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Covariant Lattices, Superconformal Invariance and Strings

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Abstract

We investigate the implications of various ways of realizing superconformal invariance on the world-sheet, in the framework of the bosonic covariant lattice approach to fermionic strings. We discuss in detail the role of a certain type of gauge interactions, and clarify their group theoretical structure. The results are relevant for lower dimensional heterotic and type II string theories. We show that (in our approach) there exists no four dimensional type II theory containing the standard model.

1. Introduction

The covariant lattice approach, originating in the work of [1] [2], and further developed in [3] [4] [5] [6], has proved to be a very convenient tool for constructing various fermionic string theories in their “critical” (or, more properly, maximal) dimension and below. Like all consistent string theories, these theories are conformally invariant, modular invariant and superconformally invariant. The details of this construction are explained in [6]. Here we will remind the reader merely of the main features.

What our method has in common with other approaches [7] [8] [9] [10] [11] [12] [13] [14], is that for the left- and right-moving sectors of the closed string one chooses independently one of the following two conformal field theories: either the bosonic one with 26 bosonic matter fields or the $N=1$ superconformal one with 15 bosonic matter fields (or equivalently, 10 bosons and 10 fermions). By assembling these in various ways one can construct the generic bosonic, heterotic [15] or type II [16] string theories. To obtain a theory in D space-time dimensions, one chooses D bosonic left- and right-moving fields as the space-time coordinates $X^\mu(z, \bar{z})$. The various methods differ in the way the remaining fields are treated. In the covariant lattice approach the remaining bosons, together with the bosonized spinor ghosts have their internal momenta quantized on an odd self-dual lorentzian lattice. Some of these correspond to D bosonized NSR-fermions ψ^μ , while the remaining ones play the role of internal degrees of freedom. In general, it is *not* appropriate to regard them as arising from a space-time compactification process.

The advantage of our approach becomes evident in discussing modular invariance: it is simply guaranteed by the self-duality of the lattice, provided [4] one assigns appropriate statistics to tensor and spinor representations of $SO(D)_{\text{lorentz}}$. By the arguments of [17] we expect the theory to be not only one-loop, but *all-loop* modular invariant¹ [19].

In this paper, we will focus on the third consistency condition, namely superconformal invariance. In [6] chiral heterotic strings in less than ten dimensions were constructed, and a consistency condition was obtained by requiring the absence of massive chiral states. This condition turned out to be equivalent to a condition derived in [11] from superconformal invariance. That condition is clearly not the most general one; for example, it is not satisfied by Narain's compactification [7].

The main purpose of this paper is to investigate generalizations of this condition. The results will be relevant for the right-moving (fermionic) sector of the heterotic string and for both sectors of the type II string. For most of this paper we will consider only one sector, which we will refer to as "right-moving". The lattice has the following structure:

$$\Gamma = \Gamma_L \times \Gamma_R. \quad (1.1)$$

Note that Γ_L and Γ_R are in general not self-dual lattices by themselves. In $D = 10 - 2n$ dimensions one has

$$\Gamma_R = \Gamma_{3n} \times \Gamma_{5-n,1}, \quad (1.2)$$

¹Note also that self-duality of the lattice of a compactified *bosonic* string is sufficient for all-loop modular invariance [18].

where $\Gamma_{5-n,1}$ is the lorentzian space-time lattice defined in [6]. In type II theories, Γ_L is of the same form as Γ_R ; in heterotic theories, $\Gamma_L = \Gamma_{16+2n}$. The lattice Γ in (1.1) is odd lorentzian self-dual, and can be mapped on an even lorentzian self-dual one by mapping [4] $\Gamma_{5-n,1}$ into D_{8-n} . Both formulations have their specific advantages, and we will freely use either one depending on which is more convenient. To distinguish them we will use superscripts (e) or (o). In either case a vector on Γ_R can be decomposed as follows:

$$w^{(o)} = (u, v^{(o)})$$

or (1.3)

$$w^{(e)} = (u, v^{(e)}),$$

where $u \in \Gamma_{3n}$, while $v^{(o)}$ belongs to $\Gamma_{5-n,1}$ and $v^{(e)}$ to D_{8-n} . As explained in [6], in the even lattice formulation massless states correspond to certain length-squared two vectors on Γ_R ; on Γ_L they correspond also to length-squared two vectors, and, in the case of heterotic strings, also to bosonic oscillator excitations. We remind the reader that because of the physical state selection rule (explained in [3] [5] [6]), the conjugacy classes (o), (v), (s) and (c) of $\Gamma_{5-n,1}$ (or D_{8-n}) contain Lorentz vectors, scalars and spinors of both chiralities, respectively. This selection rule applies to the right-moving sector of the heterotic string and to both sectors of the type II string. In particular, massless vector bosons are Lorentz vectors in one sector and Lorentz scalars in the other¹. The latter sector determines the gauge group. One has to distinguish two cases. If this sector is the bosonic

¹In type II strings, Lorentz vectors can appear also as bispinors where the spinor indices come from different left- and right-moving sectors. We will return to this issue in section 6.

string, the gauge group is the usual Frenkel-Kac group G_{FK} [20] generated by length-squared two vectors plus oscillators. If the sector in question is the NSR-string, we have massless gauge bosons for

$$\nu^{(e)} \in (v) \text{ of } D_{8-n} \quad u^2 = 1. \quad (1.4)$$

In this case, we will call the gauge group G_{NSR} . Thus, in general, in heterotic strings one can have gauge groups $G_{\text{FK}}^L \times G_{\text{NSR}}^R$; in type II strings one can have $G_{\text{NSR}}^L \times G_{\text{NSR}}^R$ (where the groups can be different). As we will see below, it turns out that G_{NSR} is not just determined by the structure of the lattice, but also on the way superconformal invariance is realized.

Gauge symmetries of this kind have already been discussed in the literature (in fermionic language) [11] [21] [22] [23]. The purpose of this paper is to show how they appear in the bosonic lattice construction. Picture changing, which is built into our formulation from the very beginning, will be seen to play a crucial role in determining the gauge interactions. In sections 2 and 3 we discuss general bosonic realizations of the supercurrent. In the rest of the paper we then turn to a somewhat simpler special case, namely a realization in terms of free fermions¹ [25] [26].

¹While writing this paper we received [24] which arrives at conclusions similar to the ones obtained in sections 4 and 6 below, although with different methods.

2. Superconformal Invariance and Picture Changing

World-sheet supersymmetry is generated by a supercurrent $T_F(z)$ which has to have the following properties

$$\begin{aligned} T_F(z) T_F(w) &= \frac{M}{(z-w)^3} - \frac{2 T(w)}{z-w} + \text{finite}, \\ T(z) T_F(w) &= \frac{3/2}{(z-w)^2} T_F(w) + \frac{1}{z-w} \partial_w T_F(w) + \text{finite}, \end{aligned} \quad (2.1)$$

where $T(z)$ is the energy momentum tensor and $M=10$. In ten dimensions the operator product algebra is satisfied by

$$T_F(z) = \psi^\mu \partial X_\mu(z), \quad (2.2)$$

where ψ^μ are the NSR fermions which also may be written in bosonized form [1]. Because of Lorentz invariance, in $10-2n$ dimensions the space-time part $T_F^{\text{st}}(z)$ of the supercurrent has the same form, but we are free to modify the remaining part $T_F^{\text{int}}(z)$. What we are looking for is the most general operator constructed out of $3n$ bosons associated with Γ_{3n} , which satisfies (2.1) with $M=2n$. The second of these equations implies that it must have conformal weight $3/2$. It must then be a linear combination of the form

$$\begin{aligned} T_F^{\text{int}}(z) &= \sum_{\vec{\ell}} A(\vec{\ell}) e^{i \vec{\ell} \vec{X}}(z) + i \sum_{\vec{\ell}} \vec{B}(\vec{\ell}) \cdot \partial \vec{X} e^{i \vec{\ell} \vec{X}}(z), \\ \vec{\ell}^2 &= 3, \quad \vec{\ell}^2 = 1, \quad \vec{\ell} \cdot \vec{B}(\vec{\ell}) = 0, \end{aligned} \quad (2.3)$$

where X^I denotes the $3n$ bosons, and where multiplication with appropriate cocycle operators is understood. The superconformal algebra (2.1) imposes certain conditions on A and B ; however, there is no need here to write these equations down. In general, there are many choices for $A(t)$ and $B(l)$ that satisfy (2.1), and we will discuss some of them below.

The supercurrent plays an important role in determining interactions. The covariant lattice approach is based on conformal field theory, and it provides us with covariant vertex operators for all physical states. In the “canonical” ghost charge vacuum, the emission vertices for the Neveu-Schwarz sector have ghost charge -1 and those for the Ramond sector ghost charge $-1/2$. In order to calculate scattering amplitudes one has to combine the vertices in such a way as to cancel the background ghost charge [1]. To calculate arbitrary amplitudes we need thus also vertex operators in different ghost charge sectors. These can be obtained by applying the picture changing operator ($T_F = T_F^{\text{st}} + T_F^{\text{int}}$)

$$P_{+1} = e^{i\phi} T_F + \text{irrelevant terms}, \quad (2.4)$$

where ϕ is the bosonized superconformal ghost as defined in¹ [1] [6]. Picture changing is achieved as follows:

$$V_{q+1}(z) = : P_{+1} V_q(z) : , \quad (2.5)$$

where V_q is a vertex operator with ghost charge q . For example, to calculate R-R-NS scattering at tree level one can consider the correlation function $\langle V_{-1/2} V_{-1/2} V_{-1} \rangle$; for (NS)³ couplings one considers $\langle V_{-1} V_{-1} V_0 \rangle$.

¹Our convention is to keep explicit factors of i in the exponentials in vertex operators.

Both expressions have the correct ghost charge -2 for the sphere. (For type II strings these considerations apply of course to both sectors separately). We will calculate such three-point functions in the next section. Notice that only in the second case does one need the picture changing operator.

One can only have a consistent string theory if both V_q and V_{q+1} are well-defined. In particular the picture changing operator should act on all vertices without introducing branch cuts. Consider for example a vertex operator $V_q(u) = \exp(iu \cdot X) \exp(iq\phi)$ with lattice momentum u on Γ_{3n} . The operator product of P_{+1} with V_q will have a branch cut unless

$$\begin{aligned} q - \vec{\alpha} \vec{t} &\in \mathbb{Z} \\ \text{and} \\ q - \vec{\alpha} \vec{l} &\in \mathbb{Z} \end{aligned} \quad (2.6)$$

where t and l are the vectors appearing in (2.3). Since odd ghost charges are part of the conjugacy class (v) of $\Gamma_{5-n,1}$, this implies that all vectors $w^{(o)} = (t, (v))$ and $w^{(o)} = (l, (v))$ should have integer dot-products with all other lattice vectors. Because the lattice is self-dual, this implies that they must lie on the lattice themselves. The corresponding even lattice must then contain all vectors $w^{(e)} = (l, (v))$ and $w^{(e)} = (t, (v))$, for all l and t appearing in the supercurrent. This constrains the choice of the supercurrent with respect to the lattice (or vice versa). The shortest vectors in the two sets have length-squared 2 and 4 respectively. We will refer to such vectors, whose existence is enforced by the supercurrent, as "constraint vectors". From the selection rules [6] it follows that every length-squared 2 vector of the form $(l, (v))$

implies the existence of a G_{NSR} gauge boson. This fact will play an important role in our discussion. The space-time part $T_F^{\text{st}} \exp(i\phi)$ of P_{+1} (associated with $w^{(e)} = (u, r)$, $u=0$, $r^2=2$) belongs to the (0) conjugacy class of D_{8-n} . Thus the length-squared two vectors required by T_F^{st} are always present¹, and imply, in fact, the existence of gravitons.

The simplest way to satisfy (2.1) is to keep the form (2.2) even below ten dimensions. In four dimensions, after bosonization one obtains then an expression of the form (2.3) with $A=0$ and six non-vanishing coefficients $B(I)$. The resulting theory will have at least six G_{NSR} gauge bosons (of course there may be gauge bosons in addition to those required by the supercurrent). This situation arises in torus compactifications, including the ones discussed in [7] (here, the six gauge bosons belong to the $N=4$ supergravity multiplet; they arise due to the usual Kaluza-Klein mechanism).

3. Three Point Interactions

In this section we will calculate three-point couplings in the right-moving sector for three (Lorentz) scalars and investigate their dependence on the supercurrent. (The scalars can of course acquire Lorentz indices from the left-moving sector to become G_{NSR} gauge bosons or space-time fermions.) In addition, we also calculate couplings between left- and right-moving sectors.

¹Therefore, the argumentation of [3] discussing modular invariance can always be applied.

Consider first the three-scalar coupling. In the canonical ghost charge vacuum, the emission vertex for a state with lattice vector (w_L, w_R) , $w_R = (u, v)$, where v is a vector weight of $\Gamma_{5-n,1}$ (or D_{8-n}), is given by¹

$$\begin{aligned} V(\vec{w}_L, \vec{w}_R, z, \bar{z}) &= V^L(\vec{w}_L, z) V^R(\vec{w}_R, \bar{z}) e^{i\vec{k}\vec{X}}(z, \bar{z}), \\ V^R(\vec{w}_R, \bar{z}) &= e^{i\vec{u}\vec{X}} e^{-i\phi}(\bar{z}) \equiv V_{-1}(\vec{u}, \bar{z}). \end{aligned} \quad (3.1)$$

The picture changed vertex operator $V_0(u, \bar{z})$ is then obtained by evaluating

$$\lim_{\bar{w} \rightarrow \bar{z}} \left((T_F e^{i\phi})(\bar{w}) \cdot V(\vec{w}_L, \vec{w}_R, z, \bar{z}) - \text{singular terms} \right) \quad (3.2)$$

and extracting the factor that depends on the right lattice. In the special (massless) case $u^2 = 1$ the result is

$$V_0(\vec{u}) = \sum_{\vec{\ell}} A(\vec{\ell}) e^{i(\vec{\ell} + \vec{u})\vec{X}} + i \vec{B}(-\vec{u}) \cdot \vec{\partial} \vec{X} - \sum_{\vec{\ell}} \vec{B}(\vec{\ell}) \cdot \vec{u} e^{i(\vec{\ell} + \vec{u})\vec{X}} + \frac{i}{2} (h_\mu \psi^\mu) e^{i\vec{u}\vec{X}}. \quad (3.3)$$

The constraint $(l+u)^2 = 2$ in the third term is a consequence of the identity $B(l) \cdot l = 0$ which follows from (2.1). The vertex operator (3.3) can be used to calculate the coupling of a G_{NSR} gauge boson to any other state which is also a Lorentz-scalar in the right sector. The correlation function in the right-moving sector to consider is

$$\left\langle V_{-1}(\vec{u}_1, \bar{z}_1) V_{-1}(\vec{u}_2, \bar{z}_2) V_0(\vec{u}_3, \bar{z}_3) \right\rangle \quad (3.4)$$

¹We use $\bar{z}$ for right-movers, z for left-movers.

with $u_3^2=1$, while u_1 and u_2 are unrestricted. To get the physical three-point function C_{u_1, u_2, u_3} one has to multiply (3.4) with the contribution of the reparametrization ghosts, the space-time factors $\exp(i\vec{k} \cdot X(z, \bar{z}))$ and all remaining contributions from the left sector. The reparametrization ghosts yield a factor $|z_{12}z_{23}z_{13}|^2$, where $z_{ij}=z_i - z_j$. The space-time exponentials contribute a factor $|z_{12}|^{k_1 k_2/2} |z_{23}|^{k_2 k_3/2} |z_{13}|^{k_1 k_3/2}$, where k_i is the space-time momentum corresponding to $V(u_i)$. The rest of the left sector gives z_{ij} -dependent factors which determine the Lorentz tensor structure T_{Lorentz} of the result, which is however not of interest to us here.

Putting all $\bar{z}$ -factors together we get for the contribution of the first term in (3.3)

$$\sum A(\vec{\epsilon}) \binom{\vec{h}_1, \vec{h}_2/4 + \vec{u}_1, \vec{u}_2}{(\vec{z}_{12})} \binom{1 + \vec{h}_1, \vec{h}_2/4 + \vec{u}_1, (\vec{\epsilon} + \vec{u}_3)}{(\vec{z}_{13})} \binom{1 + \vec{h}_2, \vec{h}_3/4 + \vec{u}_2, (\vec{\epsilon} + \vec{u}_3)}{(\vec{z}_{23})} \quad (3.5)$$

If the momenta satisfy the on-shell conditions

$$\begin{aligned} \frac{1}{4} \vec{h}_1^2 &= 1 - \vec{u}_1^2, \\ \frac{1}{4} \vec{h}_2^2 &= 1 - \vec{u}_2^2, \\ \vec{h}_3^2 &= 0, \end{aligned} \quad (3.6)$$

then one can reduce the expression to a constant by using lattice and space-time momentum conservation and the condition $(t+u_3)^2=2$. (The complete 3-point function has to be independent of $\bar{z}_i$ (and z_i) because of Möbius invariance). The result is

$$C_{u_1, u_2, u_3} = A(\vec{\epsilon}) \cdot T_{\text{Lorentz}}, \quad (3.7)$$

with $t = -u_1 - u_2 - u_3$.

In a similar way we find for the contribution of the second term in (3.3):

$$\begin{aligned}
 & - (\bar{z}_{12})^{\frac{1}{2}(\vec{u}_1 + \vec{u}_2)^2 - 1} \left(\vec{u}_1 \cdot \vec{B}(-\vec{u}_3) (\bar{z}_{13})^{\frac{1}{2}(\vec{u}_2^2 - \vec{u}_1^2)} (\bar{z}_{13})^{\frac{1}{2}(\vec{u}_1^2 - \vec{u}_2^2) + 1} \right. \\
 & \quad \left. + \vec{u}_2 \cdot \vec{B}(-\vec{u}_3) (\bar{z}_{13})^{\frac{1}{2}(\vec{u}_2^2 - \vec{u}_1^2) + 1} (\bar{z}_{13})^{\frac{1}{2}(\vec{u}_1^2 - \vec{u}_2^2)} \right) . \quad (3.8)
 \end{aligned}$$

This is a constant if and only if $u_1 = -u_2$; the result is then

$$C_{u_1, -u_1, u_3} = \vec{u}_1 \cdot \vec{B}(-\vec{u}_3) \cdot T_{\text{Lorentz}} . \quad (3.9)$$

Analogously one determines the contribution of the third term in (3.3):

$$\begin{aligned}
 & - \sum_{\substack{\vec{\ell} \\ (\vec{\ell} + \vec{u}_3)^2 = 2}} \vec{B}(\vec{\ell}) \cdot \vec{u}_3 (\bar{z}_{12})^{\frac{\vec{\ell}_1 \cdot \vec{\ell}_2}{4} + \vec{u}_1 \cdot \vec{u}_2} (\bar{z}_{13})^{1 + \frac{\vec{\ell}_1 \cdot \vec{\ell}_3}{4} + \vec{u}_1 \cdot (\vec{\ell} + \vec{u}_3)} (\bar{z}_{13})^{1 + \frac{\vec{\ell}_2 \cdot \vec{\ell}_3}{4} + \vec{u}_2 \cdot (\vec{\ell} + \vec{u}_3)} \\
 & \quad (3.10)
 \end{aligned}$$

This gives also a constant ($l = -u_1 - u_2 - u_3$):

$$C_{u_1, u_2, u_3} = - \vec{B}(\vec{\ell}) \cdot \vec{u}_3 \cdot T_{\text{Lorentz}} . \quad (3.11)$$

Finally, the momentum dependent last term in (3.3) gives no contribution in this case, where all states are Lorentz scalars in the right-moving sector.

We thus see that certain couplings (in particular, three-point G_{NSR} gauge boson couplings) depend on A and B , ie., on the way superconformal invariance is realized. (We will see below how A and B are related to the structure constants of G_{NSR} .)

From the above it is clear that the group G_{NSR} simply acts as a subgroup of the Frenkel-Kac group G_{FK}^{3n} of the Γ_{3n} -part of the right lattice (1.2). The group G_{FK}^{3n} is generated by vertex operators ∂X^I ($I=1,\dots,3n$) and $\exp(ir \cdot X)$, where $r \in \Gamma_{3n}$ with $r^2=2$. Notice that the lattice vector $w_R=(u,v)$, $v=0$, $u=r$ does not correspond to a physical state because of the projection rules of [6]. In particular, one does not have gauge bosons of the complete group G_{FK}^{3n} . Rather, the gauge bosons we *do* get correspond to vectors of the form $v^2=1$, $u^2=1$. After picture changing, their vertex operators are linear combinations of generators of G_{FK}^{3n} , as one can see from (3.3) (the last term does not contribute here). The correlation function (3.4) can thus be interpreted as the matrix element of a linear combination of G_{FK}^{3n} generators between lattice vectors. The gauge group G_{NSR} , generated by the picture changed vertex operators $V_0(u)$, must thus be a subalgebra of the “unphysical” Frenkel-Kac group G_{FK}^{3n} . The representation content of a given level of the string with respect to G_{NSR} is obtained by simply decomposing all representations of G_{FK}^{3n} at that level. The embedding of G_{NSR} in G_{FK}^{3n} depends of course on the coefficients A and B in (3.3).

The problem of determining the embedding splits into two parts. Because (in the even formulation) gauge bosons originate from vectors $v^2=1, u^2=1$, which are roots of Γ_R , the space-time lattice D_{8-n} extends to D_{8-n+m} if there are $2m$ gauge bosons [6]. Thus one has the structure

$$\Gamma_R = \Gamma_{3n-m} \times D_{8-n+m}, \quad (3.12)$$

$$G_{\text{FK}}^{3n} = G_{\text{FK}}^{3n-m} \times D_m,$$

where G_{FK}^{3n-m} is the Frenkel-Kac group of Γ_{3n-m} . It is easy to determine the embedding of G_{NSR} in D_m : clearly, the adjoint representation of G_{NSR} is embedded in the vector representation of D_m . (Notice that G_{NSR} is a conformal subalgebra¹ of D_m [27] [28]). More specific information is needed to discuss the embedding of G_{NSR} in G_{FK}^{3n-m} . Before we examine a special case of supercurrents in the next section, let us briefly consider further three-point couplings.

To what particles the couplings (3.7), (3.9) and (3.11) precisely correspond, depends on the left-moving part which determines the Lorentz structure. For instance, by choosing for two of the three vertex operators a fermion emission vertex as the left factor, one can use the expressions derived above to describe the fermion-fermion- G_{NSR} gauge boson coupling. Hence these couplings have a dependence on the supercurrent, just as those of the G_{NSR} gauge bosons. Massless fermions of this type have length-squared 2 vectors on the right lattice, just as the gauge bosons. But while for the gauge bosons these vectors are roots of D_{8-n+m} (i.e vectors of D_m and of D_{8-n}), the lattice vectors that describe fermions may also have components in Γ_{3n-m} . This is what might in principle allow complex fermion representations. The roots of D_{8-n+m} lead only to adjoints of G_{NSR} , as explained above.

In general, there exist also states which owe their Lorentz index to the *right*-moving sector. Let us consider, for instance, for type II strings the coupling of a G_{NSR}^R gauge boson to two fermions which are Lorentz scalars in the left-moving sector, and which get their spinor index from the right-moving sector (hence $V^L(z) = V_{-1}(u_i, z)$, $i=2,3$ with $u_i^2=1$). After application of the picture changing

¹This means that G_{NSR} has the same Sugawara tensor as D_m .

operator to one fermion, the relevant correlation function in the left-moving sector is

$$\langle (\psi_\mu e^{-i\phi})(z_1) V_{-1}(\vec{a}_2, z_2) V_0(\vec{a}_3, z_3) \rangle \quad (3.13)$$

which leads to a coupling proportional to the momentum k_3^μ of that fermion. It arises due to the momentum dependent last term in (3.3), and carries no dependence on the supercurrent. Clearly, this “mixed” coupling between different sectors describes a derivative and hence non-minimal interaction. From a low energy point of view these fermions are thus singlets of G_{NSR}^R . The same result is obtained also for heterotic strings. This strengthens the claim made in [6] that in heterotic strings fermions can only be chiral with respect to the gauge symmetries originating from the left sector. The three-point coupling between G_{NSR}^R and G_{NSR}^L gauge bosons turns out to be zero. This proves the assertion made in section 1 that the gauge group is always a direct product of left- and right-moving parts.

4. The Supercurrent in Terms of Free Fermions

In this section we discuss special solutions to (2.1) constructed out of free fermions. Here we make use of the fact that the most general form of the supercurrent in terms of free fermions is already known [25] [26] [23]. According to these authors, the most general expression is

$$T_F^{\text{int}}(z) = \frac{1}{6} f_{ABC} \psi^A \psi^B \psi^C, \quad (4.1)$$

where the fermion fields satisfy

$$\psi^A(z) \psi^B(w) \sim \frac{1}{z-w} \delta^{AB} \quad (4.2)$$

and f_{ABC} are structure constants of some semi-simple $N=6n$ -dimensional (where $D=10-2n$) group G_{SC} , normalized as follows:

$$f_{ABC} f_{DBC} = 2 \delta_{AB} . \quad (4.3)$$

Accordingly, there are N free fermions.

Of course (4.1) can always be cast into the bosonic form (2.3) by replacing $6n$ fermions by $3n$ bosons, using

$$\begin{aligned} \psi_{I1} &= \frac{1}{\sqrt{2}} \left(e^{i \vec{e}_I \vec{X}} + e^{-i \vec{e}_I \vec{X}} \right) , \\ \psi_{I2} &= \frac{1}{i\sqrt{2}} \left(e^{i \vec{e}_I \vec{X}} - e^{-i \vec{e}_I \vec{X}} \right) . \end{aligned} \quad (4.4)$$

where e^I is a set of $3n$ orthonormal vectors. It is then clear that the role of the structure constants of G_{SC} is played by A and B in (2.3).

Notice that there is still a lot of freedom in this construction. First of all one may change the choice of the basis e^I with respect to the lattice by $SO(3n)$ rotations. Secondly, one can rotate the ψ 's by $SO(6n)$ rotations respecting (4.2). Because G_{SC} rotations do not affect the structure constants, the true degrees of freedom are parametrized by the coset space $SO(6n)/G_{SC}$. Both kinds of rotations are restricted by the lattice. By rotating the basis e^I , one rotates the constraint vectors. Clearly "small" $SO(3n)$ rotations will therefore not be allowed, but some discrete ones may be acceptable. The number of constraint vectors is determined by the number of non-vanishing structure constants f_{ABC} . In general, $SO(6n)/G_{SC}$ rotations change that number.

Despite all this freedom, it is important to realize that one does not obtain the most general *bosonic* supercurrent in this way. For example

$$T_F^{int}(z) = \frac{1}{13} \sum_{I=1}^{3n} (e^{i\sqrt{13}X^I} + e^{-i\sqrt{13}X^I}) \quad (4.5)$$

is always a solution to (2.1), but e.g. for $n=1$ there is no choice of basis for which it is of the form (4.1).

Consider now a lattice that produces exactly $2m$ G_{NSR} gauge bosons. Then we know that the space-time lattice D_{8-n} is embedded in D_{8-n+m} ; furthermore D_{8-n+m} is the largest D-type algebra of which D_{8-n} is a subalgebra on the lattice. For the first m orthonormal vectors e^I we choose m orthonormal vector weights of D_m . We split the $6n$ fermion indices $A, B, C, \dots$ into sets labeled $a, b, c, \dots$ ($a=1, \dots, 2m$) and $i, j, k, \dots$ ($i=2m+1, \dots, 6n$), so that the ψ^a 's belong to D_m . Then the supercurrent takes the form

$$\begin{aligned} T_F^{int}(z) = \frac{1}{6} (& f_{abc} \psi^a \psi^b \psi^c + 3 f_{abi} \psi^a \psi^b \psi^i \\ & + 3 f_{aij} \psi^a \psi^i \psi^j + f_{ijk} \psi^i \psi^j \psi^k) . \end{aligned} \quad (4.6)$$

The resulting constraint vectors fall into two classes: terms which are the product of three fermions which bosonize to different X_I 's require length-squared 4 constraint vectors; if two fermions in one term bosonize to the same X_I 's, the bosonic form of that term is $\partial X_I \exp(iI \cdot X)$, which yields a length-squared 2 constraint. Decomposing vectors according to

$$\Gamma_R = \Gamma_{3n-m} \times D_m \times D_{8-n},$$

we find then that the four types of terms require the existence of the following kinds of constraint vectors

- (a) $f_{abc} \quad (0, (v), (v))$
- (b) $f_{abi} \quad (l, (0), (v))$
- (c) $f_{aij} \quad (r, (v), (v))$
- (d) $f_{ijk} \quad (t, (0), (v)) \text{ or } (l, (0), (v)),$

where $l^2=1$, $r^2=2$ and $t^2=3$. Constraints of type (a) are automatically satisfied, because the conjugacy class (v) of the D_m lattice contains all possible length 1 and 3 vectors that can appear. Constraint vectors of type (b) are not allowed, since they would enlarge D_{8-n+m} to a larger algebra. Therefore $f_{abi}=0$, and hence f_{abc} are the structure constants of some subalgebra H of G_{SC} . For the same reason the second constraint in case (d) is not allowed. Suppose now that indices i and j exist so that $f_{ija} \neq 0$ and $f_{ijk} \neq 0$ for some a and k. Then the difference of the constraint vectors implied by these two non-vanishing structure constants is of the form $(l, (v), (0))$, which is also not allowed. Therefore for every i and j either f_{ija} or f_{ijk} (or both) must vanish. Only the following possibilities are consistent with this:

- (A) H is trivial; G_{SC}/H is a group manifold
- (B) $H=G_{SC}$; G_{SC}/H is trivial
- (C) H is a factor of G_{SC} ; G_{SC}/H is a group manifold
- (D) G_{SC}/H is a symmetric space
- (E) G_{SC}/H is the product of a symmetric space and a group manifold.

Case (A) is relevant for chiral strings, for which at least one sector has to be of this type [6]. Case (B) is the one studied in [21] [23]; the compactified theories á la Narain [7] belong to class (D), with a symmetric space $(SU(2)/U(1))^6$.

The group H is precisely the gauge group G_{NSR} of the previous section. To see this more clearly, write the vertex operators (3.1) as $e^{-i\phi}\psi^a$. Picture changing with (4.6) produces then the fermionic form of (3.3):

$$V_0(\vec{e}_a, z) = f_{abc} \psi^b \psi^c + f_{aij} \psi^i \psi^j + \frac{i}{2} (h_\mu \psi^\mu) \psi^a. \quad (4.7)$$

In this fermionic formulation, the three-point G_{NSR} gauge boson couplings (3.7), (3.9) and (3.11) are then proportional to the H structure constants f_{abc} . G_{NSR} gauge boson-fermion-fermion couplings may depend on both f_{abc} and f_{aij} .

The first term in (4.7) generates a subalgebra of $SO(2m)$ isomorphic to H . The second term acts on the Γ_{3n-m} part of the lattice as a subalgebra of the tangent space group associated to G_{SC}/H generated by $\psi[i\psi j]$. (This tangent space group need not be a Frenkel-Kac symmetry of the lattice; the constraint vectors listed under (c) above only require that H be non-trivially realized.) Notice that the presence of vectors $(r, (v), (v))$ implies the presence of vectors $(r, 0, 0)$, roots of Γ_{3n-m} . Here we see once more that H acts on Γ_{3n-m} as a subalgebra of G_{FK}^{3n-m} . Because G_{SC}/H is a symmetric space, the embedding of H in the tangent space group (and hence in G_{FK}^{3n-m}) is conformal¹. As observed in section 3, $H \subset SO(2m)$ is also conformal. (The diagonal embedding $H \subset SO(2m) \times G^{3n-m}$ however is not conformal.) The third term in (4.7) corresponds to generators which mix the internal with the space-time part of D_{8-n+m} .

¹One can check this explicitly from the tables of [27] and [28]. It also follows from the results of [29] as observed in [28].

In $D = 10 - 2n$ dimensions, the possibilities for G_{SC} are limited to $SU(2)^{2n}$ [21] plus $SU(4) \times SO(3)$ and $SO(5) \times SU(3)$ if $D = 4$ ($n = 3$) [23] (there are more possibilities if $n > 3$). From the above it is clear that the only possible gauge groups $H \equiv G_{NSR}$ in four dimensions are these groups themselves, or their symmetric subgroups. (In the next section we will show that groups of type (E), i.e. a product of a factor of G_{SC} and a symmetric subgroup of the other factor, are possible only if $G_{SC} = SU(2)^6$.)

For example, in four dimensions ($\Gamma_R = \Gamma_{9-m} \times D_{5+m}$) one can obtain a string theory with gauge group $H = SU(2) \times U(1) \times SO(5)$ as a symmetric subgroup of $G_{SC} = SU(3) \times SO(5)$. There are 14 gauge bosons ($m = 7$), so that the space-time lattice D_5 must extend to D_{12} . The remaining part Γ_{3n-m} has dimension 2. The $SO(5)$ factor occupies a D_5 subalgebra of D_{12} , and does not act on the tangent space. It does not require further discussion. The $SU(2) \times U(1)$ factor is embedded in the remaining D_2 subalgebra of D_{12} , and acts non-trivially on $\Gamma_{3n-m} = \Gamma_2$. To discuss this example in detail we need the $SU(3)$ structure constants, normalized as in (4.3). Apart from this unusual normalization, we take the standard choice

$$\begin{aligned} f_{123} &= \sqrt{2/3} \ ; \\ f_{147} &= f_{146} = f_{157} = f_{345} = -f_{156} = -f_{367} = 1/\sqrt{6} \ , \\ f_{458} &= f_{678} = 1/\sqrt{2} \ . \end{aligned} \tag{4.8}$$

The $SU(2) \times U(1)$ subalgebra is generated by T^1, T^2, T^3 and T^8 . We denote the two coordinates in the D_2 subalgebra of D_{12} as X_1 and X_2 , and the two coordinates in Γ_{3n-m} as X_3 and X_4 . We can then define the free fermions as follows

$$\begin{aligned}
 \psi_1 &= \frac{1}{\sqrt{2}} (e^{iX_1} + e^{-iX_1}) C_1, & \psi_2 &= \frac{1}{i\sqrt{2}} (e^{iX_1} - e^{-iX_1}) C_1, \\
 \psi_3 &= \frac{1}{\sqrt{2}} (e^{iX_2} + e^{-iX_2}) C_2, & \psi_4 &= \frac{1}{i\sqrt{2}} (e^{iX_2} - e^{-iX_2}) C_2, \\
 \psi_5 &= \frac{1}{\sqrt{2}} (e^{iX_3} + e^{-iX_3}) C_3, & \psi_6 &= \frac{1}{i\sqrt{2}} (e^{iX_3} - e^{-iX_3}) C_3, \\
 \psi_7 &= \frac{1}{\sqrt{2}} (e^{iX_4} + e^{-iX_4}) C_4, & \psi_8 &= \frac{1}{i\sqrt{2}} (e^{iX_4} - e^{-iX_4}) C_4,
 \end{aligned} \tag{4.9}$$

where C_i is a cocycle factor. Using (4.8) and (4.9) one constructs the bosonic form of the supercurrent and the $SU(2) \times U(1)$ generators. For the latter we find

$$\begin{aligned}
 T^3 &= -\sqrt{2/3} \partial X_1 - 1/\sqrt{6} \partial X_2 + 1/\sqrt{6} \partial X_4, \\
 T^8 &= -1/\sqrt{2} \partial X_3 - 1/\sqrt{2} \partial X_4, \\
 T^1 &= -\frac{i}{2} \sqrt{2/3} (e^{i(X_1+X_2)} - e^{-i(X_1+X_2)} + e^{i(X_1-X_2)} - e^{-i(X_1-X_2)}) C_1 C_2 \\
 &\quad + \frac{i}{2} \sqrt{2/3} (e^{i(X_3-X_4)} - e^{-i(X_3-X_4)}) C_3 C_4, \\
 T^2 &= -\frac{1}{2} \sqrt{2/3} (e^{i(X_1+X_2)} + e^{-i(X_1+X_2)} + e^{i(X_1-X_2)} + e^{-i(X_1-X_2)}) C_1 C_2 \\
 &\quad + \frac{1}{2} \sqrt{2/3} (e^{i(X_3-X_4)} + e^{-i(X_3-X_4)}) C_3 C_4.
 \end{aligned} \tag{4.10}$$

It should be clear that this is certainly not the most general way to realize $SU(2) \times U(1)$, because of the freedom to change bases discussed earlier in this section. It is however the most efficient way, since it requires a Frenkel-Kac group no larger than $SU(2) \times U(1)$, generated by ∂X_3 and ∂X_4 and the roots $e_3 - e_4$ and $-e_3 + e_4$. Notice in particular that the additional $SO(4)$ roots $\pm(e_3 + e_4)$ are not required to be present. It is not difficult to construct lattices which allow this structure, and we will indeed find an example in section 6.

5. Constraint vectors

In this section we study the constraint vectors due to non-vanishing structure constants f_{ijk} for a fixed number ($2m$) of gauge bosons. All these vectors must be of the form $(t, (v))$ on the lattice $\Gamma_R = \Gamma_{3n-m} \times D_{8-n+m}$, with $t^2 = 3$. Length one vectors t are not allowed (to avoid enlargement of D_{8-n+m}), nor should any be generated by adding an odd number of constraint vectors (by adding an even number, one gets vectors of the form $(r, (o))$, r^2 even).

The set of constraint vectors can be obtained by associating the $6n - 2m$ generators of G_{SC} which are not in H with the $6n - 2m$ free fermions constructed out of the $3n - m$ bosons whose momenta lie on Γ_{3n-m} . One may exploit $SO(6n)/G_{SC}$ rotations to choose a basis for the Lie algebra which minimizes the number of structure constants, and hence the number of constraint vectors. In an arbitrary basis one gets all possible constraint vectors, which can all be satisfied if and only if the entire lattice Γ_R is D_{8+2n} . But then one automatically obtains the maximal number (i.e $6n$) of gauge bosons.

The $6n - 2m$ free fermions can be grouped into $3n - m$ pairs which are related to exponentials of the same Bose field, as in (4.5). Two fermions of the same pair cannot be contracted with the same structure constant as this would give rise to a constraint vector $(l, (v))$, $l^2 = 1$. Therefore we have to group the generators of G_{SC}/H into commuting pairs, each of which can be represented by a fermion pair. If this is not possible, the constraint cannot be satisfied, and it is impossible to find a covariant lattice theory with the gauge symmetry H . We will call two generators of G_{SC} "associated" if they are represented by one fermion pair.

A term $f_{ijk}\psi_i\psi_j\psi_k$ in the supercurrent produces constraint vectors containing $(\pm 1, \pm 1, \pm 1)$, with all eight combinations of signs. In some cases there may be a cancellation of two terms in the supercurrent, so that some of these constraint vectors do not appear. This is only possible if there is another structure constant f_{mnk} , with m, n associated to i, j , and $|f_{ijk}| = |f_{mnk}|$ (the example at the end of section 4 has this property). If there is no such cancellation, the entries of all lattice vectors in the 3-dimensional subspace of i, j and k must be integer or half-integer in order to have integer dotproducts with all constraint vectors, so that the sublattice is of the form $(D_1)^3$. The relevant part of the constraint vectors is then most conveniently written as $((v), (v), (v))$.

From now on we will specialize to four dimensions. Consider first coset spaces G_{SC}/H of type (C), as defined in section 4. We must thus satisfy the constraint vectors of a simple factor H' of dimension $2k$ of G_{SC} on a k -dimensional lattice. Then we must be able to group all generators of H' into commuting pairs. This is possible for D_n , n even (the roots $e_i + e_j$ and $e_i - e_j$ may be associated) but not for other simple Lie algebras. In particular it cannot be done for $SO(5), SU(3), SU(4)$ or $SO(3)$. Thus a solution of type (C) is only possible for $G_{SC} = SU(2)^6$, where one can always associate two $SU(2)$'s with each other. More about this below.

Next we consider cosets of type (E), which are the product of a symmetric space and a group manifold. We find again that this is only possible for $G_{SC} = SU(2)^6$. This can be seen as follows: In all other cases there exists always at least one structure constant f_{ijk} of the group manifold with two unpaired indices i and j . The corresponding group generators have to be associated with two coset generators λ_i and

λ_j . Now we make use of the fact that if G/H is a symmetric coset space, then the action of H on the tangent space is real irreducible. In particular this means that there does not exist a basis choice for which the coset generators split into two mutually commuting sets. It is easy to see that therefore at least one of the following three statements must be true:

- (i) $[\lambda_i, \lambda_j] \neq 0$
- (ii) There exists a coset generator λ_m so that $[\lambda_m, \lambda_i] \neq 0$ and $[\lambda_m, \lambda_j] \neq 0$
- (iii) there exist coset generators λ_m and λ_n so that $[\lambda_m, \lambda_i] \neq 0$,
 $[\lambda_n, \lambda_j] \neq 0$ and $[\lambda_m, \lambda_n] \neq 0$.

These commutators lead to constraint vectors of the form given under (c) in section 4. Adding all the constraint vectors implied by the non-vanishing commutators listed above to the constraint vector implied by f_{ijk} produces in each of the cases (i), (ii) and (iii) an unwanted additional gauge boson.

Next we turn to case (A), with trivial gauge group. In chiral string theories [6] at least one of the two sectors *has* to be of this kind (because non-trivial H implies enlargement of D_5 to D_{5+m} , which in turn implies that all D_5 fermions appear in non-chiral pairs). Again the only possibility appears to be $G_{SC} = SU(2)^6$, although the argument is not quite as rigorous as the previous ones. In this case one has to find a way to “overlap” the constraint vectors of $SU(4)$ and $SO(3)$ or $SO(5)$ and $SU(3)$. Separately, these four groups need at least 8, 3, 7 and 7 dimensions respectively to avoid roots of Γ_R among the constraint vectors. (e.g. in $SU(3)$ one can only find one commuting pair of generators; the remaining six have to be related to different bosons). By inspection of the simplest choices of structure constants we

find that it is not possible to overlap the constraint vectors without producing roots of Γ_R . Choosing more complicated sets of structure constants, leading to more constraint vectors, makes it even harder to overlap the constraint vectors.

If $G_{SC} = SU(2)^6$, the minimal set of structure constants consists of those of the six $SU(2)$'s, but we still have the freedom to overlap the constraint vectors in different ways. The various possibilities can be represented graphically by associating a 3-point vertex to each of the six structure constants, and connecting them with 9 lines corresponding to the 9 fermion pairs (ψ_{11}, ψ_{12}) . From these diagrams one can read off which 3 coordinates are linked to each of the six $SU(2)$ factors, and hence which constraint vectors are required. A self-contraction leads to a length 1 vector associated with (v) of D_5 , and is therefore not allowed. The graphs without self-contractions are shown in Fig 1. The simplest set of constraint vectors is the one corresponding to (a). It requires the existence of the following vectors t on Γ_9 (associated with (v) of D_5)

$$\begin{aligned}
 &((v), (v), (v), 0, 0, 0, 0, 0, 0) \\
 &(0, 0, 0, (v), (v), (v), 0, 0, 0) \\
 &(0, 0, 0, 0, 0, 0, (v), (v), (v))
 \end{aligned} \tag{5.1}$$

This is equivalent to the “triplet” constraint used in [11] precisely as discussed in [6]. Several other graphs (namely (c), (d), (e) and (h)) lead to the same three constraint vectors, plus some additional ones (these graphs are precisely the ones with three vertices which are not connected to each other).

One can also read off which vector length-squares can be obtained by closing the basic set under addition. If a diagram can be cut into two pieces by cutting n lines, then there is a vector of length² n . If n is odd it is associated to (v) of D_5 , if n is even to (0) . Clearly (g) contains a length 1 vector, and is therefore ruled out. The remaining ones, (b), (f) and (j) correspond to new constraint vectors, which are not of the type given above.

This possibility was already hinted at in [6]. By requiring that all massive excitations of a *given* massless chiral spinor be non-chiral, it was deduced that certain lattice vectors had to be present. The set given in (5.1) emerged as just one of seven possibilities, and the only one that solved the problem for *all* massive spinors. The cases (b), (f) and (j) turn out to correspond to some of the other six possibilities, but have additional constraint vectors to take care of other spinors. It is in fact possible to prove diagrammatically that in any of the nine cases depicted in Fig 1 massive spinors come in non-chiral pairs. The necessary cancellations are due to the θ function identity $\theta_2\theta_3\theta_4=2\eta^3$, as explained in [6] (this applies to all chiral string theories we know).

6. Four Dimensional Type II Theories

The results we have obtained thus far are of special interest for type II strings in four dimensions. Originally it was thought that type II strings could only give $U(1)^6$ gauge groups. Later it was realized that also the group $SU(2)^6$ could be obtained

[21] [22]. From the results of section 4 it is clear that one can get other gauge groups as well, some of which are just large enough to contain the standard model. In [23] the cases with 18 gauge bosons and $N=4$ supergravity were discussed. Various models have been discussed in [24]. In this section we will present a few other examples.

As in [6] we will consider even self-dual lorentzian lattices which are decomposable into D_n factors, and for classification purposes we will make use of the possibility to map these lorentzian lattices to even self-dual euclidean ones. In four dimensions the lattices to consider have 14 left and 14 right dimensions. They must contain a space-time lattice D_5 on both sides, and should contain one of the possible sets of constraint vectors on both sides. Of course the constraint vectors on the left and the right lattice need not be the same.

In general the lattices are thus of the form $D_5 \times (D_1)^9$ on both sides (where the factors can combine to larger D_n 's). To find out whether it is possible to satisfy a certain constraint, one has to inspect the list of conjugacy classes of the lattice. In some cases there are easier ways to determine potential constraint vectors. Constraint vectors of length-squared 2 can only exist if the D_5 factor is part of a larger D_n . If one wants to have only such constraints, at least 6 length-squared 2 vectors are needed, so that the D_5 factor must extend to D_8 . If D_5 cannot be extended at all, the constraint vectors must all have length-squared four. Furthermore (if the constraints are due to a free fermion supercurrent) they must be of the form $((v), (v), (v), (v), 0^6)$, where the first entry belongs to D_5 , and the other three (v) 's must be vector conjugacy classes of *different* D_n factors. Thus in this case the lattice must have at least three separate D_n factors in addition to D_5 .

To illustrate this we consider a class of models which has the maximum number (i.e. 18) of G_R gauge bosons, and which we therefore expect to contain those of [23]. To get 18 gauge bosons, the lattice must be $(\Gamma_{14})_L \times (D_{14})_R$, and can therefore be mapped to a 16 dimensional euclidean lattice $\Gamma_{14} \times D_2$. Thus it is either D_{16} or $E_8 \times E_8$. In the first case the lorentzian lattice is $D_{14} \times D_{14}$, and we get 18 gauge bosons from *each* side. This theory has a tachyon and no massless fermions. (Note that for D_{14} all possible constraints are automatically satisfied. On each side we can therefore have one of the three possible G_{SC} groups $SU(2)^6$, $SU(3) \times SO(5)$ or $SU(4) \times SO(3)$ as gauge group G_{NSR} .) If the lattice is $E_8 \times E_8$ there are two ways to embed D_2 , namely $E_8 \times E_8 \supset E_8 \times D_6 \times D_2$ and $E_8 \times E_8 \supset E_7 \times E_7 \times D_2$. In the first case we can embed the space-time part D_5 of the left lattice in either E_8 or D_6 . Using E_8 one gets a $N=4$ supergravity theory, which is the one discussed in [23]. The second case, as well as the second embedding of D_2 , does not yield anything new, because the constraints on the left sector cannot be satisfied.

Many properties of the spectrum follow directly from simple features of the even lattice, and in particular from its roots. For the heterotic strings this was discussed in detail in [6] and type II strings are virtually identical in most respects. Some of the more interesting aspects are listed below:

– *Tachyons*

As for heterotic strings, in the even lattice formulation there is a simple criterion for the presence of tachyons of minimal mass in type II strings: they are present if and only if the D_5 factors from left and right are both embedded in the same D_n

factor of the euclidean lattice. In this way one understands easily why the 36 gauge boson model constructed above is tachyonic. It is also easy to find a non-tachyonic model with many gauge bosons. For example, one can construct a lorentzian lattice $(D_{12} \times D_2)_L \times (D_{12} \times D_2)_R$ from the Niemeier lattice $D_{12} \times D_{12}$; the left and right D_2 factors combine to a self-dual Englert-Neveu type [30] lattice. This model has $14 + 14$ gauge bosons which may, for example, gauge the group $SO(5) \times SU(2) \times U(1)$ on both sides, as in the example discussed in section 4. There are no massless fermions.

– *Massless fermions*

One can only get massless fermions if the space-time group D_5 is embedded in a D_n -factor, with $n \leq 8$ (since otherwise the spinor weights have length-squares larger than 2). Therefore the gauge group in that sector can only have dimension 6 or less. One can only have a symmetric subgroup of G_{SC} which is this small if $G_{SC} = SU(2)^6$. This agrees with and generalizes an observation made in [26]. *Chiral* fermions can occur only if D_5 is not part of a larger D_n group (this need to be the case only in one sector).

– *Supersymmetry*

As in the heterotic case [6], the number of supersymmetries is directly related to embeddings of $(D_5)_L \times (D_5)_R$ in exceptional algebras. If in one sector D_5 is embedded in E_6 , E_7 or E_8 , one gets respectively one, two or four gravitini from this sector. By combining both sectors one can thus obtain $N=1, \dots, 6$ and $N=8$ supergravity theories; note that there are two ways to get $N=2$ and $N=4$ supersymmetry.

– *Bispinor gauge bosons*

It is possible to get gauge bosons as tensor products of massless left and right spinors, in addition to those of G_{NSR} . Upon inspecting their couplings it turns out that they can gauge only $U(1)$ -factors. Their multiplicity can be quite large since it is the product of the multiplicities of the left and right spinors. For example, if D_5 is embedded in D_8 in both sectors, there can be 32 such bosons in addition to the $6+6$ G_{NSR} gauge bosons. A model of this kind is discussed in [24].

– *Anomalies*

Before turning to the question which fermion spectra can be obtained in type II strings, we want to explain why they are free of local anomalies. As was shown in [31], modular invariance guarantees Green-Schwarz factorization [32] of the anomaly of the full Frenkel-Kac group G^{3n} in type II theories (the factor is actually only $\text{Tr}F^2$, and not $\text{Tr}F^2 - \text{Tr}R^2$ as in the heterotic string). Normally it would be enough to say that since the actual gauge group G_{NSR} is a subgroup of this group, its anomalies will also cancel. This is certainly correct for the polygon graphs, as well as for the one-loop anomaly cancelling term, calculated in [33]. Here one gets the correct answer by simply replacing group generators by subgroup generators in all traces. But it is not correct for the BAA vertex, which has a fixed normalization independent of the G_{NSR} structure constants f_{abc} . One may however check that the normalization given in (4.3) is precisely the right one to make the cancellation mechanism work. We do not know whether there exist four dimensional type II strings with non-trivial Green-Schwarz cancellation, but it is good to know that string theory has already anticipated that possibility.

– *The Standard Model ?*

Finally we turn to the important question whether type II theories can produce the standard model. Within our approach, the answer is negative. There are two symmetric subgroups of G_{SC} which are locally isomorphic to $SU(3) \times SU(2) \times U(1)$, namely $SU(3) \times SO(5) \supset SU(3) \times SO(3) \times SO(2)$ and $SU(4) \times SO(3) \supset SU(3) \times U(1) \times SO(3)$. In these two cases the groups $SU(3)$ and $SO(3)$, respectively, do not act on the tangent space of G_{SC}/H . It follows then that massless fermions can only be in the singlet or adjoint representations of these groups, and hence not in the representations of the standard model.

Although it is of little relevance, one might still ask whether it is possible to get chiral fermions at all. This possibility is severely restricted by anomaly cancellation. As we have seen in section 3, the H -representations of the massless fermions are embedded in representations with weights of length 1 of the Frenkel-Kac group. There are usually very few possibilities, so that one is forced to pair them off into real representations in order to cancel anomalies. In many cases, the possibility of getting chiral fermions can be quickly ruled out in this way, but we have not attempted to do this systematically¹.

¹We have been informed by P. Ginsparg and L. Alvarez-Gaumé that in [34] it is proved that one cannot get chiral theories with interesting gauge groups from type II theories.

7. Summary

In the covariant lattice approach, we have elucidated the role of world-sheet superconformal invariance in lower dimensional heterotic and type II string theories. In ten dimensions, there is only one Lorentz-invariant choice for the supercurrent. Below ten dimensions, Lorentz invariance is less restrictive, because one is free to modify the “internal” part of the supercurrent. It is in this respect that the covariant lattice construction is different from torus compactifications which leave the ten dimensional form of the supercurrent unchanged.

We have shown that the requirement of superconformal invariance implies the existence of certain vectors on the self-dual lattice, which (in the even lattice formulation) have to have length-squares two and/or four. These vectors represent the picture changing operator in bosonic language. As is always the case in the even lattice formulation, the existence of length-squared two vectors (roots) implies the existence of gauge particles. The length-squared two vectors arising from the space-time part of the supercurrent imply gravitons; those arising from the internal part imply gauge bosons, which in general form a subset of all G_{NSR} gauge bosons. The picture changing operator maps the vertex operators of these gauge bosons to linear combinations of the Frenkel-Kac generators of the lattice. In this way we can understand why G_{NSR} depends on both the lattice and on the choice of the picture changing operator, ie., on the way superconformal invariance is realized.

The presence of G_{NSR} gauge bosons in one left- or right-moving sector precludes the existence of chiral fermions arising from that sector. Consequently, chiral string theories must have at least one fermionic sector whose superconformal invari-

ance is realized entirely in terms of length-squared four vectors in the internal part. As already discussed in [6], the existence of such vectors severely restricts the possibility of performing continuous Lorentz rotations acting on the lattice, in contrast to the non-chiral torus-compactified case [7].

G_{NSR} gauge bosons are not of much interest in heterotic string theories, because they render these non-chiral. On the other hand, in type II theories one might get G_{NSR} gauge symmetries from one and chiral fermions from the other sector. However, as we have seen, it turns out that the possible gauge groups and fermion representations are so limited, that the standard model cannot be obtained.

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Figure Caption

Fig 1: Graphical representation of the possibilities for length-squared four constraint vectors in the case $G_{SC} = (SU(2))^6$.