CERN-TH.4679/87
MIU-THP-87/011
UMN-TH-602/87

RE-EXAMINATION OF THE STANDARD MODEL IN THE LIGHT OF B MESON MIXING

John Ellis, J.S. Hagelin^{*)}

CERN - Geneva

and

S. Rudaz

School of Physics and Astronomy
University of Minnesota, Minneapolis, MN 55455

A B S T R A C T

We examine the consistency of the standard model in the light of recent evidence for $B_d^0-\bar{B}_d^0$ and possible $B_s^0-\bar{B}_s^0$ mixing. We find that a consistent determination of the Kobayashi-Maskawa angles is possible only if $m_t \gtrsim 50$ GeV. We confirm previous expectations that $B_s^0-\bar{B}_s^0$ mixing should be large, though not necessarily maximal. The top quark contributions to $K_L \rightarrow \mu^+\mu^-$ and to the K_L-K_S mass difference are small. The observed value of the CP impurity parameter ϵ in the $K^0-\bar{K}^0$ system is now easily accommodated, while previous predictions for ϵ'/ϵ are essentially unchanged. We obtain significantly improved upper and lower bounds on the rate for $K^+ \rightarrow \pi^+\nu\bar{\nu}$, which is expected to have a branching ratio of $(1 \text{ to } 8) \times 10^{-10}$. Observation of this decay would yield significant new constraints on the standard model.

^{*)} Permanent address: Department of Physics, Maharishi International University, Fairfield, IA 52556, U.S.A.

As soon as the bottom quark was discovered,¹ it was realized that there could be observable mixing between conjugate pairs of neutral B mesons.² Subsequently, it came to be generally expected that there would be substantial, perhaps even maximal, $B_S^0-\bar{B}_S^0$ mixing, whilst $B_d^0-\bar{B}_d^0$ mixing was expected to be considerably smaller.^{3,4} Recently two experiments have reported evidence for mixing among neutral B mesons. First was UAl,⁵ which was not able to distinguish between $B_d^0-\bar{B}_d^0$ and $B_S^0-\bar{B}_S^0$ mixing. Second was ARGUS,⁶ whose studies at the $\Upsilon(4S)$ resonance yielded unambiguous evidence for $B_d^0-\bar{B}_d^0$ mixing. In this paper, we examine the implications of these results for the parameters of the Standard Model, including Kobayashi-Maskawa angles and the top quark mass. We also explore their consequences for other flavour-changing and CP-violating processes, including $K_L \rightarrow \mu^+\mu^-$, $K^+ \rightarrow \pi^+\nu\bar{\nu}$, ϵ and ϵ'/ϵ . Our results may be summarized as follows.

The observation of $B_d^0-\bar{B}_d^0$ mixing fixes the value of the V_{td} matrix element of the Kobayashi-Maskawa matrix, and hence $s_2=\sin\theta_2$, as a function of m_t . We find that s_2 is larger than previous expectations,^{7,4} though it decreases as m_t increases. Indeed, our determination of s_2 can be reconciled with the long b lifetime⁹ and the ratio $\bar{R} \equiv \Gamma(b \rightarrow u)/\Gamma(b \rightarrow c) \lesssim 0.08$ ¹⁰ only if $\cos\delta \approx -1$ and $m_t \gtrsim 50$ GeV, after allowing for uncertainties arising from our ignorance of hadronic matrix elements. The amount of $B_S^0-\bar{B}_S^0$ mixing is not directly affected by the determination of s_2 , but is sensitive to the same hadronic uncertainties. With our lower bound on m_t , we confirm the expectation^{3,4} of large $B_S^0-\bar{B}_S^0$ mixing, though this need not be maximal. Our new results on the Kobayashi-Maskawa angles allow a determination of the short-distance contribution to $K_L \rightarrow \mu^+\mu^-$,^{11,12,13} which we conclude is dominated by long-distance effects. They also permit a more precise determination of $K^+ \rightarrow \pi^+\nu\bar{\nu}$,^{12,13,7} which we now predict to have a branching ratio in the range from $(1 \text{ to } 8) \times 10^{-10}$. Turning to CP-violating parameters in the $K^0-\bar{K}^0$ system, we find that the observed value of the CP impurity parameter ϵ is easily accommodated by the larger Kobayashi-Maskawa angles and/or top quark mass required for significant $B_d^0-\bar{B}_d^0$ mixing. Previous calculations^{7,8} of ϵ'/ϵ as a function of m_t are essentially unaffected, though the fact that $B_d^0-\bar{B}_d^0$ mixing favours large values of m_t leads one to expect relatively small values of ϵ'/ϵ .

We begin by defining $r_d \equiv \Gamma(B_d^0 \rightarrow l^+X)/\Gamma(B_d^0 \rightarrow l^-X)$, which is given by¹⁴

$$r_d = \frac{(\Delta M)^2 + (\Delta\Gamma/2)^2}{2\Gamma^2 + (\Delta M)^2 - (\Delta\Gamma/2)^2} \approx \frac{(\Delta M)^2}{2\Gamma^2 + (\Delta M)^2} \quad (1)$$

where $\Delta M \approx 2\text{Re}M_{12}$, $\Delta\Gamma \approx 2\text{Re}\Gamma_{12}$ and $\Gamma^{-1} = \tau_b$ is measured⁸ to be $(1.11 \pm 0.14) \times 10^{-12} \text{ s}$. The approximation $(\Delta\Gamma/2)^2 \ll (\Delta M)^2$ is justified in ref. 3. The off-diagonal element M_{12} in the $B_d^0 - \bar{B}_d^0$ mass matrix is given by

$$M_{12} = \frac{G_F^2 f_{B_d}^2 B m_{B_d}}{12\pi^2} \left[(V_{td}^* V_{tb})^2 m_t^2 + \mathcal{O}(m_b^2) \right] \quad (2)$$

where the effects of QCD³ and of higher order terms in m_t^2/m_W^2 are not shown explicitly, but are included in our subsequent calculations.

The mixing parameter r_d (1) is determined at the $\Upsilon(4S)$ by the proportion of like-sign dilepton events¹⁵

$$r_d = \frac{\ell^{++} + \ell^{--}}{\ell^{+-}} \quad , \quad (3)$$

which the ARGUS Collaboration⁶ has observed to be $r_d \approx 0.20 \pm 0.11$. One can use this measurement with the relations (1,2) to estimate V_{td} ($=s_1 s_2$) and hence s_2 as a function of m_t :

$$s_2 = \frac{10.8 \text{ GeV}}{\tilde{m}_t} \left(\frac{r_d}{1-r_d} \right)^{1/4} \left(\frac{0.11 \text{ GeV}}{B^{1/2} f_{B_d}} \right) \left(\frac{1.11 \times 10^{-12} \text{ s}}{\tau_b} \right)^{1/2} \quad (4)$$

where $\tilde{m}_t$ is defined to "improve" the physical value of m_t by including QCD³ and higher order terms in m_t^2/m_W^2 .¹² In order to use this observation to determine s_2 as a function of m_t , it is necessary to specify the values of the B_d decay constant f_{B_d} and the correction factor B to the vacuum insertion approximation for the matrix element $\langle B_d^0 | (\bar{b} \gamma_\mu (1-\gamma_5) d)^2 | \bar{B}_d^0 \rangle$. It is generally expected that $B \approx 1$, with considerably less uncertainty than for the analogous parameter in the $K^0 - \bar{K}^0$ system. Recent estimates¹⁶ of the B_d decay constant give $100 \lesssim f_{B_d} \lesssim 130 \text{ MeV}$, while direct estimates¹⁷ of the relevant $\Delta B=2$ matrix element that do not use vacuum insertion yield similar values: $90 \lesssim (B^{1/2} f_{B_d}) \lesssim 130 \text{ MeV}$.

In Fig. 1a, s_2 is plotted as a function of m_t for our "preferred" values of $r_d = 0.20$, $\tau_b = 1.11 \times 10^{-12} \text{ s}$ and $f_{B_d} = 0.11 \text{ GeV}$, while fig. 1b represents a reasonable lower limit on s_2 obtained using $r_d = 0.09$, $\tau_b = 1.25 \times 10^{-12} \text{ s}$ and $f_{B_d} = 0.16 \text{ GeV}$ (which is considerably larger than any of the published estimates). Having determined s_2 as a function of m_t , one can then constrain⁷ the value of s_3 using the b lifetime:

$$|V_{bc}| \approx |s_3 + s_2 e^{i\delta}| \approx 0.05 \left(\frac{1.11 \times 10^{-12} \text{ s}}{\tau_b} \right)^{1/2} \quad (5)$$

For $s_2 > |V_{bc}|$, the minimum value of s_3 is obtained when $\cos \delta \approx -1$. This minimum value of s_3 is also plotted in figs. 1a and b, using consistent values of $\tau_b = 1.11 (1.25) \times 10^{-12}$ s respectively.

These minimum values of s_3 can be compared with the upper bound⁷ on s_3 obtained from the experimental non-observation of the $b \rightarrow u$ transition. The numerical value of the upper bound on $\Gamma(b \rightarrow u)/\Gamma(b \rightarrow c)$ is the subject of some debate, centered on the reliability of theoretical calculations of the lepton spectra in $b \rightarrow ul\nu$ and $b \rightarrow cl\nu$ decays. In particular, the observed spectra do not agree with simple quark model predictions, and a theoretical model¹⁹ for $b \rightarrow ul\nu$ decay based on calculable transitions to exclusive final states gives a spectrum much softer than the simple quark models close to the end-point. For these reasons, we take a more conservative view of $\Gamma(b \rightarrow u)/\Gamma(b \rightarrow c)$ than some limits quoted in the literature.

Taking $\bar{R} \equiv \Gamma(b \rightarrow u)/\Gamma(b \rightarrow c) \leq 0.08^{10}$ gives a maximum value of s_3 , which is also shown in figs. 1a and b. It is evident that this maximum value of s_3 is only consistent with the previous lower bound on s_3 derived from equation (5) if m_t is sufficiently large: our "preferred" scenario in fig. 1a yields

$$m_t \gtrsim 100 \text{ GeV} \quad (6a)$$

while our "minimum" values of s_2, s_3 in fig. 1b yield

$$m_t \gtrsim 50 \text{ GeV}. \quad (6b)$$

It is easy to check that these bounds are independent of τ_b and, apart from corrections due to higher order terms in m_t^2/m_W^2 and QCD, scale inversely with f_{B_d} . Smaller values of m_t can be obtained at the expense of larger values of $\bar{R}$, e.g., $\bar{R} = 0.13^{10}$ would yield $m_t \gtrsim 43 \text{ GeV}$.

We now explore the consequences of $B_d^0 - \bar{B}_d^0$ mixing for other flavour-changing processes and for CP violation in the Standard Model. First we discuss $B_s^0 - \bar{B}_s^0$ mixing, whose description is identical to equations (1-3) with d subscripts replaced by s subscripts. The relevant Kobayashi-Maskawa angle

$$V_{ts}^* V_{tb} = (c_1 s_2 c_3 + c_2 s_3 e^{-i\delta})(c_1 s_1 s_3 - c_2 c_3 e^{i\delta}) \approx -(s_3 + s_2 e^{i\delta}) \quad (7)$$

is already fixed by the b lifetime (5) and is therefore insensitive to the new determination of s_2 . Hence previous predictions^{3,4} of large $B_S^0-\bar{B}_S^0$ mixing remain valid. For completeness, we include in fig. 1 the values of r_s as functions of m_t in our "preferred" scenario with $f_{B_S} = 0.11$ to 0.15 GeV (fig. 1a) and in our "minimal" scenario with $f_{B_S} = 0.16$ to 0.22 GeV (fig. 1b). For these two cases, we find $r_s > (0.90, 0.70)$ for m_t greater than the lower bounds (6). Hence $B_S^0-\bar{B}_S^0$ mixing is expected to be much larger than the observed $B_d^0-\bar{B}_d^0$ mixing.

Note that there is also an upper bound on neutral B meson mixing from Mark II¹⁸ which might appear to be inconsistent with the combination of maximal $B_S^0-\bar{B}_S^0$ mixing and the magnitude of $B_d^0-\bar{B}_d^0$ mixing seen by ARGUS. However, any such conclusion would depend on an experimentally untested hypothesis for the relative amounts of B_u^- , B_d^0 , B_S^0 and b -baryon production. In any case, we see from fig. 1 that $B_S^0-\bar{B}_S^0$ mixing can be less than maximal for m_t close to the lower bounds (6).

Finally, we note that the observation of mixing among neutral B mesons in principle permits a measurement of the CP-violating charge asymmetry¹⁵ $a \equiv (1^{++} - 1^{--})/(1^{++} + 1^{--})$. Since this charge asymmetry is necessarily small in the Standard Model whenever $B_d^0-\bar{B}_d^0$ mixing is appreciable,³ the observation of a charge asymmetry $\gtrsim$ few percent would be clear evidence for new physics in $B^0-\bar{B}^0$ mixing.

We now leave the neutral B mesons and consider the $K^0-\bar{K}^0$ system. The short-distance contribution to $\text{Re}M_{12}^K$ arising from the top quark is proportional to $(V_{td}^* V_{ts})^2 m_t^2$, where the magnitude of V_{ts} is fixed by the b lifetime as in equation (7). The reciprocal relation (4) between V_{td} and m_t imposed by $B_d^0-\bar{B}_d^0$ mixing implies that the top quark contribution to $\text{Re}M_{12}^K$ is independent of m_t and much smaller than the measured K_L-K_S mass difference. One thereby confirms that the K_L-K_S mass difference is predominantly due to the charm quark and/or long-distance contributions, in accordance with previous expectations.⁷ We also recall that for $m_t < 50$ GeV the CP impurity parameter ϵ was somewhat difficult to accommodate in the Standard Model for the small values of s_2 and s_3 favoured by τ_b and $\bar{R}$.²⁰ The larger values of s_2 (and hence s_3) and/or m_t imposed by $B_d^0-\bar{B}_d^0$ mixing removes any apparent discrepancy.

A detailed calculation shows that previous theoretical estimates^{7,8} of ϵ'/ϵ are relatively unaffected by the new information on $B_d^0-\bar{B}_d^0$ mixing. This is because previous calculations of ϵ'/ϵ were performed by fitting the CP impurity parameter ϵ in conjunction with the upper bounds on s_2 and s_3 arising from τ_b

and $\bar{R}$. For $m_t > 50$ GeV, the lower bounds on s_2 and s_3 that result from $B_d^0-\bar{B}_d^0$ mixing are compatible with these upper bounds, and previous estimates still apply. Values of $m_t < 50$ GeV and correspondingly larger values of s_2, s_3 (4,5) are still possible provided $\bar{R} > 0.08$, but this corresponds to a domain where ϵ'/ϵ is least sensitive to s_2 . Thus previous estimates of ϵ'/ϵ as a function of m_t are relatively unaffected, but the fact that larger values of m_t are favoured by $B_d^0-\bar{B}_d^0$ mixing corresponds to smaller values of ϵ'/ϵ .^{7,8}

Turning our attention to rare K decays, we first note that the short-distance contribution to $K_L \rightarrow \mu^+ \mu^-$ from the top quark can now be estimated reliably. The short-distance contribution¹⁰ to the $K_L \rightarrow \mu^+ \mu^-$ amplitude is proportional to $(V_{td}^* V_{ts}) m_t^2$, and is thus effectively linear in m_t . Including higher order terms in m_t^2/m_W^2 and QCD,¹² we find for $m_t \leq 250$ GeV that the top-quark contribution is small compared to the experimental rate, which we therefore conclude is due to long-distance physics. Indeed, the recent measurement²¹ of $BR(K_L \rightarrow \gamma\gamma) = 6.05 \pm 0.04 \pm 0.08 \pm 0.20$ is consistent with the assumption that $K_L \rightarrow \mu^+ \mu^-$ is entirely due to the absorptive part of the 2γ intermediate state, i.e. from $K_L \rightarrow \gamma\gamma \rightarrow \mu^+ \mu^-$.

Perhaps the most interesting implication of $B_d^0-\bar{B}_d^0$ mixing for physics outside the $B_d^0-\bar{B}_d^0$ system concerns the rare decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$. The short-distance contribution to the branching ratio for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ per neutrino flavour is¹²

$$\begin{aligned} BR(K^+ \rightarrow \pi^+ \nu_i \bar{\nu}_i) &= \frac{0.61 \times 10^{-6}}{|V_{us}|^2} \left| \sum_{j=c,t} V_{js}^* V_{jd} D(x_j) \right|^2 \\ &= 0.61 \times 10^{-6} \left| D(x_c) + s_2(s_2 + s_3 e^{i\delta}) D(x_t) \right|^2 \end{aligned} \quad (8a)$$

in the approximation of small Kobayashi-Maskawa angles, where $x_i = m_i^2/m_W^2$ and

$$D(x) = \frac{1}{8} \left[1 + \frac{3}{(1-x)^2} - \frac{(4-x)^2}{(1-x)^3} \right] x \log x + \frac{x}{4} - \frac{3}{4} \frac{x}{1-x}, \quad (8b)$$

to which we add the effects of QCD as described in ref. 13. It is easy to convince oneself that long-distance contributions are much smaller than these short-distance contributions from the charm and top quarks. Previously, an upper bound on $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ has been given¹² based on the requirement that the short-distance contribution from the top quark not give too large a rate for $K_L \rightarrow \mu^+ \mu^-$, and lower bounds on $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ have been given⁷ using τ_b , $\bar{R}$, and consistency with the CP impurity parameter ϵ in the $K^0-\bar{K}^0$ system. Both the upper and lower bounds on $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ can now be significantly improved, as shown in fig. 2.

The shaded regions in fig. 2a,b correspond to allowed values of $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ (assuming three generations of neutrinos) for $r_d = 0.20$ and 0.09 respectively. The lower bounds are given for $\tau_b = 1.25 \times 10^{-12}$ s and $f_{B_d} = 0.16$. The upper bounds are obtained using $\tau_b = 0.97 \times 10^{-12}$ s and the minimum value of f_{B_d} consistent with $r_d = 0.20$ (0.09), $\bar{R} \leq 0.08$, and m_t . As m_t increases, smaller values of f_{B_d} are allowed for fixed r_d and $\bar{R}$ (down to a minimum value which we take to be $f_{B_d} = 0.080$ GeV), resulting in larger values of s_2 and hence larger values for $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$. For purposes of comparison, the dashed regions in fig. 2 correspond to $\bar{R} \leq 0.13$, which we observe does not significantly affect the upper or lower bounds on $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$. We find that

$$BR(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (1 \text{ to } 1.5) \times 10^{-10} \text{ for } m_t = 50 \text{ GeV}, \quad (9a)$$

and

$$BR(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (1 \text{ to } 8) \times 10^{-10} \text{ for } m_t = 100\text{--}200 \text{ GeV}. \quad (9b)$$

This range of branching ratios (9) is much narrower than before,^{7,13} and appears accessible to experiment. Observation of this decay would give useful information on m_t and the Kobayashi-Maskawa angle V_{td} which, unlike $B_d^0 - \bar{B}_d^0$ mixing, is not subject to large uncertainties in light hadronic matrix elements.

In conclusion, we find that the surprisingly large amount of $B_d^0 - \bar{B}_d^0$ mixing reported by ARGUS is compatible with the Standard Model, provided either m_t is sufficiently large ($m_t \gtrsim 50$ GeV) or the ratio $\bar{R} = \Gamma(b \rightarrow u)/\Gamma(b \rightarrow c)$ is larger than presently believed. The most interesting implication of our analysis for other processes in the Standard Model is for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, for which we are able to give significantly improved upper and lower bounds. This process should be observable in a forthcoming experiment,²² and would give us useful information on m_t and on the Kobayashi-Maskawa angle V_{td} .

We would like to acknowledge valuable conversations with D. Cassel, M. Ferro-Luzzi, H. Fritzsch, K. Kleinknecht, W. Schmidt-Parzeval, H. Schröder and E. Thorndike. One of us (SR) acknowledges the research support afforded by the U.S. Department of Energy under contract No. DE-AC02-83ER40105 and by a Presidential Young Investigator Award supplemented by a grant from the Exxon Education Foundation.

References

1. S.W. Herb et al., Phys. Rev. Lett. 39 (1977) 252.
2. J. Ellis, M.K. Gaillard, D.V. Nanopoulos and S. Rudaz, Nucl. Phys. B131 (1977) 285.
3. J.S. Hagelin, Nucl. Phys. B193 (1981) 123.
4. T. Brown and S. Pakvasa, Phys. Rev. D31 (1985) 1661; I.I. Bigi and A.I. Sanda, Phys. Rev. D29 (1984) 1393; V. Barger and R.J.N. Phillips, Phys. Lett. 143B (1984) 259; A. Ali and C.Jarlskog, Phys. Lett. 144B (1984) 266; A. Ali, preprints DESY 85-107 and DESY-86-108.
5. UAl Collaboration, C. Albajar et al., CERN-EP 209/86 (1986).
6. ARGUS Collaboration, H. Albrecht et al., DESY preprint (to be published) as reported by H. Schröder at the XXII Rencontre de Moriond, Les Arcs, France, March 1987.
7. F.J. Gilman and J.S. Hagelin, Phys. Lett. 133B (1983) 443.
8. L. Wolfenstein, Comments Nucl. Part. Phys. 14 (1985) 135, and Ann. Rev. Nucl. Part. Sci. 36 (to be published); J.F. Donoghue, B.R. Holstein and G. Valencia, UMHEP 272 (to be published in J. Mod. Phys. A); A.J. Buras, W. Slominski and H. Steger, Nucl. Phys. B238 (1984) 529.
9. M. Gilchrese, Proceedings of the XXII International Conference on High Energy Physics, Berkeley, CA (1986).
10. E. Thorndike, Talk presented at the Salt Lake City meeting of the Division of Particles and Fields of the A.P.S., University of Rochester preprint UR-987 (1987) and private communication; K. Wachs, Talk given at the XXII Rencontre de Moriond, Les Arcs, France, March 1987.
11. R.E. Shrock and M.B. Voloshin, Phys. Lett. 87B (1979) 375.
12. T. Inami and C.S. Lim, Prog. Theor. Phys. 65 (1981) 297.
13. J. Ellis and J.S. Hagelin, Nucl. Phys. B217 (1983) 189.
14. A. Pais and S.B. Treiman, Phys. Rev. D12 (1975) 2744.
15. L.B. Okun, V.I. Zakharov and B.M. Pontecorvo, Lett. Nuovo Cimento 13 (1975) 218.
16. M. Suzuki, Lawrence Berkeley Laboratory Report No. 19983, (1985) unpublished; E. Golowich, Phys. Lett. 91B (1980) 271; H. Krasemann, Phys. Lett. 96B (1980) 397; see also M. Claudson, Harvard University Report No. HUTP-81/A016 (1981) unpublished.
17. M.A. Shifman and M.V. Voloshin, ITEP Reports 86-54 and 86-83 (1986) to be published; I.I. Bigi and A.I. Sanda, Phys. Rev. D29 (1984) 1393; P. Colic et al., Max Planck Institute Report No. MPI-PAE/P Th 39/82 (1982) unpublished; N. Isgur and G. Karl, Phys. Lett. 74B (1978) 353; Phys. Rev. D18 (1978) 4187; 20 (1979) 1191.
18. Mark II Collaboration, T. Schaad et al., Phys. Lett. 160B (1985) 188.
19. B. Grinstein, N. Isgur and M.B. Wise, Phys. Rev. Lett. 56 (1986) 298.
20. P.H. Ginsparg, S.L. Glashow and M.B. Wise, Phys. Rev. Lett. 50 (1983) 1415.
21. A.M. Lutz, Talk given at the XXII Rencontre de Moriond, Les Arcs, France, March 1987.
22. Brookhaven AGS experiment #787, Brookhaven-Princeton-TRIUMPF Collaboration, T.F. Kycia, spokesman.

Figure Captions

1. The values of s_2 and minimal and maximal values of s_3 deduced from the observed $B_d^0-\bar{B}_d^0$ mixing, τ_b , and $\bar{R} = \Gamma(b \rightarrow u)/\Gamma(b \rightarrow c)$ respectively: (a) using our "preferred" values $r_d = 0.20$, $\tau_b = 1.11 \times 10^{-12}$ s, and $f_{B_d} = 0.11$ GeV, and (b) using the "minimal" scenario $r_d = 0.09$, $\tau_b = 1.25 \times 10^{-12}$ s, and $f_{B_d} = 0.16$ GeV. Shaded are the corresponding values of r_s for $B_s^0-\bar{B}_s^0$ mixing computed for the same values of τ_b and (a) $f_{B_s} = 0.11$ to 0.15 GeV, and (b) $f_{B_s} = 0.16$ to 0.22 GeV.

2. Allowed branching ratios for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, using $f_{B_d} = 0.08$ to 0.16 GeV, $\tau_b = (0.97 \text{ to } 1.25) \times 10^{-12}$ s, and assuming (a) $r_d = 0.20$, and (b) $r_d = 0.09$. Shaded regions correspond to allowed branching ratios for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ assuming $\bar{R} = \Gamma(b \rightarrow u)/\Gamma(b \rightarrow c) \leq 0.08$, whereas dashed regions correspond to $\bar{R} \leq 0.13$.

Figure 1

Figure 2