LOOP CORRECTIONS TO THE ρ -PARAMETER; THE MAGNETIC MOMENT OF
THE $W^\pm$ AND $e^+e^- \rightarrow W^+W^-$)

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A B S T R A C T

Vector-boson loop corrections to the ρ -parameter are considered for vector-boson self-interactions following from global $SU(2)_{WI}$ broken by electromagnetism. Minimizing the degree of divergence of the ρ -parameter restricts the free vector-boson self-couplings to a single one, the anomalous magnetic dipole moment, κ , of the $W^\pm$. The analysis of $e^+e^- \rightarrow W^+W^-$ shows that this process is well suited for accurately determining this remaining free parameter, κ .

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After the discovery¹⁾ of the intermediate vector bosons the experimental determination of their self-couplings, which are precisely predicted by the gauge principle underlying the $SU(2)_L \times U(1)_Y$ spontaneously broken gauge theory²⁾, will be one of the very important tasks for the years to come. In particular LEP 200, as well as an even more energetic e^+e^- collider, but also $p(\bar{p})$ interactions at sufficiently high energies will be well suited for this purpose.

Vector boson self-couplings which differ from the $SU(2)_L \times U(1)_Y$ predictions are by no means excluded by the past success of the spontaneously broken $SU(2)_L \times U(1)_Y$ gauge theory. All its empirically verified predictions can also be derived within a framework^{3,4,5)} based on global weak isospin symmetry, $SU(2)_{WI}$ broken by electromagnetism, by evaluating this effective Lagrangian at the tree level and incorporating⁵⁾ fermion loop corrections to the $W^\pm$ and Z_0 masses in the leading log approximation. Within this more general approach the vector boson self couplings need not necessarily, but may in fact be different^{6,7,F1)} from the $SU(2)_L \times U(1)_Y$ gauge theory predictions.

If one restricts oneself to dimension-four terms in the Lagrangian and does not allow for P or C violating electromagnetic interactions of the vector bosons, the most general Lagrangian based on global $SU(2)_{WI}$ broken by electromagnetism^{6,7)} contains two free trilinear coupling constants of the vector bosons, $\hat{g}$ and κ , and two quadrilinear ones, $\hat{g}$ and $\tilde{g}$. Here, $\hat{g}$ denotes the trilinear self-coupling of the weak isospin triplet, $W^\pm W^3$, and κ the anomalous magnetic dipole moment of the $W^\pm$. In terms of κ the magnetic moment and the electric quadrupole moment of the $W^\pm$ are given by

$$\mu_W = \frac{e}{2M_W} (1 + \kappa) \quad (1a)$$

and

$$Q_W = - \frac{e}{M_W^2} \kappa. \quad (1b)$$

The $SU(2)_L \times U(1)_Y$ self-interactions are obtained as a special case with $\hat{g} = e/\sin\theta_W$ and $\kappa=1$, as well as $\tilde{g} = \hat{g}^2 = e^2/\sin^2\theta_W$ and $\tilde{g} = 0$. The more general framework based on global $SU(2)_{WI}$ broken by electromagnetism thus constitutes an excellent testing ground for the $SU(2)_L \times U(1)_Y$ gauge theory.

F1) The $SU(2)_L \times U(1)_Y$ vector boson self-couplings are obtained if W-dominance⁴⁾ (which within the above approach must be required to hold for interactions of photons with leptons and quarks) is extended^{8,9)} to also hold for the intermediate vector bosons, $W^\pm$. Within the present paper we do not adopt this assumption. For a different point of view, compare⁹⁾.

The purpose of the present note is twofold:

- i) to derive the restrictions on the above mentioned trilinear and quadrilinear self-couplings which follow by considering vector boson loop corrections to the ratio of the neutral to charged current strengths, the experimentally well-determined ρ -parameter, and
- ii) to display the information to be gained on trilinear vector boson self-couplings in future measurements of $e^+e^- \rightarrow W^+W^-$, provided the restrictions from the ρ -parameter on the trilinear couplings are taken into account.

For the underlying Lagrangian we refer to KMSS⁶⁾. For later usage we only note that the $Z_0 W^+ W^-$ coupling, g_{ZWW} , and the weak anomalous magnetic moment of the $W^\pm$, commonly denoted by κ_Z , are related to $\hat{g}$ and κ via

$$g_{ZWW}(\hat{g}) = -e \tan \theta_W + \frac{\hat{g}}{\cos \theta_W} \quad (2)$$

and

$$\kappa_Z(\hat{g}, \kappa) = \frac{-e \tan \theta_W + \frac{\hat{g}}{\cos \theta_W}}{-e \tan \theta_W + \frac{\hat{g}}{\cos \theta_W}} \quad (\text{for } \hat{g} \neq e \sin \theta_W). \quad (3)$$

The fact that κ_Z is not an independent third parameter besides $\hat{g}$ (or, equivalently, g_{ZWW}) and the anomalous magnetic dipole moment κ , is intimately connected with the underlying principle of allowing for no intrinsic breaking of $SU(2)_{WI}$ symmetry, i.e., $SU(2)_{WI}$ symmetry is assumed to be restored in the limit of vanishing electromagnetic interactions, $e \rightarrow 0$, an assumption well motivated by the empirically established features of electroweak interactions.

The self energy contributions generated by vector boson loops are shown in the diagrams of fig. 1. When evaluating these diagrams we expect to obtain integrals which diverge as Λ^4 or even Λ^6 , where Λ denotes an ultraviolet cut-off. The cut-off Λ within the present context is to be interpreted as the mass scale at which new physical phenomena (e.g., due to compositeness of the $W^\pm$ and Z_0) set in. Evaluating the transverse parts of the vector boson self-energies by employing the Feynman rules given in (6) one finds for the Z_0

$$\frac{\Pi_{ZZ}(0)}{M_Z^2} = \frac{3}{4 \cdot 32 \pi^2} \left(2(\hat{g} - \hat{g}^2) + \frac{14}{3} \hat{g} \right) \frac{\Lambda^4}{M_W^4} + O(\Lambda^2) \quad (4)$$

and for the $W^\pm$

$$\begin{aligned} \frac{\Pi_{WW}(0)}{M_W^2} = & \frac{3}{4 \cdot 32\pi^2} \left[2e(\hat{g} - \frac{e}{\sin \theta_W}) \kappa \frac{\sin \theta_W}{\cos^2 \theta_W} - \frac{e^2}{\cos^2 \theta_W} (\kappa - 1)^2 \right. \\ & \left. - (\hat{g}^2 - \frac{e^2}{\sin^2 \theta_W}) (\frac{1 + \cos^2 \theta_W}{\cos^2 \theta_W}) + 2(\hat{g} - \frac{e}{\sin^2 \theta_W}) + \frac{14}{3} \tilde{g} \right] \frac{\Lambda^4}{M_W^4} + O(\Lambda^2). \end{aligned} \quad (5)$$

The leading terms are of order Λ^4 and evidently vanish if standard $SU(2)_L \times U(1)_Y$ couplings are inserted for $\hat{g}$, κ , $\hat{g}$ and $\tilde{g}$.

The vector boson vacuum polarization contributions (4) and (5) will affect the ρ -parameter defined by the ratio of the neutral to charged current strengths. In tree approximation (within the present framework as well as within the $SU(2)_L \times U(1)_Y$ gauge theory with Higgs doublet) we have

$$\rho = \frac{G_{NC}}{G_\mu} = 1, \quad (6)$$

where G_μ denotes the Fermi coupling and G_{NC} its neutral current equivalent. By explicit calculation one convinces oneself that vertex correction contributions do not arise in order Λ^4 . The correction to the tree level result $\delta\rho = \rho - 1$ is thus given by the difference¹⁰⁾ of (4) and (5)

$$\begin{aligned} \delta\rho = & \frac{\Pi_{ZZ}(0)}{M_Z^2} - \frac{\Pi_{WW}(0)}{M_W^2} = \\ = & \frac{3}{4 \cdot 32\pi^2} \frac{\sin^2 \theta_W}{\cos^2 \theta_W} (\hat{g} - \frac{e}{\sin \theta_W} \kappa)^2 \frac{\Lambda^4}{M_W^4} + O(\Lambda^2) = \\ = & 6.89 \cdot 10^{-4} (\hat{g} - \frac{e}{\sin \theta_W} \kappa)^2 \frac{\Lambda^4}{M_W^4} + O(\Lambda^2). \end{aligned} \quad (7)$$

The dependence on the quadrilinear couplings has dropped out.

From measurements of the neutral to charged current ratio in ν -scattering we know¹¹⁾ that $\rho = 1$ with a conservative error of $\delta\rho = \pm 0.05$. Assuming that Λ , the onset of possible new phenomena (e.g. due to compositeness), is not much smaller than $\Lambda \approx 1$ TeV, according to (7) we have to require that

$$\hat{g} \sin \theta_W = e\kappa \quad (8)$$

be fulfilled in very good approximation. The empirical value of ρ thus implies

a relationship, (8), between κ and $\hat{g}$, but it does not constrain these couplings individually. Any value of $\hat{g}$ and κ is allowed^{F2)} provided (8) is fulfilled. We remark that the stronger requirement of individual vanishing of Π_{ZZ} and Π_{WW} in order Λ^4 does not lead to a restriction on $\hat{g}$ and κ stronger than given by (8). From (4) one obtains an additional relation between $\hat{g}$ and the quadrilinear couplings, $\hat{g}$ and $\tilde{g}$,

$$\tilde{g} = \frac{3}{7}(\hat{g}^2 - \hat{g}), \quad (9)$$

which again leads to (8) when inserted in (5).

We now turn to a brief discussion of the physical meaning of relation (8):

(i) First of all, according to (8), any deviation in κ from $\kappa = 1$ must be accompanied by an appropriate deviation of $\hat{g}$ from its $SU(2)_L \times U(1)_Y$ value, $\hat{g} = e/\sin\theta_W$, if $\rho \approx 1$ is to hold.

(ii) Interpreted in terms of the underlying Lagrangian⁶⁾ before diagonalization of the γW^3 mixing term, requirement (8) with $\kappa \neq 0$ means that W-dominance must hold for the electromagnetic form factor of the $W^\pm$ associated with its anomalous magnetic dipole moment, while W-dominance for the charge form factor need not necessarily be valid in order to protect $\rho \approx 1$. If W-dominance is required to hold for the anomalous magnetic moment as well as the charge form factor, $\kappa = 1$ is obtained, as (8) as well as $\hat{g}\sin\theta_W = e$ has to be fulfilled. This is nothing else but the well-known result that global $SU(2)_{WI}$ broken by the photon in such a manner that W-dominance holds for leptons, quarks as well as the charged vector bosons, implies^{8,9)} gauge theory couplings.

(iii) A special situation occurs if $\kappa = 0$ and, consequently, $\hat{g} = 0$. The underlying Lagrangian for this case is obtained⁶⁾, if the electromagnetic interaction of the $W^\pm$ is introduced solely by applying the minimal substitution principle to the $W^\pm$ kinetic energy term in the Lagrangian (without adding an additional trilinear vector boson interaction, as $\hat{g} = 0$). The full $Z_0 W^+ W^-$ interaction then originates from γW^3 mixing and g_{ZWW} according to (2) is given by $g_{ZWW} = -e \tan\theta_W$. Interpreted in terms of the underlying Lagrangian before diagonalization of the γW^3 mixing term, the case $\kappa = 0$ ($\hat{g} = 0$) corresponds to $\gamma W^\pm$ interactions which are pointlike compared with the scale $1/M_W$, which is relevant for photon lepton (quark) interactions. As we will see below, this case of $\kappa = \hat{g} = 0$ may be well discriminated from the case of $\kappa = 1$ and $\hat{g} = e/\sin\theta_W$ by measuring $e^+e^- \rightarrow W^+W^-$.

^{F2)} The additional Λ^2 contributions present in (7) will eventually limit possible values of $\hat{g}$ and κ . A priori these are suppressed relative to the Λ^4 contributions by a factor M_W^2/Λ^2 .

It is instructive to estimate the accuracy with which (8) has to be fulfilled in order to avoid large deviations from $\rho = 1$. Assuming $\Lambda \approx 10 M_W \approx 0.8 \text{ TeV}$ and $\delta\rho \leq 0.05$ for this purpose, we obtain

$$|\kappa - \hat{g} \frac{\sin\theta_W}{e}| = \left(\frac{4 \cdot 32\pi^2 \cos^2\theta_W}{3e^2} \delta\rho \right)^{1/2} \frac{M_W^2}{\Lambda^2} \leq 0.13, \quad (10)$$

i.e. (8) has to be fulfilled fairly accurately (using, e.g., $\kappa = 1$ as a scale).

For comparison with a bound on κ already given in the literature^{12,13)} we evaluate (10) for the particular case of $\hat{g} = e/\sin\theta_W$ corresponding to the $SU(2)_L \times U(1)_Y$ value of $\hat{g}$. For $\hat{g} = e/\sin\theta_W$, (10) becomes

$$|\kappa - 1| \leq 0.13. \quad (11)$$

This bound is approximately one order of magnitude weaker than a bound obtained by Suzuki¹²⁾, which in contrast to (10) is of fourth order in M_W/Λ and given by^{F3)}

$$|\kappa - 1| \leq \frac{2 \cdot 32\pi^2}{3e^2} \delta\rho \frac{M_W^4}{\Lambda^4} \leq 0.012. \quad (12)$$

When numerically evaluating (12) the same input has been used as in (10) and (11). The difference between (11) and (12) is due to the constraint (3) on κ_Z which enters the derivation of (7) and thus (11). It implies $\kappa_Z \neq 1$ as soon as $\kappa \neq 1$. The stronger bound (12) follows from the assumption that κ is the only parameter allowed to differ from its $SU(2)_L \times U(1)_Y$ value, i.e., in particular one assumes $\kappa_Z = 1$ for arbitrary $\kappa \neq 1$. There is no reason, however, to restrict oneself to this specific deviation from the standard model. Even more so, this hypothesis means that one allows only for such deviations from the standard model which also intrinsically violate global $SU(2)_{WI}$ (as (3) is violated).

We now turn to the analysis of the process $e^+e^- \rightarrow W^+W^-$. Assuming that the scale Λ where new physical phenomena appear is sufficiently large ($\Lambda \approx 1 \text{ TeV}$), relation (8) has to be fulfilled in good approximation. Let us remark that this hypothesis on Λ can in principle at least be tested at future colliders, such as the projected SSC. The amplitude for the process $e^+e^- \rightarrow W^+W^-$ is then dependent on the single parameter κ only, which determines the other trilinear couplings according to

$$g_{ZWW}(\kappa) = e \frac{\kappa - \sin^2\theta_W}{\sin\theta_W \cos\theta_W}, \quad (13)$$

and

F3) The inequality (12) differs by an (unimportant) factor of 3/2 from Suzuki's result.

$$\kappa_Z(\kappa) = \frac{\kappa \cos^2 \theta_W}{\kappa - \sin^2 \theta_W}, \quad (14)$$

where (8) has been substituted in (2) and (3). (For $\kappa = \sin^2 \theta_W$, the coupling g_{ZWW} vanishes and we are left with a weak magnetic coupling of strength $g_{ZWW} \kappa_Z = e \kappa \tan \theta_W$). The sensitivity of cross section and angular distribution of $e^+e^- \rightarrow W^+W^-$ under variation of κ , provided (13) and (14) are valid, will be discussed next.

In fig. 2 we show^{F4)} the total cross section as a function of the e^+e^- energy for different values of κ including the $SU(2)_L \times U(1)_Y$ prediction. In particular the case of $\kappa = \hat{g} = 0$ can be well discriminated from the standard model, while it will be difficult to rule out values of κ near $\kappa = 1$ from measurements of the total cross section at LEP 200, provided such measurements are consistent with standard model predictions within errors. As seen in figure 2, deviations of κ from $\kappa = 1$ can lead to strong deviations in the total cross section for $e^+e^- \rightarrow W^+W^-$ from the standard model predictions, even though consistency with the measured value of $\rho \approx 1$ is assured via (8), (13) and (14) (in so far as the most dangerous contribution of order Λ^4 is vanishing). The forward backward asymmetry, A_{FB} , shown in fig. 3 may be used to discriminate between $\kappa < 1$ and $\kappa > 1$.

Finally, fig. 4 addresses the question of how well one can determine $\hat{g}$ and κ provided measurements show agreement with standard model predictions within 5 % in the cross section as well as the forward backward asymmetry. We first of all show the contour in the $\kappa, \hat{g}$ plane which is obtained⁷⁾, if both parameters are allowed to vary freely and future experiments agree with the $SU(2)_L \times U(1)_Y$ predictions within 5 %. Imposing relation (8), due to protecting $\rho \approx 1$, one notices that one can strongly restrict the allowed domain in the $\hat{g}, \kappa$ plane. The accuracy in determining $\hat{g}$ and κ becomes of the order of 10 % to 20 %.

We end with the following summarizing conclusions:

(i) No stringent limit on the anomalous magnetic dipole moment, κ , of the $W^\pm$ can be deduced from the agreement of the measured value of the ρ -parameter, $\rho \approx 1$, with the $SU(2)_L \times U(1)_Y$ prediction. Deviations in κ from its $SU(2)_L \times U(1)_Y$ value, $\kappa = 1$, can always be compensated in the ρ -parameter (at least in leading, Λ^4 order) by appropriate changes in $\hat{g}$ (or equivalently g_{ZWW}).

(ii) The relation between $\hat{g}$ and κ which is necessary and sufficient to protect $\rho \approx 1$ (in order Λ^4) is given by (8), $\hat{g} \sin \theta_W = e \kappa$. In terms of the underlying γW^3 mixing Lagrangian (for $\kappa \neq 0$) it means nothing else but the requirement of W -dominance for the anomalous magnetic dipole form factor of the $W^\pm$.

F4) We are indebted to F.M. Renard for the numerical evaluation which led to figures 2 to 4 during one of the authors' (D.S.) stay at the U.S.T.L. in Montpellier.

(iii) Requiring (8) reduces the number of free trilinear self-couplings of the vector bosons within an electroweak theory based on global $SU(2)_{WI}$ symmetry broken by electromagnetism to a single one, κ . This single parameter, κ , can then be fairly accurately determined by future measurements of $e^+e^- \rightarrow W^+W^-$ at LEP 200, thus resolving a previously noted ambiguity in the $\hat{g}, \kappa$ plane in the interpretation of such future measurements.

(iv) Let us finally note that relation (8) also assures¹⁴⁾ absence of the most strongly increasing and unitarity violating contributions (proportional to s^2) when evaluating vector boson scattering ($WW \rightarrow WW$, $ZZ \rightarrow WW$, $\gamma\gamma \rightarrow WW$, $\gamma Z \rightarrow WW$) at the tree level.

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Figure Captions

- Fig. 1: Vector boson loop contributions to the Z_0 and $W^\pm$ propagators. The dependence on the trilinear ($\hat{g}$, κ) and quadrilinear ($\hat{g}$, $\tilde{g}$) couplings is indicated below the corresponding diagrams.
- Fig. 2: The total cross section for $e^+e^- \rightarrow W^+W^-$ as a function of the center of mass energy $\sqrt{s}$. The only free parameter is κ , as g_{ZWW} and κ_Z are functions of κ given by (13) and (14).
- Fig. 3: The forward backward asymmetry as a function of the center of mass energy, $\sqrt{s}$, for different values of κ .
- Fig. 4: Restrictions on $\hat{g}$ and κ from measurements of $e^+e^- \rightarrow W^+W^-$ for $\hat{g}$ and κ treated as free parameters, as well as under the restriction (8) which yields the straight line $\kappa = \hat{g}\sin\theta_W/e$.

- Figure 1 -

- Figure 2 -

- Figure 3 -

- Figure 4 -