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EFFECTIVE SUPER-HIGGS AND $\text{Str}m^2$ FROM FOUR-DIMENSIONAL STRINGS^{*})

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A B S T R A C T

We introduce supersymmetry breaking terms into the superpotential of the supergravity theories recently derived as the point field limit of the four-dimensional $N = 1$ superstrings. The terms we discuss have the special feature of giving mass to the gravitino without inducing one-loop quadratic divergences in the effective theory. It is shown that this property of the effective theory is due to the special geometrical structure shared by the scalar Kähler manifolds of the four-dimensional superstring-induced models.

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String theories with world sheet supersymmetry [1,2] provide a framework for a consistent quantum theory unifying all known interactions: in this respect, they are at present the only serious candidates for the superunification program. One can obtain a variety of four-dimensional string models with realistic particle spectra either by compactifying [3,4] the known ten-dimensional superstrings or by constructing superstring models defined directly in four dimensions [5,6,7,8,9]. For stability reasons, one requires by construction four dimensional models with at least one space-time supersymmetry. Indeed, for the recently proposed four-dimensional superstring models it has been proven that the flat space-time background is stable [8] at the level of genus one surfaces, and this is probably true at any higher genus [10]. However, as a physical requirement space-time supersymmetry has to be broken by the vacuum. It is then questionable if the stability of the flat background remains valid when supersymmetry is broken. One can also construct non-supersymmetric $d = 4$ string models [7,8], but in that case the would-be supersymmetry is broken (either spontaneously or explicitly¹) at the Planck mass scale. In all known models where supersymmetry is broken by construction, the flat background turns out to be unstable at the one-loop level. This *does not* mean that there is no hope in that direction, but such a situation is uncomfortable.

In general, starting from a four-dimensional string model (supersymmetric or not), and following the Polyakov approach [11], the cosmological constant at the level of genus one surfaces is obtained by integrating over the appropriate modular parameter τ the contribution to the partition function of the space-time bosons minus the one of the space-time fermions

$$\Lambda_{cosm} \propto \int_{fund.reg.} \frac{d\tau d\bar{\tau}}{(Im\tau)^2} [Z_B(\tau, \bar{\tau}) - Z_F(\tau, \bar{\tau})]. \quad (1)$$

For the recently introduced four-dimensional superstring models, $\Lambda_{cosm} = 0$ [8] even before integration over τ . This implies separate cancellations in the massless sector and at each mass level of the massive sector. In general, however, one should not exclude the possibility of non-trivial cancellations

¹Two-dimensional world-sheet supersymmetry does not necessarily imply that the breaking of space-time supersymmetry is spontaneous: indeed, there are examples of $d = 4$ string models with a purely bosonic spectrum.

between massless and massive degrees of freedom, once the integration over τ is performed [12].

Let us assume the existence of a four-dimensional string model with *spontaneously broken* supersymmetry. This assumption allows us to organize the string states in the following way: we can denote by S_0 the set of states (bosons and fermions) which are massless in the supersymmetric limit, and by S_i ($i = 1, 2, \dots$) the set of states at the i^{th} mass level in the supersymmetric limit; similarly, we can denote by Z_{S_0} and Z_{S_i} the corresponding contributions to the partition function. In the supersymmetric limit, $Z_{S_0} = 0$ and $Z_{S_i} = 0$ separately and before integration over τ . If the theory with spontaneously broken supersymmetry is stable for $\Lambda_{cosm} = 0$, then there are two possibilities:

1. Z_{S_0} and Z_{S_i} are separately vanishing. This means that the mass splitting between bosons and fermions in S_0 (and in each of the S_i) is such that the different contributions independently cancel at each mass level.
2. Z_{S_0} and Z_{S_i} give non-zero contributions, but at the end a miraculous cancellation among the contributions of the different levels occurs.

Here we will examine the first possibility. The reason is that it can be easily treated in the framework of the effective theory involving only the massless modes in S_0 . Even in this case the precise knowledge of all their interactions is necessary. Therefore we restrict the present analysis to the models where these interactions are known. Working in the effective supergravity theory defined by S_0 , the contribution of the states in S_0 to the cosmological constant, $\Lambda_{cosm}^{S_0}$, is given by

$$\Lambda_{cosm}^{S_0} = -\frac{1}{128\pi^2}(Str M^0)\Lambda^4 + \frac{1}{64\pi^2}(Str M^2)\Lambda^2 + \dots, \quad (2)$$

where Λ is a cut-off scale which characterizes the tower of massive string states in $S_1, S_2, \dots$. In fact, the overall contribution $\Lambda_{cosm} = \Lambda_{cosm}^{S_0} + \sum_i \Lambda_{cosm}^{S_i}$ has to be finite, and therefore the scale Λ must be proportional to the mass scale M_{string} defined by the inverse of the slope of the string trajectories. The contribution to $\Lambda_{cosm}^{S_0}$ proportional to Λ^4 vanishes by construction, since $Str M^0$ is equal to the number of bosons n_B^0 minus the number of fermions n_F^0 in S_0 . The next leading contribution does not vanish in a generic $N = 1$

supergravity model. The question is whether in the string generated supergravities $StrM^2$ vanishes or not: this is the issue that will be addressed in the present paper.

In a general $N = 1$ supergravity [13], $StrM^2$ is given by [14]

$$StrM^2 = 2e^{\mathcal{G}}[N - 1 - \mathcal{G}^i(\mathcal{G}^{-1})_i{}^m R_m{}^n(\mathcal{G}^{-1})_n{}^j \mathcal{G}_j] + \dots, \quad (3)$$

where $\mathcal{G} = J(y, \bar{y}) + \log |g(y)|^2$, the real function $J(y, \bar{y})$ is the Kähler potential and the analytic function $g(y)$ is the superpotential. In our notation, y denotes collectively the scalar components of the chiral superfields and $\bar{y}$ their complex conjugates. Following the standard conventions, we define $\mathcal{G}_i \equiv \frac{\partial \mathcal{G}}{\partial y^i}$, $\mathcal{G}^i \equiv \frac{\partial \mathcal{G}}{\partial \bar{y}_i}$, so that the Kähler metric is

$$\mathcal{G}_i{}^j \equiv \frac{\partial}{\partial y^i} \frac{\partial}{\partial \bar{y}_j} \mathcal{G} = J_i{}^j. \quad (4)$$

A very important geometrical quantity, which plays a crucial role in the evaluation of $StrM^2$, is the Ricci tensor $R_m{}^n$ of the scalar manifold, defined as

$$R_m{}^n \equiv \frac{\partial}{\partial y^m} \frac{\partial}{\partial \bar{y}_n} \log[\det \mathcal{G}_i{}^j]. \quad (5)$$

It must be stressed that $R_m{}^n$ involves the fourth derivatives of the function $\mathcal{G}$: this means that the simple knowledge of the quadratic fluctuations around the symmetric vacuum $\langle y \rangle = 0$ in the kinetic function $J(y, \bar{y})$ is not enough to discuss the quadratically divergent contributions to $\Lambda_{cosm}^{S_0}$. Another important thing to note is that $R_m{}^n$ depends *only* on the kinetic function J and not on the superpotential g . To complete the illustration of eq.(3), the dots stand for the D-term contributions, $\langle e^{\mathcal{G}} \rangle \equiv m_{3/2}^2$ is the gravitino mass squared and N is the total number of chiral supermultiplets.

Recently, the complete low-energy effective action of a class of four dimensional superstring models [7,8] has been derived [15,16,17]. This is not the case for the corresponding models obtained by Calabi-Yau or orbifold compactification [18,19]. This forces us to consider only the 4-d models constructed by using world-sheet fermions with periodic and antiperiodic boundary conditions [8]. A variety of such models with $N = 1$ supersymmetry can be constructed. In view of the following discussion, we classify them in two categories:

1. $N = 1$ superstring models whose entire massless spectrum and interactions are obtained by a 2-d fermionic projection ($Z_2 \times Z'_2$) of a $N = 4$ superstring model.
2. $N = 1$ superstring models whose massless sector contains additional $Z_2 \times Z'_2$ symmetric states, which cannot be obtained by $Z_2 \times Z'_2$ projection from a $N = 4$ model.

This additional sector will be called ‘twisted’ sector (TS), while the $Z_2 \times Z'_2$, $N = 4$ truncated sector will be called ‘untwisted’ sector (UTS). The physically interesting models are those of class 2., because they contain chiral ‘families’ under some gauge group $H \supset SU(3) \times SU(2) \times U(1)$.

The complete effective theory of class 1. models was derived in ref.[15,16], where it was shown that the scalar Kähler manifold $\mathcal{M}$ is a product of Einstein spaces, namely $\mathcal{M} = \prod_{A=0}^3 \mathcal{M}_A$, with

$$\mathcal{M}_0 = \frac{SU(1,1)}{U(1)}, \quad \mathcal{M}_A = \frac{SO(2, n_A)}{SO(2) \times SO(n_A)} \quad (A = 1, 2, 3), \quad (6)$$

where $\mathcal{M}_0$ is the ‘dilaton’ submanifold and the $\mathcal{M}_A$ are submanifolds associated to n_A chiral superfields, respectively. A convenient parametrization of the manifold $\mathcal{M}$ is given by the Kähler potential [15] $J_{UTS} = \sum_{A=0}^4 J_A$, where

$$J_0 = -\log(S + S^*), \quad (7)$$

$$J_A = -\log\left(1 - \sum_{i_A} |y^{i_A}|^2 + \frac{1}{4} \left| \sum_{i_A} y^{i_A} y^{i_A} \right|^2\right) \quad (A = 1, 2, 3). \quad (8)$$

The superpotential is given by

$$g = g^{UTS} = c_{i_1 i_2 i_3} y^{i_1} y^{i_2} y^{i_3}. \quad (9)$$

In eq.(9) the coefficients $c_{i_1 i_2 i_3}$ are proportional to the structure constants of the $N = 4$ underlying gauge group G and $y^{i_1}, y^{i_2}, y^{i_3}$ transform as elements of the coset G/H , where H is the $N = 1$ gauge group. The $\mathcal{G}$ function of the model is therefore

$$\mathcal{G} = J_{UTS} + \log |g^{UTS}|^2. \quad (10)$$

The gauge kinetic function of the model depends only on the S field (dilaton + pseudoscalar) and is given by

$$f_{ab} = \delta_{ab} S. \quad (11)$$

In the case where massless twisted states are also present, the $N = 1$ effective theory cannot be obtained simply by $Z_2 \times Z'_2$ projections of a $N = 4$ theory. However, by ‘integrating out’ the string massive spin one states it is possible [17] to obtain the effective $N = 1$ theory as a Z_2 projection of an underlying $N = 2$ model. As shown in ref.[17], for one twisted sector (for instance, in the case of $H = SO(10) \times SO(6) \times SO(14) \times SO(14)'$ at the $N = 1$ level) the complete theory is known, namely

$$\mathcal{G} = J + \log |g|^2, \quad (12)$$

$$J = J_{UTS} + J_{TS}, \quad (13)$$

$$g = g^{UTS} + g^{TS}, \quad (14)$$

where J_{UTS} , g^{UTS} and the kinetic function f_{ab} are the same as before, while

$$J_{TS} = -2 \log \left(1 - \frac{1}{2} \sum_{i_{T1}} |y^{i_{T1}}|^2 e^{\frac{1}{2}(J_2 + J_3)} \right) \quad (15)$$

and

$$g^{TS} = t_{i_1 j_{T1} k_{T1}} y^{i_1} y^{j_{T1}} y^{k_{T1}}, \quad (16)$$

where $y^{i_{T1}}$ are fields in the 16 of $SO(10)$, transforming as $\underline{4} + \overline{\underline{4}}$ of $SO(6)$ and as singlets of $SO(14) \times SO(14)'$. In total this model contains sixteen $SO(10)$ families ($2 \times \underline{4}$ and $2 \times \overline{\underline{4}}$). The $t_{i_1 j_{T1} k_{T1}}$ are (up to a normalization factor) the Clebsch-Gordan coefficients of the corresponding representation of the gauge group.

In the general case when more twisted sectors are present, the complete expression of J_{TS} is not known. However, the Kähler potential is known in the limit of small fluctuations ($|y^{i_{T\alpha}}| \ll 1$). For our purposes (see later), this is already sufficient to evaluate $Str M^2$ around $y^{i_{T\alpha}} = 0$. Indeed, this region is most likely the physically relevant one, since the $y^{i_{T\alpha}}$ are the scalar partners of the family fermions². Within this approximation we have [17] (for instance, in the case of $H = SO(10) \times SO(6)_1 \times SO(6)_2 \times SO(6)_3 \times E_8$ at the $N = 1$ level)

$$J_{TS} = J_{TS1} + J_{TS2} + J_{TS3}, \quad (17)$$

²We do not consider here the possible effects of large vacuum expectation values for the standard model singlets contained in the 16 of $SO(10)$.

with

$$J_{TS1} = \sum_{i_{T1}} |y^{i_{T1}}|^2 e^{\frac{1}{2}(J_2+J_3)}, \quad (18)$$

$$J_{TS2} = \sum_{i_{T2}} |y^{i_{T2}}|^2 e^{\frac{1}{2}(J_1+J_3)}, \quad (19)$$

$$J_{TS3} = \sum_{i_{T3}} |y^{i_{T3}}|^2 e^{\frac{1}{2}(J_1+J_2)}. \quad (20)$$

The kinetic function f_{ab} is the same as before. Following always ref.[17], g^{TS} is completely known and is given by

$$g^{TS} = t_{i_1 j_{T1} k_{T1}} y^{i_1} y^{j_{T1}} y^{k_{T1}} + t_{i_2 j_{T2} k_{T2}} y^{i_2} y^{j_{T2}} y^{k_{T2}} + t_{i_3 j_{T3} k_{T3}} y^{i_3} y^{j_{T3}} y^{k_{T3}}. \quad (21)$$

Observe that, since J_{TS} contains interference terms involving fields both from the twisted and from the untwisted sector (eqs.(15) and (17-20)), now the scalar Kähler manifold is not given as a product of symmetric spaces. As we will see later, this non-trivial interference between UTS and TS is of primary importance and guarantees the vanishing of $Str M^2$ even in the presence of supersymmetry breaking.

As we will show immediately, the models defined by eqs.(7-11), (12-16) and (17-21) have a positive semi-definite scalar potential. Any possible vacuum state is characterized by zero cosmological constant and unbroken supersymmetry ($m_{3/2}^2 \equiv \langle e^G \rangle = 0$). This fact follows from the homogeneity properties of the superpotential and from the presence of the S field. Namely

$$\text{Case 1. : } g - y^{i_A} g_{i_A} = 0 \quad (A = 1, 2, 3); \quad (22)$$

$$\text{Case 2. : } g - y^{i_1} g_{i_1} - \frac{1}{2} y^{i_{T2}} g_{i_{T2}} - \frac{1}{2} y^{i_{T3}} g_{i_{T3}} = 0 \quad (1 \rightarrow 2 \rightarrow 3 \text{ cyclic}). \quad (23)$$

Indeed, when eqs.(22) and/or (23) are valid then the scalar potential is positive semi-definite and takes a very simple form.

Case 1.:

$$V = V_0 + \sum_{A=1}^3 V_A + V_{D\text{-terms}}, \quad (24)$$

$$V_0 = e^{J_{UTS}} |g|^2, \quad (25)$$

$$V_A = e^{J_{UTS}} [e^{-J_A} |g_{i_A}|^2 + |\bar{y}_{i_A} g_{i_A}|^2] \quad (A = 1, 2, 3), \quad (26)$$

$$\begin{aligned} V_{D\text{-terms}} &= \frac{1}{S + S^*} (\mathcal{G}_i T^{ai}_{\ j} y^j) (\mathcal{G}_k T^{ak}_{\ i} y^i) \\ &= e^{J_0} \left[\sum_{A=1}^3 e^{J_A} \bar{y}_{i_A} T^{ai_A}_{\ j_A} y^{j_A} \right] \left[\sum_{B=1}^3 e^{J_B} \bar{y}_{i_B} T^{ai_B}_{\ j_B} y^{j_B} \right]. \end{aligned} \quad (27)$$

Note the particular form of $V_{D\text{-terms}}$ and the fact that e^{J_0} can be factorized out of the whole potential V .

All the minima of the potential (24) have zero vacuum energy and correspond to $\langle g_{i_A} \rangle = 0$ ($\Rightarrow \langle g \rangle = 0$), $\langle V_{D\text{-terms}} \rangle = 0$. In general, however, there are many flat directions along which the gauge group H is spontaneously broken. Indeed, the D-terms contribution to the potential vanishes for every direction which is purely real or imaginary. Also, for purely real directions the chiral part of the potential is flat for every direction along which some of the factor subgroups in H remain unbroken. For instance, in the case of the minimal $N = 1$ model based on a product of four $SO(m_I)$ groups it is sufficient that one of the $SO(m_I)$ remains unbroken. For any acceptable minimum of V , $\langle g \rangle = 0$ so that $m_{3/2}^2 = \langle |g|^2 e^{J_{UTS}} \rangle = 0$ and supersymmetry is unbroken.

Case 2.:

Thanks to eq.(23), also in this case the potential takes a manifestly positive semi-definite form:

$$V = V_0 + \sum_{A=1}^3 V_A + \sum_{\alpha=1}^3 V_{TS\alpha} + V_{D\text{-terms}}, \quad (28)$$

$$V_0 = e^J |g|^2, \quad (29)$$

$$V_1 = e^{J-\frac{1}{2}(J_{TS2}+J_{TS3})}[e^{-J_1}|g_{i_1}|^2 + |\bar{y}_{i_1}g_{i_1}|^2], \quad (30)$$

$$V_2, V_3 = (\text{cyclic permutations } 1 \rightarrow 2 \rightarrow 3), \quad (31)$$

$$V_{TS1} = e^{J-\frac{1}{2}(J_2+J_3)-\frac{1}{2}J_{TS1}}|g_{i_{T1}}|^2, \quad (32)$$

$$V_{TS2}, V_{TS3} = (\text{cyclic permutations } 1 \rightarrow 2 \rightarrow 3), \quad (33)$$

$$V_{D\text{-terms}} = e^{J_0} \left[\sum_{A=1}^3 e^{J_A} \bar{y}_{i_A} T^{ai_A}_{j_A} y^{j_A} + e^{\frac{1}{2}(J_2+J_3)} \bar{y}_{i_{T1}} T^{ai_{T1}}_{j_{T1}} y^{j_{T1}} \right. \\ \left. + e^{\frac{1}{2}(J_1+J_3)} \bar{y}_{i_{T2}} T^{ai_{T2}}_{j_{T2}} y^{j_{T2}} + e^{\frac{1}{2}(J_1+J_2)} \bar{y}_{i_{T3}} T^{ai_{T3}}_{j_{T3}} y^{j_{T3}} \right]^2. \quad (34)$$

Also here all the minima correspond to zero vacuum energy, with $\langle g_{i_A} \rangle = \langle g_{i_{T\alpha}} \rangle = 0$ ($\Rightarrow \langle g \rangle = 0$), $\langle V_{D\text{-terms}} \rangle = 0$. There are many flat directions along which H is spontaneously broken. All the flat directions we discussed in case 1. are also present in case 2. when $\langle y^{i_{T\alpha}} \rangle = 0$. Observe also that for $y^{i_{T\alpha}} = 0$ the potential of case 2. is identical to that of case 1..

Before moving to the case of spontaneously broken supersymmetry, a final remark is in order. It is conceivable that the fermionic construction of the $d = 4$, $N = 1$ superstring models considered above might be interpreted as a toroidal compactification of the known ten-dimensional strings on T^6/h , with a fixed radius R of the compactified dimensions: the special choice of the radius is responsible for the enlargement of the gauge group and the presence of the twisted sectors in the $N = 2$ and $N = 1$ cases. Starting for instance from the $d = 10$, $SO(32)$ heterotic string, dimensional reduction on T^6 gives a $N = 4$, $SO(32) \times U(1)^6$ model in $d = 4$. This group is enlarged to $G \subset SO(44)$ when the radius of the six-dimensional torus is fixed to a convenient value³ R . Then the action of h consists in a $Z_2 \times Z'_2 \times \prod_k Z_2^{(k)}$ truncation which breaks $N = 4$ to $N = 1$ and G to H ($Z_2 \times Z'_2$ acts on the gravitinos and on the gauge group G , while $\prod_k Z_2^{(k)}$ acts only on the gauge group G). As we saw, the resulting $d = 4$, $N = 1$ effective theory with gauge group H has exact supersymmetry.

We would like now to discuss a possible modification of the scheme described above which may lead to spontaneously broken supersymmetry. We require to have a 'minimal' modification, such that the $N = 1$, $d = 4$ effective theory retains the same degrees of freedom and has the same geometrical

³This enlargement of the gauge group is a purely stringy phenomenon, and cannot be seen in the field theory limit.

structure as in the supersymmetric case. From the point of view of field theory, a possible modification may correspond to a generalized dimensional reduction *a la* Scherk and Schwarz [20]. This would extend the T^6 compactification to a compactification which depends on some internal coordinates ($T^6 \rightarrow T^6(z)$). This scenario would be similar to the ‘gauging’ [21] of a $N = 4$ theory with some non semisimple group, mixing the graviphotons and the $N = 4$ matter vector fields and yielding vanishing cosmological constant. A relevant feature of this possible compactification is that it induces a modification of the superpotential, $g \rightarrow g + \Delta g$, but it keeps invariant the Kähler potential, $J \rightarrow J + \Delta J = J$, of the $N = 1$, $d = 4$ effective theory. Guided by this reasoning, we are led to introduce a term Δg which modifies the superpotential g and induces supersymmetry breaking, preserving however the positivity of the $N = 1$ chiral potential ($\langle V \rangle = 0$). This requirement by itself severely restricts the possible Δg modifications. Together with the stability requirement ($V \geq 0$), an additional condition must be satisfied by the underlying $N = 4$ gauge group G (before the $Z_2 \times Z'_2$ projection), so that the spectrum of the $N = 1$ effective theory contains at least four complex scalar singlets under the gauge group H : one is the S field, while the other three y_A^0 ($A = 1, 2, 3$) are members of the untwisted Kähler manifold $\frac{SO(2, n_A)}{SO(2) \times SO(n_A)}$ ($A = 1, 2, 3$). This extra condition is necessary and follows from the form of the Δg modification satisfying the stability requirement $V \geq 0$. If this condition is not satisfied, it is not possible to define a Δg which is consistent with the gauge symmetry H and which gives rise to a stable potential. $N = 1$ models which fulfill this condition can be easily constructed using the general rules of ref.[8], e.g. the $N = 1$ models with $H = SO(m_0) \times SO(m_1 - 2) \times SO(m_2 - 2) \times SO(m_3 - 2)$, $m_0 = 2 + 4k$, $m_0 + m_1 = 0 \pmod{8}$ and $m_0 + m_1 + m_2 + m_3 = 44$. The underlying $N = 4$ model is based on the gauge group $G = SO(38) \times U(1)^3$. A common feature of all models is the $U(1)^3 \subset G$: the three singlets y_A^0 ($A = 1, 2, 3$) are in one to one correspondence with the three $U(1)$ factors of the underlying G symmetry.

The form of Δg is dictated by the general $N = 1$ potential. We will first consider the case in which the TS is absent (case 1.). For a general g the

potential of eq.(24) is not manifestly positive semi-definite:

$$V = V_0 + \sum_{A=1}^3 V_A + V_{D\text{-terms}}, \quad (35)$$

$$V_0 = e^{J_{UTS}} |g - (S + S^*)g_S|^2, \quad (36)$$

$$V_A = e^{J_{UTS}} \left[e^{-J_A} (|g_{i_A}|^2 - |y^{i_A} g_{i_A} - g|^2) + \left| \bar{y}_{i_A} g_{i_A} - \frac{1}{2} \bar{y}_{i_A} \bar{y}_{i_A} (y^{j_A} g_{j_A} - g) \right|^2 \right] \quad (A = 1, 2, 3), \quad (37)$$

and $V_{D\text{-terms}}$ as in eq.(27). V has to be positive semi-definite. This is the case if, for $S = \langle S \rangle$

$$|g_{y_A^0}| = |y^{i_A} g_{i_A} - g| \quad (A = 1, 2, 3), \quad (38)$$

where y_A^0 are the three singlets discussed above and the indices i_A describe the remaining fields in the manifold $\mathcal{M}_A$. From eq.(38) one finds the structure of the modified potential

$$g|_{S=\langle S \rangle} = C(1 + y_1^0 e^{i\phi_1})(1 + y_2^0 e^{i\phi_2})(1 + y_3^0 e^{i\phi_3}) + g_{susy}, \quad (39)$$

where g_{susy} is as in eq.(9), C is a complex non zero number and ϕ_1, ϕ_2, ϕ_3 are real phases. Remarkably enough, the superpotential modification we are considering here comes from a gravitino mass term which is cubic in terms of the representatives of $\prod_{A=1}^3 \frac{SO(2, n_A)}{SO(2) \times SO(n_A)}$, once we interpret the inhomogeneous term of eq.(39) as the contribution of the compensator fields. This forces the compensators $y_A^{(1)}, y_A^{(2)}$ to appear in g only in the form $y_A^+ \equiv \frac{1}{\sqrt{2}}(y_A^{(1)} + i y_A^{(2)})|_{S=\langle S \rangle}$, the only combinations which give field-independent results.

If g is independent of S , the resulting potential (35) is manifestly positive semidefinite and has stationary points when $g_{i_A} = 0$ for $i_A \neq 0$. However, for all $g_{i_A} = 0$, $g = \text{constant} \neq 0$, so that the only minimum is at $S \rightarrow \infty$. In the general case, g is also a function of S (for instance, due to non-perturbative quantum corrections like gaugino condensation [22]). Then it is possible to have $|g - (S + S^*)g_S|^2 = 0$, which determines the coupling constant of the theory

$$g_{gauge}^2 = \frac{2}{\langle S + S^* \rangle} = 2 \langle \frac{g_S}{g} \rangle. \quad (40)$$

Any $\langle S + S^* \rangle \neq 0$ will break spontaneously the dilatation symmetry of the theory and determine the universal gauge coupling constant. Other sources of S -dependence are also conceivable, because in the present case the gauge group G of the underlying $N = 4$ supersymmetry is given by a product of factor groups. In that case it is possible [21] to assign different S -dependent ‘coupling constants’ for the gaugings involving the graviphotons (the same is true for the $N = 2$ ‘gaugings’). This generates additional D-terms in the underlying $N = 4$ theory. This is analogous to the anomalous $U(1)$ S -gauging discussed in ref.[23]. The difference here is that the D-terms appearing in the $N = 4$ theory, when truncated by $Z_2 \times Z'_2$ to obtain the relevant $N = 1$ model, appear as a modification to the chiral part of the potential ($g \rightarrow g(S)$), rather than a D-term modification.

We are now in a position to evaluate $StrM^2$. As we have already mentioned at the beginning of this work, in the absence of massless twisted states the Kähler manifold is a product of Einstein spaces, with

$$R_{i_A}{}^{j_A} = R_A \mathcal{G}_{i_A}{}^{j_A} = R_A J_{i_A}{}^{j_A} \quad (A = 0, 1, 2, 3). \quad (41)$$

The curvatures R_A ($A = 1, 2, 3$) are equal to the number of complex scalar fields of the corresponding submanifolds $\mathcal{M}_A$,

$$R_A = n_A \quad (A = 1, 2, 3), \quad (42)$$

while the curvature of the dilaton submanifold is

$$R_0 = 2. \quad (43)$$

In these cases $StrM^2$ has a very simple expression in terms of the curvatures R_A of the submanifolds,

$$StrM^2 = 2(N - 1 - \sum_{A=0}^4 a_A R_A) e^g + \dots, \quad (44)$$

where the dots stand for D-term contributions, N is the total number of chiral superfields and

$$a_A = \mathcal{G}^{i_A}(\mathcal{G}^{-1})_{i_A}{}^{j_A} \mathcal{G}_{j_A} \quad (A = 0, 1, 2, 3). \quad (45)$$

If the D-terms are vanishing, as it is the case for the models we consider, and supersymmetry is spontaneously broken, the goldstino is just given by

$$\eta_L = - \langle e^{\mathcal{G}/2} \mathcal{G}_i (\mathcal{G}^{-1/2})^i_j \rangle \chi_L^j, \quad (46)$$

where χ_L^j is the fermionic partner of y^j , and a_A can be interpreted as the fraction of supersymmetry breaking associated to the manifold $\mathcal{M}_A$. The scalar potential of the theory is given by

$$V = e^{\mathcal{G}} [\sum_A a_A - 3] + V_{D\text{-terms}}, \quad (47)$$

so for any acceptable minimum $\sum_A a_A = 3$. Using g of eq.(39) one finds

$$a_A = 1 + V_A e^{-\mathcal{G}}, \quad (48)$$

$$a_0 = V_0 e^{-\mathcal{G}} = \left| 1 - (S + S^*) \frac{gS}{g} \right|^2. \quad (49)$$

At any minimum of the potential

$$a_A = 1 \quad (A = 1, 2, 3), \quad a_0 = 0, \quad (50)$$

so that $Str M^2 = 0$ after supersymmetry breaking. Note that this situation is *independent* of the S modification to the full $\mathcal{G}$ function.

In the more interesting situation where massless TS with chiral families are present (case 2.), the vanishing of $Str M^2$ is still valid, as we will show immediately. In that case, when g is arbitrary the potential of eq.(28) gets modified as follows

$$V = V_0 + \sum_{A=1}^3 V_A + \sum_{\alpha=1}^3 V_{TS\alpha} + V_{D\text{-terms}}. \quad (51)$$

V_0 , $V_{TS\alpha}$ and $V_{D\text{-terms}}$ are as in eqs.(36), (32-33) and (34), respectively, while V_A ($A = 1, 2, 3$) modify as

$$\begin{aligned} V_1 &= e^{J - \frac{1}{2}(J_{TS2} + J_{TS3})} \\ &\times [e^{-J_1} (|g_{i1}|^2 - |g - y^{i1} g_{i1} - \frac{1}{2} y^{iT2} g_{iT2} - \frac{1}{2} y^{iT3} g_{iT3}|^2) \\ &+ |\bar{y}_{i1} g_{i1} + \frac{1}{2} \bar{y}_{j1} \bar{y}_{j1} (g - y^{i1} g_{i1} - \frac{1}{2} y^{iT2} g_{iT2} - \frac{1}{2} y^{iT3} g_{iT3})|^2], \\ V_2, V_3 &= (\text{cyclic permutations } 1 \rightarrow 2 \rightarrow 3). \end{aligned} \quad (52)$$

As in case 1., V is manifestly positive semi-definite, provided that for $S = \langle S \rangle$

$$|g_{y_1^0}| = |g - y^{i_1} g_{i_1} - \frac{1}{2} y^{i_{T_2}} g_{i_{T_2}} - \frac{1}{2} y^{i_{T_3}} g_{i_{T_3}}|. \quad (53)$$

Interesting enough, the modification $\Delta g|_{S=\langle S \rangle}$ in the presence of the TS is *identical* to that of eq.(39) obtained in the absence of the TS:

$$[g - g_{susy}]_{S=\langle S \rangle} = C(1 + y_1^0 e^{i\phi_1})(1 + y_2^0 e^{i\phi_2})(1 + y_3^0 e^{i\phi_3}). \quad (54)$$

This is due to the presence of $\frac{1}{2} y^{i_{T_2}} g_{i_{T_2}} + \frac{1}{2} y^{i_{T_3}} g_{i_{T_3}}$ in the right hand side of eq.(53).

Assuming as before a non trivial S -dependence of g , such that $V_0|_{S=\langle S \rangle} = 0$, the modification of eq.(54) gives rise to a manifestly positive semidefinite potential with $\langle g_{i_A} \rangle = 0$ ($i_A \neq 0$, $A = 1, 2, 3$) at the minimum.

We will examine first the case of only one twisted sector (see eqs.(12-16)), where the Kähler potential is known exactly (not only for small $y^{i_{T_\alpha}}$). In that case the Kähler manifold is given by $\mathcal{M} = \frac{SU(1,1)}{U(1)} \times \frac{SO(2, n_1)}{SO(2) \times SO(n_1)} \times \mathcal{M}'$, where now $\mathcal{M}'$ is *not* a product of symmetric spaces. In the expression of the curvature R_m^n , the components associated to the first two factors are the same as in case 1., while the components associated to $\mathcal{M}'$ are found to be:

$$R_m^n = \frac{n_2 + n_3 + n_T + 1}{2} \mathcal{G}_m^n + \Delta_m^n, \quad (55)$$

where

$$\Delta_m^n = \begin{cases} \frac{n_2 - n_3 - 1}{2} J_{2m}^n & \text{for } m, n \in \{i_2\} \\ \frac{n_3 - n_2 - 1}{2} J_{3m}^n & \text{for } m, n \in \{i_3\} \\ 0 & \text{otherwise.} \end{cases} \quad (56)$$

Using the fact that $g_{i_{T_1}} = 0$ at the minimum, one obtains the useful identity $[i_{\mathcal{M}'} \equiv (i_2, i_3, i_{T_1}), i_{UT} \equiv (i_2, i_3)]$:

$$\begin{aligned} \mathcal{G}^{i_{\mathcal{M}'}} \mathcal{G}_{i_{\mathcal{M}'}}^{-1}{}^{i_{UT}} &= e^{-\frac{1}{2} J_{TS}} [\mathcal{G}^{i_{UT}} J_{UTS}^{-1}{}^{i_{UT}}] \\ &= e^{-\frac{1}{2} J_{TS}} [\mathcal{G}^{i_2} (J_2^{-1})_{i_2}{}^{j_2} + \mathcal{G}^{i_3} (J_3^{-1})_{i_3}{}^{j_3}]. \end{aligned} \quad (57)$$

Using eqs.(55) and (56) and the fact that $a_0 = 0$, $a_1 = 1$ as in case 1., one finds

$$Str M^2 = (e^{J_{TS}} - 1) e^{\mathcal{G}}|_{\langle y^{i_{T_1}} \rangle = 0} = 0. \quad (58)$$

Note that $\langle y^{i_{T1}} \rangle = 0$ corresponds to the case where the $y^{i_{T1}}$ fields do not participate in the breakdown of supersymmetry [$\mathcal{G}_{i_{T1}} = 0$].

The vanishing of $StrM^2$ is also true in the general case where many twisted sectors are present [see eqs.(17-21)], as long as $y^{i_{T\alpha}} = 0$ at the minimum, which is precisely the region where we can use the expression of J_{TS} given by eqs.(17-20). The vanishing of $StrM^2$ in the presence of the TS is a consequence of the non trivial interference between the UTS - TS Kähler manifolds, coming through the rescalings $|y^{i_{T1}}|^2 e^{\frac{1}{2}(J_2+J_3)}$, etc.. In the absence of these rescalings, $StrM^2$ would not have been zero, but equal to $2n_T m_{3/2}^2$, where n_T is the number of chiral multiplets in the twisted sectors. This remark is of the utmost importance and shows that an adequate knowledge of the scalar manifold as well as the complete knowledge of all sectors (hidden or not hidden) of the theory is necessary in order to be able to say something about one loop quadratic divergences in a spontaneously broken supergravity theory. It is also important that our result on the vanishing of $StrM^2$ (which implies the absence of one loop quadratic divergences in the effective $N = 1$ supergravity theory) is not sensitive to the details of supersymmetry breaking. Indeed, $StrM^2 = 0$ assuming only that $\Delta J = 0$, $\Delta g \neq 0$ but such that $V \geq 0$.

Our result has its own interest in supergravity. It represents a unique example in which the one loop quadratic divergencies are absent because of the geometry of the scalar manifold of the theory. At the string level, it suggests that theories might exist where supersymmetry is spontaneously broken and the contributions to the cosmological constant are independently zero for the massless and the massive levels. An explicit construction of such string models along the lines suggested above would be of main importance.

In conclusion, we have shown that the recently proposed four dimensional superstring models have a massless sector based on a Kähler manifold whose geometry implies the vanishing of $StrM^2$ in the presence of supersymmetry breaking. The only assumption we used in the parametrization of the spontaneous breakdown of supersymmetry is the requirement of positive definite potential. We presented a minimal modification of the superpotential which gives rise to that desired result, and we justified it in terms of the generalized (internal coordinate dependent) toroidal compactification *a la* Scherk and Schwarz.

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