ANTIHYDROGEN FORMATION THROUGH
POSITRON-ANTIPROTON RADIATIVE COMBINATION

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ABSTRACT

In this paper the production of a beam of antihydrogen atoms through the capture of positrons by antiprotons in an arrangement of merged parallel beams is considered. The achievable luminosity is estimated on the basis of available antiproton and positron intensities. The scaling of the luminosity with various parameters is investigated. It is argued that an intense beam of antihydrogen can be obtained if antiprotons and positrons are stored, recirculated and cooled. While the number of stored antiprotons which can be achieved at present is already high, schemes for the accumulation of slow and cooled positrons have to be developed. Possible ways are pointed out. The fact that a high number of both species of charged particles can be accumulated, stored, and recirculated, compensates largely for the low capture cross-section.

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1. INTRODUCTION
The production and investigation of antihydrogen has recently gained considerable interest [1–18]. Antihydrogen—not yet detected in nature—constitutes a complete entity of genuine antimatter. As a consequence, the artificial production of antihydrogen provides the possibility of performing a variety of fundamental experiments, which range from the test of fundamental symmetries to the study of antimatter–matter interaction. A comprehensive discussion of possible experiments can be found in ref. [17]. Recently a proposal [19] to produce antihydrogen was made, in which an initial antihydrogen production rate of a few atoms per hour was aimed at. In this proposal perspectives for the production of several thousands of antiatoms per second were given on the basis of the expected development of experimental techniques concerning the production of thermalized positrons and the evolution of high-power lasers. In this paper the relevant aspects of antihydrogen formation through radiative positron capture in a merged-beam arrangement are summarized and general relations are derived which should allow the determination of the optimal experimental parameters.

2. ANTIHYDROGEN PRODUCTION
Antihydrogen $\bar{H}$ can be produced through spontaneous radiative capture of a positron by an antiproton:

$$\text{e}^+ + \bar{p} \rightarrow \bar{H} + h\nu$$

(1)

in an arrangement similar to electron cooling of protons [20]. The energy gained is released through the emission of a photon of energy $h\nu$.

Moreover, it was shown theoretically that induced capture

$$n\ h\nu + \text{e}^+ + \bar{p} \rightarrow \bar{H} + h\nu + n\ h\nu$$

(2)

may be used to enhance the antihydrogen formation by more than two orders of magnitude [7].

Also three-body processes may be considered to form atomic antimatter. These are reactions such as

$$\text{e}^+ + \text{e}^+ + \bar{p} \rightarrow \bar{H}^+ + h\nu$$

(3)

and

$$\text{e}^+ + \bar{p} + \bar{p} \rightarrow \bar{H}_2^- + h\nu$$

(4)

and

$$\text{Ps} + \bar{p} \rightarrow \bar{H} + \text{e}^-,$$

(5)

where Ps stands for positronium atoms.

Another reaction would be the direct pair production in high-energy nuclear collisions

$$p + A \rightarrow \bar{H} + H + \text{anything}$$

(6)

and the formation of antihydrogen in antibaryon decays such as
\[ \Lambda, \bar{n} \rightarrow \bar{H} + \nu_e . \]  

(7)

In this paper we consider only the first two reactions. The third and fourth reactions are unlikely to become important because of the low density achievable for stored antiprotons or positrons; reaction (5) is discussed elsewhere [21]. The last two reactions certainly yield too-low rates and are therefore not considered.

3. CAPTURE CROSS-SECTION

The cross-section for spontaneous radiative capture of a positron into an atomic state with principal quantum number \( n \) is given by [22]

\[ \sigma_n = 2\pi^2 r_e^2 \alpha^{-1} n^{-7}(E_0 + n^2 E_e)^{-1} E_0/E_e , \]

(8)

where \( E_0 = 13.6 \) eV is the binding energy of the ground state of antihydrogen, \( r_e \) is the classical electron radius, \( \alpha \) is the fine structure constant, and \( E_e \) is the energy of the positrons in the antiproton rest frame. This cross-section is diverging quadratically for positron velocities \( v_e \) approaching zero. Hence the product \( v_e^2 \sigma_n = K_n \) is constant for small velocities and assumes the value of about [7]

\[ K_n = 11 \text{ mb \, c}^2/n . \]

(9)

4. LUMINOSITY

It is very difficult to produce and maintain a sufficiently dense two-component plasma of antiprotons and positrons at low temperature. Yet charged particles can be stored and compressed in phase space by being circulated and cooled in a storage ring. Small positron-antiproton energies can then be realized by a collinear beam arrangement where both beams are merged and matched in velocity. The luminosity \( L \) for merged parallel beams in the laboratory frame is:

\[ L = N_p N_e \kappa \ell A(C_p C_e A_p A_e)^{-1} |\beta - \beta_e| c , \]

(10)

where \( N_p \) (\( N_e \)) is the number of antiprotons (positrons) circulating in \( k \) bunches with the velocity \( c \beta \) (\( c \beta_e \)) in a ring of circumference \( C_p \) (\( C_e \)). The quantities \( A_p \) and \( A_e \) define the cross-sectional area of the particle beams, \( A \) the total overlap area, and \( \ell \) the length of the overlap region. In the antiproton rest frame the difference \( \beta - \beta_e \) becomes \( \beta^* \gamma^{-2} \), where \( \beta^* \) is the positron velocity in that frame and \( \gamma = (1 - \beta^2)^{-1/2} \).

5. SPONTANEOUS CAPTURE RATE

Should both beams be monoenergetic, the product of capture cross-section and luminosity would yield the formation rate \( R \):

\[ R = N_p N_e \kappa \ell \gamma^{-2} A(C_p C_e A_p A_e)^{-1} \beta^* c \sigma . \]

(11)

With the approximation of (9) one obtains

\[ R_n = N_p N_e \kappa \ell \gamma^{-2} A(C_p C_e A_p A_e)^{-1} (K_n / \beta^* c) . \]

(12)

However, since both beams usually have a velocity distribution (finite temperature) the cross-section has to be averaged over this distribution. For equal beam temperatures one can neglect
the antiproton velocity distribution. Because of the large antiproton to positron mass ratio only the positron velocity distribution matters. Then the product $\beta^* c \sigma_\alpha$ is replaced by the recombination coefficient $\alpha_{r,n} = (\beta^* c, \sigma_\alpha)$, which was evaluated, elsewhere [7], for a Maxwellian velocity distribution and for conditions as found in a typical electron cooling beam:

$$\alpha_{r,n} = 2(\Delta v_e \sqrt{\pi})^{-1} K_n \,,$$  

(13)

where $\Delta v_e$ is the width of the positron velocity distribution. In order to account for all captures the recombination coefficient has to be multiplied by a factor $f_c$, which lies between 2 and 3: $\alpha_r = f_c \alpha_{r,1}$. Under these conditions the formation rate is given by

$$ R = N_p N_e k \ell \gamma^{-2} A (C_p C_e A_p A_e)^{-1} \alpha_r \,.$$  

(14)

6. ANTIPROTON INTENSITY

In the following we want to work out how this rate can be maximized. Let us first consider the antiproton part. From the beam rigidity $B_Q [T \cdot m] = 3.13 \beta \gamma$ one can calculate the bending radius for maximal magnetic field (say 1.56 T): $q [m] = 2 \beta \gamma$. This would give a ring circumference of $4 \pi \beta \gamma$ in metres. However, one has to allow for injection space, beam optics, and controls, which in practice at least doubles this circumference:

$$C_p [m] = 8 \pi \beta \gamma \,.$$  

(15)

Under ideal conditions such a storage ring can be filled up to its instability limit, which we take as

$$N_{\text{max}} = \pi \epsilon \beta^2 \gamma^3 \Delta Q / r_p \,,$$  

(16)

where $\epsilon$ is the beam emittance (in rad m) and $r_p$ is the classical proton radius ($1.54 \times 10^{-18}$ m). A beam tune shift of $\Delta Q = 0.01$ might be admissible. Owing to the limited antiproton accumulation rate, $N_{\text{max}}$ will hardly exceed $10^{13}$ in the foreseeable future at existing antiproton sources.

The antiproton beam area in the overlap region is related to the beam emittance through $A_p = \epsilon \beta'$, where $\beta'$ is the focusing function in the overlap region. The value of this quantity should be small. However, it can hardly be smaller than $\ell$, the length of the overlap region. Inserting this in (14) we get

$$ R = 0.25 N_q k \ell A (\beta' C_p A_e C_p)^{-1} \alpha_r \Delta Q \,.$$  

(17)

7. POSITRON PRODUCTION

Now we turn to the positron part. There are two important ways to obtain positrons for the purpose discussed here; one is from radioactive sources, the other is from pair creation.

Radioactive $\beta^+$ emitters deliver positrons at a rate of

$$ R_e = 3.3 \times 10^{11} C_\beta f_{\beta} [s^{-1}] \,,$$  

(18)

where $C_\beta$ is the source activity in curie and $f_{\beta}$ is the $\beta^+$ branching ratio. The energy spectrum of positrons from such sources is broad; therefore they have to be thermalized in a moderator in order to provide cold positrons. A moderation efficiency $\eta_m$ of about $10^{-3}$ and a positron temperature $T_e$ of about 0.1 eV can be achieved [13, 23, 24]. These positrons move with an average velocity of $\beta_1 =$
\( \sqrt{2T_e/mc^2} \). For the purpose of decreasing the final positron beam size it is probably required to remoderate the positrons, which can be done with an efficiency \( \eta_m \) of about 0.3 [13, 25]. Acceleration of the positrons to the final velocity \( \beta c \) decreases their spatial density by \( \beta \eta_c / \beta \). Hence the achievable positron density is

\[
n_e = (R_e/A_e) \eta_m \eta_m (\beta \eta_c / \beta c). \tag{19}
\]

This equation shows that positron densities \( n_e \) of a few per \( \text{cm}^3 \) may be achieved for continuous beams from radioactive sources of manageable strength. Using such a beam to fill a positron ring of circumference \( C_e \) yields only a very small number of stored positrons:

\[
N_e = n_e C_e A_e. \tag{20}
\]

On the other hand, slow positrons can be produced in batches from pulsed high-energy electron beams [26, 27]. A rate of

\[
R_e \eta_m = 10^{-5} I_e / e \tag{21}
\]

for electron beam current \( I_e \) and for energies above 200 MeV seems achievable [28]. The rate limitation is probably given by the thermal load on the production target. This method allows the achievement of higher positron densities (about 1000 \( \text{cm}^{-3} \)); moreover, the positron beam is bunched, which allows stacking and accumulation.

High-energy electron-positron colliders use positron storage and damping rings to accumulate positrons and compress their phase space. In the Electron-Positron Accumulator (EPA) ring at CERN, for instance, 10 bunches of \( 10^{10} \) positrons are accumulated and cooled by synchrotron radiation at 600 MeV [29]. These positrons are too energetic to be useful for the purpose discussed here. They have to be decelerated (which increases beam size and velocity spread) and cooled in order that use may be made of them for the antihydrogen production.

From what was stated above it became clear that positrons have to be accumulated if one wants to achieve antihydrogen production rates of more than, say, a few hundred per second. Bunched beams can be stacked in a ring. However, stacking increases the phase space occupied by the positrons; the beam gets larger and smeared out in space and time. Hence this procedure is of limited value. Positrons delivered by radioactive sources are even more difficult to accumulate since they are produced continuously. They have to be bunched to enable their stacking. The only way to overcome this problem is to compress the phase space occupied by the positrons emitted from a source-moderator arrangement. For antiprotons phase-space compression is achieved through beam cooling; for positrons one has to look into similar methods to circumvent Liouville's theorem. The moderation provides such a possibility to compress phase space although it is connected with a loss of intensity.

A method for the bunching of a continuous flow of positrons from a radioactive source was proposed and developed some time ago [24, 30]. Positrons were trapped in a longitudinal magnetic field with a bottleneck by increasing their transverse energy once they had crossed the magnetic ridge. This was achieved by driving a radiofrequency cavity with the cyclotron resonance frequency of the positrons. At the other end of the bottle the positrons were reflected by an electric potential. It was estimated [30] that a captured positron would leave the bottle again only after about 400 bounces if its transverse energy was increased to the double of its longitudinal value upon the first traversal of the cavity. It was also thought that a cooling of the positrons could increase this value further. At a given moment the positrons in filling a cylindrical volume inside the longitudinal magnetic field were
accelerated by a parabolic electrical potential so that they all arrived at a determined point simultaneously. A scheme like this would certainly be the first step in accumulating a large number of positrons in a storage device. In the next section some schemes are examined.

9. POSITRON RECIRCULATION

Once positrons have been accumulated they have to be merged with antiprotons of approximately the same velocity so that the relative velocity is minimized. In the following we discuss how this might be achieved for beams of relativistic energies.

A beam of antiprotons circulating in a storage ring is merged with a positron beam in a straight section of the ring. In order to obtain high formation rates both beams have to be cold, i.e. they have to have a small momentum spread, beam radius, and divergence. This is achieved for the antiprotons through beam cooling. In the previous section it was discussed how positrons can be thermalized and cooled. Adiabatic acceleration to velocities equal to that of the antiprotons would reduce the longitudinal temperature of the positrons further. Since the antihydrogen formation is rather rare the stored particles hardly interact in one passage so that a recirculation scheme is opportune. The antiproton storage ring provides this possibility. In the following we discuss a recirculator for the positrons. At relatively low energies ($\gamma = 1$) this could be similar to the beam transport in an electron-cooling device, consisting essentially of toroidal and solenoidal magnetic-field sections [31].

The simplest scheme would be a 'race track' consisting of four toroids, two short, and two long solenoids. One of the long solenoids serves as the common straight section with the antiproton ring. The longitudinal field has the advantage that the transverse positron motion is translated into a rotation about the field lines. For an appropriate field strength the positron beam can be made small and angular acceptance large at the same time. Such a storage ring is not difficult to realize; the more difficult problem is to inject the positrons. This has to be done through one of the toroid sections. The positron beam has to have a pulse structure in order to make a pulsed injection. Another possibility might be to inject the positron beam with the help of a remoderator inside the magnet. It should be possible to recirculate the positron beam in such a storage device many times. The lifetime of the positron beam would probably be determined by the interaction with the residual gas, which would increase the transverse velocity on account of the longitudinal one. The radius of the spiral would grow until the beam is lost on the walls. For intense positron beams also intra-beam scattering might lead to the same effect. This might partially be compensated by beam cooling, and the longitudinal energy loss corrected by applying a small radio-frequency field, which also could serve to maintain the bunch structure.

Another possibility would be to have a classical storage-ring lattice with focusing and deflecting elements. Depending on the energy of the positrons they could either be of magnetical or electrical nature. Pulsed injection would be simpler owing to more space being available. However, it might be more difficult to prevent the beam from heating up.

No matter what scheme is used it should be possible to circulate the positrons for at least a few seconds if an ultrahigh vacuum ($10^{-12}$ Torr) is achieved.

10. ANTIHYDROGEN FORMATION THROUGH SPONTANEOUS CAPTURE

We now come back to the formula for the antihydrogen formation rate given in (17). It is reasonable to assume that the positron beam can be made as small as the antiproton beam, hence $A_e = A$. Furthermore, for the cases considered here one can suppose the positron ring circumference to be of the order of $C_e = 3\ell$. Inserting these values into (17) one arrives at a simple formula for the maximum formation rate:

$$R = 8 \times 10^{-4} N_e \beta \gamma (\beta' r_p)^{-1} \alpha \epsilon \cdot$$ (22)
Assuming $\alpha_r = 1.54 \times 10^{-18} \text{ m}^3\text{s}^{-1}$ [(9) and (13)] and $\beta' = 2 \text{ m}$ this reduces, for coasting beams $(k = 1)$, to

$$R = 2 \times 10^{-4} N_e \beta \text{ [s}^{-1}] .$$

(23)

With a value of $10^8$ for $N_e$, as estimated above, we arrive at

$$R = 2 \times 10^6 \beta \text{ [s}^{-1}] ,$$

(24)

which is probably conservative, given the fact that the performance of a dedicated positron accumulator was most likely underestimated. The relation (24) shows only a weak dependence on the particle velocities $\beta$. However, it indicates that one should work at relativistic energies $(\beta \approx 1)$ to achieve the highest rates.

8. POSITRON ACCUMULATION

8.1 Accumulation through bunching, remoderation, and stacking

Let us first discuss the technique of ref. [30], where positrons are gathered in a sort of magnetic bottle (the buncher). We assume 3 m length for it. The low-energy positrons $(v = 3 \times 10^7 \text{ cm s}^{-1})$ can be stored on an average for 4 ms (400 bounces). During this time about $10^6$ slow positrons can be collected from a 1 Ci positron source. They will be ejected from the bottle by a parabolic potential and sent onto a foil for remoderation. The narrow time structure allows one to extract the remoderated positrons (let us assume about 10% of the originally collected ones) from the foil and to inject them into a device with a longitudinal magnetic field and an electric potential wall at each end (the accumulator). During the injection into this device one wall is lowered to let the positron bunch in and thereafter raised to the trapping voltage. In the buncher, collection is continued and after 4 ms its content is dumped again into the accumulator. Meanwhile the trapping voltage has been increased so that the new pulse can be stacked without ejecting the first one. If the velocity spread of the ejected bunch can be made small enough, it should be possible to accumulate at least $10^3$ bunches within a few seconds, which means a total of about $10^8$ positrons. By adding another stage of the same type, and repeating the process, one might be able to accumulate $10^{10}$ positrons. The density of positrons might be increased further by damping their motion in the storage device through synchrotron radiation or resistive cooling.

8.2 Accumulation from a pulsed source at low energies

Positrons produced by conversion from pulsed electron beams maintain the time structure through the moderation. In ref. [26] it was reported that pulses of less than 20 ns could be produced containing $6 \times 10^5$ positrons with a repetition rate of 1.44 kHz. Such bunches would have a spatial extension of the order of centimetres (depending on the positron energy). They could be directly injected into a storage device. Stacking should then allow collection of a large number of such pulses, perhaps $10^3$. Now one can envisage performing a remoderation in order to compress phase space and continue stacking. A final fill would then be ejected, remoderated, and injected into the positron ring for antihydrogen formation. With this scheme one could probably accumulate $10^{10}$ slow positrons in a few minutes.

8.3 Accumulation at relativistic energies

Naturally one can also profit from the accumulation scheme as used for high-energy colliders. For instance, in the EPA positron damping ring $10^{11}$ positrons will be collected and cooled within a few minutes. Although a deceleration would increase the emittance and momentum spread of the
beam, cooling of the beam could be done at intermediate energies before it is further decelerated. In order to make efficient use of such a beam it should be decelerated to the energy of optimal remoderation and thermalized in a remoderation foil. Again at least $10^8$ positrons could be stored in this way at low energy.

In conclusion, we feel that with one of the methods sketched above, $10^4$ to $10^{10}$ positrons could be accumulated. They would thereafter be accelerated to the desired velocity and stored in the positron recirculator. Hence for the following estimate of the antihydrogen production we will use $N_e = 10^8$.

One should keep in mind that accumulation and storage of positrons as well as antiprotons is only possible because these particles are charged. An accumulation scheme is unfortunately not applicable in the case of positronium which would be needed for reaction (5). Moreover, charged particles’ beams can be recirculated, which is very important for reactions of small cross-sections.

11. ANTIHYDROGEN FORMATION THROUGH INDUCED CAPTURE

As mentioned before, positron capture by antiprotons can be stimulated through irradiation of the merged beams with light of appropriate wavelength [7]. From formula (33) of ref. [7] one finds [15] for a typical arrangement an enhancement factor $G$ of

$$G = 2Pn^5\left(\frac{hc}{13.6 \text{ eV}}\right)^3/(a^2mc\Delta_1\Delta_\perp)$$

where $P$ is the radiation power, $a$ is the radius of the photon beam, and $\Delta_1$ and $\Delta_\perp$ are the longitudinal and transversal velocity spreads of the positrons in the particle rest frame. Capture into an excited state would yield a gain factor of a hundred or more with commercially available lasers. The benefit of the induced capture depends very much on the available laser and defines the regime of operation. If only pulsed lasers can be used in order to achieve the required radiation power, the antiproton and positron beams have to be matched in time structure and repetition rate; otherwise the duty cycle is so low that this process becomes unimportant. Given the flexibility one has in choosing the particle beam energy and the final state of capture, it should be possible to reach a duty cycle close to unity. In this case the hydrogen formation rate via induced capture becomes:

$$R_{\text{ind}} = GR$$

Under these optimal conditions the antiproton beam lifetime with respect to recombination is of the order of 1 day, which means that nearly all antiprotons can be converted into antihydrogen (since the antiproton beam lifetime from residual gas scattering can certainly be made longer).

12. CONCLUSIONS

Antihydrogen can be produced through radiative capture of a positron by an antiproton in an arrangement where an antiproton beam is merged with a positron beam and the velocities of both beams are made equal. Owing to the low interaction rate both beams should be stored and recirculated in rings having in common a straight section as the interaction region. Accumulation of antiprotons and positrons is desirable.

For this overlapping ring arrangement the scaling of the luminosity with various parameters was examined and a simple formula for the maximal antihydrogen production rate in this arrangement was derived:

$$R = 2 \times 10^4 \dot{\rho} \; [s^{-1}]$$
It was assumed that at the chosen energy the antiproton ring can be filled to the stability limit; however the stored intensity would in no case exceed $10^{13}$. Moreover, the ring circumference is minimized. Compared to earlier estimates, the antihydrogen production rate is several orders of magnitude higher; this is due to the fact that here an accumulation of slow positrons was considered and it was assumed that at least $10^8$ can be collected and prepared for the experiment (cooled, accelerated, and injected) within the positron lifetime in the storage ring. The development of an appropriate positron accumulation and storage scheme is the next step in achieving high antihydrogen rates. The relation given above shows that the dedicated antihydrogen production through radiative capture should be done at relativistic energies: this means that it can be performed at any antiproton storage ring provided a suitable positron ring is matched to it.

As pointed out earlier [7], induced capture may enhance the formation rate by several orders of magnitude. In order to make full use of this fact one has to work in a regime where one can apply continuous lasers. In the case where one has to work with pulsed lasers in order to obtain the required radiation power, the particle beams also have to be bunched and one has to match the laser repetition rate and pulse length to the time structure of the stored particles. Then antihydrogen production rates in the realm of $10^6 \text{s}^{-1}$ and more should be achievable.

Recently several methods to polarize antiprotons via the intermediate formation of antihydrogen were proposed [13, 14, 18]. So far no beams of polarized antiprotons exist; therefore even a beam of a few thousand polarized antiprotons derived from antihydrogen would be very interesting for particle- and nuclear-physics experiments. Moreover, in such a scheme the beam extraction would be automatically implemented and depolarization problems of antiprotons in the storage ring would be circumvented.

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