EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN-EP/87-34

February 17th, 1987

BOUNDS ON NEW CONTACT INTERACTIONS FROM FUTURE e^+e^- COLLIDERS

B. Schrempp and F. Schrempp*
Sektion Physik, Theoretische Physik, Universität München
D-8000 München, Fed. Rep. of Germany

N. Wermes
CERN, CH-1211 Geneva 23, Switzerland

D. Zeppenfeld
DESY, D-2000 Hamburg 52, Notkestr. 85, Fed. Rep. of Germany
and
Physics Dept., Univ. of Wisconsin, Madison, Wisc. 53706, USA**

ABSTRACT

A systematic study of the "reach" for contact interactions as a probe for new physics at future e^+e^- colliders is performed. A large energy range is considered, from PETRA/PEP energies at the lower end, via the Z^0 peak (SLC/LEP-I) and LEP-II, up to the TeV regime of a potential future linear e^+e^- collider. Lower bounds on the compositeness scale Λ for leptons are calculated for Bhabha scattering and μ -pair production using realistic assumptions on the expected luminosities and detector performances. The impact of longitudinal and transverse polarization is studied in detail. The "reach" corresponding to high rates at the Z^0 peak is compared to the one obtained from running at higher energies. At LEP-II energies, a sensitivity to (lepton) substructure at the level of $\Lambda \sim 10$ TeV is to be expected. At a 2 TeV e^+e^- collider the sensitivity can be as large as 100 TeV.

* Supported by Deutsche Forschungsgemeinschaft (DFG)

** Present address

1. Introduction

A major objective of experiments at future high energy accelerators will be to search for new particles and interactions announcing the onset of physics beyond the Standard Model.

While open thresholds provide the most unambiguous signals of new particles the mass range accessible in this case is strictly limited by the available beam energy. A considerably larger mass range may be explored, however, through the study of effects from virtual new particle exchanges. For masses much larger than the beam energy such indirect signatures may be conveniently and model independently investigated by adding general contact interaction terms to the Standard Model Lagrangian, quite analogously to Fermi's early description of weak interactions.

The potential of such a strategy is particularly emphasized by Eichten, Lane, and Peskin [1] in the context of quark and lepton substructure [2]. It should be noted, however, that (experimental) limits on the strength of contact interactions provide information on possible new physics in a far more general sense.

The purpose of this paper is a systematic study of the 'reach' for possible new contact interactions at e^+e^- colliders under construction (TRISTAN, SLC, LEP-I) and under consideration (LEP-II or a linear e^+e^- collider in the TeV range as presently discussed on both sides of the Atlantic Ocean [3]). We concentrate on Bhabha scattering and μ -pair production, reactions which are particularly suitable in this context. Throughout this paper we follow ref. [1] and express our results in the familiar language of quark and lepton substructure, not forgetting however the wider range of applicability of contact interactions.

The entire energy range of interest, from 40 GeV (PETRA/PEP) up to 4 TeV, is included in this study based on realistic assumptions about the expected detector performances and machine luminosities. Particular attention is devoted to the Z^0 -peak where the benefits from higher statistics are strongly reduced by the absence of interference between the imaginary Z^0 amplitude and the real contact interaction amplitudes.

An important issue in the discussion about future machines is the question of beam polarization. We investigate in detail the impact of longitudinally and transversely polarized beams on the 'reach' for new contact interactions.

The paper is organized as follows. In chapter 2 we start out by recalling the formalism given by Eichten, Lane and Peskin [1] for unpolarized beams and specify various ingredients of our analysis. Lower bounds on the compositeness scale Λ are calculated for expected measurements of σ_{tot} , $d\sigma/d\cos\theta$, and the Forward - Backward asymmetry A_{FB} . The entire energy range of future e^+e^- machines is studied and the special role of the Z^0 is discussed. Some analytical insight is provided by an approximate formula for the energy and luminosity dependence of the bounds on Λ . The formalism is extended for polarized beams in chapter 3 and the role of polarization to improve the sensitivity both on and off the Z^0 is investigated. Chapter 4 deals with aspects of radiative corrections. Chapter 5 finally contains a summary of our results.

2. Unpolarized Beams

In writing down the contact interaction between fermions with flavours a, b we follow the convention of ref. [1], in particular demanding chirality conservation for each flavour independently. Two points are worth being recalled.

1. If electrons are composite contact interactions will appear in $e^+e^- \rightarrow e^+e^-$. If e and μ have constituents in common contact interactions will also appear in $e^+e^- \rightarrow \mu^+\mu^-$.
2. We shall consider only chirality (i.e. helicity) conserving contact interactions. They are the most likely ones in any scheme. In the context of quark, lepton substructure they are conform with the requirement of chiral symmetry which is a necessary prerequisite for naturally small quark and lepton masses.

The four fermion interaction then takes the form

$$\begin{aligned}
 \mathcal{L}_{ab} = & k_{ab} \frac{g^2}{\Lambda_{ab}^2} \cdot [\eta_{LL} (\bar{\psi}_L^a \gamma_\mu \psi_L^a) (\bar{\psi}_L^b \gamma^\mu \psi_L^b) + \eta_{RR} (\bar{\psi}_R^a \gamma_\mu \psi_R^a) (\bar{\psi}_R^b \gamma^\mu \psi_R^b) \\
 & + \eta_{RL} (\bar{\psi}_R^a \gamma_\mu \psi_R^a) (\bar{\psi}_L^b \gamma^\mu \psi_L^b) + \eta_{LR} (\bar{\psi}_L^a \gamma_\mu \psi_L^a) (\bar{\psi}_R^b \gamma^\mu \psi_R^b)]. \quad (1)
 \end{aligned}$$

Here $\psi_L = 1/2 (1 - \gamma_5) \psi$ and $\psi_R = 1/2 (1 + \gamma_5) \psi$ are the standard chirality projections of the fermion spinors. The compositeness scale $\Lambda_{ab} = \Lambda$ for contact interactions between flavours a and b is traditionally defined with strong effective coupling $g^2/4\pi = 1$ and the largest $|\eta_{if}| = 1$. Finally, k_{ab} is a statistical factor with $k_{ab} = 1/2$ for $a=b$ and $k_{ab} = 1$ otherwise.

L_{ab} produces interference terms in the differential cross sections of lepton-lepton scattering of order $(g^2/\Lambda^2)/(4\pi\alpha/s) = s/(\alpha\Lambda^2)$, where α is the QED coupling constant. Particular choices of η_{if} define classes of different helicity configurations as listed in Table 1. In the following we distinguish scenarios where $L_{\psi\psi}$ reduces to couplings of two left-handed (LL), two right-handed (RR), two vector (VV), or two axial-vector (AA) currents, corresponding to appropriate choices of η_{if} in Table 1. For each of these configurations one also has the choice of positive or negative interference with the electroweak amplitude, denoted by a '+' or a '-' sign. In addition we have studied a particular choice [4] of η_{if} values which, as will be shown below, renders a very meager sensitivity and serves as a worst case study (WCS). This choice has $\eta_{LL} = -\eta_{RR} = \pm 1$, $\eta_{RL} = 0$, chosen such that the interference with the QED photon exchange amplitude exactly cancels. It should be pointed out that this helicity configuration is perhaps not naturally expected for the case of Bhabha scattering, since for exchanges of heavy vector particles η_{RR} and η_{LL} represent sums of coupling constants squared and therefore cannot have different signs.

In the following we study contact interactions for the processes $e^+e^- \rightarrow e^+e^-$ and $e^+e^- \rightarrow \mu^+\mu^-$. The lowest order diagrams are shown in Fig. 1. The unpolarized differential cross section for Bhabha scattering in lowest order, including γ and Z^0 exchanges as well as contact terms is given by

$$\frac{d\sigma}{d\cos\theta} = (\pi\alpha^2/4s) \cdot [4A_0 + A_-(1 - \cos\theta)^2 + A_+(1 + \cos\theta)^2] \quad (2)$$

where

$$A_0 = \left(\frac{s}{t}\right)^2 \cdot \left| 1 + \frac{g_R g_L}{e^2} \frac{t}{t_Z} + \frac{\eta_{RL} \cdot t}{\alpha\Lambda_c^2} \right|^2$$

$$A_- = \left| 1 + \frac{g_R g_L}{e^2} \frac{s}{s_z} + \frac{\eta_{RL} s}{\alpha \Lambda_c^2} \right|^2$$

$$A_+ = \frac{1}{2} \left| 1 + \frac{s}{t} + \frac{g_R^2}{e^2} \left(\frac{s}{s_z} + \frac{s}{t_z} \right) + \frac{2\eta_{RR} s}{\alpha \Lambda_c^2} \right|^2$$

$$+ \frac{1}{2} \left| 1 + \frac{s}{t} + \frac{g_L^2}{e^2} \left(\frac{s}{s_z} + \frac{s}{t_z} \right) + \frac{2\eta_{LL} s}{\alpha \Lambda_c^2} \right|^2$$

with

$$t = -s(1 - \cos\theta)/2; \quad s_z = s - m_Z^2 + im_Z \Gamma_Z,$$

$$t_z = t - m_Z^2, \quad g_R/e = \tan\theta_W, \quad g_L/e = -\cot 2\theta_W.$$

Throughout this paper we use the following Standard Model parameters: $m_Z = 92 \text{ GeV}$, $\Gamma_Z = 2.78 \text{ GeV}$, $\sin^2\theta_W = 0.23$.

<i>Table 1: Standard helicity configurations for contact interactions</i>			
	η_{LL}	η_{RR}	η_{RL}
$LL_{\pm}$	± 1	0	0
$RR_{\pm}$	0	± 1	0
$VV_{\pm}$	± 1	± 1	± 1
$AA_{\pm}$	± 1	± 1	∓ 1
$WCS_{\pm}$	± 1	∓ 1	0

Note that contact terms contribute to both, s-channel and t-channel particle exchanges. For $e^+e^- \rightarrow \mu^+\mu^-$ scattering the differential cross section is easily obtained from eq. (2) by setting all terms

which contain t or t_z to zero and replacing the factor 2 in front of η_{RR} and η_{LL} by one. Some care is required in comparing the results of this paper with experimental limits obtained at PETRA and PEP [5], [6], [7], [8], [9], [10]. In several analyses the factor 2 mentioned above has not been properly taken into account. It should be included in the cross section for Bhabha scattering, but not in $e^+e^- \rightarrow \mu^+\mu^-$. It should also be pointed out that $\eta_{RL} = \eta_{LR}$ is, in all generality, only true for Bhabha scattering but not for other final state fermions. A more explicit account of this fact is given in chapter 3.

Figure 2 illustrates the interference effects of contact interactions in the total cross section (a) and in the forward-backward asymmetry A_{FB} (b) for $e^+e^- \rightarrow \mu^+\mu^-$ for a typical substructure scale $\Lambda = 5$ TeV and various helicity combinations. A_{FB} is here defined as

$$A_{FB} = \frac{\int_0^1 \frac{d\sigma}{d\cos\theta} d\cos\theta - \int_{-1}^0 \frac{d\sigma}{d\cos\theta} d\cos\theta}{\int_{-1}^1 \frac{d\sigma}{d\cos\theta} d\cos\theta} \quad (3)$$

The 'data' points indicate the expected statistical accuracy corresponding to an integrated luminosity of 500 pb^{-1} , our reference value for LEP-II [13]. In Fig. 3 the quantity

$$\Delta_c = \frac{(\frac{d\sigma}{d\cos\theta})_{\text{composite}}}{(\frac{d\sigma}{d\cos\theta})_{EW}} - 1 \quad (4)$$

is plotted for $e^+e^- \rightarrow \mu^+\mu^-$ and $e^+e^- \rightarrow e^+e^-$, respectively, where $(\frac{d\sigma}{d\cos\theta})_{EW}$ is the Standard Model expectation, i.e. $\Lambda = \infty$.

Several comments are in order here: (1) In general, models with VV or AA couplings produce greater sensitivities in σ_{tot} , A_{FB} , and $d\sigma/d\cos\theta$ than models with LL or RR couplings; (2) RR and LL type couplings are almost indistinguishable from each other. (3) Given the fact that A_{FB} is much less affected by systematic uncertainties than σ_{tot} one obtains great sensitivity from A_{FB} for the AA helicity combination and to some extent also for RR, LL. Of course, the greatest sensitivity always arises from a complete analysis of $d\sigma/d\cos\theta$. Because Δ_c of eq.(4) is similar for positive and negative interferences of the contact terms with the electroweak amplitude we usually omit this distinction in the following.

To calculate lower bounds on Λ from $d\sigma/d\cos\theta$ we use the standard definition of χ^2 using the 'data' points and the model expectations, respectively. In addition to the expected statistical error for a given exposure a systematic error of 3% coming from the error on the luminosity measurement is taken into account. This error introduces an overall normalization uncertainty to the total cross section and acts on all data points of the angular distribution in a correlated way. The 95% C.L. lower bound on Λ is then given by the curve of a given helicity combination which corresponds to a χ^2 increase of 4 with respect to the null hypothesis, i.e. the Standard Model expectation. Note that the 'data' points used in the fit are not the ones shown in Fig. 3. Instead we use the exact Standard Model value, as the most probable value of a (future) measurement, with the error representing the expected range of fluctuations for different experiments.

To produce realistic estimates we assume a uniform experimental acceptance function within $|\cos\theta| < 0.95$ (< 0.90 for the TeV regime) and a binwidth of $\Delta\cos\theta = 0.1$. Restricting oneself to $|\cos\theta| < 0.90$ at $\sqrt{s} = 190$ GeV would change the resulting bounds on Λ by at most 3%.

To calibrate our procedure we compare the Λ bounds, that we obtain with this procedure, with the experimental limits from PETRA and PEP experiments [5] – [10]. Our calculations agree with these limits (c.f. Table 2), within the range allowed for by the various experimental measurements. Table 2 presents the obtained 95% confidence level bounds on Λ expected for various energies and integrated luminosities. Figure 4 shows the dependence of the limits on the integrated luminosity for 190 GeV centre of mass energy, that is the peak energy for the planned LEP upgrade LEP-II. Figure 5 shows the same for an e^+e^- collider in the TeV regime [12].

It is important to determine the observable which is most sensitive to effects of new contact interactions. An illustration of the situation is presented in Fig. 6 where lower bounds on Λ , as expected from $d\sigma/d\cos\theta$, σ_{tot} , or A_{FB} , are plotted versus the integrated luminosity for (a) AA and (b) VV models, respectively. The Forward-Backward asymmetry A_{FB} is expected to be much less affected by systematic uncertainties than the total cross section. We assumed here for the sake of the argument that $d\sigma/d\cos\theta$ and σ_{tot} can be measured with a 3% systematic uncertainty, coming mainly from the luminosity measurement, whereas A_{FB} has no systematic error. Figure 6 demonstrates that for contact interactions which produce very asymmetric $\cos\theta$ distributions (e.g. for AA interactions) the bounds

obtained from A_{FB} are of the same quality as the ones obtained from $d\sigma/d\cos\theta$. On the other hand, in the absence of large systematic errors the total cross section σ_{tot} approaches the sensitivity of $d\sigma/d\cos\theta$ when the asymmetry does not differ much from the Standard Model asymmetry. Large systematic errors mostly affect σ_{tot} . Of course, both σ_{tot} and the asymmetry are contained in $d\sigma/d\cos\theta$ which therefore always is the most sensitive observable if it can be measured.

The different response of A_{FB} and σ_{tot} to e.g. VV and AA contact terms together with the systematic error of the normalization (i.e. σ_{tot}) is the reason also for the crossing of the sensitivity curves in Figs. 4 and 5.

Next, we present an approximate analytical discussion [4] of the energy and luminosity dependence of the bounds on Λ . This is a very important aspect in the evaluation of the physics potential of future accelerators and is particularly interesting when one wants to compare LEP-II with LEP-I or SLC. We restrict ourselves to $d\sigma/d\cos\theta$ and note that Δ_C , the fractional deviation from the Standard Model cross section (eq. 4), is, for $s \neq m_Z^2$, of order $s/\alpha\Lambda^2$ [1], [11], i.e. the relative size of the interference between Standard Model amplitude ($4\pi\alpha/s$) and contact interaction amplitude (g^2/Λ^2). The statistical uncertainty, $\delta\sigma/\sigma$, with which a measurement can be made is proportional to $N^{-1/2}$, where N is the number of events, that is

$$\frac{\delta\sigma}{\sigma} \sim \left(\frac{\alpha^2}{s} \cdot R \cdot \int L dt\right)^{-1/2} \quad (6)$$

where R is defined as usual as the ratio of the actual cross section and $\sigma_{\mu\mu}$. By relating eq. (6), the measurement uncertainty, with Δ_C , the size of the effect, one obtains the desired relation between the bound on Λ as a function of s and $\int L dt$

$$\text{limit } (\Lambda) \sim (R \cdot \int L dt \cdot s)^{1/4} \quad (7)$$

Eq. (7) allows to take into account the different conditions at different e^+e^- machines. Ideally the integrated luminosity should be proportional to s to make up for the decreasing cross sections at high energies. Therefore, dividing out the 'patience factor' $(1/s \cdot \int L dt)^{1/4}$ of eq. (7), we can define a reduced

Λ which increases proportional to $\sqrt{s}$ and which gives a first estimate on the scales that can be probed at a machine of a given energy,

$$\Lambda_{\text{red}} = \text{limit } (\Lambda) \cdot \left[\frac{s}{(190 \text{ GeV})^2} \frac{500 \text{ pb}^{-1}}{\int L dt} \right]^{1/4} \quad (8)$$

In eq. (8) we have normalized to the conditions expected at LEP – II within a few years of operation [13].

Figure 7 a),b) shows Λ_{red} as a function of the centre of mass energy of the colliding beams from 30 GeV to 5 TeV. The solid lines correspond to the two extreme helicity combinations VV and WCS, the dashed lines show the linear dependence on $\sqrt{s}$ of the asymptotic LL model.

From this figure one may obtain a first guess on the requirements for the luminosity performance of a considered e^+e^- collider if one wants to probe substructure in electrons (muons). For example, at a 2 TeV linear collider [12] an integrated luminosity of roughly 50 fb^{-1} would be necessary to probe the substructure scale Λ_{ee} to about 100 TeV, a factor 10 times the limits expected at LEP – II. Deviation from the “ $\sqrt{s}$ scaling” occurs in particular at the Z^0 resonance. Exactly on the Z^0 peak the interference effects are minimal (except for WCS which is minimal slightly off m_Z) because the s – channel Breit – Wigner amplitude is purely imaginary at m_Z . Interference with the real contact term is thus greatly reduced. The exact behaviour close to m_Z depends on the type of contact interaction considered. In the tails of the Z^0 peak additional dips can occur as a result of interference with the strongly fluctuating real part of the Z^0 amplitude in this region (see Fig. 7). The actual limits expected for energies between the PETRA/PEP energy regime and 2 TeV (CLIC) are presented in Table 2 (with 3% systematic errors included).

The sensitivity for contact interactions increases by a factor $\approx 4-5$ going from PETRA/PEP to LEP – II. Even a large exposure right on the Z^0 peak will not significantly improve the limits presently available from PETRA/PEP. The situation changes drastically when one considers an exposure of 300 pb^{-1} at 110 GeV an energy which can still be reached with LEP – I and where the luminosity performance of the machine is expected to be even better. Here the improvement in sensitivity is already about a factor 2 – 3 with respect to PETRA/PEP.

3. Polarized Beams

The interest in polarized beams for the study of contact terms in the search for electron (muon) substructure arises for the following reasons.

- One may hope to increase the sensitivity using longitudinal polarization, especially for contact interactions with a LL, RR or WCS helicity structure [14].
- Transverse polarization allows to exploit the dependence of $d\sigma/d\phi$ on the azimuthal angle ϕ of the outgoing lepton as a further tool in the search for new contact interactions.
- Transverse polarization allows in particular to measure the imaginary part of the interference between the contact term and the complex Z^0 - pole contribution which is maximal on the Z^0 peak.

The doubly differential cross section for polarized beams, following the same ansatz for the contact interactions as in the unpolarized case, can conveniently be calculated using the helicity formalism [15]. Denoting the s-channel and t-channel matrix elements for $e^+(\vec{k}, \bar{\sigma}) + e^-(k, \sigma) \rightarrow f(p, \tau) + \bar{f}(\bar{p}, \bar{\tau})$, as

$$\begin{aligned}
 M_s(\sigma, \bar{\sigma}, \tau, \bar{\tau}) = & \left[\sum_{V=\gamma, Z_0} \frac{s}{s - m_V^2 + i m_V \Gamma_V} g_\sigma^{Vee} g_\tau^{Vff} + \frac{g^2 s}{\Lambda^2} \eta_{\sigma, \tau} \right] \\
 & \cdot \delta_{\sigma, -\bar{\sigma}} \delta_{\tau, -\bar{\tau}} \cdot \sigma \tau (1 + \sigma \tau \cos \theta) e^{-i \bar{\sigma} \phi} \\
 M_t(\sigma, \bar{\sigma}, \tau, \bar{\tau}) = & \left[\sum_{V=\gamma, Z_0} \frac{s}{t - m_V^2} g_\sigma^{Vee} g_{-\bar{\sigma}}^{Vee} + \frac{g^2 s}{\Lambda^2} \eta_{\sigma, -\bar{\sigma}} \right] \\
 & \cdot \delta_{\sigma, \tau} \delta_{\bar{\sigma}, \bar{\tau}} \cdot ((1 + \cos \theta) \delta_{\sigma, -\bar{\sigma}} + 2 \delta_{\sigma, \bar{\sigma}}) e^{-i \bar{\sigma} \phi}
 \end{aligned} \tag{9}$$

where

$V = \gamma, Z_0$ indicates the exchange of γ or Z^0

$$g_{\pm}^{\gamma ff} = -e$$

$$g_{-}^{Zff} = e \left(\tan \theta_w - \frac{1}{2 \sin \theta_w \cos \theta_w} \right)$$

$$g_{+}^{Zff} = e \cdot \tan \theta_w$$

$\sigma, \bar{\sigma}, \tau, \bar{\tau} = \pm 1 = \text{spin indices}$

$k, \bar{k}, p, \bar{p} = \text{particle 4-momenta}$

with $M = M_s + M_t$, the differential cross section for arbitrary initial state polarization is given by

$$\begin{aligned} \frac{d\sigma}{d\Omega} &= \frac{1}{64\pi^2 s} \sum_{\text{pol}} |M|^2 \\ &= \frac{1}{64\pi^2 s} \sum_{\tau, \bar{\tau}} \sum_{\sigma, \bar{\sigma}} \sum_{\chi, \bar{\chi}} M(\sigma, \bar{\sigma}; \tau, \bar{\tau}) P_{\bar{\sigma}, \bar{\chi}}^{+} M^{*}(\chi, \bar{\chi}, \tau, \bar{\tau}) P_{\chi, \sigma}^{-} \end{aligned} \quad (10)$$

where the polarization information is contained in the matrix

$$P_{\sigma_1, \sigma_2}^{\pm} = \frac{1}{2} \begin{bmatrix} 1 - P_L^{\pm} & P_T^{\pm} e^{\pm i \alpha_{\pm}} \\ P_T^{\pm} e^{\mp i \alpha_{\pm}} & 1 + P_L^{\pm} \end{bmatrix} \quad (11)$$

Here

$P_L^{\pm} = \text{degree of long. polarization of initial } e^{\pm},$

each oriented along its momentum

$P_T^\pm$ = degree of transv. polarization of initial $e^\pm$,

$\alpha_\pm = \pm\pi/2$ in a reference frame as defined below.

We choose in a right-handed x,y,z coordinate system the z -axis to be the e^- beam direction, the $x-z$ plane to be the collider plane (in case of a linear collider any other well defined plane may be chosen), the transverse polarization of the $e^\pm$ beams to be in the $\pm y$ -direction. Finally, θ and ϕ are the polar and azimuthal angles of the outgoing e^- , respectively. Evaluating eq. (10) for $e^+e^- \rightarrow e^+e^-$ we find

$$\begin{aligned} \frac{d\sigma}{d\Omega} = \frac{\alpha^2 s}{16} \cdot \left\{ \right. & |G_{LL}(t) + G_{LL}(s)|^2 (1 + P_L^+)(1 - P_L^-) \\ & + |G_{RR}(t) + G_{RR}(s)|^2 (1 - P_L^+)(1 + P_L^-) \cdot (1 + \cos\theta)^2 \\ & + 8 |G_{LR}(t)|^2 (1 + P_L^+ P_L^-) \\ & + 2 |G_{LR}(s)|^2 (1 - P_L^+ P_L^-) \cdot (1 - \cos\theta)^2 \\ & + 2 \operatorname{Re} [(G_{LL}(s) + G_{RR}(s) + G_{LL}(t) + G_{RR}(t)) G_{LR}^*(s)] \\ & \cdot P_T^+ P_T^- \cos(2\phi) \sin^2\theta \\ & + 2 \operatorname{Im} [(G_{LL}(s) - G_{RR}(s) + G_{LL}(t) - G_{RR}(t)) G_{LR}^*(s)] \\ & \cdot P_T^+ P_T^- \sin(2\phi) \sin^2\theta \left. \right\} \quad (12) \end{aligned}$$

with

$$G_{AB}(x) = \frac{g_A g_B / e^2}{x_Z} + \frac{1}{x} + \frac{1}{\alpha \Lambda_c^2} \eta_{AB}, \quad A, B = L, R$$

$$x = s, t \quad x_Z = s_Z, t_Z \quad \text{as defined in eq. (2)}$$

$$\eta_{RL} = \eta_{LR} \quad (\text{for } e^+e^- \rightarrow e^+e^-)$$

and g_R, g_L, s_Z, t_Z , and η_{AB} are defined as in eq. (2) .

For $e^+e^- \rightarrow \mu^+\mu^-$ (and, more generally, for $e^+e^- \rightarrow f^+f^-$ with $f \neq e$) only s -channel contributions occur and η_{LR} may not be equal to η_{RL} . The cross section is given as

$$\begin{aligned} \frac{d\sigma}{d\Omega} (e^+e^- \rightarrow \mu^+\mu^-) = & \frac{\alpha^2 s}{16} \left\{ \left[|G_{LL}(s)|^2 (1 + P_L^+)(1 - P_L^-) + |G_{RR}(s)|^2 (1 - P_L^+)(1 + P_L^-) \right] (1 + \cos\theta)^2 \right. \\ & + \left[|G_{LR}(s)|^2 (1 + P_L^+)(1 - P_L^-) + |G_{RL}(s)|^2 (1 - P_L^+)(1 + P_L^-) \right] (1 - \cos\theta)^2 \\ & + 2\text{Re} [G_{LL}(s) G_{RL}^*(s) + G_{RR}(s) G_{LR}^*(s)] P_T^+ P_T^- \cos(2\phi) \sin^2\theta \\ & \left. + 2\text{Im} [G_{LL}(s) G_{RL}^*(s) - G_{RR}(s) G_{LR}^*(s)] P_T^+ P_T^- \sin(2\phi) \sin^2\theta \right\} \quad (13) \end{aligned}$$

First, we study the influence of longitudinal beam polarization at $\sqrt{s} = 190$ GeV. Figure 8 a) is an enlarged section of Fig. 2 a) showing $\sigma_{\text{tot}}^{\mu\mu}$ for unpolarized beams; in Fig. 8 b) $\sigma_{\text{tot}}^{\mu\mu}$ for left-handed electrons scattering on right-handed positrons with 100% polarized beams is displayed for comparison. As expected, the interference between electroweak amplitude and contact interaction amplitude increases relative to the unpolarized case for LL-models, while it vanished for RR-models with this choice of longitudinal polarization. Vice versa RR-models will be more sensitively tested when the polarizations for e^+ and e^- are reversed. Oppositely (long.) polarized beams cannot be realistically obtained in symmetric circular colliders and are also difficult to produce in linear colliders [16]. For more realistic estimates we derive lower bounds on Λ considering only one (the electron) beam with 50% longitudinal polarization. The dashed lines in Fig. 4 and Fig. 5 indicate the larger sensitivity for LL-models if the electron beam is 50% (left-handed) longitudinally polarized. The limits obtained from $d\sigma/d\cos\theta$ with polarized beams under different conditions are included in Table 2.

Given longitudinal polarization, a further observable which is most sensitive to new contact interactions is the left-right asymmetry A_{LR} defined as

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}$$

where $\sigma_{L,R}$ denote the cross sections for left and right handed polarized (electron) beams scattering off an unpolarized positron beam. The left – right asymmetry is known to be the most important observable to complement sensitive Standard Model tests obtained from experiments using unpolarized beams. Because it is affected by radiative corrections in a different way [17], A_{LR} may expose new phenomena in a cleaner way than other observables. We study the sensitivity of A_{LR} for a 50% polarized electron beam in Fig. 9. As before, it is evident that LL and RR models which in the case of unpolarized beams produce smaller effects than VV, AA models are very sensitive to the left – right asymmetry with longitudinal polarization. On the other hand, VV and AA type interactions are almost not felt by A_{LR} . Even more remarkable is the fact that A_{LR} is most sensitive to the worst case choice of η_{if} parameters (WCS). This is due to the fact that (for $\sin^2\theta_W = 1/4$) the interference term between contact interactions and γ, Z^0 exchange is proportional to $\eta_{RR} - \eta_{LL}$ in A_{LR} , while being proportional to $\eta_{RR} + \eta_{LL}$ in the unpolarized cross section. Thus WCS produces the largest possible interference term in A_{LR} . The limits that can be obtained from A_{LR} are shown in Fig. 10 and are included in Table 2. An error on the measurement of the degree of polarization of 5% has been assumed. The improvement achieved for LL, RR combinations from A_{LR} are similar to the ones obtained from $d\sigma/d\cos\theta$ with polarization. The sensitivity to the WCS combination, however, is dramatically boosted by measuring A_{LR} . Being an asymmetry, the systematic uncertainty of A_{LR} is entirely governed by the error on the degree of polarization obtained.

We conclude that longitudinal beam polarization enhances the sensitivity for LL, RR, and WCS helicity configurations almost to the level of sensitivity for the VV, AA combinations. Exploiting the capabilities of (long.) polarized beams therefore allows one to probe a much wider range of new contact interactions to a similar level of sensitivity which would not be possible with unpolarized beams only. Furthermore, longitudinal beam polarization allows to pin down the specific helicity structure of a new contact interaction, once a signal has been observed.

Next, we turn to the case of transverse polarization. We consider the double differential cross section $d^2\sigma/d\cos\theta d\phi$ using 4 bins in $\cos\theta$ and 12 bins in ϕ . Fig. 11 (a), shows the deviation from the Standard Model in $d\sigma/d\phi$ defined analogously to Δ_c in eq. (4) integrated over $\cos\theta$, for $\sqrt{s} = 190$ GeV. For unpolarized beams the contact term curves would be constant lines with the same average offset as the predictions for (trans.) polarized beams. Apparently, the improvement on the sensitivity expected from transversely polarized beams is marginal. The corresponding bounds are included in Table 2. We conclude that transverse beam polarization off-resonance does not increase the sensitivity on contact interactions noticeably when compared to the unpolarized case.

From eqs. (12) and (13) it is obvious that transverse polarization introduces an imaginary part to the contact interaction amplitude which on the Z^0 must interfere with the purely imaginary Z^0 Breit-Wigner amplitude. The size of the change in Δ_c from the interference with the electroweak amplitude is expected to be

$$\sim \frac{\Gamma_{Z^0}^m}{\alpha\Lambda_c^2} \sim 0.04 \cdot \Lambda_c^{-2}, \quad \Lambda_c \text{ in TeV}$$

Figure 11 (b) shows the resulting net effect to the ϕ distribution for $\Lambda = 2$ TeV. The WCS curve in particular shows a large variation as a function of ϕ . The corresponding lower bounds are included in Table 2. With 100% transverse polarization the sensitivity is increased by up to a factor 2–3 on the Z^0 depending on the helicity configuration. Since the effect is proportional to $P_T^+ P_T^-$ it rapidly decreases however when a more realistic degree of polarization is assumed.

4. The influence of radiative corrections

The studies described in the previous chapters were made neglecting the influence of radiative corrections. In this chapter we show that the reach of future colliders for contact interactions remains unchanged by the influence of radiative corrections.

Radiative corrections are particularly important in the vicinity of the Z^0 -peak where they produce large deviations from the tree level expectations. At the centre of mass energies of LEP-II (190 GeV) it is mainly the radiative return of the Z^0 which has to be taken into account. Experiments will of course compare the measured differential cross sections with the theoretical prediction including radiative corrections. Only the remaining deviations may then be interpreted as being due to novel contact interactions. For compositeness scales as large as the bounds given in the last two chapters, any interference between contact amplitudes and virtual corrections may safely be neglected. We hence believe that the sensitivity of future experiments to contact interactions is quantitatively described by the results presented earlier.

In order to check this assumption we have considered the effect of initial state radiation, i.e. the emission of hard photons from the incoming $e^\pm$ which is certainly the most important correction to be included. We simulate the effects of initial state bremsstrahlung plus virtual photon correction to the incoming $e^\pm$ legs using a Monte Carlo event generator [18] which generates events with initial-state radiation for an arbitrary final state.

The net result of initial-state radiation is a sizeable increase in the total cross section due to events where a hard photon is radiated. Figure 12 shows a scatterplot of the energy of the radiative photon versus the acolinearity angle between the final state leptons (here muons) for $\sqrt{s} = 190$ GeV. The projections are also shown. It is evident that a large fraction of the events is in fact coming from the Z^0 . These events would diminish the effect of contact terms according to Fig. 7. From Fig. 12 it is obvious that with strait-forward cuts in these observables one is able to eliminate most of the radiative events, in particular those coming from Z^0 decay. The resulting limits are then identical to the ones obtained above without radiative corrections. Even better, when only the Z^0 events are eliminated but the events with a radiative photon of energy less than 50 GeV are retained, the relative contribu-

tion of the contact interactions stays the same but the statistical significance of the measurement will be increased by 20% to 30% .

Concluding this chapter we state that initial state radiation does not decrease the bounds obtained above without radiation. A small increase in sensitivity might be obtained due to the increased statistics.

5. Conclusions

A systematic investigation of the sensitivity of future e^+e^- colliders to effects of contact interactions as a probe of new physics processes has been performed. The reactions $e^+e^- \rightarrow e^+e^-$ and $e^+e^- \rightarrow \mu^+\mu^-$ have been studied.

For unpolarized beams the main results are:

1. Among the studied observables – σ_{tot} , A_{FB} , and $d\sigma/d\cos\theta$ – $d\sigma/d\cos\theta$ is clearly the most sensitive one (in absence of large systematic effects).
2. The reach for contact interactions, expressed in form of a bound on the compositeness scale Λ , is

at LEP – II for 500 pb^{-1}

$$\text{bound}(\Lambda) \approx 3.7 \text{ TeV} - 13.5 \text{ TeV for } e^+e^- \rightarrow e^+e^-$$

$$\text{bound}(\Lambda) \approx 3.1 \text{ TeV} - 11.0 \text{ TeV for } e^+e^- \rightarrow \mu^+\mu^-$$

at a hypothetical 2 TeV e^+e^- collider for 20 fb^{-1}

$$\text{bound}(\Lambda) \approx 34 \text{ TeV} - 105 \text{ TeV for } e^+e^- \rightarrow e^+e^-$$

$$\text{bound}(\Lambda) \approx 32 \text{ TeV} - 95 \text{ TeV for } e^+e^- \rightarrow \mu^+\mu^- .$$

The quoted values cover the range between the most unfavourable (WCS) and the most favourable (VV,AA) helicity combinations of the contact term choice. The bounds are based on realistic assumptions for statistical and systematic errors and expected detector performances.

3. Right on the Z^0 peak the sensitivity to contact interactions is strongly reduced because the real contact interaction amplitude does not interfere with the imaginary Z^0 – pole contribution. At the wings of the Z^0 peak the sensitivity is restored leading to bounds on Λ of the order of 5 to 7 TeV for VV or AA configurations.
4. We point out that the bound on Λ approximately scales with the fourth root of the integrated luminosity and the fourth root of s . This leads us to define a reduced mass scale Λ_{red} which is a machine independent measure of the sensitivity to the compositeness scale (provided $\int L dt/s \approx \text{const.}$).

An important issue was to study the impact of beam polarization on the sensitivity to contact terms.

The results are

1. Longitudinal polarization boosts the reach for those helicity configurations which were unfavourable with unpolarized beams to about the level of the most favourable (VV, AA) configurations. We therefore conclude that by exploiting both, unpolarized and polarized beams, one can probe the compositeness scale Λ to

$$\Lambda \approx 10 \text{ TeV at LEP-II}$$

and

$$\Lambda \approx 100 \text{ TeV for a 2 TeV } e^+e^- \text{ collider.}$$

The different sensitivities of different helicity configurations with respect to polarization allow in addition to pin down the specific helicity structure of a contact interaction once a signal has been observed. The most sensitive observables are A_{LR} and $d\sigma/d\cos\theta$.

2. Transverse polarization off the Z^0 resonance has only marginal impact on the sensitivity for contact terms. On the peak of the Z^0 the sensitivity is strongly enhanced for a high degree of polarization.

Finally, we showed that radiative corrections do not change our conclusions.

Acknowledgments

This paper emerged from studies for the LEP200 Workshop in Aachen, Germany, October 86. We would like to thank all members of the 'Compositeness' group for stimulating discussions in a fruitful atmosphere. We are indebted in particular to L. Levinson and R. Kleiss for help on the radiative correction calculations.

This research was supported in part by the University of Wisconsin Research Committee with funds granted by the Wisconsin Alumni Research Foundation, and in part by the U.S. Department of Energy under contract DE-AC02-76ER00881.

References

- [1] E. J. Eichten, K. D. Lane, and M. E. Peskin, *Phys. Rev. Lett.* 50, 811 (1983).
- [2] Reviews about compositeness are for instance: M. E. Peskin, *Proceedings of the International Symposium on Lepton and Photon Interactions at High Energies*, Bonn, 1981, edited by W. Pfeil (*Physik. Inst. Universitaet Bonn*), p. 880; W. Buchmueller, *Proceedings of the XXIV. Internationale Universitaetswochen fuer Kernphysik*, Schladming, Austria, Febr. 20 to March 1, (1985), and Cern Preprint CERN-TH-4189 (1985).
- [3] Workshop on Physics at Future Accelerators, 7. - 10. Jan. 1987, La Thuile, Aosta Valley, Italy, *Proceedings*; B. Richter, *Panel Discussion on Laboratory Accelerator Programs - Present and Future*, Inv. Talk presented at the 23rd Int. Conf. on High Energy Physics, Berkeley, 1986, and SLAC-PUB-4081, 1986; B. Richter, *Invited Paper presented at the 13th Int. Conf. on High Energy Accelerators*, Novosibirsk, USSR, August 1986, and SLAC-PUB-4076, 1986.
- [4] see also : F. Schrempp, *Invited Talk given at the 23rd Int. Conf. on High Energy Physics*, Berkeley, 1986, *Proceedings*, and Max-Planck-Inst. Munich preprint MPI-PAE/PTH 69/86, (1986).
- [5] TASSO Collaboration, M. Althoff et al., *Z. Phys.* C22, 13, (1984).
- [6] HRS Collaboration, D. Bender et al., *Phys. Rev.* D30, 515, (1984).
- [7] PLUTO Collaboration, C. Berger et al., *Z. Phys.* C27, 341, (1985).
- [8] JADE Collaboration, W. Bartel et al., *Z. Phys.* C30, 371, (1986).
- [9] MAC Collaboration, E. Fernandez et al., SLAC-PUB-4005, (1986), submitted to *Phys. Rev. D*.

- [10] S. Komamiya, Proceedings of the 1985 International Symposium on Lepton and Photon Interactions at High Energies, Kyoto, August 1985, (1985).
- [11] J. Preskill, Invited talk presented at the Conference on Novel Results in Particle Physics, Vanderbilt University, (1982), unpublished.
- [12] As a possible realisation of a high energy e^+e^- machine a 2 TeV linear collider is discussed, c.f. Ref. 3.
- [13] The canonical values for energy and integrated luminosity have been defined as 190 GeV and 500 pb^{-1} at the LEP200 Workshop, organized by the European Committee for Future Accelerators (ECFA), Aachen, Sept. 29 – Oct. 1, 1986, Proceedings.
- [14] see also: I. Bars, J. F. Gunion and M. Kwan, Phys. Rev. D34, 122, (1986).
- [15] M. Jacob and G. Wick, Ann. Phys. 7, 404, (1959); see also: K. Hagiwara and D. Zeppenfeld, Nucl. Phys. B274, 1, (1986), and references therein.
- [16] see e.g. C. Y. Prescott, Inv. Talk presented at the Workshop on Tests of Electroweak Theories, Polarized Processes, and Other Phenomena, Trieste, Italy, June 10–12, 1985, and SLAC – PUB – 3718.
- [17] M. Boehm and W. Hollik, Nucl. Phys. B204, 45, (1982); B. W. Lynn, M. E. Peskin and R. G. Stuart, in "Physics at LEP", ed. J. Ellis and R. D. Peccei, CERN 86–02, Vol.1, (1986).
- [18] F. A. Behrends and R. Kleiss, Nucl. Phys. B260, 32, (1985).

TABLE 2a

Lower bounds on Λ_{ee} for $e^+e^- \rightarrow e^+e^-$

$\sqrt{s}$	$\int L dt$ [pb^{-1}]	Polarisation			Interf. ob- tained from	95% C.L. Limit [TeV]				
		P_L^+	P_L^-	P_T		RR	LL	VV	AA	WCS
34.5 GeV	74.7	unpolarized			pos.	TASSO	0.7	0.7	1.9	2.0
					neg.	ref. 5	1.9	1.9	2.9	2.3
12-46.8 GeV	103	unpolarized			pos.	JADE	1.1	1.1	2.5	2.4
					neg.	ref. 8	1.4	1.4	3.1	2.3
35 GeV	70	unpolarized			pos.	$\frac{d\sigma}{d\cos\theta}$	1.4	1.4	3.3	2.4
					neg.	$\frac{d\sigma}{d\cos\theta}$	1.4	1.4	3.3	2.3
$m_Z - \Gamma_Z$	150	unpolarised			pos.	$\frac{d\sigma}{d\cos\theta}$	3.2	3.6	6.4	4.3
					neg.	$\frac{d\sigma}{d\cos\theta}$	3.2	3.6	6.4	4.4
m_Z	150	unpolarised			pos.	"	2.5	2.6	4.5	2.9
					neg.	"	2.6	2.6	4.4	2.7
m_Z	150	0% -100%	-	-	pos.	"	0.0	3.5	4.3	2.8
		0% -50%	-	-	pos.	"	1.8	3.1	4.4	2.8
m_Z	75+75	0% -100%	-	-	pos.	A_{LR}	1.4	1.6	0.9	0.7
		0% -50%	-	-	pos.	A_{LR}	1.4	1.5	0.8	0.7
m_Z	150	-	-	+100%	pos.	$\frac{d^2\sigma}{d\cos\theta d\phi}$	3.6	3.6	4.8	3.1
		-	-	+50%	pos.	$\frac{d^2\sigma}{d\cos\theta d\phi}$	2.6	2.7	4.6	3.0
$m_Z + \Gamma_Z$	150	unpolarised			pos.	$\frac{d\sigma}{d\cos\theta}$	3.3	3.8	4.5	7.3
					neg.	$\frac{d\sigma}{d\cos\theta}$	3.2	3.7	4.5	7.3
110 GeV	300	unpolarised			pos.	"	3.4	3.3	9.2	9.1
					neg.	"	3.5	3.4	9.3	8.9
190 GeV	500	unpolarized			pos.	"	5.9	6.0	13.4	10.0
					neg.	"	6.2	6.3	13.5	9.3
190 GeV	100	unpolarised			pos.	"	3.9	4.0	9.0	6.9
					neg.	"	4.3	4.4	9.2	5.8
190 GeV	500	0% -100%	-	-	pos.	"	0.0	8.6	13.4	10.0
		0% -50%	-	-	pos.	"	4.0	7.4	13.4	10.0
		0% +50%	-	-	pos.	"	7.3	4.1	13.4	10.0
		0% -50%	-	-	neg.	"	4.5	7.6	13.5	9.3
190 GeV	250+250	0% $\pm$ 100%	-	-	pos.	A_{LR}	7.5	7.5	0.0	0.0
		0% $\pm$ 50%	-	-	pos.	A_{LR}	5.3	5.3	0.0	0.0
190 GeV	500	-	-	+100%	pos.	$\frac{d^2\sigma}{d\cos\theta d\phi}$	5.9	6.0	13.5	10.3
		-	-	+50%	pos.	$\frac{d^2\sigma}{d\cos\theta d\phi}$	5.9	6.0	13.4	10.0
2 TeV	20000	unpolarised			pos.	$\frac{d\sigma}{d\cos\theta}$	49	52	104	76
					neg.	$\frac{d\sigma}{d\cos\theta}$	52	54	105	64

TABLE 2b

Lower bounds on $\Lambda_{e\mu}$ for $e^+e^- \rightarrow \mu^+\mu^-$

$\sqrt{s}$	$\int L dt$ [pb^{-1}]	Polarisation			Interf. ob- tained from	95% C.L. Limit [TeV]					
		P_L^+	P_L^-	P_T		RR	LL	VV	AA	WCS	
34.7 GeV	41.8	unpolarized			pos. neg.	PLUTO ref. 7	2.4 0.9	2.9 0.9	2.4 1.6	4.5 1.5	- -
35 GeV	42	unpolarized			pos. neg.	$\frac{d\sigma}{d\cos\theta}$	1.5 1.4	1.5 1.4	2.2 2.1	2.6 2.6	0.8 0.8
$m_Z - \Gamma_Z$	150	unpolarised			pos. neg.	$\frac{d\sigma}{d\cos\theta}$	2.5 2.5	3.1 3.1	5.3 5.3	2.6 2.6	2.0 1.4
m_Z	150	unpolarised			pos. neg.	"	1.0 0.6	1.0 0.6	0.8 0.5	1.7 1.7	1.0 1.0
m_Z	150	0% -100%	-	-	pos.	"	0.0	1.3	0.9	1.6	0.6
		0% -50%	-	-	pos.	"	0.8	1.2	0.8	1.7	0.8
m_Z	75+75	0% -100%	-	-	pos.	A_{LR}	0.7	0.8	0.6	0.5	0.8
		0% -50%	-	-	pos.		0.7	0.7	0.6	0.5	0.7
m_Z	150	-	-	+100%	pos.	$\frac{d^2\sigma}{d\cos\theta d\phi}$	2.4	2.3	2.4	1.8	3.3
		-	-	+50%	pos.		1.2	1.2	1.2	1.7	1.5
$m_Z + \Gamma_Z$	150	unpolarised			pos. neg.	$\frac{d\sigma}{d\cos\theta}$	2.4 2.4	2.8 2.8	5.5 5.5	2.8 2.7	0.7 1.5
110 GeV	300	unpolarised			pos. neg.	"	3.3 3.1	3.6 3.5	5.3 5.8	5.7 5.3	1.5 2.0
190 GeV	500	unpolarised			pos. neg.	"	6.3 6.0	6.6 6.4	10.7 10.6	10.6 11.0	3.1 3.7
190 GeV	100	unpolarised			pos. neg.	"	4.9 4.5	5.1 4.8	8.2 8.0	7.2 7.7	2.8 3.2
190 GeV	500	0% -100%	-	-	pos.	"	0.0	8.9	10.6	10.5	8.8
		0% -50%	-	-	pos.		4.4	7.9	10.7	10.6	6.3
		0% +50%	-	-	pos.		7.8	4.8	10.8	10.6	6.4
		0% -50%	-	-	neg.	"	4.1	7.7	10.5	11.0	6.7
190 GeV	250+250	0% $\pm 100\%$	-	-	pos.	A_{LR}	7.2	7.5	1.9	1.6	10.6
		0% $\pm 50\%$	-	-	pos.		5.0	5.3	0.0	0.0	7.5
190 GeV	500	-	-	+100%	pos.	$\frac{d^2\sigma}{d\cos\theta d\phi}$	6.6	7.0	10.7	13.3	3.4
		-	-	+50%	pos.		6.0	6.5	10.3	10.8	3.1
2 TeV	20000	unpolarised			pos. neg.	$\frac{d\sigma}{d\cos\theta}$	56 53	58 55	95 93	86 90	32 35

FIGURE CAPTIONS

- Fig. 1

Lowest order diagrams for $e^+e^- \rightarrow e^+e^-$ and $e^+e^- \rightarrow \mu^+\mu^-$, including contact terms.

- Fig. 2

The effect of contact terms on (a) σ_{tot} and (b) A_{FB} for $e^+e^- \rightarrow \mu^+\mu^-$ as a function of $\sqrt{s}$. In (a) the curves above and below the GSW-line correspond to positive and negative interference with the Standard Model amplitude, respectively. In (b) positive interference reduces the asymmetry while negative interference enhances it. The 'data' points indicate the expected measurement accuracy for $\int L dt = 500 \text{ pb}^{-1}$.

- Fig. 3

The effect of contact terms on $d\sigma/d\cos\theta$ for $e^+e^- \rightarrow \mu^+\mu^-$ (a),(b) and $e^+e^- \rightarrow e^+e^-$ (c),(d) with positive and negative interference. Δ_c is the relative deviation from the Standard Model cross section. The data points represent a typical measurement at $\sqrt{s} = 190 \text{ GeV}$ for 500 pb^{-1} .

- Fig. 4

Lower bounds on the compositeness scale Λ for $e^+e^- \rightarrow e^+e^-$ (a),(c) and $e^+e^- \rightarrow \mu^+\mu^-$ (b),(d) obtained from $d\sigma/d\cos\theta$ as a function of integrated luminosity at $\sqrt{s}=190 \text{ GeV}$. The various helicity combinations are explained in the text. The dashed line represents the LL combination with a 50% long. polarized e^- beam.

- Fig. 5

Lower bounds on the compositeness scale Λ for $e^+e^- \rightarrow e^+e^-$ (a),(c) and $e^+e^- \rightarrow \mu^+\mu^-$ (b),(d) obtained from $d\sigma/d\cos\theta$ as a function of integrated luminosity at $\sqrt{s}=2$ TeV. The various helicity combinations are explained in the text. The dashed line represents the LL combination with a 50% long. polarized e^- beam. Crossing of lines occurs due to the different response of these helicity configurations to the systematic error.

- Fig. 6

Comparison of the sensitivity of $d\sigma/d\cos\theta$ (solid line), A_{FB} (dashed), and σ_{tot} (dashed-dotted) to contact terms with (a) VV and (b) AA helicity combinations for $e^+e^- \rightarrow \mu^+\mu^-$. A_{FB} has been assumed without systematic error in this study.

- Fig. 7

The 'reach' of e^+e^- colliders, $\Lambda_{red} = \text{limit}(\Lambda) \cdot \{ s/(190 \text{ GeV})^2 \cdot 500\text{pb}^{-1}/\int L dt \}^{1/4}$ as a function of $\sqrt{s}$ showing approximate scaling (dashed line). The most favourable (VV) and the most disfavourable (WCS) helicity combination as well as LL are displayed. The inserts display the region around the Z^0 peak. For WCS the exponent of Λ_{red} is 1/8 instead of 1/4 in order to obtain approximate scaling also for this helicity combination (see text). (a) $e^+e^- \rightarrow e^+e^-$, (b) $e^+e^- \rightarrow \mu^+\mu^-$.

- Fig. 8

Total cross section for $e^+e^- \rightarrow \mu^+\mu^-$ including contact terms (a) without polarization and (b) with 100% long. polarization ($P_L^\pm = \pm 1$) of both beams. The data point indicates a typical measurement with 500 pb^{-1} . The resulting increased (decreased) sensitivity for LL (RR) helicity combinations is evident.

- Fig. 9

The effect of contact terms on A_{LR} for a 50% long. polarized e^- beam. The 'data' points correspond to 500 pb^{-1} . (a) $e^+e^- \rightarrow \mu^+\mu^-$, (b) $e^+e^- \rightarrow e^+e^-$.

- Fig. 10

Lower bounds on Λ obtained from A_{LR} for (a) $e^+e^- \rightarrow \mu^+\mu^-$, pos. interference, and (b) $e^+e^- \rightarrow e^+e^-$, neg. interference as a function of the degree of polarisation. An error of 5% on the polarization measurement is assumed.

- Fig. 11

The effect of transverse polarization on $d\sigma/d\phi$ in $e^+e^- \rightarrow \mu^+\mu^-$ at (a) $\sqrt{s} = 190 \text{ GeV}$, and (b) $\sqrt{s} = m_Z$ for 100% polarization. The 'data' points indicate the measurement accuracy for a typical experiment with 500 pb^{-1} in (a) and 150 pb^{-1} in (b). The variations as function of ϕ are to be compared with constant lines with the same average offset for unpolarized beams.

- Fig. 12

Scatterplot of the photon energy versus the acolinearity angle between the outgoing muons as well as the projections, for $e^+e^- \rightarrow \mu^+\mu^-$ with initial state radiation at $\sqrt{s} = 190 \text{ GeV}$. Clean cuts in these observables allow to eliminate radiative events with Z^0 decays.

$$e^+e^- \rightarrow e^+e^-$$

$$e^+e^- \rightarrow \mu^+\mu^-$$

FIG. 1

FIG. 2

FIG. 3

FIG. 4

FIG. 5

FIG. 6

FIG. 7

FIG. 8

FIG. 9

FIG. 10

FIG. 11

FIG. 12