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PARITY-DOUBLING CHIRAL SOLITONS WITH VECTOR MESONS

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A B S T R A C T

We argue the presence of new classical solutions with spontaneous parity-violation in chiral soliton models coupled with vector fields. Using a variational method, we find that these solutions become stable minima when a coupling of fourth-order stabilizing term is larger than a critical value. These form an almost degenerate parity-doublet via tunnelling effects at the semi-classical level.

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Recent developments in chiral soliton models¹⁾ have pointed to the importance of the low-lying vector (and axial-vector) mesons. Incorporating such mesons²⁻⁴⁾ is certainly necessary for meson-based descriptions of hadrons originating from the pioneering work of Skyrme⁵⁾ to be more complete.

The purpose of this note is to report a new aspect of chiral solitons (Skyrmions) coupled with vector fields; the occurrence of spontaneous parity-violation at the classical level or of an almost degenerate parity-doublet at the semi-classical level. In the Lagrangian constructed to be parity invariant, such a new class of soliton solutions is obtained with the most general spherically symmetric ansatz for the ρ -field. The extended ansatz is a mixture of parity even terms and the 't Hooft-Polyakov type (parity odd) term. It leads to two classes of stationary solutions; one is the parity-conserving (PC) solutions only with the parity odd term, another is the spontaneously parity-violating (SPV) ones with both parity even and odd contributions. The latter solutions appear as a pair transformed each other under the parity transformation, being degenerate in masses at the classical level. It is found in the ρ -coupled chiral model with a fourth-order derivative term that the SPV solutions become stable minima when a coupling of the stabilizing term is larger than a critical value, $e > e_c$, while the conventional PC solution is the minimum for $e < e_c$. A simple variational calculation estimates $e_c = 5 \sim 7$ with other parameters fixed from low-energy relations. The exact degeneracy in the SPV states for $e > e_c$ will be resolved via tunnelling effects at the semi-classical level, restoring the manifest parity invariance. This leads then to an almost degenerate parity-doublet state, as the mixing transition is found to be small.

The Lagrangian we start with is that of a chiral $SU(2)_L * SU(2)_R$ non-linear σ -model coupled with ρ -mesons in the massless pion limit:

$$\mathcal{L} = \frac{f_\pi^2}{4} \text{Tr}[\partial_\mu U \cdot \partial^\mu U^\dagger] - a \frac{f_\pi^2}{4} \text{Tr}[D_\mu \xi \cdot \xi^\dagger + D_\mu \xi^\dagger \cdot \xi]^2 - \frac{1}{2g^2} \text{Tr}[F_{\mu\nu}^2] + \mathcal{L}_4, \quad (1)$$

where $f_\pi = 93$ MeV is the pion decay constant. The chiral field $U = (\xi^\dagger)^2 = \exp[2i\pi(x)/f_\pi]$ is coupled with the ρ -field V_μ through the covariant derivative, $D_\mu = \partial_\mu - iV_\mu$. The field strength is given by $F_{\mu\nu} = i[D_\mu, D_\nu]$. We have used the matrix-valued notations, $\pi(x) = \pi^a(x) \cdot \tau^a/2$, $V_\mu(x) = gV_\mu^a(x) \cdot \tau^a/2$. The fourth-order derivative term $\mathcal{L}_4$ is needed to stabilize soliton solutions⁶⁾. It will be sufficient for our purpose to consider two models with

$$\begin{aligned}
 (A) \quad \mathcal{L}_4 &= \frac{1}{32e_A^2} \text{Tr} [\partial_\mu U \cdot U^\dagger, \partial_\nu U \cdot U^\dagger]^2 \\
 (B) \quad \mathcal{L}_4 &= -\frac{1}{32e_B^2} \text{Tr} [D_\mu \xi \cdot \xi^\dagger + D_\mu \xi^\dagger \cdot \xi, D_\nu \xi \cdot \xi^\dagger + D_\nu \xi^\dagger \cdot \xi]^2.
 \end{aligned} \tag{2}$$

The stabilizer in (A) is the Skyrme term, and that in (B) contains the vector field^{*)}. The Lagrangian (1),(2) may be derived on the basis of the recently proposed scenario of hidden local symmetry in which the ρ is considered as a dynamical gauge boson⁷⁾. The hidden gauge has been fixed to be unitary here, $\xi_L^\dagger = \xi_R = \xi^\dagger$. One may also construct (1), (2) as an extension of the Weinberg Lagrangian⁸⁾ by adding higher derivative terms.

To find soliton solutions, we take the standard hedgehog ansatz for the chiral field

$$U(\vec{x}) = \exp[i(\hat{x} \cdot \tau) F(r)], \tag{3}$$

where $(\hat{x} \cdot \tau) = \vec{x} \cdot \vec{\tau}/r$. For the vector field, however, we employ the most general spherically symmetric one^{**) invariant under combined spatial and isospin rotations:}

$$\begin{aligned}
 V_i(\vec{x}) &= \epsilon_{ijk} \hat{x}_j \tau_k \frac{G(r)}{2r} + \{ \tau_i - \hat{x}_i (\hat{x} \cdot \tau) \} \frac{H(r)}{2r} \\
 &\quad + \hat{x}_i (\hat{x} \cdot \tau) \frac{K(r)}{2r},
 \end{aligned} \tag{4}$$

and $V_0(\vec{x}) = 0$. This leads to the static energy-functional

$$\begin{aligned}
 M[F, G, H, K] &= - \int \mathcal{L} d^3x = M_0 + M_4, \\
 M_0 &= \frac{4\pi f_\pi^2}{m_\rho} \int_0^\infty d\eta \left[\frac{1}{2} (\eta^2 F'^2 + 2 \sin^2 F) \right. \\
 &\quad + a \{ (\hat{G} + \cos F)^2 + H^2 + \frac{1}{2} K^2 \\
 &\quad + \frac{1}{2\eta^2} (\hat{G}^2 + H^2 - 1)^2 + (\hat{G}' - \frac{HK}{\eta})^2 \\
 &\quad \left. + (H' + \frac{\hat{G}K}{\eta})^2 \} \right],
 \end{aligned} \tag{5}$$

*) The overall sign of the terms in (2) is chosen in such a way that their contributions to the static energies are positive definite.

**) Note that the Euler-Lagrange equations for the ansatz (3), (4) coincide with the equations obtained as the conditions for the static energy given in (5), (6) to be stationary with respect to variations of F, G, H, K .

and

$$M_4 = \frac{2\pi m_p}{e_A^2} \int_0^\infty d\eta \left\{ \frac{1}{\eta^2} \sin^4 F + 2 (F' \sin F)^2 \right\} \quad \text{for (A)}$$

(6)

$$M_4 = \frac{2\pi m_p}{e_B^2} \int_0^\infty d\eta \frac{1}{\eta^2} \left[\{ (\hat{G} + \cos F)^2 + H^2 + K^2 \}^2 - K^4 \right] \quad \text{for (B)}$$

where $\hat{G} = G-1$, and the primes refer to $\eta (=m_\rho r)$ derivatives. We take for simplicity $a=2$ for which the Kawarabayashi-Suzuki-Riazuddin-Fayyazudin (KSRF) relation $m_\rho^2 = 2g^2 f_\pi^2$ holds. The observed value $m_\rho = 770$ MeV fixes $g = g_{\rho\pi\pi} = 5.9$. The only free parameter left is then $e_{A(B)}$.

One finds in (5) and (6) that there exists a discrete symmetry, $M[F, G, H, K] = M[F, G, -H, -K]$. Hence, $H(r) \equiv 0$ and $K(r) \equiv 0$, for which the vector field (4) has definite parity, solve the stationary equations with respect to H and K , $\delta M / \delta H|_{H=K=0} = \delta M / \delta K|_{H=K=0} = 0$. These PC solutions were discussed in a previous paper⁹⁾.

Let us investigate SPV solutions with non-vanishing $H(r)$ and $K(r)$. We may redefine with the above discrete symmetry a parity transformation as $\vec{x} \rightarrow -\vec{x}$, $G \rightarrow G$, $H \rightarrow -H$, $K \rightarrow -K$, so that $V_i(\vec{x}) \rightarrow -V_i(-\vec{x})$. It transforms the degenerate states (F, G, H, K) and $(F, G, -H, -K)$ each other. We confine ourselves here to the topological (baryon) number $Q = B = 1$ sector. The boundary conditions are given by $F(0) = \pi$, $G(0) = 2$, $H(0) = K(0) = 0$, and $F(\infty) = G(\infty) = H(\infty) = K(\infty) = 0$. We will not solve numerically the stationary equations for the energy-functional. Instead, we shall make here a variational calculation, because it may give some insights into both the PC and SPV solutions in a simple manner. We take the trial functions^{*)}:

$$\begin{aligned} \cos F(\eta) &= \frac{\eta^4 + 2R^2\eta^2 - R^4}{(\eta^2 + R^2)^2}, \\ G(\eta) &= \frac{2B^4}{(\eta^2 + B^2)^2}, \\ H(\eta) &= \frac{2\delta B^2 \eta (\eta^2 + 2B^2)^{\frac{1}{2}}}{(\eta^2 + B^2)^2} \end{aligned} \quad (7)$$

*) The trial function for $F(\eta)$ is a simplified form of the one given in Ref. 10).

Note that the function $K(\eta)$ is not an independent variable, because the energy-functional (5) and (6) contains no derivative terms of K . It can be then expressed in terms of $F(\eta)$, $G(\eta)$ and $H(\eta)$ through a constraint equation, which we use below. The functions (7) satisfy the required boundary conditions at the origin and infinity. The parameters R , B specify the extension of $F(\eta)$, $G(\eta)$ in η space, and δ the size of $H(\eta)$ and $K(\eta)$ contributions. The substitution of (7) into (5) and (6) reduces the energy-functional to a variational-energy $M(R, B, \delta)$, a function of three parameters. The static mass M is obtained by minimizing $M(R, B, \delta)$ with respect to these parameters. The variational solutions and resulting masses are shown in Figs. 1 and 2. The masses and the parameters for the solutions are also given in Table I. We find that both in (A) and (B) the PC solutions located at $\delta = 0$ are stable minima for $e_{A(B)} < e_{A(B),c}$. When $e_{A(B)}$ increases, however, there appear the SPV solutions with finite values of $\delta = \pm\delta_0$. For $e_{A(B)} > e_{A(B),c}$ they become stable minima, while the PC ones turn to local minima or saddle points. Our variational calculation estimates the critical couplings $e_{A,c} = 6.9$ and $e_{B,c} = 5.7^{*)}$. As a test of the numerical accuracy of our trial functions, we compared the $F(\eta)$ and $G(\eta)$ in the variational solutions with those in the exact solutions of the stationary equations. It turns out that the masses obtained from the variational solutions agree with those from the exact ones within 1% both in (A) and (B).

To study further the "phase transition" between the PC and SPV solutions, let us consider a variational mass $M(\delta)$ obtained at the middle stage of the minimization procedure; we solve the minimization equations for R and B , $\partial M(R, B, \delta)/\partial R = \partial M(R, B, \delta)/\partial B = 0$, which determine functions $R = R(\delta)$, $B = B(\delta)$. The $M(\delta)$ is obtained by substituting these into $M(R, B, \delta)$, and is shown in Fig. 3. The SPV solutions consist of a pair of degenerate states with $\delta = \pm\delta_0$. At the semi-classical level, a tunnelling transition will give rise to a mass difference between them; the classically degenerate states with mass M , denoted as $|+\rangle$ and $|-\rangle$, form via the mixing the parity eigenstates, $|+\rangle \pm |-\rangle$, with mass $M \mp (\Delta M/2)$. We now give an estimation of the ΔM using the variational parameter δ . The soliton states are assumed to mix each other via the path specified by δ as shown in Fig. 3. We may estimate the probability for the "coordinate" δ to tunnel from $\delta = -\delta_0$ to $\delta = \delta_0$ by introducing the time dependence in δ . Substituting (7) with $R = R(\delta)$ and $B = B(\delta)^{**})$ into the Lagrangian density (1), (2), and integrating over x -space, we obtain the

*) If the exact numerical solutions are found, they may have smaller values of the critical couplings than those given here.

**) In the calculation below, we approximate $R = R(\delta)$, $B = B(\delta)$, by linear functions, for simplicity.

Lagrangian for $\delta(t)$: $L = N(\delta)\dot{\delta}^2/2 - M(\delta)$, where $M(\delta)$ is defined above, and $N(\delta)$ emerges from those terms which contain time derivatives in the Lagrangian (1), (2). In the WKB approximation¹¹⁾ ΔM is expressed in terms of the tunnelling probability calculated with $N(\delta)$, $M(\delta)$. It is found that $\Delta M/M$ is less than 1% for $7 \lesssim e_A \lesssim 30$ in the model (A). It increases from 1% up to 10% as e_B decreases from 30 to 7.

Finally, we make a few remarks on the SPV solutions and the parity-doubling states:

(i) It will be interesting to compare the structure of the soliton solution here with the 't Hooft-Polyakov monopoles. The crucial difference between them is that the local gauge freedom can be used in the monopole case to make the hedgehog ansatz parity invariant, while no such a gauge freedom is left in the chiral model. In other words, it is not possible to arrange the hidden local gauge in such a way that the ansatz respects the parity invariance.

(ii) The occurrence of the SPV and of the parity doublets will be not specific to the models considered here. Rather, it is likely to be a generic feature of the chiral models coupled with the ρ -mesons, though whether the solutions become the absolute minima depends on coupling strength of the soliton stabilizing terms. For instance, such solutions would appear in the ω -stabilized models with the ρ (and A_1)⁴⁾. Although the only PC states are relevant to the effective low-energy theories to QCD, the SPV or parity-doubling states would play a role in effective theories at TeV or much higher energy regions.

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FIGURE CAPTIONS

Fig. 1: a) The variational solutions for the functions of (7) in the model (A) for $e_A = 10$;
b) Those in the model (B) for $e_B = 10$.

Fig. 2: a) The static mass versus the coupling e_A in the model (A);
b) That in the model (B);
The solid lines represent the PC solutions and the SPV ones for $e_A < e_{A,c} = 6.9$, $e_B < e_{B,c} = 5.7$ and for $e_A > e_{A,c}$, $e_B > e_{B,c}$, respectively. The dashed lines represent the PC solutions which are local minima or saddle points in the latter regions.

Fig. 3: a) The variational mass $M(\delta) = M(-\delta)$ versus the parameter δ for several values of e_A in the model (A);
b) Those in the model (B).

TABLE CAPTION

Table I: a) The static masses and the parameters, R , B , δ for the variational solutions in the model (A);
b) Those in the model (B).

Fig. 1a

Fig. 1b

Fig. 2b

Fig. 3a

Fig. 3b

	PC				SPV			
	M(MeV)	δ	R	B	M(MeV)	δ	R	B
$e_A = 4$	2054	0	3.2	3.6	—	—	—	—
6	1593	0	2.4	2.9	1687	± 0.9	1.3	0.8
8	1391	0	2.0	2.6	1288	± 1.0	0.9	0.4
10	1283	0	1.8	2.4	1035	± 1.0	0.7	0.3
15	1163	0	1.6	2.3	694	± 1.0	0.5	0.2
20	1116	0	1.5	2.15	525	± 1.0	0.4	0.15

Table I (a)

	PC				SPV			
	M(MeV)	δ	R	B	M(MeV)	δ	R	B
$e_B = 4$	1060	0	1.4	2.0	—	—	—	—
6	1055	0	1.4	2.1	1045	± 0.5	1.1	1.4
8	1053	0	1.4	2.1	968	± 0.7	0.8	0.9
10	1052	0	1.3	2.0	874	± 0.8	0.6	0.7
15	1051	0	1.3	2.0	683	± 0.9	0.4	0.5
20	1050	0	1.3	2.0	559	± 0.9	0.3	0.3

Table I(b)