the effective mass is an always increasing function of the chemical potential and no first-order phase transition occurs. This can be checked in Fig. 1b.

The equation of state, for constant positive Γ is, a priori, different from that of a scalar plasma. However, the same results hold qualitatively. In Fig. 2a, the pressure has been plotted as a function of the chemical potential: it shows that the transition is first-order. Starting from massive quarks at low density^{*)}, it leads to massless quarks at high density. Hence a glance at (2.20a,b) shows that at high density, the behaviour of the quark plasma will be that of an ideal gas of massless particles, as one would expect from perturbative QCD. In Figs. 2b to 2d, the low-density behaviour is studied. First, as explained in detail in Ref. 66), at zero temperature, when the pressure is plotted against

*) Strictly speaking at $T = 0$, the massive quark phase has zero numerical density. However, this is no longer the case as T is slightly increased (and the transition still exists).

the inverse density of quarks minus antiquarks - see Fig. 2b - the Maxwell construction cannot be carried out. But as soon as the temperature is raised a little - which is more realistic - the area above the $p = 0$ axis becomes infinite and the equality of areas can be achieved. Then, finally, for constant positive Γ , the energy per particle is shown as a function of the particle density in Fig. 2a: as in Refs. 64)-66), at low densities, the plasma is in a collective bound state. Finally, in Fig. 2d, the equation of state - pressure versus energy density - has been plotted for various values of Γ . Thus the overall picture which is obtained is satisfying to describe quark matter at low density (where confinement occurs) and high density (where ideal gas behaviour is expected).

For constant negative Γ , the energy density as a function of particle density is represented in Fig. 3a (no collective bound state may exist) and the equation of state - pressure versus energy density - is drawn in Fig. 3b.

Next, one can plot $\Gamma(\mu)$ as given by (3.1b). Note that, in order for Γ not to be infinite, only the case with $U_s^H/m = -2\mu/m$ and not $+2\mu/m$ is allowed, otherwise p_F^r is equal to zero. As can be seen in Fig. 4a, $\Gamma(\mu)$ is always positive so a phase transition may occur. Also, Γ has three infinite branches, corresponding to either $(1+U_s^H/m)|_{U_s^H/m=-2\mu/m}$ or p_F^r equal to zero. However, only the left part of the curve (dotted line for the decreasing part, and dashed line for the increasing part, here and up to Fig. 5b) is allowed, as it corresponds to positive masses, i.e., $(1+U_s^H/m)|_{U_s^H/m=-2\mu/m} > 0$. The continuous part corresponds to $(1+U_s^H/m)|_{U_s^H/m=-2\mu/m} < 0$. Note that, even when only the dashed and dotted parts of the curve are considered, to a given value of Γ , there correspond two values of the chemical potential. It is, however, possible to choose between these two values as follows. Inserting our results for Γ into (3.1a), one gets the behaviour of the effective mass depicted in Fig. 4b: the meaning of the dotted and dashed lines is as mentioned above and the continuous line corresponds to $U_s^H = 0$ (precisely, $p_F^r = 0$ and Γ are undetermined). In this figure, only the dashed line satisfies $|U_s^H| < 2\mu$ all along and so the dotted line may be ignored.

The interesting features of the constant positive Γ case at low density (first-order phase transition, collective bound state) are preserved as can be seen for instance in Figs. 5a-5b. However, owing to the fact that our computation of Γ is valid only for values of $\mu < 1/2$ (see Fig. 4a), our knowledge of $p(\mu)$,

is restricted to finite intervals of μ . The same remark holds for $p(\epsilon)$. However, these curves can be continued, at high density, with the corresponding ones for an ideal gas of massless particles. This prolongation should be very smooth since, in principle, the endpoints in Figs. 5a-5b correspond to $m_H = 0$, i.e., to an ideal gas of massless particles. In addition, we introduced a very sharp screening when $|U_s^H| < 2\mu$; this is why our curves terminate so abruptly. In reality, the screening is probably smoother and our curves should terminate less abruptly.

4. - POTENTIALS WITH SCALAR AND VECTOR EFFECTS IN THE HARTREE APPROXIMATION

For type II potentials, two self-consistent equations must be solved to get U_s^H and U_0^H . In order to understand the general case, it is useful to examine briefly the hypothetical case where the confining potential would be purely of a vector type. So one must solve:

$$\frac{U_0^H}{m} = \begin{cases} \Delta \frac{P_F^3}{3} & \text{if } \frac{\mu}{m} - \frac{U_0^H}{m} \geq 1 \\ -\Delta \frac{P_F^3}{3} & \text{if } \frac{U_0^H}{m} - \frac{\mu}{m} \geq 1 \\ 0 & \text{otherwise} \end{cases} \quad (4.1a)$$

where

$$P_F = \sqrt{\left(\frac{U_0^H}{m} - \frac{\mu}{m}\right)^2 - 1}$$

and

$$\Delta = \begin{cases} \frac{\frac{U_0^H}{m}}{\pm P_F^3 / 3} \end{cases} \quad \left| \frac{U_0^H}{m} = \pm 2 \left| \frac{U_0^H}{m} - \frac{\mu}{m} \right| \right| \quad (4.1b)$$

where

$$P_F^\Delta = \begin{cases} \sqrt{\left(\frac{U_0^H}{m} - \frac{\mu}{m}\right)^2 - 1} & \left| \frac{U_0^H}{m} = \pm 2 \left| \frac{U_0^H}{m} - \frac{\mu}{m} \right| \right| \text{ if } \left(\frac{U_0^H}{m} - \frac{\mu}{m}\right)^2 \left| \frac{U_0^H}{m} \pm 2 \left| \frac{U_0^H}{m} - \frac{\mu}{m} \right| \right| \\ 0 & \text{otherwise} \end{cases}$$

Again let us suppose that Δ is constant. In the case of constant negative Δ , as shown in Fig. 6a, U_0^H possesses three branches, the upper one corresponds to antiquarks (here and in Figs. 7a and 7b in dashed lines), the lowest one to quarks (here and in Figs. 7a and 7b in dotted lines) and the intermediate one arises when p_F equals zero (here and in Figs. 7a and 7b in continuous lines). In the case of constant positive Δ , $U_0^H/m = -\Delta p_F^3/3$ with $U_0^H/m = \mu/m > 1$ cannot be satisfied, so U_0^H is an always increasing function of μ and no energy transition may occur. This can be checked in Fig. 6b. For constant negative Δ , the pressure is plotted as a function of the chemical potential in Fig. 7a: a first-order phase transition might occur. However, in Fig. 7b, the pressure is represented as a function of the inverse of the density of quarks minus antiquarks. It may be seen that the quark (dotted) branch is mechanically unstable while the dashed line is stable but corresponds to antiquarks at $T = 0$ - which is usually considered as not very physical. (This is not too annoying though since this case is purely hypothetical.)

For constant positive Δ , the energy per particle is plotted as a function of the density of particles plus antiparticles in Fig. 8a and the pressure is plotted as a function of the energy density in Fig. 8b.

Finally, one can plot $\Delta(\mu)$ as given by (4.1b). Contrary to the pure scalar case, both Δ positive and negative are possible (as can be seen by studying the solution of $|U_0^H/m| = 2|U_0^H/m - \mu/m|$). The solution for U_0^H/m as a function of μ is rather similar to the constant Δ case so we do not reproduce it (the main difference is that the mechanically unstable quark branch disappears as can be checked analytically).

Next, the coupled self-consistent equations (2.25a,b) for U_s^H and U_0^H may be solved simultaneously. If one assumes Γ and Δ to be constant, many cases could be investigated: $\Gamma > \Delta > 0$, $\Gamma > 0 > \Delta$, $\Delta > \Gamma > 0$, $\Delta > 0 > \Gamma$, etc. As an example, when $r > \Delta > 0$, Eqs. (2.25a,b) are formally similar to those studied by Walecka⁵⁹⁾, Chin and Walecka⁶⁰⁾, Chin⁶⁸⁾ and Bowers, Gleeson and Pedigo⁴⁰⁾. These authors adjusted the precise values of Γ and Δ to fit nuclear matter data (binding energy and saturation density of nuclear matter data or nucleon-nucleon scattering). They obtained $\Gamma \sim 27.04$ and $\Delta \sim 19.83$. According to what precedes, it may be inferred that, at low density, this corresponds to a mass transition (and a possible collective bound state) and no strange behaviour of the zero vectorial component of the Hartree field. In addition, in their case, because

$U_0^H \neq 0$ when $\mu \rightarrow \infty$, the high density behaviour is not that of an ideal gas. Physically, this means that the vector meson exchange potential is not turned off at small distances.

We now turn to the case when Γ and Δ are determined by (2.25c,d). For the same reason as in the pure scalar case, only $U_0^H = -2(\mu - U_0^H)$ is allowed, thus leading to a positive Γ . Similarly to the pure vector case, both Δ positive and negative are allowed. However, since on the one hand $\Gamma \equiv 3m^2/\pi^2 \int d^3z [\alpha V_c(z)]$ and $\Delta \equiv 3m^2/\pi^2 \int d^3z [-\beta V_c(z)]$ and on the other hand, the scalar and vector screening lengths are likely to be of the same order of magnitude, it seems reasonable to think that Γ and Δ will have opposite signs. In this case, one would expect a mass transition and an energy transition. It turns out, numerically, that only the mass transition is allowed and $U_0^H = 0$ (the solution is very similar to that of Section 3 where it is shown). Hence the interesting results of the pure scalar case might survive when the interquark potential has a more complicated structure such as scalar and vector components.

5. - CONCLUSIONS

In this paper, the methods of relativistic quantum many-body theory have been applied to the study of quark matter interacting through phenomenological potentials at zero temperature. It was shown that if the chosen potential has a scalar Lorentz structure, then the quark plasma undergoes a mass transition from a massive collective bound state at low density to a massless perfect gas state at high density. In the case where this potential has a more complicated structure such as scalar-vectorial, if Γ and Δ have opposite signs - which seems natural - such a mass transition would still occur. Moreover, all the computed quantities are independent of the shape of the interquark potential, because of the way screening through pair creation has been implemented. Thus a reasonable description for quark matter seems to emerge from this model.

However, some reservations must be made. First, in this paper, quarks were assumed to be at zero temperature. But zero-temperature cases may be pathologic: for instance, as mentioned in Section 3, for purely scalar potentials, the Maxwell construction cannot be carried at zero temperature while it may be achieved for an infinitesimally small temperature. So the non-zero temperature case will be studied in a following paper.

Next, the interquark potential may have a more complicated structure than just scalar or scalar-vectorial; also the confining part was assumed to be colour-independent - and this is not clear as discussed in Section 2.

Finally it is by no means obvious which kind of mass must be used for m . Looking at Eq. (2.3) for instance, one might favour a current mass - indeed this is what Olive⁴⁶⁾ did for example - because one would expect that, as a result of interactions with other quarks, this current mass is increased to the value of a constituent mass. However, it is not so: in both the scalar case (Fig. 4b) and the scalar plus vector case (Fig. 10a), the Hartree field contribution to the quark mass is negative and $m_H/m \rightarrow 0$ as μ increases while $m_H/m \rightarrow 1$ as μ decreases. Also it may be shown that if one introduces non-zero temperatures in a gap equation such as (2.25b), at a given density, m_H/m decreases as the temperature increases⁶¹⁾, so that, in order that chirality be restored at high density or temperature, the choice of a constituent mass for m is most reasonable - note incidentally that this is also the mass that must be used in potential models of quarkonia and hadrons.

It is also easy to imagine some possible improvements to this model. First Fock exchange terms are known to be neglectable in the equation of state for nuclear matter and small for neutron matter. Since these terms are independent of the shape of the potential, it would be interesting to check if they are also negligible for our quark matter model. This point will be studied in a forthcoming paper.

Second and last, in the Hartree approximation, one replaces the interaction of a number of quarks on a given quark by an exterior field acting on this quark. As a consequence of this, we only get a collective bound state at low densities and not hadrons. In order to get, if not baryons, at least mesons, an approximation of a different type should be used.

ACKNOWLEDGEMENTS

The author is greatly indebted to R. Hakim for his continuous support, freely-shared insights and numerous comments. She wishes also to thank E. Alvarez, F. Karsch, O. Pène and E. Ribeiro for stimulating discussions.

REFERENCES

- 1) G. Lasher - Phys.Rev.Lett. 42 (1979) 1646.
- 2) C.J. Hogan - Ap.J. 252 (1982) 418.
- 3) M. Crawford and D.N. Schramm - Nature 298 (1982) 538.
- 4) E. Witten - Phys.Rev. D30 (1984) 272.
- 5) J.H. Applegate and C.J. Hogan - Phys.Rev. D31 (1985) 3037.
- 6) S.A. Bonometto, A.P. Marchetti and S. Matarrese - Phys.Lett. 157B (1985) 216.
- 7) C. Alcock and E. Farhi - Phys.Rev. D32 (1985) 1273.
- 8) R. Schaeffer, P. Delbourgo-Salvador and J. Audouze - Nature 317 (1985) 407.
- 9) S. Midorikawa - Phys.Lett. 158B (1985) 107.
- 10) C.J. Hogan - Mon.Not.Roy.Soc. A218 (1985) 629.
- 11) W.B. Fechner and P.C. Joss - Nature 274 (1978) 347.
- 12) N. Iwamoto - Phys.Rev.Lett. 24 (1980) 1637.
- 13) J.D. Bjorken and L.D. McLerran - Phys.Rev. D20 (1979) 2353.
- 14) M.B. Kislinger and P.D. Morley - Phys.Lett. 67B (1977) 371; Ap.J. 219 (1978) 1017.
- 15) B. Freedman and L.D. McLerran - Phys.Rev. D17 (1978) 1109.
- 16) V. Baluni - Phys.Lett. 72B (1978) 381.
- 17) J.C. Collins and M.J. Perry - Phys.Rev.Lett. 34 (1975) 1353.
- 18) G. Baym and S.A. Chin - Nucl.Phys. A262 (1976) 527.
- 19) S.A. Chin - Phys.Lett. 78B (1978) 552.
- 20) B.A. Freedman and L.D. McLerran - Phys.Rev. D16 (1977) 1130; Phys.Rev. D16 (1977) 1147; Phys.Rev. D16 (1977) 1169.
- 21) V. Baluni - Phys.Rev. D17 (1978) 2092.
- 22) E.V. Shuryak - Soviet Phys. JETP 47 (1978) 212.
- 23) J.I. Kapusta - Nucl. Phys. B148 (1978) 461.
- 24) T. Toimela - Phys.Lett. 142B (1983) 407; Int.J.Th.Phys. 24 (1985) 901.
- 25) C.G. Källman and T. Toimela - Phys.Lett. 11B (1983) 409.
- 26) A.D. Linde - Rep.Prog.Phys. 42 (1979) 390.

- 27) E.V. Shuryak - Physics Reports 61 (1980) 71.
- 28) D.J. Gross, R.D. Pisarski and L.G. Yaffe - Rev.Mod.Phys. 53 (1981) 43.
- 29) G. Baym and S.A. Chin - Phys.Lett. 62B (1976) 241.
- 30) P. Haensel, J.L. Zudnik and R. Schaeffer - Astr. and Astrophys. 160 (1986) 121.
- 31) J. Kuti, B. Lukacs, J. Polonyi and K. Szlachanyi - Phys.Lett. 95B (1980) 75.
- 32) E. Suhonen - Phys.Lett. 119B (1982) 81.
- 33) M. Gyulassy and T. Matsui - Phys.Rev. D29 (1984) 419.
- 34) M. Gyulassy, K. Kajantie, H. Kurki-Suonio and L. McLerran - Nucl.Phys. B237 (1984) 477.
- 35) H. Kurki-Suonio - Nucl.Phys. B255 (1985) 231.
- 36) H.W. Barz, L.P. Csennai, B. Kämpfer and B. Lukacs - Phys.Rev. D32 (1985) 115.
- 37) T. DeGrand and K. Kajantie - Phys.Lett. B147 (1984) 273.
- 38) G. Chapline and M. Nauenberg - Phys.Rev. D16 (1977) 450.
- 39) B. Müller and J. Rafelski - Phys.Lett. 101B (1981) 111.
- 40) R.L. Bowers, A.M. Gleeson and R.D. Pedigo - Phys.Rev. D12 (1975) 3043; Phys.Rev. D12 (1975) 3056; Ap.J. 213 (1977) 840.
- 41) E. Alvarez and R. Hakim - Phys.Rev. D19 (1979) 1696; Astr. and Astrophys. 80 (1979) 71.
- 42) P. Vinciarelli - Lett. Nuovo Cimento 4 (1972) 905.
- 43) W.A. Bardeen, M.S. Chanowitz, S.D. Drell, M. Weinstein and T.M. Yan - Phys.Rev. D11 (1975) 1094.
- 44) R. Friedberg and T.D. Lee - Phys.Rev. D15 (1977) 1694; Phys.Rev. D16 (1977) 1096.
- 45) R.V. Wagoner and G. Steigman - Phys.Rev. D20 (1979) 825.
- 46) K.A. Olive - Nucl.Phys. B190 (1982) 213.
- 47) C.G. Källman - Phys.Lett. 112B (1982) 213.
- 48) D.N. Schramm and K.A. Olive - Nucl.Phys. A418 (1984) 289.
- 49) R. Hakim and S. Collin - in "Formation and Evolution of Galaxies and Large Structures in the Universe", J. Audouze and J. Tran Thanh Van eds. (Reidel, 1984).
- 50) A.B. Henriques, B.H. Kellett and R.G. Moorhouse - Phys.Lett. 64B (1976) 85.
- 51) H. Krasemann - Phys.Lett. 101B (1981) 259.

- 52) D. Flamm and F. Schöberl - "Quantum Numbers, Gauge Theories and Hadron Spectroscopy", (Gordon and Breach, 1982), Chapter 3.
- 53) D. Gromes, Z.Phys. C22 (1984) 265; Z.Phys.C. 26 (1984) 401.
See also: E. Eichten and F. Feinberg - Phys.Rev. D23 (1981) 2724.
- 54) H.W. Crater and P. Van Alstine - Phys.Rev.Lett. 53 (1984) 1525.
- 55) G. Feinberg and J. Sucher - Phys.Rev. D20 (1979) 1717.
- 56) D.P. Stanley and D. Robson - Phys.Rev.Lett. 45 (1980) 235.
- 57) Y.C. Leung - "Physics of Dense Matter" (World Scientific, 1984) 41, Chapter 18.
- 58) E. Alvarez - Phys.Rev. D23 (1981) 1715.
- 59) C. Kittel - "Quantum Theory of Solids" (Wiley, 1963).
- 60) L.P. Kadanoff and G. Baym - "Quantum Statistical Mechanics" (Benjamin, 1962).
- 61) J.D. Walecka - Ann.Phys. 83 (1974) 491; Phys.Lett. 59B (1975) 109.
- 62) S.A. Chin and J.D. Walecka - Phys.Lett. 52B (1974) 24.
- 63) S.A. Chin - Ann.Phys. 108 (1977) 301.
- 64) J. Diaz Alonso and R. Hakim - Phys.Rev. D29 (1984) 2690.
- 65) G. Kalman - Phys.Rev. D9 (1974) 1656.
- 66) J. Diaz Alonso and R. Hakim - Phys.Lett. 66A (1978) 476.
- 67) J. Kogut and L. Susskind - Phys.Rev.Lett. 34 (1975) 767.

FIGURE CAPTIONS

- Fig. 1a Plot of the effective mass as a function of the chemical potential for different values of constant positive Γ . A phase transition occurs if $\Gamma > 2.5$. A glance at Fig. 2b allows one to determine which states are not mechanically stable (dotted line) and which states are physically accessible (dashed line).
- Fig. 1b Same as Fig. 1a for constant negative Γ 's. No phase transition takes place.
- Fig. 2a Plot of the pressure as a function of the chemical potential for different values of constant positive Γ . The transition is seen to be first order. See Fig. 1a for curve designations.
- Fig. 2b Plot of the pressure as a function of the logarithm of the inverse of the particle density for constant positive Γ 's. For large values, the Maxwell construction cannot be carried out (see text). See Fig. 1a for curve designations.
- Fig. 2c Plot of the energy per particle as a function of the density of particles plus antiparticles for constant positive Γ 's. There exists a range of densities, for which the plasma is in a collective bound state. See Fig. 1a for curve designations.
- Fig. 2d Plot of the pressure as a function of the energy density for a constant positive Γ 's. The equation of state exhibits an ideal gas behaviour at high densities.
- Fig. 3a Plot of the energy per particle as a function of the density of particles plus antiparticles for different values of constant negative Γ . No collective bound state may exist in this case.
- Fig. 3b Plot of the pressure as a function of the energy density for constant negative Γ 's. The equation of state does not tend asymptotically towards that of a perfect gas.

- Fig. 4a Plot of Γ as a function of the chemical potential when screening is assumed to take place at $U_s^H/m = -2\mu/m$. The continuous line corresponds to $1 + U_s^H/m|_{U_s^H/m=-2\mu} \leq 0$ and the dotted and dashed lines correspond to $1 + U_s^H/m|_{U_s^H/m=-2\mu} > 0$.
- Fig. 4b Plot of the effective mass as a function of the chemical potential when Γ is given by the left part of Fig. 4a. See Fig. 4a for curve designations.
- Fig. 5a Plot of the pressure as a function of the chemical potential with Γ given as in Fig. 4a. See Fig. 4a for curve designations.
- Fig. 5b Plot of the pressure as a function of the energy density with Γ given as in Fig. 4a. See Fig. 4a for curve designations.
- Fig. 6a Plot of the zero-vectorial component of the Hartree field for different values of constant negative Δ . An energy transition may occur. The dashed line represents an antiquark excess and the dotted line a quark excess.
- Fig. 6b Same as Fig. 6a for constant positive Δ 's. No energy transition is possible.
- Fig. 7a Plot of the pressure as a function of the chemical potential for different values of constant negative Δ . The transition is seen to be first order. See Fig. 6a for curve designations.
- Fig. 7b Plot of the pressure as a function of the logarithm of the inverse of the particle density for constant negative Δ 's. See Fig. 6a for curve designations: the quark excess branch is mechanically unstable.
- Fig. 8a Plot of the energy per particle as a function of the particle density for constant positive Δ 's.
- Fig. 8b Plot of the pressure as a function of energy density for constant positive Δ 's.

Figure 1a

Figure 1b

Figure 2a

Figure 2b

Figure 2c

Figure 2d

Figure 3a

Figure 3b

Figure 4a

Figure 4b

Figure 5a

Figure 5b

Figure 6a

Figure 6b

Figure 7a

Figure 7b

Figure 8a

Figure 8b