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COMMENTS ON THE POSSIBILITY OF ELECTROWEAK BARYON NUMBER VIOLATION
AT HIGH TEMPERATURES

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A B S T R A C T

Anomalous baryon number violation in the standard electroweak theory was first discussed by 't Hooft, who found it to be suppressed by a large factor $\exp(-8\pi^2/g^2)$ at zero temperature, due to a large energy barrier separating vacua with different baryon number. One might have expected that in the early Universe or in high energy collisions, this process would become unsuppressed when the energies involved became comparable to the barrier height. We argue here that in both cases processes violating baryon number are likely to remain suppressed by at least the same zero-temperature factor.

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Ever since the discovery by 't Hooft¹⁾ that anomalous baryon and lepton number violating interactions must occur in the standard electroweak theory, various authors²⁾⁻⁸⁾ have raised the possibility that such processes could be important at finite temperature, or in high energy collisions. The multiple-fermion effective interaction violating baryon and lepton number in the presence of instantons was calculated by 't Hooft¹⁾, who found it to be suppressed by the familiar factor of $\exp(-8\pi^2/g^2)$. Instantons are solutions of the classical Euclidean equations of motion which correspond to tunnelling WKB solutions in Minkowski space⁹⁾. The Euclidean action $S_E = 8\pi^2/g^2$ corresponds to the action for tunnelling through a barrier of height $U = (3\pi^2/g^2)1/\rho$ in winding number space, where ρ is the instanton size. As was shown by 't Hooft¹⁾, in the standard electroweak theory, the vacuum expectation value of the Higgs scalar doublet $\langle 0|H|0\rangle = (1/\sqrt{2})(0,v)$ provides an infra-red cut-off for large instantons, with those of size $\rho \gtrsim 1/v$ being exponentially suppressed. Therefore one would expect the minimum barrier height to be $U \sim (3\pi^2/g^2)v$.

This energy is of the same order as the mass m_{sph} of the sphaleron⁶⁾, which is a static saddle point solution to the classical equations of motion of the electroweak theory, first discussed by Manton¹⁰⁾ in the context of an $SU(2)$ gauge group, and subsequently by Klinkamer and Manton⁶⁾ in the context of $SU(2)_L \times U(1)_Y$. The properties of the sphaleron do not seem to be essentially altered by the inclusion of the $U(1)_Y$ factor with a (relatively) small mixing angle θ_W ⁶⁾, so we neglect it as an inessential complication in what follows. Since the minimum barrier height for $\Delta B = \Delta L = 1$ processes is then $U = O(m_{\text{sph}})$, one might have expected that at sufficiently high temperatures in the early Universe, $T \sim m_{\text{sph}}$, or in high energy collisions at centre-of-mass energies $E_{\text{cm}} \sim m_{\text{sph}}$, one would have enough energy to overcome the barrier without tunnelling. The expectation that one would thereby avoid any exponential damping factor has been expressed in several papers²⁾⁻⁸⁾. The purpose of this article is to present some arguments which indicate that there are problems with this naïve expectation.

We discuss below the rates for anomalous $\Delta B = \Delta L = 0$ processes and suggest that they are always suppressed by an exponential factor at least as large as that for zero-temperature instantons. The zero-temperature sphaleron solution is significantly modified at finite temperatures $T = O(m_W)$. Gross, Pisarski and Yaffe¹¹⁾ have shown that large instantons $\rho \gtrsim 1/T$ are cut off at finite temperature because the electric gauge field acquires a mass $m_E \sim gT$. Since

$\Delta B = \Delta L \neq 0$ processes necessarily involve both electric and magnetic fields, we expect a similar suppression at high temperatures for any topologically non-trivial solution whose $t = 0$ slice is a sphaleron. Hereafter we call any such configuration a "sphaleron". At intermediate temperatures we believe that the action for configurations inducing $\Delta B = \Delta L \neq 0$ transitions is always bounded below by the instanton action. We argue also that estimates of $\Delta B = \Delta L \neq 0$ cross-sections in high energy collisions have also omitted an $\exp(-8\pi^2/g^2)$ factor which expresses the difficulty of making an extended, coherent non-perturbative gauge field configuration out of a high energy perturbative configuration.

Quite generally, non-perturbative violation of baryon number conservation in the electroweak theory requires some flow in the spectrum of fermionic energy levels¹²⁾, and this in turn requires gauge field configurations with non-zero Pontryagin number ν . The status of such configurations in a gauge theory which is at zero temperature and not spontaneously broken is relatively clear [Ref. 11] and references therein]. The action in each $|\Delta\nu| = 1, 2, \dots$ sector is minimized by (multi-) instanton solutions with action $A = 8\pi^2/g^2(\rho)$, where ρ is the size of the instanton and $g(\rho)$ is the running gauge coupling. The case which interests us is different in two respects: (i) the electroweak gauge theory is spontaneously broken at zero temperature, and (ii) we want to know what happens in the Universe at finite temperatures, both above and below the electroweak critical temperature T_c . The situation when $T > T_c$ is again relatively clear: since the Higgs order parameter vanishes in this case, the non-perturbative field configurations correspond to those of an unbroken theory, which have been classified by Gross, Pisarski and Yaffe¹¹⁾. The only finite-action solutions of $SU(N)$ with $|\Delta\nu| \neq 0$ that they find are finite-temperature instantons whose contribution to the free energy is suppressed by the exponential of the modified action

$$\exp(-A(T)) : A(T) = \frac{8\pi^2}{g^2(\rho)} + \frac{1}{3}(\pi\rho T)^2(2N+N_f) + \dots \quad (1)$$

where N_f is the number of fermions in fundamental representations of $SU(N)$. The second term in (1) is due to the non-zero electric mass $m_{el} = gT\sqrt{2N+N_f}/6$. It has the effect of cutting off the instanton size parameter ρ at a size of order $1/T$:

$$\rho' = \frac{\rho}{\sqrt{1 + \frac{1}{3}(\pi^2\rho^2 T^2)}} \quad (2)$$

is the size of the zero-temperature instanton whose superposition best fits the exact finite-temperature instanton solution of size ρ . The maximum value of ρ' (2) is

$$\rho'_{\max} = \frac{\sqrt{3}}{\pi T_c} \quad (3)$$

at the critical temperature T_c .

The situation at $T < T_c$ is less clear, because no complete configuration of the spontaneously broken gauge-Higgs system is known which has $|\Delta v| = 1$ and interpolates between different vacua at times $t = -\infty$ and $t = +\infty$. What is known is a static solution of the $SU(2)_L$ [or $SU(2)_L \times U(1)_Y$] gauge-Higgs system at zero temperature, called the sphaleron. Considered as a static solution, the sphaleron is not topologically stable. However, static solutions do not cause spectral flow and hence cannot lead to $\Delta B \neq 0$ interactions. Our interest in the sphaleron centres on its possible rôle as an (approximate) $t = 0$ slice of a minimum action gauge-Higgs field configuration defined on the full range $-\infty < t < \infty$ which has $|\Delta v| = 1$ and interpolates between different vacua as in Fig. 1a, causing spectral flow and hence $\Delta B \neq 0$. No complete configurations with these properties (which we refer to as "sphalerons") are known, but they are presumed to exist. Just as the existence of the instanton reflects the fact that $\pi_3(SU(2)) = \mathbb{Z}$, i.e., there is a winding number for mappings of S_3 into $SU(2)$, there is expected to be a corresponding integer winding number for mappings of $S_1 \times S_2$ into $SU(2)$, and hence also some associated gauge-Higgs configurations, though this expectation has not been made rigorous for $\theta_W \neq 0$ or $T \neq 0$. We are not interested in configurations like that in Fig. 1b, which may involve the sphaleron but have $\Delta v = 0$ and hence $\Delta B = 0$. The energy of the sphaleron is⁶⁾

$$E = \frac{B m_W}{\alpha} \quad ; \quad B = O(3) \quad (4)$$

and the classical solution is expressed in terms of functions of the dimensionless variable gvr , giving it a classical radius $\langle r \rangle \sim 1/gv$. One can presumably take over (4) as a reasonable estimate of the free energy of the "sphaleron" configuration at low temperatures $T \ll gv$, and accept that at such low temperatures $\Delta B \neq 0$ processes mediated by "sphalerons" are suppressed by a typical Boltzmann factor

$$\exp(-A_S) : A_S = \frac{B m_W(T)}{T \cdot \alpha} : m_W(T) = \frac{g}{2} v(T) = \frac{g v}{2} \sqrt{1 - \frac{T^2}{T_c^2}} \quad (5)$$

where the last equality follows from the assumption of a second-order phase transition. The key problem is now to interpolate between formula (1) which is valid when $T > T_c$, and formula (5) which is valid when $T \ll T_c$. Is there any

region in between where there is no exponential suppression of $|\Delta v| = 1$ and hence $\Delta B \neq 0$ processes?

It is difficult to be sure what happens in the intermediate region of T , because no explicit solutions are known and our discussion must therefore be heuristic. Figure 2 shows the suppression factors as functions of the temperature for various $|\Delta v| = 1$ configurations, including a small instanton (the curve labelled A_0), an instanton of size $\rho \sim 1/gv$ (the A_v curve) and the estimate of formula (9) for the "sphaleron" (labelled A_s). We notice that A_s and A_0 become equal when the temperature is

$$T_s \sim \frac{gv}{2} = m_W \quad (6)$$

However, it is doubtful whether the "sphaleron" curve ever falls below A_0 when finite temperature effects are treated correctly, for the following reasons.

(1) The instanton solutions are well-known to provide lower bounds on the possible actions of zero-temperature gauge configurations with $|\Delta v| = 1$, and the same argument can be made at finite temperature. The additional terms in the action associated with Higgs fields, namely

$$\int d^4x \left[|D\phi|^2 + \lambda^2 \left(|\phi|^2 - \frac{v^2}{2} \right)^2 \right] \quad (7)$$

are positive semi-definite. Therefore the argument that the instanton action is a lower bound on the action of any configuration with $|\Delta v| = 1$ still applies. There is a caveat, namely that this argument is based on the imaginary time formalism, which is applicable if the gauge and Higgs fields are in thermal equilibrium.

(2) As was already mentioned, the $T = 0$ sphaleron solution depends on the dimensionless quantity gvr . We expect that the radial dependence becomes $f(gvr, Tr)$ at finite T , and that the corrections of $O(Tr)$ become $O(1)$ when $T = O(gv) = O(T_s)$. This could well change $A_s(T)$ from the naïve formula (5). In particular, we might expect the electric mass correction to the instanton action (1) to have a counterpart for "sphalerons" at finite temperature. Since the sphaleron size $\langle r \rangle \sim (gv)^{-1}$, we think it is reasonable to replace ρ in Eq. (1) by $1/gv$, which would give an explicit correction to the "sphaleron" action which is of order T^2/g^2v^2 . This probable correction even suggests that the zero-temperature sphaleron might extrapolate to the small instanton curve A_v in Fig. 2.

(3) Topologically non-trivial $|\Delta v| = 1$ solutions to the zero-temperature electroweak field equations presumably exist and look like instantons as long as their size parameters ρ are much smaller than the electroweak scale, as measured by v or gv . Since the correlation length of the electroweak field $\lambda \sim (gv)^{-1}$, presumably $\rho \times gv \ll 1$ is the appropriate condition for a $|\Delta v| = 1$ solution to resemble an instanton, and some size $\rho \sim 1/gv$ is a good estimate of the maximum size an electroweak instanton could have. Since the sphaleron has a radius $\langle r \rangle \sim 1/gv$, it therefore seems reasonable that the "sphaleron" should extrapolate to the A_v curve in Fig. 2.

(4) As was already mentioned in connection with Eq. (1), Gross, Pisarski and Yaffe¹¹⁾ have argued that the only $|\Delta v| \neq 0$ solutions to the pure gauge equations at finite temperature are instantons, and that their sizes $\rho \lesssim 1/T$. Figure 3a is a pictorial representation of the effect on possible instanton sizes of increasing T . For $T \neq 0$, the Euclidean field theory is formulated using periodic boundary conditions in the time direction with period $\beta = 1/T$, and the instanton solution is made periodic by introducing "mirror" instantons at $\Delta t = \pm n\beta$. As T increases the allowed instanton, $\rho < \beta$, get smaller and smaller, in spatial extent as well as in the time direction. What happens to the allowed "sphaleron" solutions at finite T ? If we represent a "sphaleron" configuration by a contour of energy density, it has a cylindrical symmetry which looks like Fig. 3b at zero temperature. As long as β is much longer than the length ℓ of the sausage-like cylinder, the "sphaleron" is not much changed. But if $\beta \ll \ell$, one of two things may happen. Either the sausage is sliced as in Fig. 3c, or it is compressed in the time direction, as in Fig. 3d. (Remember that all finite-temperature boson configurations must be periodic, and have $|\Delta v| = 0, 1, 2, \dots$.) In the sausage slice case of Fig. 3c, the configuration becomes completely static, has $\Delta v = 0$, induces no spectral flow and hence no baryon number violation. Indeed, this configuration is topologically trivial and is (probably) an unimportant high action configuration in the trivial vacuum sector. In the compressed case of Fig. 3d, the "sphaleron" configuration resembles the instantons of Fig. 3a. Indeed, since $\langle r \rangle \lesssim \ell < 1/\beta$, the sphaleron looks more and more like a small instanton as T increases.

The conclusion of this discussion is that even at finite T the "sphaleron" action A_s is bounded below by the instanton action A_0 . Since the "sphaleron" configuration required for a $\Delta B \neq 0$ transition must be coherent over a large ($\gtrsim 1/T$) spatial and temporal extent, the exponential suppression may be seen as reflecting the large cost in entropy that such configurations demand. Therefore, non-perturbative electroweak $\Delta B \neq 0$ processes are negligible throughout the

expansion of the Universe. This means a fortiori that baryogenesis by non-perturbative electroweak effects¹³⁾ is not feasible.

We would also like to add a few comments on the possibility of non-perturbative electroweak baryon number violation in high energy collisions. The rate for this has been estimated recently⁸⁾ by first guessing the probability that a colliding $q\bar{q}$ pair with centre-of-mass energy $O(m_{sph})$ will radiate $O(55)$ weak vector bosons, and then assuming that these vector bosons have a probability which is $O(1)$ to produce a sphaleron configuration, which then has a probability of $O(1)$ to decay with $\Delta B \neq 0$. In our view, the weak point of this approach is the assumption that a large number of perturbative particles have a significant probability to produce a non-trivial non-perturbative field configuration. The correct way to estimate $\Delta B \neq 0$ processes at low energies is presumably to start from 't Hooft's effective interaction which has the form^{1),5)}

$$\mathcal{L}_{eff} \propto \rho^{14} q^9 l^3 \exp(-A_0) \quad (8)$$

where we have included a factor of the exponential of the minimal action in the $|\Delta v| = 1$ sector, and used dimensional analysis to fix the appropriate power of the instanton size. The effective interaction (8) gives a cross-section

$$\sigma(E) \sim \rho^{28} E^{26} \exp(-2A_0) \quad (9)$$

when $E \lesssim 1/\rho$, and the maximum of the cross-section is

$$\sigma_{max} \sim \rho^2 \exp(-2A_0) \quad (10)$$

when $E \sim 1/\rho$. Above this energy one expects a form factor $F(\rho^2 E^2)$ to cut off the rise in the cross-section. The largest possible size for an electroweak instanton size is $\rho \sim 1/gv \sim 1/m_W$, so the largest possible $\Delta B \neq 0$ cross-section for instanton-mediated processes is

$$\sigma_{max} \sim \frac{1}{m_W^2} \exp(-2A_0) \quad (11)$$

The exponential damping factor in (11) was missing from the cross-section estimates of Ref. 8). We believe that it is a correct bound on the suppression factor which is inevitable when one makes any non-perturbative gauge field configuration with $|\Delta v| = 1$. Since the instanton action is a lower bound on the action for any field configuration with $|\Delta v| = 1$, we also believe that (11) is an upper bound on the $\Delta B \neq 0$ cross-sections mediated by sphalerons. Going to higher energies

$E \gg m_W$ does not help because the wavelength $\lambda \sim 1/E$ of the incoming particles becomes much smaller than the radius $\langle r \rangle \sim 1/gv$ of the sphaleron, and there is little probability of producing the necessary non-perturbative coherence of the bosonic fields over a distance $\langle r \rangle \gg \lambda$. Thus, non-perturbative electroweak baryon number violation is likely to be unobservably small at high energies as well as at high temperatures.

It is worthwhile pausing for a moment, before concluding, to discuss briefly how the situation discussed here differs from the monopole catalysis problem¹⁴⁾. Monopoles can affect the Fermi vacuum over surprisingly large regions¹⁴⁾ of size m_f^{-1} if the theory contains light fermions of mass m_f , thus inducing baryon number violating processes with surprisingly large cross-sections. There, however, the monopole is an asymptotic state. If one calculates a $\Delta B \neq 0$ scattering matrix with only fermions as external lines, on the other hand, one will not obtain such large cross-sections. Sphalerons, unlike monopoles, are classically unstable objects⁶⁾, and therefore they cannot appear as asymptotic states. Moreover, even if one could attach meaning to the concept of an S-matrix element with external asymptotic sphaleron states, and if some low-lying excitation of energy $\ll m_{\text{sph}}$ were available to violate baryon number, it would not be relevant when calculating the effective interaction with only fermions as external particles.

Our conclusion is that non-perturbative electroweak baryon number violation is likely to be unobservably small at high energies as well as at high temperatures. Our arguments cannot be rigorous in the absence of explicit solutions for finite-temperature, time-dependent gauge-Higgs field configurations with spectral flow. However, we believe that we have raised important issues that are relevant to the problem, and that the expectations expressed elsewhere about these processes do not seem to accord with what is known at $T \ll T_c$ and $T > T_c$. The arguments given so far²⁾⁻⁸⁾ in favour of large rates for $\Delta B \neq 0$ processes in the electroweak theory are, in our opinion, not necessarily correct. If electroweak baryon number violation does occur at high rates, it is doubtful that it is for the reasons given so far²⁾⁻⁸⁾.

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FIGURE CAPTIONS

Fig. 1 Spectral flow in winding number space. The bell-shaped curve is the $T = 0$ energy barrier between vacua with winding numbers $v = 0$ and $v = 1$ ⁹⁾. The sphaleron is the classical solution to the electroweak equations of motion with energy equal to the height of the barrier and $v = \frac{1}{2}$ ⁶⁾. Two cases are considered. (a) Spectral flow: the curved arrow represents a gauge-Higgs field configuration that starts out in the $v = 0$ vacuum at time $t = -\infty$, goes through the sphaleron configuration at $t = 0$ and ends up in the $v = 1$ vacuum at $t = +\infty$. (b) No spectral flow: the gauge-Higgs field configuration begins and ends in the $v = 0$ vacuum, going through the sphaleron at $t = 0$.

Fig. 2 Actions A of various $|\Delta v| = 1$ configurations, as functions of the temperature T . A_0 is the finite temperature action of a small $|\Delta v| = 1$ instanton of size $\rho = (gv)^{-1}$, whilst A_s is the estimate of Eq. (5) for the "sphaleron" action, which falls to A_0 at $T = T_s \sim M_W$. The dotted line shows the continuation of the solid line all the way to T_c according to Eq. (5) and the dashed lines show the extrapolations argued in the text.

Fig. 3 Energy-density contours representing field configurations in Euclidean space-time. The horizontal axis is $|\vec{x}|$ and the vertical axis is imaginary time. (a) Periodic, multi- ("mirror") instanton solution of maximum size $\rho \lesssim 1/T$ as a function of increasing temperature. (b) Energy-density contour representation of the "sphaleron" configuration at $T = 0$. (c) and (d); possible high temperature behaviour of the "sphaleron" configuration, as described in the text.

Figure 1

Figure 2

Figure 3