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SUPERSTRING MODIFICATIONS OF 4D-SUPERGRAVITY ACTIONS

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ABSTRACT

It is known that superstring theories yield to effective Lagrangians for the massless modes, which modify the Einstein action and ordinary particle couplings by adding infinitely many higher curvature terms and higher derivative interactions. Chern-Simons modifications to the 4D-supergravity action, couplings of antisymmetric tensors, 4D-Wess-Zumino terms and other issues on higher derivative point-field locally supersymmetric Lagrangians are reviewed.

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I. INTRODUCTION

The developments of elementary particle physics over the last decades have led to a better and deeper understanding of elementary particle phenomena in terms of invariance principles.

In particular, the success of the "standard model" has led to a revival of the ideas of an ultimate unified theory encompassing all known forces of nature by enforcing gauge symmetry principles. In the last decade the gauge invariance principle on symmetries relating fermions to bosons¹ (supersymmetry) has led to the development of supergravity theories² and to their interaction with matter. In this framework, a major role has been played by the spontaneous breakdown of supersymmetry³ (super Higgs effect), in understanding the pattern of gauge symmetry breaking and the consequent particle mass spectrum.

In the last years the unification program has merged in an underlying unified theory called superstring,⁴ which reproduces, at large distances compared to the Planck scale, Einstein theory, Yang-Mills interactions and supergravity forces.

Superstrings are theories which describe the dynamics of one-dimensional extended objects whose extension is the Planck length.

The stunning fact is that the very existence of these extended objects implies the presence of a massless spin-2 excitation with a self-coupling which at large scales exactly reproduces the Einstein-Hilbert Lagrangian. More importantly, strings also need other massless excitations which can be identified with ordinary vector gauge bosons, spin-1/2 fermions and spin-0 scalars. On top of a finite set of massless states, the vibrational modes of string theory describe an infinite set of massive excitations with higher spin and higher masses multiple of the Planck mass. At energy scales much lower than M_p these particles are "decoupled" and string dynamics reduces to an ordinary point-field theory for the massless modes.⁵ However, integration over the infinitely many massive modes of arbitrary spin, gives new interactions for the massless modes which appear as a modification of the standard (non-renormalizable) Einstein action and the standard (renormalizable) Yang-Mills action.⁵ These modifications involve infinitely many higher curvature terms and higher derivative interactions.

Some of these terms have an alternative field theory interpretation as coming from the anomaly cancellation mechanism discovered by Green and Schwarz⁶ in the framework of 10-dimensional supergravity. A crucial role in the 10-dimensional anomaly cancellation mechanism is played by an antisymmetric tensor field, whose non-trivial Lorentz and Yang-Mills transformations allow the existence of Wess-Zumino terms cancelling the quantum Yang-Mills and gravitational anomalies.

Different arguments⁷ led to the conclusion that the 4-dimensional physics should have an approximate supersymmetry which may become exact for energies approaching the Planck scale. Under this assumption it is of major relevance to understand the superstring modifications to the standard supergravity Lagrangians and to explore their consequences.

In this review we will consider, after some generalities, the Chern-Simons modifications to ordinary supergravity, the coupling to antisymmetric tensor fields, the 4D-reduction of the Wess-Zumino term and the restriction on higher curvature invariant from Kahler geometry.

II. GENERALITIES OF $D = 4$, N -EXTENDED MATTER-COUPLED SUPERGRAVITIES

It is known that compactification of 10-dimensional superstrings⁸ or superstrings directly formulated in four dimensions⁹ can exist with an extended supersymmetry algebra up to $N = 4$ or up to $N = 8$ fermionic generators. It is therefore relevant to recall some properties of N -extended supergravity theories. $N \leq 4$ supergravities occur in all

compactifications of heterotic strings or type I strings while $N \leq 8$ supergravities can only occur for type II superstrings.¹⁰

Although in supergravity theories all values of $1 \leq N \leq 8$ are possible, the relevant cases for superstrings are $N = 1, 2, 4$ and 8 .

Supergravity theories² describe the locally supersymmetric interaction of matter multiplets with the graviton multiplet. It is therefore appropriate to recall the structure of these multiplets.

Field multiplets are associated with particle multiplets. The latter are described by irreducible massless or massive representations of the N -extended supersymmetry algebra.¹¹

The spin and multiplicity content of each supermultiplet depends on the number of spinorial charges Q_α^i ($i = 1, \dots, N$). In $D = 4$ dimension the highest N for which a graviton multiplet exists is the $N = 8$ superalgebra --instead the highest N for which a gauge vector multiplet exists is the $N = 4$ superalgebra.

Through dimensional reduction, the $N = 8$ superalgebra corresponds to the $N = 1$ super-Poincaré algebra in $D = 11$ dimensions or to the $N = 2$ superalgebras in $D = 10$ dimensions. Instead the $N = 4$ algebra corresponds to the $N = 1$ case in $D = 10$ dimensions.

Since N -extended super-Poincaré algebras have $SU(N)$ as a group of automorphism, particle states inside each supermultiplet can be classified according to irreducible (antisymmetric) representations of $SU(N)$. This is true provided the highest helicity state is a $SU(N)$ singlet. In addition to the $SU(N)$ index, particle supermultiplets can also be associated with some representation of a group G which commutes with the supercharges Q_α^i . This is, for instance, what happens in super Yang-Mills theories.

The most important field representations are those which correspond to massless particles. Massive representations can be obtained in a similar way by appropriately combining massless representations.¹¹

The generic massless representation of an N -extended supersymmetry algebra contains helicity states going from $\lambda_{\max}$ to $\lambda_{\max}^{-N/2}$. The multiplicity of the state with helicity $\lambda_{\max}^{-K/2}$ being $\begin{bmatrix} N \\ K \end{bmatrix}$. This state belongs to the K -fold antisymmetric irrep. of $SU(N)$.

This collection of states corresponds to 2^N degrees of freedom. To include antiparticle states a doubling of the representation is necessary unless

$$\lambda_{\max} = \frac{N}{2} - \lambda_{\max}, \quad \text{i.e.,} \quad \lambda_{\max} = \frac{N}{4}, \quad (2.1)$$

where N is a multiple of 4. These exceptional cases, corresponding to CPT

self-conjugate supermultiplets, exist for $N = 4$ ($\lambda_{\max} = 1$) and for $N = 8$ ($\lambda_{\max} = 2$). They correspond respectively to the Yang-Mills multiplet and to the gravitational multiplet.

Massless multiplets with non-trivial interactions can only exist up to $N = 8$ for $\lambda_{\max} = 2$, $N = 4$ for $\lambda_{\max} = 1$ and $N = 2$ for $\lambda_{\max} = 1/2$. Therefore in theories with $N > 4$ we have only one interesting massless multiplet, the gravity multiplet. For $4 \geq N > 2$, we have in addition the Yang-Mills multiplet and for $N = 1, 2$ we have a third multiplet which does not contain gauge particles and only describes spin- $1/2$ fermions and scalar bosons.

An important role in supergravity theories is played by the scalar sector. Supersymmetry arguments imply that the derivative self-interactions of the scalar fields describe a specific σ -model

$$e^{-1} \mathcal{L}(\phi_i) = g_{ij}(\phi) \partial_\mu \phi^i \partial^\mu \phi^j \quad (2.2)$$

where the structure of the scalar manifold and its isometries depend on N and on the nature of the particular supermultiplet to which ϕ^i belong.

Let us first consider the $SU(N)$ properties of the scalar fields. Since in order to go from $\lambda_{\max}$ to $\lambda = 0$ one needs to apply $2\lambda_{\max}$ supercharges, the scalars will generally belong to the $2\lambda_{\max}$ -antisymmetric representation of $SU(N)$ ($N \geq 2$). Note that scalar fields are present in the graviton multiplet only for $N \geq 4$ and in the Yang-Mills multiplet for $N \geq 2$.

In all supergravity multiplets¹² the scalar fields parametrize a manifold which is a coset space K/H in which $H = U(N)$ (for $N = 8$ H reduces to $SU(8)$ and for $N = 4$ H reduces to $U(1)$).

In addition, the group K has the property that its decomposition under H contains the adjoint and the four-fold antisymmetric representation of $SU(N)$.¹³ K is also a non-compact group of which H is a maximal compact subgroup. This last requirement is mandatory for having scalar field variables with non-compact domain of positive kinetic energy. With these requirements in mind one easily see that the appropriate coset spaces for the gravity multiplets are¹² for $N = 4$, $SU(1,1)/U(1)$; for $N = 5$, $SU(5,1)/U(5)$; for $N = 6$, $SO^*(12)/U(6)$; for $N = 8$, $E_{7,7}/SU(8)$. For $N \leq 4$, we have also to discuss the scalar fields contained in the $\lambda_{\max} = 1$ and $\lambda_{\max} = 1/2$ multiplets. For $N = 4$ and 3, the Yang-Mills multiplet contains scalars which are sextets and triplets under $SU(4)$ and $SU(3)$ respectively. However, since the multiplet carries an adjoint index of the gauge group G , there are $\dim G$ copies of them. In those cases the appropriate scalar-manifold for the matter-coupled supergravity Lagrangians is^{14,13,15}

$$\frac{SU(1,1)}{U(1)} \times \frac{SO(6, \dim G)}{SO(6) \times SO(\dim G)} \quad \text{for } N = 4 \quad \text{and} \quad \frac{SU(3, \dim G)}{U(3, \dim G)} \quad \text{for } N = 3 .$$

In both cases, as in the case of $N > 4$, pure supergravities, the non-compact group K decomposes under H in $\text{adj } H$ and a representation which is the adjoint under G and the two-fold antisymmetric under $SU(N)$. The $N = 2$ and 1 theories are exceptional because the scalar manifold M is no longer required by supersymmetry to be a coset space. This means that the metric g_{ij} is not determined by supersymmetry. However, supersymmetry still gives important restrictions on the allowed manifolds.

For $N = 2$, we have scalar fields of the Yang-Mills sector, which are complex $SU(2)$ singlets. The scalar fields of the $\lambda_{\max} = 1/2$ multiplet, called hypermultiplet, are instead $SU(2)$ doublets, which must belong to some symplectic (pseudo-real) representation R_p of the group G . The corresponding manifolds have therefore dimension $2 \dim G$ (for the vector multiplet) and $2 \dim R_p$ for the hypermultiplets. The first manifold turns out to be a Kahler manifold¹⁶ (of complex dimension $\dim G$) while the second is a quaternionic manifold.¹⁷

Kahler manifolds can be parameterized by complex coordinates z^i, z_j^* with a metric locally derivable from a Kahler potential

$$g_{ij} = \frac{\partial^2}{\partial z^i \partial z_j^*} J(z, z^*) . \quad (2.3)$$

However, in the case of $N = 2$ vector multiplets, $J(z, z^*)$ is further restricted to be derivable from an analytic function $f(z)$ through the relation¹⁶

$$J = -\log Y \quad (2.4)$$

with

$$Y = f + f^* - \frac{1}{2} (f_i - f_i^*) (z_i - z_i^*) .$$

Finally in $N = 1$ supergravity the only scalar fields are those belonging to Wess-Zumino $(1/2, 0^+, 0^-)$ multiplets. The two scalar partners of the left-handed (Weyl) fermion form a complex field which parametrizes a Kahler manifold¹⁸⁻²¹ (with no local restriction on the Kahler potential J) of complex dimension $\dim R$, in which R is an arbitrary representation of the gauge group G .

In all $N > 2$ theories the standard supergravity Lagrangian, up to two-derivative interactions, is completely specified by the manifold M and the gauge group G . In $N = 2$ we have the additional freedom in the choice of

the Kahler and quaternionic manifolds for vector multiplets and hypermultiplets.

In $N = 1$ theories we have two further arbitrarinesses,²⁰ the gauge kinetic function $f_{AB}(z)$ and the superpotential $g(z)$, both analytic in the complex scalar field variables, the first function being symmetric in the adjoint indices (AB) of the gauge group G . In matter coupled extended supergravities, an important role is played by the vector fields of the theory. There are two kinds of vector fields, the vectors belonging to the supergravity multiplet which are in the two-fold antisymmetric representation of $SU(N)$ and those in the Yang-Mills multiplet which are $SU(N)$ singlets. The vector field strengths and their dual transform according to a linear representation of the isometry group K of the non-linear σ -model. This is a particular case of a general result due to Gaillard and Zumino²² on coupling of vector fields to nonlinear σ -models. So the linear representation of K must contain the two-fold antisymmetric representation of $SU(N)$ and the adj. rep. of G . This gives a restriction on the possible groups G' which can be gauged in N -extended supergravities. Actually the restriction is more severe since G' must be contained in a subgroup K' of K which does not mix field strengths with their dual.²³ Since K is a non-compact group in general the gauge group G' can be non-compact. This situation occurs if the graviphotons (vectors of the supergravity multiplets) participate in the gauging and are mixed with the matter multiplets.

This is the interesting situation in which a spontaneous breakdown of supersymmetry may occur with a complete or partial super-Higgs effect in Minkowski space.²⁴

If the graviphotons gauge a $SO(N)$ group, then spontaneous supersymmetry breaking generally occurs in anti-de Sitter space. The only exception seems to be $N = 2$ matter-coupled supergravity where a gauging may give broken supersymmetry with vanishing cosmological constant.²⁵

In extended supergravities derived from effective Lagrangians for superstrings the graviphotons generally gauge a trivial $U(1)$ group. In this situation the extrema of the scalar potential correspond to unbroken supersymmetry in Minkowski space.

III. MATTER COUPLING IN 4D, $N = 1$ SUPERGRAVITY WITH LINEAR MULTIPLICETS AND DUALITY TRANSFORMATIONS

In the effective Lagrangians for the massless modes of superstrings, extended supergravities play a significant role for two reasons: there are superstring compactifications which have an extended supersymmetry in $D = 4$ dimensions. Moreover, some $N = 1$ supergravity effective Lagrangians can

be obtained by a suitable truncation of an extended supergravity Lagrangian.

The very first example, due to Witten,²⁶ of a $N = 1$ supergravity Lagrangian inspired by 10-dimensional superstring compactification may be obtained from a consistent truncation of a $N = 4$ Lagrangian. Other interesting Lagrangians, related to the effective theory of orbifold compactifications or 4-dimensional superstrings may be obtained in a similar way.²⁷

$N = 1$ effective Lagrangians obtained with this procedure will correspond to a special choice of the would be arbitrary f_{AB} and g functions introduced in the previous section. However, the interesting theory at low-energies compared to the Planck scale, must have at most a $N = 1$ supersymmetry so we will confine our discussion to the $N = 1$ supergravity Lagrangian coupled to the matter fields which describe the massless modes of superstrings compactified in 4 dimensions.

A generic $N = 1$ supergravity theory, in its standard form,²⁰ describes the coupling of $N = 1$ supergravity to matter and Yang-Mills multiplets.

If we denote by V the (Lie algebra valued) vector multiplets and generically by Z^i the matter multiplets, the standard supergravity Lagrangian is given by the sum of three terms

$$\int d^8\Omega \Phi(Z, \bar{Z}e^V) + \text{Re} \int d^6\Omega g(Z) + \text{Re} \int d^6\Omega f_{AB}(Z) W_a^A W_b^B \epsilon^{ab} \quad (3.1)$$

where $d^8\Omega$ and $d^6\Omega$ are appropriate full superspace and chiral measures respectively and W_a^A is the covariant field strength supermultiplet.

This is the Lagrangian written in the superspace geometry of Wess and Zumino and is the superspace equivalent²⁸ of the standard supergravity Lagrangian in component forms. The Φ function is related to the Kahler potential J by the relation¹⁸

$$J(z, z^*) = -3 \log \Phi(z, z^*) \quad (3.2)$$

where $z^i = Z^i(\theta=0)$. In component form the Kahler invariance of the action (3.1) correspond to the following transformation:

$$J(z, z^*) \rightarrow J(z, z^*) + f(z) + f^*(z^*) ; \quad g(z) \rightarrow g(z) e^{-f(z)} . \quad (3.3)$$

This transformation acts as a chiral rotation (of phase f/f^*) on the matter fermions as well as on the gravitino. However, we can make a Kahler gauge choice (which is singular at the points in which the superpotential vanishes) such that all fermions become Kahler inert. This is obtained by

making a Weyl redefinition¹⁸ on the supervierbein such that the superpotential is set equal to one.

Under this transformation the Φ function becomes

$$\Phi = e^{-\mathcal{G}/3} \quad (3.4)$$

where

$$\mathcal{G} = J + \log |g|^2 \quad (3.5)$$

and the entire Lagrangian depends only on $\mathcal{G}$ and the f_{AB} function. Note that the Yang-Mills term in Eq. (3.1) is invariant under a super-Weyl rescaling of the metric, because of superconformal invariance.

We now consider a special form of the Φ , g and f functions which allow the introduction of linear multiplets, associated with antisymmetric tensor fields. Let us group the set of chiral fields as follows:

$$Z^i = (X^i, S^i) \quad (3.6)$$

and let us assume the Φ , g , and f functions to be of the following particular form

$$\Phi(z, z^*) = \Phi(x, x^*, s^i, \bar{s}^i) \quad (3.7)$$

$$g(z) = e^{a_i s^i} \hat{g}(x) \quad (3.8)$$

$$f_{AB}(z) = C_{AB} (b_i s^i) + \hat{f}_{AB}(x^i) \quad (3.9)$$

$$(a_i, b_i \text{ real coefficients and } C_{AB} = C_{BA})$$

Using a super-Weyl rescaling of the supervielbein, the s -dependent part in (3.8) can be reabsorbed in the Φ function. For the f_{AB} function let us temporarily set $b_i = 0$.

Let us now introduce a set of real multiplets L_i , for each S_i , satisfying the constraints

$$\Sigma(L) = \bar{\Sigma}(L) = 0, \quad \Sigma(L) = (\bar{D}\bar{D} - 8R)L \quad (3.10)$$

where Σ denotes the chiral projection of the L multiplet.¹ A real multiplet subject to (3.10) is called a linear multiplet. It was first introduced in Ref. (29) and then further discussed in the literature,³⁰ in

connection with antisymmetric tensor fields occurring in supergravity theories and superstrings.

In terms of L_i we can rewrite a "classically" equivalent Lagrangian by replacing $S^i + \bar{S}^i \rightarrow U^i$, where U^i are now unconstrained real superfields and by introducing the Lagrange multiplier

$$-\int d^8\Omega U^i L_i. \quad (3.11)$$

By variation with respect to L_i we get $U^i = S^i + \bar{S}^i$. However, if we instead vary with respect to U^i we obtain the dual Lagrangian, expressed in terms of L_i through the Legendre transform

$$\frac{\partial \Phi}{\partial U^i} = L_i.$$

Note that we can express U^i as functions of L_i only if the transformation is invertible.

Let us consider the specific case which is relevant for superstring compactifications. In this case we have exactly one antisymmetric tensor, which is embedded in a linear multiplet L . This is the S chiral multiplet, related to the dilaton, in the notation of Ref. 26. The Φ function, for the simplest superstring low-energy effective Lagrangian considered by Witten²⁶ is

$$\Phi = (S + \bar{S})^{1/3} (T + \bar{T} - C^i \bar{C}_i)^{1/3} \tilde{\Phi} \quad (3.13)$$

in which T is the Wess-Zumino multiplet related to the breathing mode of the Calabi-Yau manifold and C^i are the charged-matter fields of the theory. By performing the duality transformation on Eq. (3.13)

$$\frac{\partial \Phi}{\partial U} = L \rightarrow U = \left(\frac{3L}{\tilde{\Phi}} \right)^{-3/2} \quad (3.14)$$

we get the Lagrangian expressed in terms of the linear multiplet:

$$\int d^8\Omega 2(L)^{-1/2} \hat{\Phi}^{3/2} \quad (3.15)$$

where

$$\hat{\Phi} = \tilde{\Phi}/3 = \frac{T + \bar{T} - C^i \bar{C}_i}{3}.$$

From Eq. (3.14) we now have that the scalar partner of the $b_{\mu\nu}$ field contained in L is

$$L(\theta=0) = \Phi_S(\theta=0) . \quad (3.16)$$

The linear multiplet can be solved in terms of a spinor multiplet as follows³⁰

$$L = \mathcal{D}^\alpha L_\alpha + \bar{\mathcal{D}}_{\dot{\alpha}} \bar{L}^{\dot{\alpha}} \quad (3.17)$$

where

$$\bar{\mathcal{D}}_{\dot{\alpha}} L_\alpha = 0 . \quad (3.18)$$

L is gauge invariant under the transformation

$$L_\alpha \rightarrow L_\alpha + i(\bar{\mathcal{D}}\bar{\mathcal{D}}-8R) \mathcal{D}_\alpha \Sigma \quad (3.19)$$

where Σ is an arbitrary real superfield. This is a consequence of the identity

$$\mathcal{D}^\alpha (\bar{\mathcal{D}}\bar{\mathcal{D}}-8R) \mathcal{D}_\alpha \Sigma = \bar{\mathcal{D}}_{\dot{\alpha}} (\mathcal{D}\mathcal{D}-8\bar{R}) \bar{\mathcal{D}}^{\dot{\alpha}} \Sigma . \quad (3.20)$$

IV. CHERN-SIMONS COUPLINGS

In the present section we would like to discuss the situation in which the b_i coefficients are not vanishing. In this case one gets an S dependent coupling to the Yang-Mills field strength of the form

$$\text{Re} \int d^6\Omega b_i S_i W^2 . \quad (4.1)$$

Using several properties of the field strength multiplet W_α^A one can show that each term can be written as a full super-space integral as follows³¹

$$\text{Re} \int d^6\Omega S_i W^2 = \int d^8\Omega (S_i + \bar{S}_i) \Omega_3(V) \quad (4.2)$$

when $\Omega_3(V)$ is a real gauge-dependent multiplet. Eq. (4.2) is crucial in order to perform a duality transformation since, as before, we replace $S_i + \bar{S}_i \rightarrow U_i$. Then Eq. (3.12) is modified as follows:

$$\frac{\partial \Phi}{\partial U^i} = L_i - b_i \Omega_3(V) = \hat{L}_i . \quad (4.3)$$

As a consequence of Eq. (4.2) the duality transformed Lagrangian will only involve the "invariant" linear multiplets $\hat{L}_i$.

From Eq. (4.2) we have that the gauge variation of $\Omega_3(V)$ must be a linear multiplet since

$$\int d^8\Omega(s_i + \bar{s}_i) \delta\Omega_3(V) = 0 . \quad (4.4)$$

This is also seen by using the property³¹

$$(\bar{D}\bar{D}-8\mathcal{R}) \Omega_3^{YM}(V) = W^2(V) , \quad (W^2 = \text{Tr } WW) \quad (4.5)$$

and its complex conjugate. From (4.3) and (4.4) it follows that

$$\delta\Omega_3^{YM}(V) = \mathcal{D}^\alpha \delta_\alpha(V, \Lambda, \bar{\Lambda}) + \bar{\mathcal{D}}_{\dot{\alpha}} \delta^{\dot{\alpha}}(V, \Lambda, \bar{\Lambda}) , \quad (4.6)$$

where δ_α is a chiral multiplet and Λ is the gauge transformation chiral parameter. Because of Eq. (4.6) it follows that L_α must transform non-trivially under Yang-Mills transformations

$$\delta_{YM}^{L_i \alpha} = b_i \delta_\alpha(V, \Lambda, \bar{\Lambda}) . \quad (4.7)$$

Under the above transformation the linear multiplet L_i transforms accordingly

$$\delta_{YM}^{L_i} = b_i \delta\Omega_3^{YM}(V, \Lambda, \bar{\Lambda}) \quad (4.8)$$

so that the $\hat{L}_i$ field strength is invariant. Eqs. (4.7) and (4.8) are the 4-dimensional superspace analogue of the Yang-Mills transformation under which the antisymmetric tensor field $b_{\mu\nu}$ transforms in the 10-dimensional supergravity. Indeed the dimensional reduction of the 10-dimensional supergravity Lagrangian³² gives precisely a coupling of the form³³

$$\text{Re} \int d^6\Omega S W^2 \quad (4.9)$$

where S is the chiral multiplet containing the dilaton and the pseudoscalar field b which is dual to the 4-dimensional antisymmetric tensor field $b_{\mu\nu}$. The $\Omega_3^{YM}(V)$ supermultiplet is the Yang-Mills Chern-Simons supermultiplet. It contains in its vector $(\theta\sigma_\mu\bar{\theta})$ component the dual of the three-form ω_{3YM} , among other terms. Its superfield and component expressions have recently been derived.^{31,34} The $\hat{L}$ field strength multiplet contains, in its vector-component, the dual of the covariant field strength three-form

$$H = dB + b\omega_{3YM} \quad (4.10)$$

among other terms.

The anomaly freedom in ten-dimensional supergravity, as discovered by Green and Schwarz,⁶ needs a further modification of the H three-form since $b_{\mu\nu}$ gauge transforms under local Lorentz transformations. In the chiral multiplet language this amounts to add to the standard supergravity Lagrangian a new higher derivative coupling of the form³⁵

$$\text{Re} \int d^6\Omega a S W_{\alpha\beta\gamma}^2 \quad (a = \text{real coefficient}) \quad (4.11)$$

where $W_{\alpha\beta\gamma}$ is the Weyl supermultiplet.¹ This multiplet has the property of containing the Hirzebruch signature term

$$\text{Tr}(R \wedge R) \quad (4.12)$$

in its last component. The component expression of Eq. (4.11) contains therefore a term

$$\text{Im } s \text{Tr}(R \wedge R) \quad (4.13)$$

which is the duality transformed from the Chern-Simons coupling in $D = 4$ dimensions.³⁵

In order to perform the superspace duality transformation, one has to use the property³¹

$$\text{Re} \int d^6\Omega S W_{\alpha\beta\gamma}^2 = \int d^8\Omega (S + \bar{S}) \Omega_3^L \quad (4.14)$$

where Ω_3^L is now a gauge-variant multiplet which will contain, in its vector component, the dual of the Chern-Simons Lorentz three-form $\omega_3^L(\omega_\mu)$, where ω_μ is the spin connection.

As before Eq. (4.14) will give a new modification of the $\hat{L}$ field strength multiplet

$$\hat{L} = L - b\Omega_3^{YM} - a\Omega_3^L. \quad (4.15)$$

From (4.14) it follows that

$$(\bar{D}\bar{D}-8R) \Omega_3^L = W_{\alpha\beta\gamma}^2. \quad (4.16)$$

This implies that $\delta_L \Omega_3^L$ is a linear multiplet so that $\delta_L \Omega_3^L = \mathcal{D}^\alpha \delta_\alpha^L$

$$+ \bar{\mathcal{D}}_{\alpha} \dot{\delta}^{\alpha L}.$$

From Eq. (4.16) it follows that $\hat{L}$ satisfy the "Bianchi identity"³¹

$$(\bar{\mathcal{D}}\bar{\mathcal{D}}-8R) \hat{L} = -b w_{YM}^2 - a w_{\alpha\beta\gamma}^2. \quad (4.17)$$

The "prepotential" chiral superfield will now transform as

$$\delta L_{\alpha} = b \delta_{\alpha}^{YM} + a \delta_{\alpha}^L \quad (4.18)$$

such that $\hat{L}$, as defined by Eq. (4.15) will be invariant. It is worthwhile to notice that unlike the Yang-Mills case, the Lorentz Chern-Simons form is not unique. In fact we can add any invariant combination of the other curvature square supermultiplets^{28,35}

$$\bar{R}\bar{R} \text{ and } E_{\alpha\dot{\alpha}} \cdot E^{\alpha\dot{\alpha}} \quad (4.19)$$

without affecting Eq. (4.13). Here R and $E_{\alpha\dot{\alpha}}$ are the scalar curvature and the Einstein multiplet respectively.^{1,28} The addition of these new terms modifies the scalar supersymmetric counterpart of Eq. (4.13). For the superinvariant (4.11) the term is given by

$$\text{Re } s \, C_{\mu\nu\rho\sigma}^2 \sqrt{-g} \quad (4.20)$$

when $C_{\mu\nu\rho\sigma}$ is the Weyl tensor. However, if we use the super Gauss-Bonnet combination³⁶ given by

$$w_{\alpha\beta\gamma}^2 + (\bar{\mathcal{D}}\bar{\mathcal{D}}-8R) \left[\frac{1}{2} E_{\alpha\dot{\alpha}}^2 - 2\bar{R}\bar{R} \right]. \quad (4.21)$$

Eq. (4.20) is modified into

$$\text{Re } s \, \varepsilon_{abcd} R^{ab} \wedge R^{cd} \quad (4.22)$$

where the curvature square invariant is the Euler characteristic of the manifold. This corresponds to a new Chern-Simons supermultiplet given by

$$\Omega_3^2 + \frac{1}{2} E_{\alpha\dot{\alpha}}^2 - 2\bar{R}\bar{R}. \quad (4.23)$$

Because the last two terms are invariant under local Lorentz transformations in superspace, it follows that the new covariant field strength

$$\hat{L}^{\text{new}} = \hat{L} - a(1/2 E_{\alpha\alpha}^2 - 2RR) \quad (4.24)$$

is still invariant under the same gauge transformations as before.

The combination given by Eq. (4.21) corresponds to the supersymmetric completion of the 10-dimensional ghost-free combination considered by Zwiebach³⁷

$$R_{\mu\nu\rho\sigma}^2 - 4R_{\mu\nu}^2 + R^2 \quad (4.25)$$

which upon reduction to $D = 4$, reproduces the expression given in (4.22).

The previous example is the simplest case of departure from the standard supergravity Lagrangian given by Eq. (3.1), because of superstring effects. Other examples of higher curvature modifications will be given in Section VI.

The higher curvature terms considered in this section have an alternative derivation, since they can be deduced from 10-dimensional anomaly-free supergravity, without any reference to strings. An interesting phenomenon happens when the component Lagrangian with the inclusion of the Lorentz Chern-Simons term is worked out.³⁸

It turns out that the supersymmetric completion of the Chern-Simons 3-form contains, because of supersymmetry, an additional Chern-Simons form of the axial vector field A_μ of conformal supergravity.

In the component Lagrangian, the equations of motion of the supergravity auxiliary fields are no longer algebraic. They can only be solved by iteration starting from the lowest order solution ($a = 0$). This indicates that a component expression of $D = 10$ anomaly-free supergravity necessarily involves infinitely many terms in the basic massless fields. This is indeed what happens as shown by recent derivations of the ten-dimensional anomaly-free Lagrangians,³⁹ by showing the existence of an exact solution for the modified superspace Bianchi identities.⁴⁰

The presence of Kahler transformations as a transformation of the type

$$J \rightarrow J + A + \bar{A} \quad (4.26)$$

on the Kahler superfield $J = -3 \log \Phi$ suggests an additional possible Chern-Simons form connected with the Kahler geometry.

Indeed one can add to the standard Lagrangian a higher-derivative term in the σ -model variables of the form

$$\text{Re} \int d^6 \Omega \, s w^2 \quad (4.27)$$

where W_α is the composite σ -model (Abelian) field strength introduced in Ref. 35:

$$W_\alpha = (\bar{D}\bar{D}-8R) \mathcal{D}_\alpha (\log \tilde{\Phi}) . \quad (4.28)$$

It is obvious that this is a quartic derivative interaction in the σ -model variables. As before, by using the previous manipulation, we can write Eq. (4.27) as follows:

$$c \int d^8\Omega(S+\bar{S}) \Omega_3^K(\tilde{\Phi}) . \quad (4.29)$$

Therefore, by performing the duality transformation, we get a new possible Chern-Simons modification to $\hat{L}$

$$\hat{L}' = \hat{L} - c\Omega_3^K(\tilde{\Phi}) . \quad (4.30)$$

V. THE 4D--WESS-ZUMINO ANOMALY TERM

In superstring effective actions there are several modifications to the standard supergravity form. One type of modification consists in higher- curvature terms, which are already present at the string tree level as a consequence of integration over the massive string modes. Other terms present in the superstring effective actions are superstring loop effects. Because superstrings are believed to be finite, order by order in perturbation theory, these are calculable effects, and in general will modify both the standard supergravity terms and the higher curvature and higher derivative terms.

Recently^{41,42,43} some of the one-loop effects have been calculated, in particular those coming from the Green-Schwarz Wess-Zumino term, which are a pure one-loop string effect. From the string point of view different string loops scale with different powers of the dilaton field. The one-loop effects are globally scale invariant under a dilaton shift

$$\tilde{\Phi} \rightarrow \tilde{\Phi} + c . \quad (5.1)$$

In the 4-dimensional effective theory, the classical Lagrangian scale as²⁶

$$\mathcal{L} \sqrt{-g} \rightarrow \lambda \mathcal{L} \sqrt{-g} \quad (5.2)$$

under a global scale transformation of the metric and of the S field

$$g_{\mu\nu} \rightarrow \lambda g_{\mu\nu} , \quad S \rightarrow \lambda S , \quad (5.3)$$

the other bosonic fields remaining inert. In higher loops the Lagrangian scale as⁴²

$$\mathcal{L}_{n\text{-loop}} \sqrt{-g} \rightarrow \lambda^{(1-n)} \mathcal{L}_{n\text{-loop}} \sqrt{-g} \quad (5.4)$$

under the same transformation. This can be translated in superfield language by saying that the S field scale as λS while $\int d^8\Omega$ scale as $\int d^8\Omega \lambda^{2/3}$. The rescaling of the metric actually correspond to a constant Weyl superscaling of the supervierbein

$$E \rightarrow E \lambda^{2/3} . \quad (5.5)$$

Under the same rescaling the chiral measure rescales as $\int d^6\Omega \lambda$ which therefore implies that all terms in the standard supergravity action rescale in the same way. The rescaling properties are best expressed in the language of superconformal geometry,⁴⁴ where a compensating multiplet S_0 is introduced.

In superconformal geometry the global scale transformation acts on S_0 and S but not on any other superfield. Then the scale transformation of S_0 is assumed to be $S_0 \rightarrow S_0 \lambda^{1/3}$ and the standard supergravity action takes the form⁴⁴

$$\int d^8\Omega \Phi S_0 \bar{S}_0 + \text{Re} \int d^6\Omega f_{AB}(Z) W^A W^B + \text{Re} \int d^6\Omega g(Z) S_0^3 . \quad (5.6)$$

At one-loop the action should be scale-invariant. This means that

$$\Phi_{1\text{-loop}} = \varepsilon (S+\bar{S})^{-2/3} \Phi_1(X) , \quad (\Phi_1 = \tilde{\Phi}^2 + \dots) \quad (5.7)$$

$$f_{AB1\text{-loop}} = f_{AB}(X) . \quad (5.8)$$

Eq. (5.7) gives a modification to the Kahler potential (Eq. (3.2)) of the form

$$J_{1\text{-loop}} = - \frac{3\varepsilon}{S+\bar{S}} \tilde{\Phi} . \quad (5.9)$$

In the dual formulation, the one-loop contribution to the standard supergravity action is

$$L\tilde{\Phi} \quad (5.10)$$

the only term which is conformal invariant and scale invariant.

In higher n-loop order the L dependence is

$$\left(\frac{L}{s_0 \bar{s}_0} \right)^{1/2(3n-1)} s_0 \bar{s}_0 .$$

Another important one-loop effect is derived in the presence of 4-dimensional compactifications with a U(1) with anomalous fermion representations.⁴³ In this case the Wess-Zumino term survives down to D = 4 and gives⁴⁵

$$cLV \tag{5.12}$$

where V is the vector multiplet of the anomalous U(1) gauge group. Such situations have recently been discussed and found to occur in SO(32) superstring compactification⁴⁶ as well as in 4-dimensional superstrings.⁹ For a U(1) gauge group the anomalous gauge transformation of the prepotential multiplet is

$$\delta_{\text{gauge}}^L \alpha = i\Lambda W_\alpha \tag{5.13}$$

which gives rise to the anomaly term

$$c \int d^8 \Omega \delta(LV) = c \text{Re} \int d^6 \Omega (i\Lambda W^2) . \tag{5.14}$$

It is easily seen that the term in Eq. (5.12) gives rise to a mass term for the vector field, whose longitudinal component is precisely the antisymmetric tensor $b_{\mu\nu}$.

It is convenient for this purpose to make a duality transformation. The inverse duality transformation is obtained by starting with a function of L, then taking L unrestricted and by adding the Lagrange multiplier

$$\int d^8 \Omega L(S+\bar{S}) \tag{5.15}$$

where S is chiral. In the presence of the anomaly term given by Eq. (5.12), we see that the effect is of replacing $S+\bar{S}$ with $(S+\bar{S}+cV)$ so that the duality transformed Lagrangian will contain a Kahler potential for the S field of the form

$$J(S, X) = J(S+\bar{S}+cV, X, \bar{X} e^V) . \tag{5.16}$$

Eq. (5.16) indicates that S transforms under U(1) gauge transformations as

follows⁴³

$$S \rightarrow S - icA . \quad (5.17)$$

The multiplet $S+\bar{S}+cV$ (or more precisely its exponential) becomes in fact a massive vector multiplet. The field $\text{Im } s$ becomes the longitudinal component of V_μ while $\text{Re } s$ is its superpartner, which in absence of supersymmetry breaking, will have the same mass as the vector field V_μ .

Since the field $\text{Re } s$ is related to the dilaton mode, this mechanism shows that already at one loop, the dilaton may acquire a mass.

VI. HIGHER-ORDER CURVATURE TERMS AND KÄHLER GEOMETRY

In the present section we discuss further higher curvature modifications⁴⁷ to the 4-dimensional standard supergravity form. We have seen in Section IV that curvature square terms are "geometrical" in the sense that they amount to modify the Bianchi identity of the three-form field strength multiplet containing the antisymmetric tensor field $B_{\mu\nu}$. The Bianchi identity of the $\hat{L}$ superfield is given by

$$(\bar{\mathcal{D}}\bar{\mathcal{D}}-8R)\hat{L} = -bW_{YM}^2 - aW_{\alpha\beta\gamma}^2 + (\bar{\mathcal{D}}\bar{\mathcal{D}}-8R)X , \quad (6.1)$$

where X is a covariant term. The first two terms on the right-hand side of Eq. (6.1) are related to the supersymmetric version of the Green-Schwarz anomaly cancellation mechanism. Higher order modifications^{47,48} can be described both in the formalism using $\hat{L}$ or in the dual formulation which makes use of the chiral multiplet S .

We will consider here an example of higher order terms which includes 4-derivative interactions in the Yang-Mills fields and eight-derivative interactions in the gravitational field. In the framework of the superconformal tensor calculus we can use the global scale invariance of the classical action to fix the powers of the several fields involved. In particular we noticed that the $\hat{L}$ multiplet is inert under the global symmetry under which the classical action rescales as $A \rightarrow \lambda A$.

Since $S_0 \rightarrow S_0 \lambda^{1/3}$, this fixes the L and S_0 dependence in standard supergravity to be

$$(S_0 \bar{S}_0)^{3/2} L^{-1/2} \quad (6.2)$$

because of superconformal invariance. The $\hat{\Phi}$ dependence in the classical action can be either fixed by demanding Kähler invariance or by using an extra symmetry related to the breathing modes of the manifold.⁴⁹ For the

standard supergravity form this uniquely gives the superconformal version of Eq. (3.15), i.e.

$$\int d^8\Omega \hat{L}^{-1/2} (S_0 \bar{S}_0 \hat{\Phi})^{3/2} . \quad (6.3)$$

A quartic curvature term can be easily constructed by multiplying the same expression which occur in Eq. (6.3) by the superfield

$$\frac{W^2 \bar{W}^2}{\hat{L}^3} . \quad (6.4)$$

This superfield has conformal weight zero and is inert under the global symmetry. In Eq. (6.4) W generically denotes supercurvature, either Yang-Mills, gravitational or σ -model field strength. Higher curvature terms may also be constructed.⁵⁰ Beyond the quartic curvature term, this construction requires the $(\bar{D}\bar{D}-8R)$ operation in order to construct superfields whose first components contain bosonic curvature terms. The number of terms increases considerably if one takes care of possible different Yang-Mills and Lorentz- index contractions.

A basic restriction on the classical Lagrangian comes by demanding Kahler invariance of the higher order terms,⁵⁰ which requires the $\hat{\Phi}$ function to appear in the combination $\hat{\Phi} S_0 \bar{S}_0$, with the compensating multiplet. The Kahler geometry of the tree-level Lagrangian gets modified by higher string loop connections. This is best seen by noticing that dimensional reduction of the Wess-Zumino anomaly term produces a D-type contribution to the one- loop effective action. Eq. (5.10) can be confronted with the results of Ref. 41 for some of the bosonic components. Its form is given by:

$$a \hat{L} (T + \bar{T} - C\bar{C}) \quad (6.5)$$

which in turn leads, in the dual version, to a modification of the Kahler potential of the type

$$J = -\log\{S + \bar{S} + a[(T + \bar{T}) - C\bar{C}]\} - 3 \log(T + \bar{T} - C\bar{C}) \quad (6.6)$$

where actually only the first term in the Taylor expansion of the "a" dependent part has to be retained in Eq. (6.6).

VII. CONCLUSIONS

In the present review we have discussed several aspects of the effective Lagrangians for superstrings with a surviving $N = 1$

supersymmetry in 4- dimension. We have seen that the use of superspace techniques, superconformal geometry and linear multiplets considerably simplify the analysis of the string corrections to the standard supergravity Lagrangian.

An important role in examining both the higher curvature terms as well as the string loop effects is played by the string global scale symmetries.

Another important issue has been the supersymmetric formulation of the Green-Schwarz mechanism, the introduction of super--Chern-Simons terms and their influence in higher-loop corrections.

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