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Determination of the $\bar{p}$ -neutron scattering amplitude from
 $\bar{p}$ -nucleus elastic scattering data

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Abstract

The elastic scattering data of $\bar{p}$ on ^{12}C , ^{16}O and ^{40}Ca nuclei at incident energies of 47 and 179 MeV are fitted with a Glauber model. The best fit procedure determines both the parameters of the $\bar{p}$ -neutron scattering amplitude and the degree of accuracy of the model. It is found that the theory holds well at 179 MeV, whereas it seems to be slightly inaccurate at 47 MeV. The comparison with the results on the $\bar{p}$ -neutron interaction obtained in other works is also performed.

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1. Introduction

In the recent years many new $\bar{p}$ -p and $\bar{p}$ -nucleus data at low energy ($E < 200$ MeV), among which high precision elastic scattering data⁽¹⁻⁴⁾, have been obtained in some laboratories, particularly at the new LEAR facility of CERN.

These data have been extensively analyzed with optical potential, microscopic and other phenomenological models, obtaining quantitative information on the $\bar{p}$ -nucleon interaction and on the $\bar{p}$ -nucleus potential. Now the existence of a $\bar{p}$ -nucleus diffuse and deep imaginary potential, larger than the real one, as well as the validity of microscopic models based on elementary $\bar{p}$ -nucleon interactions deduced from the Paris and Dover and Richard potentials have been definitively established, (for a review of these topics see refs.(5,6)).

Also the Glauber theory revealed to be an effective means of data analysis. The agreement between the $\bar{p}$ -nucleus elastic scattering distributions and the predictions of the Glauber theory is put in evidence in ref.(7), while in refs.(8-10) extensive analyses of the $\bar{p}$ -nucleus total reaction cross sections lead to the determination of the $\bar{p}$ -neutron ($\bar{p}$ -n) total cross section. However, in our opinion the possibility to obtain with this model quantitative information on the elementary $\bar{p}$ -nucleon interaction has not been fully explored. In particular, two important aspects have to be still clarified: i) the degree of accuracy of the Glauber theory as a function of the $\bar{p}$ incident energy; ii) the possibility to derive completely from the $\bar{p}$ -nucleus elastic scattering data the $\bar{p}$ -n scattering amplitude.

This paper deals with these two points, performing the investigation as follows. In sect.2 we discuss briefly the foundations of the Glauber theory and present the basic formulas of the model; in sect.3 the input parameters relative to the $\bar{p}$ - p scattering amplitude and to the shape of the nucleus are determined; in sect.4 the physical content of the model is discussed; in sect.5 the model is fitted to $\bar{p}$ - ^{12}C , ^{40}Ca and to $\bar{p}$ - ^{12}C , ^{16}O , ^{40}Ca elastic scattering data at 47 and 179 MeV, respectively. The fit is performed considering the total $\bar{p}$ - n cross section, the slope and the real to imaginary ratio of the $\bar{p}$ - n scattering amplitude to be free parameters. Finally, in sect.6 the conclusions are drawn.

2. Theory

The Glauber treatment of the hadron nucleus scattering is based on the projectile free nucleon amplitudes, neglecting off-shell and rescattering (i.e. multiple scattering on the same nucleon) effects. The limits of validity of this approach have been extensively studied in the past⁽¹¹⁻¹³⁾. Recently, it has been definitely established that, when the eikonal propagator is employed in the Watson multiple scattering expansion of the on-shell transition amplitude, there is a complete cancellation between off-shell and rescattering effects if the nucleon spin is neglected, and one obtains the Glauber multiple scattering expansion^(14,15). Hence, the Glauber theory practically is the eikonal approximation to the first order and can be used whenever the elementary projectile-nucleon amplitude is very strongly peaked at small transfer-

red momenta. This is the case, for instance, of the $\bar{p}$ -nucleus scattering at high energy (> 1 GeV) and also of the $\bar{p}$ -nucleus scattering at low energy, where the large annihilation probability gives a distinct diffractive structure to the elementary $\bar{p}$ -nucleon scattering amplitude. Rather detailed analyses^(16,17), performed on this line, led to the conclusion that the Glauber theory is applicable with good accuracy to the $\bar{p}$ -nucleus scattering down to energies around 50 MeV (300 MeV/c in momentum).

For this reason, we use here the same Glauber model which describes successfully the $\bar{p}$ -nucleus scattering above 1 GeV^(18,19). In this approach, assuming for the nucleus the independent particle model, in which the ground state wave function is expressed as a product of the wave functions of A nucleons without antisymmetrization, the $\bar{p}$ -nucleus scattering amplitude is given by⁽¹⁹⁾:

$$F_N(q) = iK \int J_0(qb) [1 - D(b)] b db \quad (1)$$

with

$$D(b) = \prod_{j=1}^A \left[1 - \frac{1}{iK} \int J_0(tb) S_j(t) f_j(t) t dt \right] \quad (2)$$

In these equations b is the impact parameter, K and q are the laboratory incident and transferred momenta, respectively, J_0 is the Bessel function, S_j is the nuclear form factor of the j -th nucleon and $f_j(q)$ the scattering amplitude of the $\bar{p}$ on that nucleon.

The form factor is expressed by:

$$S_j(q) = \int \exp(i\mathbf{q} \cdot \mathbf{r}_j) \rho(\mathbf{r}_j) d^3r_j \quad (3)$$

where $\rho(\underline{r}_j)$ is the density of the j -th nucleon. For light nuclei ($A \leq 16$) we use single particle densities deduced from a harmonic oscillator potential well. The form factors for 4 nucleons in the S-shell and for $(A-4)$ nucleons in the P-shell are in this case (20):

$$\left. \begin{aligned} S_S(q) &= e^{-\frac{1}{4} R^2 q^2} \\ S_P(q) &= (1 - \frac{1}{6} R^2 q^2) e^{-\frac{1}{4} R^2 q^2} \end{aligned} \right\} \quad (4)$$

where R is the nuclear radius.

For the heavier nuclei ($A > 16$) we suppose that all the nucleons are in the same state, and use a density $\rho(\underline{r}_j) \equiv \rho(r)$ of the Woods-Saxon type. Hence, eq.(3) assumes the form

$$S_j(q) \equiv S(q) = \frac{4\pi}{q} \int \rho(r) \sin(qr) r dr \quad (5)$$

which can be integrated numerically.

The $\bar{p}$ -nucleon scattering amplitude is parametrized in the usual form (18,19):

$$f_j(q) = \frac{K}{4\pi} \sigma_j (\alpha_j + i) e^{-\frac{1}{2} \beta_j^2 q^2} \quad (6)$$

where j stands for neutron or proton ($j=n,p$), σ_j is the $\bar{p}$ -nucleon total cross section, β_j^2 is the slope parameter and $\alpha_j = \text{Re} f_j(0) / \text{Im} f_j(0)$. Considering that we have introduced two nucleon indices, i.e. a shell index ($k=S,P$) for the form factors of the light nuclei and a neutron or proton index ($j=n,p$) for the $\bar{p}$ -nucleon scattering amplitude, we define the integral

$$G_{jk}(b) = \frac{\sigma_j}{4\pi} \int J_0(qb) e^{-\frac{1}{2} \beta_j^2 q^2} S_k(q) q dq \quad (7)$$

so that eq.(2) becomes:

$$D(b) = \Pi_{jk} [1 - (1 - i\alpha_j) G_{jk}(b)] \quad (8)$$

Closed formulae for $G_{jn}(b)$ and $G_{jp}(b)$ for the light nuclei are given in ref.(19). (In that work the same scattering amplitude for neutron and proton is employed). For heavy nuclei, we obtain $G_{jk}(b) \equiv G_j(b)$ from eqs.(5) and (7) by numerical integration.

To improve the accuracy of the model we must introduce now the Coulomb interaction. It is possible to accomplish this in a satisfactory way using the average Coulomb field approximation (18,19,21). The result is an amplitude of the form

$$F(q) = F_C(q) + F_{NC}(q) \quad (9)$$

where $F_C(q)$ is the Coulomb amplitude for the scattering of a particle of charge Z and velocity β by a point like nucleus:

$$\left. \begin{aligned} F_C(q) &= -\frac{2\gamma K}{q^2} \exp(i\phi_C) \\ \phi_C &= -2\gamma \ln(q/2K) + 2\eta \\ \eta &= \arg\Gamma(1+i\gamma), \\ \gamma &= Z/(137\beta) \end{aligned} \right\} \quad (10)$$

and $F_{NC}(q)$ is the nuclear scattering amplitude defined above in eqs.(1,2) modified for the effect of the Coulomb interaction:

$$\begin{aligned}
F_{NC}(q) &= iK \int J_0(qb) [e^{i\chi_{pt}(b)} - e^{i\chi_c(b)} D(b)] b db \\
\chi_{pt}(b) &= 2\gamma \ln(Kb) \\
\chi_c(b) &= 4\pi\gamma [\ln(Kb) \int_0^b T(t) t dt + \int_b^\infty \ln(Kt) T(t) t dt]
\end{aligned}
\tag{11}$$

Here $T(b)$ is the transparency of the charge density $\rho_{ch}(r)$:

$$T(b) = \int \rho_{ch}(b+z) dz, \quad r = b + z \tag{12}$$

For the light nuclei, we use the charge density corresponding to the harmonic potential well⁽²⁰⁾:

$$\begin{aligned}
\rho_{ch}(r) &= \frac{2}{\pi^{3/2} Z R_{ch}^3} [1 + \delta (\epsilon + \mu r^2)] e^{-(r/R_{ch})^2} \\
\delta &= (Z-2)/3 \\
\epsilon &= 3(R_{ch}^2 - R^2)/(4R_{ch}^2) \\
\mu &= R^2/R_{ch}^4
\end{aligned}
\tag{13}$$

where R_{ch} and R are the radii of the charge and nucleon distributions, respectively.

For the heavy nuclei, we assume for $\rho_{ch}(r)$ the Woods-Saxon density with radius R_{ch} and diffuseness a_{ch} .

Finally, we include in the scattering amplitude the effects on the motion of the nucleons due to the nuclear center of mass constraint. Following refs. (18,19) we write:

$$F_G(q) = \exp[\langle r^2 \rangle q^2 / (6A)] F(q) \tag{14}$$

where $\langle r^2 \rangle$ is the m.s. radius of the nucleus and q^2 the transferred four-momentum. The exponential factor in eq.(14) is exact for the light nuclei, for which we use shell model harmonic oscillator (gaussian) wave functions without antisymmetrization. In this case, we have^(18,22)

$$\langle r^2 \rangle = R^2 (5/2 - 4/A) \quad (15)$$

Note that the commonly used value^(19,23) $\langle r^2 \rangle = 3/2 R^2$, is valid only for antisymmetrized gaussian wave functions and is inappropriate in this case. In effect, it has been shown that its use increases sensibly the inaccuracy of the model⁽²²⁾ (see also sect.4). For the heavy nuclei, the exponential factor in eq.(14) is only approximate; however, in this case the center of mass effects are small (see also sect.4). For the Woods-Saxon density, we use the well known formula:

$$\langle r^2 \rangle = \frac{3}{5} R^2 + \frac{7}{5} \pi^2 a^2 \quad (16)$$

3. Choice of the input parameters of the model

In our model the relevant physical quantities are the $\bar{p}$ -nucleon scattering amplitudes, determined by the parameters σ_p , β_p^2 , α_p , σ_n , β_n^2 , α_n (see eq.(6)) and the shape of the nucleus, determined by the parameters R and R_{ch} for the light nuclei and by R , a , R_{ch} and a_{ch} for the heavy ones. The determination of the neutron parameters σ_n , β_n^2 and α_n is one of the objective of this paper; therefore they are left free and their values will be found by fitting the experimental data in sect.5. The remaining parame-

ters are fixed as follows.

The parameters of the $\bar{p}$ -p scattering amplitude come from $\bar{p}$ -p scattering experiments; the results obtained at momenta between 0.3 and 1.5 GeV/c are summarized by the following empirical formulae, taken from refs.(9,24):

$$\sigma_p = 61.2 + 53.4/p \quad (\text{mb}) \quad (17)$$

$$B_p^2 = 65.6 - 129.4 p + 109.0 p^2 - 30.2 p^3 \quad (\text{GeV/c})^{-2} \quad (18)$$

$$\alpha_p = 1.33 - 10.34 p + 22.28 p^2 - 13.63 p^3 \quad (19)$$

where p is the incident momentum in GeV/c. The results of these formulae for the two energies discussed here are reported in Tab.I. We notice that the values of σ_p from eq.(17) are about 5% higher than those coming from a recent LEAR experiment⁽²⁵⁾.

The geometry of the target nucleus is defined through the density $\rho(r)$ of eqs.(3), (4) and (5), which describes the distribution of the nucleon centers. Indeed, the Glauber model implies a folding just of the distribution of the nucleon centers with the nucleon scattering amplitudes, so giving nuclear shapes equivalent to the potential shapes obtained in the framework of the optical model⁽²⁶⁾. For the light nuclei, the radius R of the point-like nucleon distribution and the radius R_{ch} of the charge density, to be used in eqs.(4), (13) and (15), are⁽²⁰⁾:

$$R = \frac{\langle r^2 \rangle_{ch} - \langle r^2 \rangle_p}{(Z-2)/Z + 3(1-1/A)/2} \quad (20)$$

$$R_{ch}^2 = R^2 + \frac{2}{3} \langle r^2 \rangle_p \quad (21)$$

where $\langle r^2 \rangle_{\text{ch}}$ and $\langle r^2 \rangle_p = 0.69 \text{ fm}^2$ are the m.s. radii of the charge density of the nucleus and of the proton, respectively. In eqs.(20,21) the center of mass effects are excluded, because they are already taken into account in eq.(14).

For the heavy nuclei, we assume that the observed charge density is the convolution between the point-like proton distribution and the proton charge density:

$$\langle r^2 \rangle_{\text{ch}} = \langle r^2 \rangle + \langle r^2 \rangle_p \quad (22)$$

Hence from eq.(16) we obtain:

$$a^2 = \frac{5}{7\pi^2} (\langle r^2 \rangle_{\text{ch}} - \langle r^2 \rangle_p - \frac{3}{5} R^2) \quad (23)$$

The quantities R_{ch} , a_{ch} and $\langle r^2 \rangle_{\text{ch}}$ are drawn from electromagnetic scattering experiments and are reported for many nuclei in ref.(28). Therefore, with the reasonable assumption⁽²⁷⁾ $R=R_{\text{ch}}$, eq.(23) permits to determine the diffuseness of the point-like nucleon distribution.

The values of the nuclear parameters obtained with this procedure for the nuclei considered in this work are reported in Tab.II.

4. Accuracy of the model

Before showing the results, we discuss briefly the approximations and the limitations of our model. The most relevant effects not accounted for are non eikonal effects, Pauli correlations, spin effects and possible effects due to the approximate treatment

of the Coulomb interaction and of the center of mass constraint for the heavy nuclei.

First, we note that the relative weight of the non eikonal effects on the $\bar{p}$ -nucleus elastic differential cross section is⁽¹³⁾ ($a \simeq 1\text{fm}$)

$$w(q) \simeq \left[(2R^2/(a^2+R^2) - q^2 R^2/4)/(2KR) \right]^2 \quad (24)$$

Considering the range of transferred momenta of the $\bar{p}$ -nucleus data used here, this weight amounts on the average to no more than 5-10% except at 179 MeV for angles above 40° for ^{40}Ca ($w \simeq 50\%$) and above 50° for ^{12}C and ^{16}O ($w \simeq 15\%$).

For what concerns spin effects, we note that they have not yet been detected in $\bar{p}$ -p elastic scattering experiments⁽²⁵⁾ and that the current $\bar{p}$ -nucleon potential models predict very small spin-flip amplitudes⁽²⁹⁾. When these amplitudes are inserted in the Glauber model, they give completely negligible effects on the $\bar{p}$ -nucleus elastic scattering distributions⁽²⁹⁾.

To take into account the Pauli exclusion principle, one should utilize in the Glauber model antisymmetrized instead of product wave functions^(21,23). The antisymmetrized wave functions make the calculations much more cumbersome, but the results differ sensibly from those obtained with product wave functions only at high transferred momenta, provided that the correct value of eq.(15) for the m.s. radius of the nucleus is used^(18,22). We tested directly this point by calculating the p - ^{16}O differential elastic cross section at 1 GeV and comparing the result to that of ref.(23), obtained with antisymmetrized wave functions. We ob-

tained the confirmation that the curves calculated in the two different ways are graphically indistinguishable up to the second diffraction minimum; for higher transferred momenta the average difference is around 10-20%.

Finally, we mention the inaccuracy due to the approximate treatment of the Coulomb interaction and of the center of mass constraint for heavy nuclei. These (and other) effects are examined in detail in refs. (18,22) where the elastic scattering of 1 GeV protons on even-even nuclei is considered. In these works it is shown that the difference between models like our one and more exact approaches are significative only at high transferred momenta and are not greater than 5-10%.

From these considerations we conclude that the deficiencies of our model affect the elastic scattering distributions only at high transferred momenta, by an amount comparable to the errors of the available $\bar{p}$ -nucleus data. Although small, these effects will play some role in our data analysis; since they are very difficult to be evaluated, in what follows we will refer globally to them as to the "systematic errors of the model".

5. Data analysis

As explained in the introduction, our aim is to determine the $\bar{p}$ -n scattering amplitude from the $\bar{p}$ -nucleus elastic scattering data obtained recently at LEAR. Here we consider the ^{12}C and ^{40}Ca data taken at 46.8 and 47.8 MeV, respectively (~ 302 MeV/c) and the ^{12}C , ^{16}O and ^{40}Ca data taken at 179.7, 178.4 and 179.8 MeV, respectively (~ 607 MeV/c). These data have a good accuracy, the stati-

stical error ranging from 5% to 30% from small to high transferred momenta, the angular acceptance being $2(\Delta\theta) \approx 3^\circ$ and the absolute uncertainty on the overall normalization being around 10%.

We have calculated the theoretical differential elastic cross section $(d\sigma_{th}(\theta)/d\Omega)_{lab}$ as the square modulus of the amplitude $F_G(\theta)$ by eq.(14), have performed the relativistic lab.-c.m. transformation and have averaged the result over the angular experimental acceptance as in ref.(30):

$$\sigma_{th}(\theta) = \frac{1}{3} \left[\frac{d\sigma_{th}(\theta - \Delta\theta/\sqrt{2})}{d\Omega} + \frac{d\sigma_{th}(\theta)}{d\Omega} + \frac{d\sigma_{th}(\theta + \Delta\theta/\sqrt{2})}{d\Omega} \right] \quad (25)$$

In this equation all the quantities are referred to the center of mass system.

Then, the comparison with the data was made by minimizing with respect to the three parameters σ_n , β_n^2 and α_n the χ^2 given by:

$$\chi^2 = \sum_{i=1}^N \frac{[\sigma_{th}(\theta_i) - \sigma_{exp}(\theta_i)]^2}{[\Delta\sigma_{exp}(\theta_i)]^2} \quad (26)$$

where $\sigma_{exp}(\theta)$ is the experimental cross section, $\Delta\sigma_{exp}(\theta)$ its statistical error and N the number of the experimental points.

We note that eq.(26) gives a χ^2 slightly overestimated, because the additional error due to the angular acceptance is neglected.

In order to avoid in the calculation of $\sigma_{th}(\theta)$ spurious effects due to inaccurate computational techniques (a fact often present in the broad literature on the Glauber theory; for details see the review of ref.(18)), we performed the calculations in double precision using analytical formulae whenever possible and

solving numerically the basic formulae of the model in the other cases. The χ^2 minimization has been done by means of the standard program MINUIT of the CERN library⁽³¹⁾.

Since the "a priori" evaluation of the systematic errors of our model is difficult, we decided to fit at first each nucleus separately, then different nuclei together. In this way the presence of systematic errors can be detected by the statistical incompatibility among the best fit parameters determined from different nuclei.

The results are shown in Tab.III.

At 179 MeV the fit on each nucleus is good and there is a good consistency among the different groups of best fit parameters. Hence, we think that the overall fit (row (e) of Tab.III) gives a reliable estimate of the p-n parameters at this energy, considering also the acceptable χ^2 value obtained with 126 points and only three free parameters. The curves relative to this case are shown in fig.1.

At 47 MeV a very good fit is achieved for ^{12}C and ^{40}Ca separately, and the best fit values of σ_n and β_n^2 are consistent each other, though with low probability. On the contrary, the values of α_n are incompatible (rows (f) and (g) of Tab.III). Moreover, if the α_n value found for ^{12}C (^{40}Ca) is kept fixed in fitting the ^{40}Ca (^{12}C) data, a worse reduced χ^2 is obtained ($\chi^2/\text{DF}=1.9$ for ^{12}C and $\chi^2/\text{DF}=3.7$ for ^{40}Ca).

In Tab.III we present also the fit of ^{12}C and ^{40}Ca together at 47 MeV keeping α_n fixed at the two best fit values found on the two nuclei separately (row (h)) and with α_n as a free parameter (row (i)). The comparison between these two fits shows that, in

spite of the indetermination on α_n , a fairly accurate determination of σ_n and β_n^2 is still possible.

The result of the fit with σ_n , β_n^2 and α_n as free parameters is also shown in fig.2, where the effects due to the inaccuracy of the model are particularly evident around the minimum of the ^{12}C distribution.

To test further the dependence of the results on the systematic errors of the model at large angles, we limited the fit only to the data below an angle, calculated from eq.(24), corresponding to a relative weight of the non eikonal effects less than 5%. The results obtained in this way agree very well statistically with those reported in Tab.III. In particular, at 47 MeV the discrepancy concerning the estimate of α_n remains unchanged. We think that the inefficiency of the model at this lower energy could be due to the inadequate treatment of the strong Coulomb effects, that influence mainly the depth of the diffraction minima and consequently the value of α_n (18,22).

In fig.3 the contours of constant χ^2 at 179 and 47 MeV are shown for ^{40}Ca , where the correlations between the free parameters are more evident than in the other nuclei. From these plots it results that at 179 MeV β_n^2 and α_n are inversely correlated with σ_n , whereas β_n^2 and α_n are nearly independent each other; at 47 MeV σ_n and β_n^2 are again inversely correlated, but α_n is independent of these two parameters. As previously noted, in this last case the inaccurate determination of α_n does not affect sensibly the estimate of σ_n and β_n^2 .

The diffraction theory can explain in part the above-

mentioned correlations between the parameters. Indeed, σ_n and β_n^2 determine the nucleon size effective in the scattering process, that is the effective size of the nucleus, which is connected to the location of the minima of the elastic angular distribution. Hence, an increase of σ_n during the fitting procedure causes a decrease of β_n^2 and vice versa. On the other hand, since α_n is the relative strength of the real to the imaginary part of the $\bar{p}$ -n amplitude, it is related to the amount of diffraction, that is to the depth of the minima. Hence, a decrease of α_n causes an increase of the depth of the minima which is compensated in part by an increase of σ_n .

The contours of fig.3 are representative of all our fits and show also that only one minimum exists in the χ^2 space, which is found unambiguously by the fitting procedure.

In Tab.IV the $\bar{p}$ -n parameters determined in this work are compared with those deduced from other analyses and models. In order to compare homogeneous quantities and to compensate for small discrepancies present in these analyses about the choice of the $\bar{p}$ -p parameters, we use here the averages between the $\bar{p}$ -p and $\bar{p}$ -n parameters. From the table we see that there is a reasonable overall agreement between the σ and β^2 values. In particular, the agreement between our results on σ and those obtained from the analyses of $\bar{p}$ -nucleus total cross sections is very good. On the contrary, the estimates of α at 179 MeV are in distinct disagreement.

6. Conclusions.

We have shown the validity of the Glauber theory for describing the $\bar{p}$ -nucleus scattering at 179 MeV (~ 600 MeV/c), and have determined at this energy the $\bar{p}$ -n parameters analogous to the $\bar{p}$ -p ones measured in elastic scattering experiments.

At 47 MeV (~ 300 MeV/c) we have detected the presence of systematic errors in the model which make the theoretical prediction inaccurate around the diffraction minima of the differential elastic cross sections, and prevent the determination of the parameter α_n . Hence, at 300 MeV/c and below the Glauber model must be considered as a tool for parametrizing in an effective way the diffractive behaviour of the scattering distributions, but care is required in the attribution of a physical significance to the real to imaginary ratio of the $\bar{p}$ -nucleon amplitude derived from the data⁽⁷⁾.

Concerning the total cross sections, we find $\sigma_n/\sigma_p = 0.91 \pm 0.01$ and $\sigma_n/\sigma_p = 0.75 \pm 0.03$ at 179 and 47 MeV, respectively, in good agreement with the results of refs.(9,10), obtained analyzing $\bar{p}$ -nucleus total reaction cross sections.

Concerning the slope parameter, we find that the ratio β_n^2/β_p^2 is equal to 1.06 ± 0.03 at 179 MeV and to 0.91 ± 0.07 at 47 MeV, respectively (from Tab.I and rows (e) and (i) of Tab.III). Therefore the condition $\sigma_n/\sigma_p = \beta_n^2/\beta_p^2$, often made in the literature on the basis of the diffractive nature of the $\bar{p}$ -nucleon interaction^(8,9,10,17), seems to be valid, within the errors, only at 47 MeV and not at 179 MeV.

Finally, we note that at 179 MeV the parameter α , which is of

great importance in the dispersion theory because of its connection with the existence of $\bar{p}$ -nucleon resonances around threshold⁽²⁴⁾, assumes completely different values for the proton and the neutron ($\alpha_p \approx 0.2$, $\alpha_n \approx 0.03$).

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Table Captions

Table I:

Parameters of the $\bar{p}$ -p scattering amplitude at the two energies discussed in the text.

Table II:

Radius R and diffuseness a of the point-like nucleon distributions for the nuclei considered in the text. The radius R_{ch} of the charge density is calculated from eq.(21) for ^{12}C and ^{16}O , and taken from the experiment (28) for ^{40}Ca . The values of the r.m.s. radius $\langle r^2 \rangle^{1/2}_{ch}$ of the charge distributions are taken from ref.(28). All the quantities are in fm.

Table III:

Best fit values of σ_n , β_n^2 and α_n obtained at 47 and 179 MeV. The row indices in the energy columns indicates that, in the fit on more nuclei, each nucleus has been considered at the proper energy of the incident $\bar{p}$. In the overall fit at 47 MeV (row (h)) the α_n values were fixed for each nucleus at the best fit values obtained in the individual fit (rows(f) and (g)). The error on the best fit values of the $\bar{p}$ -n parameters is the first standard deviation found by the minimization program taking fully into account the correlations between the parameters. The symbol DF stands for degrees of freedom.

Table IV:

Comparison between the averaged $\bar{p}$ -p and $\bar{p}$ -n parameters obtained in this and other recent works. This work: average between the $\bar{p}$ -p

values from eqs.(17-19) and the $\bar{p}$ -n values from the rows (e) and (i) of Tab.III; $\bar{p}$ -nucleus: $\bar{p}$ -nucleus total reaction cross section analysis of ref.(9); $\bar{p}$ - ^4He : total reaction cross section analysis of ^4He data of ref.(10); $\bar{p}$ -n: average between σ_p from eq.(17) and $\sigma_n=214\pm 19$ mb measured in ref.(33) from $\bar{p}$ -n scattering ; OM: optical model analysis of ref.(32) of $\bar{p}$ -nucleus elastic scattering data; Paris: deduced theoretically from the Paris potential (reported in ref.(33) at 47 MeV, in ref.(29) at 179 MeV); DR: as the Paris one for the Dover and Richard potential; NIJ: average between σ_p from eq.(17) and the value $\sigma_n=186$ mb reported in ref.(33) as the result of a calculation with the Nijmegen potential.

TABLE I

Energy (MeV)	Momentum MeV/c	σ_p (mb)	B_p^2 (GeV/c) ⁻²	α_p
47.4	302	238.0	35.6	-0.132
179.2	607	149.2	20.5	0.216

TABLE II

	^{12}C	^{16}O	^{40}Ca
R	1.62	1.76	3.70
a	-	-	0.507
R_{ch}	1.75	1.89	3.70
a_{ch}	-	-	0.550
$\langle r^2 \rangle_{\text{ch}}^{1/2}$	2.46	2.72	3.53

TABLE III

row	nucleus	Energy (MeV)	σ_n (mb)	β_n^2 (GeV/c) ⁻²	α_n	χ^2/DF	DF
a	¹² C	179.7	136.7±3.2	21.5±0.9	0.016±0.029	1.09	46
b	¹⁶ O	178.4	129.8±3.4	19.8±0.9	0.044±0.034	1.45	26
c	⁴⁰ Ca	179.4	137.8±5.3	25.9±1.6	0.063±0.032	1.51	45
d	¹² C, ¹⁶ O	a, b	133.0±2.3	20.5±0.6	0.037±0.021	1.25	75
e	¹² C, ¹⁶ O, ⁴⁰ Ca	a, b, c	135.2±2.0	21.8±0.6	0.035±0.017	1.53	123
f	¹² C	46.8	180.5±9.7	32.3±2.8	0.245±0.050	1.18	41
g	⁴⁰ Ca	47.8	144.1±12.6	46.7±6.3	0.159±0.063	1.15	21
h	¹² C, ⁴⁰ Ca	f, g	171.4±7.4	35.1±2.5	f, g	1.19	66
i	¹² C, ⁴⁰ Ca	f, g	177.7±8.2	32.4±2.5	0.080±0.064	1.67	65

TABLE IV

	47 MeV			179 MeV		
	σ	β^2	α	σ	β^2	α
	(mb)	(GeV/c) ⁻²		(mb)	(GeV/c) ⁻²	
This work	207.8±8.1	34.0±2.5	-	142.2±2.0	21.1±0.6	0.125±0.017
$\bar{p}$ -nucleus	217.7	-	-	141.3	-	-
$\bar{p}$ - ⁴ He	200.4±19.2	-	-	142.5±9.0	-	-
p- $\bar{n}$	226.0±19	-	-	-	-	-
OM	197	27.0	0.45	158	17.3	0.39
Paris	219	-	-	145	25.7	0.015
DR	225	-	-	155	24.4	±0.04
NIJ	212	-	-	-	-	-

Figure Captions

Fig.1: Best fit curves, σ_n, β_n^2 and α_n best fit values and experimental data at 179 MeV.

The $\bar{p}$ -p parameters are calculated from eqs.(17-19).

The nuclear radii and diffusenesses are taken from Tab.II.

Fig.2: As in fig.1 for ^{12}C and ^{40}Ca at 47 MeV.

The ^{12}C data around the minimum have been measured at 45 MeV. The small bump in the theoretical curve reflects this change in energy.

Fig.3: χ^2 contours as a function of the three free parameters for ^{40}Ca at 179 MeV (a),b),c)) and at 47 MeV (d),e),f)).

Starting from the cross at the centre, the curves of constant χ^2 per degree of freedom refer to the values: 1.51 (cross), 1.54, 1.60, 1.71, 1.87, 2.01, 2.31, 2.60 at 179 MeV; 1.15 (cross), 1.20, 1.34, 1.51, 1.91, 2.34, 2.87, 3.48, at 47 MeV.

FIG. 1

FIG. 2

FIG. 3