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INTERMITTENCY MODEL FOR RAPIDITY PARTICLE DENSITY FLUCTUATIONS

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A B S T R A C T

The one-dimensional model of intermittency is applied to particle density fluctuations in rapidity. Intermittency may explain the "anomalous" events with large density of particles in small rapidity windows and the, approximately energy-independent, linear relation between the average maximum rapidity density and the multiplicity. The connection of "hot-spots" in rapidity with large transverse energy jets and the formation of the quark-gluon plasma is briefly discussed.

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Events in multiparticle production with large concentration of particles in small windows in rapidity have been known for a long time^{1),2)}. Recently, the interest in large fluctuations in rapidity was revived by the discovery of more events of that type^{3),4)}. In general, the concentration in rapidity (or pseudorapidity) is associated with a random azimuthal distribution, and thus the events do not show jet-like features. On the other hand, normal cluster production models, without complicated correlations, are not expected to be able to generate, with a non-negligible probability, events like the "anomalous" event of the NA22 collaboration⁵⁾. One may then think that the observed large rapidity particle density fluctuations reflect non-trivial fluctuations in hadronic matter density during hadronic collisions.

There is a mechanism, known in turbulence and dynamical system theory as intermittency, that is able to generate large fluctuations. In this note, I would like to discuss rapidity particle density fluctuations in the framework of the one dimension mathematical picture of intermittency⁶⁾⁻⁸⁾. The aim is to obtain large particle density fluctuations within one event and to obtain reasonable fixed multiplicity rapidity density distributions. The connection with conventional models will be considered at the end. The suggestion of using intermittency to explain density fluctuations in multiparticle production was previously made in Refs. 9) and 5).

Intermittency is characterized by the presence of two regimes: the laminar regime with observation of regular localized signals and the turbulent regime with the registration of dispersed chaotic bursts. A rigorous theory of intermittency can be established by studying iterative maps, $x_n = f(x_{n-1})$, in the approach of a tangent bifurcation. In Fig. 1, an example is shown: x_n remains for $v = 7$ iterations in the intermittency region, defined by using a gate Δ around the tangent bifurcation point b , then leaves the region to come back later, etc. Points entering the gate to the left of b go through the "narrow channel", remain for many iterations in the region, and correspond to the laminar contribution. Points entering the gate to the right of b go only through a "wide channel", remain for a few iterations in the region and correspond to the bursts. A parameter h measures the distance of the curve to the diagonal. For a given h there is a maximum number of possible steps, v_M , given by the iteration of the critical point c . As h decreases, the number of iterations in the "narrow channel" and v_M increases, while the number of iterations in the "wide channel" is not much affected.

One now arrives to the point of making the connection between the intermittency theory and rapidity density fluctuations. During a collision, partons travel through interacting hadronic matter, find regions of high matter density ("narrow channels") and emit many particles and find also regions of low matter density "wide channel" and emit few particles. The schematic situation is shown in Fig. 2. The number n of particles in one event depends on the overall average density of the region visited by partons (depends on the parameter h in the case of maps). The following identifications seem then natural:

- a) the number v of steps within a gate Δ should be identified with the number v of particles falling in a rapidity window Δ ;
- b) the parameter h^{-1} , controlling matter density or the narrowness of the channels, should be identified with the number n of produced particles in a given event.

I state now the main theoretical intermittency results of interest to our discussion⁸⁾:

- i) for a given multiplicity n , the probability distribution $P(n, v)$ of the number v of particles falling inside a rapidity window Δ , $v = 1, 2, \dots$, can be written as a sum of two terms: one corresponding to the small v burst contribution $P_B(n, v)$ and the other corresponding to the large v laminar contribution $P_L(n, v)$,

$$P(n, v) = P_B(n, v) + P_L(n, v) \quad (1)$$

- ii) the burst distribution is asymptotically independent of n :

$$P_B(n, v) \xrightarrow{n \text{ large}} \phi(v) \quad (2)$$

with $\phi(v)$ exponentially decreasing with v , at small v , $v \gtrsim 1$;

- iii) the laminar distribution has a scaling behaviour which can be approximately written in the form

$$P_L(n, v) \xrightarrow{n \text{ large}} \frac{1}{v_M} \psi(v/v_M) \quad (3)$$

with ψ increasing and being effectively cut-off at large $v \sim v_M$;

iv) the maximal density of particles v_M is proportional to n ,

$$v_M = \frac{1}{\kappa} n \quad (4)$$

with κ being a constant;

v) v_M is gate dependent,

$$v_M \sim \Delta^\gamma, \quad (5)$$

with $0 \leq \gamma \leq 1$ and γ changing rapidly from 1 to 0 at some critical gate size¹⁰⁾;

vi) the functions ϕ and ψ qualitatively are not strongly dependent on the gate size Δ .

These statements, i) to vi), define the intermittency model for particle density distributions in rapidity.

The role of energy in the present picture still has to be specified. Density variations in hadronic matter density originate variations in n , the number of produced particles. Overall rapidity and multiplicity distributions are obtained by summing fixed n distributions weighted by the cross-section ratios $p_n \equiv \sigma_n / \Sigma \sigma_n$. When the energy changes the $\{p_n\}$ change. As the energy increases, the probability of reaching denser regions of hadronic matter (large multiplicities) increases. Overall distributions are then energy-dependent. However, the event by event physics is not determined by the energy of the reaction. In other words, statements i) to vi) above are, at first approximation at least, independent of the energy.

Before a qualitative comparison with experiment, one should notice that what is used in experiments is not the distribution $P(v)$, averaged or not over n , but the distribution $\tilde{P}(v)$ of the maximal density of particles per event. The two distributions are related by

$$\tilde{P}(v) \sim P(v) \left(1 - \int_v^{v_M} P(v') dv' \right) \quad (6)$$

As $P(v)$ is exponentially dominated by the small v region, I shall make the approximation in a qualitative discussion:

$$\tilde{P}(\nu) \sim P(\nu) . \quad (7)$$

Clearly (7) has to be violated at small $\nu \approx 1, 2$ with $\tilde{P}(\nu)$ being flatter than $P(\nu)$, but is a reasonable approximation for larger ν .

I state next the consequences of the model, i) to iv) above, for particle density distributions:

i') because of i, ii) and iii) , the particle density distribution may have two peaks, one at small ν and the other at large ν , $\nu \approx \nu_M$. This may explain why one multiplicity $n = 26$ event with 10 particles in a rapidity window $\Delta = 0.1$ was observed in the NA22 experiment and none was observed with 8 or 9;

ii') because of i), iii), ii) and iv), the average particle density is a linear function of the multiplicity n :

$$\begin{aligned} \langle \nu \rangle_n &= \int \nu P_B(n, \nu) d\nu + \int \nu P_L(n, \nu) d\nu \\ &= \int \nu \phi(\nu) d\nu + \nu_M \int \frac{\nu}{\nu_M} \psi\left(\frac{\nu}{\nu_M}\right) d\left(\frac{\nu}{\nu_M}\right) \end{aligned}$$

and

$$\langle \nu \rangle_n = A + B n , \quad (8)$$

with A and B , for a given gate, independent of the energy. Equation (8) is well satisfied experimentally, with A and B fairly independent of energy in the range $22 < \sqrt{s} < 900 \text{ GeV}^{3), 4)}$, $A \approx 0.70$ and $B \approx 2.2$ for $\Delta = 0.5$;

iii') as the main change that occurs, when the rapidity window Δ is changed, is in ν_M , ν) and vi) above, if a gate Δ' is used one has

$$\langle \nu' \rangle_n = A' + B' n \quad (9)$$

with

$$A' \approx A \quad (10)$$

and

$$B' \approx B (\Delta'/\Delta)^\gamma. \quad (11)$$

From the data of Ref. 4), we have $A' (\Delta'=0.1) = 0.78$ and $A (\Delta=0.5) \approx 0.70$, and $B/B' \approx 2.2$, corresponding to $\gamma \approx 0.5$ [recall statement v)];

iv') I write the particle (maximum) density distribution in a form consistent with i), ii) and iii),

$$P_n(v) = P_B(v) + P_L(n, v)$$

namely

$$P_n(v) = a v e^{-bv} + \frac{c}{v_M} \left(\frac{v}{v_M} \right) e^{-dv/v_M}. \quad (12)$$

The first term, $P_B(v)$, should peak at small $v \lesssim 1$, which requires $b \gtrsim 1$, and the second term, $P_L(n, v)$, should peak around $v/v_M \lesssim 1$ which requires $d \gtrsim 1$. The normalization condition is given by

$$a/b^2 + c/d^2 = 1, \quad (13)$$

and the fixed n average density by

$$\langle v \rangle_n = \frac{2}{b} a/b^2 + \frac{2}{c} c/d^2 v_M, \quad (14)$$

with v_M related to n by (4). Equation (14) thus agrees with (8).

Considering the "anomalous" $n = 26$ event of Ref. 6) as an event with the maximal $v \approx v_M$ density for that multiplicity, we have $v_M = 10$ (or $v_M = 8$ if two of the particles in the rapidity window of $\Delta = 0.1$ were misidentified as pions) which gives $\kappa = 2.6$ (or $\kappa = 3.2$), see Eq. (4). In calculations I shall take $\kappa = 3.2$.

From (12), knowing the prong cross-section probabilities $p_n \equiv \sigma_n / \Sigma \sigma_n$, one can calculate the full density distribution $P(v)$:

$$P(v) = a v e^{-bv} + c \kappa^2 v \sum_{n \geq \kappa v} \frac{p_n}{n^2} e^{-\frac{\kappa d v}{n}} \quad (15)$$

The set $\{p_n\}$ was taken from Ref. 11).

Data exist for $\langle v \rangle_n$ and $P(v)$ at $\sqrt{s} = 22$ GeV for π^+p , κ^+p and pp scattering⁴⁾. The parameters a , b , c and d appearing in Eqs. (13)-(15) were then adjusted in order to obtain reasonable agreement with data. The result is shown in Figs. 3 and 4. The values of the parameters are: $a = 3.08$, $b = 2$, $c = 0.475$ and $d = 1.43$. In Fig. 4 the probability $P(v)$ was multiplied by the reported number of events $N \approx 37 \times 10^3(\pi^+p)$.

v') The key features of the intermittency model are the fixed n two-peak distribution $P_n(v)$, statement i) above, and the linear law for $\langle v \rangle_n$, $\langle v \rangle_n = A+Bn$, statements ii), iii) and iv) above. The constructed fixed n distributions, $P_n(v)$, are shown in Fig. 5 for $n = 10, 20, 40, 80$. For small values of n the large v peak is buried under the small v dominant exponential. The two-peak shape is, however, already apparent for $n \approx 20$. It is the fixed n large v tail that makes the $P(v)$ distribution to deviate from the $\sim e^{-2v}$ exponential behaviour at small v . As seen in Fig. 5, "anomalous" events with large density occur at a rate of 3-4% for $n \gtrsim 20$. The NA22 finding an event with 10(8) particles in a window $\Delta = 0.1$, out of 20 $n = 26$ events is consistent with such a rate.

I would finally like to make some remarks.

1) As just mentioned, statements i)-iv) and Figs. 3 and 5 contain the essence of the intermittency model for rapidity particle densities. One should, however, keep in mind the qualitative nature of our discussion. For instance, only the qualitative features should be retained in Fig. 5. Variations in the parametrization and small changes in the parameters may appreciably modify the large v distribution. In any case, it would be of great interest to have high statistics of fixed n maximum rapidity density distributions. As fixed n distributions are not strongly reaction and energy dependent, statistics can be increased by putting together data from different reactions at different energies.

2) The fact that energy does not significantly affect fixed n distribution was known long ago from ISR studies of correlations¹²⁾, and confirmed by UA5 collaboration at the SppS collider¹³⁾. In particular, the two-particle correlation $C_n(\Delta y=0)$ was shown to be fairly independent of the energy and to increase with the multiplicity n .

3) Conventional models with clusters and multicluster production can presumably explain the approximate linear relation between the (maximum) particle density $\langle v \rangle_n$ and the multiplicity n . This is so because larger multiplicities trigger clusters with larger multiplicities and multicluster production with clusters superimposed in rapidity. In Eq. (12), the first term can thus be interpreted as due to normal resonances and the second one to heavier clusters or multicluster production. However, one would expect in such models at most two decaying exponentials, the flatter one being n dependent and not the large n two-peak structures seen in Fig. 5. Again, fixed n rapidity particle density distributions are of great interest to distinguish models.

4) As the energy does not play an explicit role (only through the $\{p_n\}$), the rapidity "hot spots" are not necessarily related to high E_T jets or mini-jets. However, as high rapidity densities, similarly to high E_T jets, are triggered by large multiplicities or larger matter densities (small impact parameters), large rapidity densities should in fact preferably occur in the background of large E_T jets.

5) It is not clear that "hot spots" in rapidity are related to the formation of the quark gluon plasma. The intermittency model may correspond to the presence of a mixed phase with two mechanisms of production at the event level. These local fluctuations in rapidity, rather than quantities averaged in n as multiparticle and transverse energy distributions, may give indications on new structures in interacting hadronic matter. In particular, it would be interesting to test the validity of relations (4) and (8) in high energy heavy ion collisions experiments.

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REFERENCES

- 1) K.I. Aleksejeva et al., Izv. Akad. Nauk. SSR 26 (1962) 128; J. Phys. Soc. Jpn. 17 (1962) A-III, 409;
N. Arata, Nuovo Cimento 43A (1978) 455;
N.A. Marutyan et al., Yad. Fiz. 29 (1979) 1566 [Sov. J. Nucl. Phys. 29 (1979) 804];
G.V. Apanasenko et al., Pis'ma Zh. Eksp. Teor. Fiz. 30 (1979) 157 [JETP Lett. 30 (1979) 145].
- 2) JACEE Collaboration, T.H. Burnett et al., Phys. Rev. Lett. 50 (1983) 2062.
- 3) J.G. Rushmore, Invited Talk at the Workshop on "pp Options for Supercollider (SSC)", Chicago 1984, CERN preprint CERN/EP84-34 (1984);
Ch. Geich-Gimbel, in "The Quark Structure of Matter", Eds. M. Jacob and K. Winter, World Scientific, p. 465, (1986);
D.R. Ward, Invited talk at the Workshop on "Physics Simulation at High Energies", Madison 1986, CERN preprint CERN/EP86-80 (1986).
- 4) NA22 Collaboration, M. Adamus et al., Phys. Lett. B185 (1987) 200.
- 5) I.M. Dremin, CERN preprint CERN-TH.4693/87 (1987).
- 6) P. Manneville and Y. Pomeau, Phys. Lett. A75 (1979) 1;
Y. Pomeau and P. Manneville, Comm. Math. Phys. 74 (1980) 189.
- 7) J.E. Hirsch, B.A. Huberman and D.J. Scalapino, Phys. Rev. A25 (1982) 519.
- 8) J. Dias de Deus and A. Noronha da Costa, Phys. Lett. A120 (1987) 19.
- 9) A. Bialas and R. Peschanski, Nucl. Phys. B273 (1986) 703.
- 10) A. Noronha da Costa, Ph.D. Thesis, IST Technical, University of Lisbon, 1987.
- 11) NA22 Collaboration, M. Adamus et al., Z. Phys. C32 (1986) 475.
- 12) L. Foà, Physics Reports 22 (1975) 1.
- 13) K. Böckmann and B. Eckart, in XV International Symposium on "Multiparticle Dynamics 1984", Eds. G. Gustafson and C. Peterson, World Scientific 1984;
D.R. Ward, in "Physics in Collision 5", Eds. B. Aubert and L. Montanet, Frontières 1985.

FIGURE CAPTIONS

Fig. 1 : Intermittency in one-dimensional map $x_{n+1} = f_h(x_n)$. The point remains in the gate Δ for several iterations if it enters the gate to the left of b ("narrow channel", $v = 7$), and remains for few iterations if it enters the gate to the right of b ("wide channel", $v = 1$). Iterations starting from the critical point c remain in the gate region Δ the maximum possible number of steps v_M ($v_M \approx 9$, in the example). The distance of the curve to the diagonal is controlled by the function parameter h .

Fig. 2 : Simulated motion in rapidity of a parton in hadronic matter with varying densities. In the high density regions ("narrow channels") it emits many particles, in the low density regions ("wide channels") it emits few particles. The plot of the number of steps, inside the gate, as a function of iterating time for the map of Fig. 1, would give a similar figure.

Fig. 3 : The maximum average number of particles $\langle v \rangle_n$ falling in a rapidity window $\Delta = 0.1$ as a function of the charge multiplicity n . Data are from Ref. 6). The straight line is obtained in the intermittency model, Eqs. (4) and (14).

Fig. 4 : Number of events with maximum rapidity density v as a function of v . Data are from Ref. 6). The curve was obtained with the parametrization (15). $\Delta = 0.1$.

Fig. 5 : Fixed multiplicity probability distributions of maximum rapidity density v as a function of v . For large n the intermittency two-peak structure, one at small v and the other at $v \approx v_M$, is clearly seen. $\Delta = 0.1$.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5