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TENSOR MESON-GLUONIUM MIXING FROM QCD-DUALITY SUM RULES^{*)}

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A B S T R A C T

We estimate, using QCD duality sum rules, the 2^{++} meson-gluonium mixing angle to be about 10° . We discuss the phenomenological implications of this result to the f - f' - θ systems and briefly speculate on the possible nature of the $\theta(1.7)$.

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1. - MESON-GLUONIUM MIXING ANGLE

QCD-duality sum rules have been applied quite successfully to hadron physics¹⁾. In this paper, we examine the mixing between the old $f \equiv f_2(1.27)$, $f' \equiv f_2'(1.52)$ 2^{++} mesons with the new tensor gluonium of QCD from the off-diagonal two-point correlator:

$$\begin{aligned} \Psi_{gg}^{\mu\nu\rho\sigma}(q^2) &= i \int d^4x e^{iqx} \langle 0 | T J_g^{\mu\nu}(x) (J_g^{\rho\sigma})^+ | 0 \rangle \\ &\equiv \frac{1}{2} \left\{ \eta^{\mu\rho} \eta^{\nu\sigma} + \eta^{\mu\sigma} \eta^{\nu\rho} - \frac{2}{3} \eta^{\mu\nu} \eta^{\rho\sigma} \right\} \Psi_{gg}(q^2), \end{aligned} \quad (1)$$

where

$$\begin{aligned} \eta^{\mu\rho} &= -g^{\mu\rho} + \frac{q^\mu q^\rho}{q^2}, \\ J_g^{\mu\nu}(x) &= i : \bar{q}(x) (\gamma^\mu \overleftrightarrow{D}^\nu + \gamma^\nu \overleftrightarrow{D}^\mu) q(x) :, \\ J_g^{\rho\sigma}(x) &= : -G^{\alpha\rho} G_\alpha^\sigma + \frac{1}{4} g^{\rho\sigma} G^{\alpha\beta} G_{\alpha\beta} :. \end{aligned} \quad (2)$$

$\overleftrightarrow{D}_\mu \equiv \overrightarrow{D}_\mu - \overleftarrow{D}_\mu$ is the covariant derivative. Following Ref. 2), we first study the mixing of the two operators under renormalization. The renormalized currents read:

$$\begin{aligned} J_{g,R}^{\mu\nu}(x) &= Z_{11} J_{g,B}^{\mu\nu}(x) + Z_{12} J_{g,B}^{\rho\sigma}(x), \\ J_{g,R}^{\rho\sigma}(x) &= Z_{21} J_{g,B}^{\mu\nu}(x) + Z_{22} J_{g,B}^{\rho\sigma}(x), \end{aligned} \quad (3)$$

where the renormalization constants Z_{ij} in the $\overline{MS}$ scheme are:

$$\begin{aligned} Z_{11} &= 1 + \frac{4}{3} C_2(R) \frac{1}{\epsilon} \left(\frac{\alpha_s}{\pi} \right); \quad Z_{12} = -\frac{8}{3} T(R) \frac{1}{\epsilon} \left(\frac{\alpha_s}{\pi} \right), \\ Z_{21} &= -\frac{1}{2} C_2(R) \frac{1}{\epsilon} \left(\frac{\alpha_s}{\pi} \right); \quad Z_{22} = 1 + \frac{n_f}{3\epsilon} \left(\frac{\alpha_s}{\pi} \right), \\ \frac{1}{\epsilon} &= \frac{1}{\epsilon} + \frac{1}{2} (\log 4\pi - \gamma). \end{aligned} \quad (4)$$

Here $d = 4 - \epsilon$ is the space-time dimension, n_f is the number of flavours, $C_2(R) = (N^2 - 1)/2N$ for $SU(N)_C$ and $T(R) = \frac{1}{2}$. Equation (3) is the master equation from which we shall start. The QCD expression for $\Psi_{gq}(q^2)$ comes from the diagrams

shown in Fig. 1, where the contributions of Fig. 1b, due to the mixing of operators, are necessary to remove the non-local $1/\epsilon \log q^2/v^2$ poles which come from Fig. 1a. We obtain, in the chiral limit:

$$\Psi_{gg}(-q^2 = Q^2 \gg \Lambda^2) \approx \frac{Q^4}{15\pi^2} \left(\frac{\alpha_0}{\pi}\right) \left\{ \log^2 \frac{Q^2}{v^2} - \frac{91}{15} \log \frac{Q^2}{v^2} \right\} - \frac{7}{36} \left(\log \frac{Q^2}{v^2} \right) \left(\frac{\alpha_3}{\pi}\right) \langle G^2 \rangle \quad (5)$$

The resonance contribution to the spectral function is introduced using a two-component mixing formalism between the gluonium G and the generic quarkonium F, where $F = f$ for $1/2(\bar{u}u + \bar{d}d)$ and $F = f'$ for $\bar{s}s$. Then, we obtain:

$$\frac{1}{\pi} \text{Im} \Psi_{gg}(t) \approx \sin 2\theta \frac{M_G^2}{f_G} \frac{1}{f_F} \delta(t - M^2) + \text{«QCD continuum»}, \quad (6)$$

where θ is the mixing angle; $M_G = 1.7 \text{ GeV}$, $f_G = (81 \pm 23) \text{ MeV}^3$ for the normalization in Eqs. (1) and (6); $M_F = 1.27 \text{ GeV}$ or 1.52 GeV , $f_F \approx 0.11 M_F^4$; M^2 is the average of the F and G mass squared. We estimate the mixing angle θ using a FESR analysis. In fact, a Laplace sum rule approach is inadequate here due to the almost degenerate value of $M_F \approx M_G$ which would imply a spurious enhancement of the $\sin 2\theta$ value^{1a)}. It is easy to derive the FESR:

$$\int_0^{t_c} dt \frac{1}{\pi} \text{Im} \Psi_{gg}(t) = - \frac{101}{675\pi^2} \left(\frac{\alpha_0}{\pi}\right) t_c^3 \left\{ 1 + \frac{525\pi^2}{404} \frac{\langle G^2 \rangle}{t_c^2} \right\}, \quad (7)$$

where $t_c \approx \frac{1}{2}(t_c^q + t_c^g) \approx 1.8 \text{ GeV}^2$. We have used $t_c^q \approx 2.5 \text{ GeV}^2$ from Ref. 4) and $t_c^g \approx 3 \text{ GeV}^2$ from Ref. 3). We also use the ratio of FESR:

$$\frac{\int_0^{t_c} dt t \text{Im} \Psi_{gg}(t)}{\int_0^{t_c} dt \text{Im} \Psi_{gg}(t)} \approx \langle t_{eff}^c \rangle, \quad (8)$$

which can provide the mean value of the energy where the duality between the QCD and resonance sides is expected to be obtained³⁾. This corresponds to

$$\langle t_{eff}^c \rangle \approx \frac{591}{108} t_c \approx 2 \text{ GeV}^2, \quad (9)$$

which implies from (6) and (7):

$$\theta \simeq -10^\circ \quad (10)$$

for both f and f' channels. Equation (10) shows that the mixing angle between the gluonium and lowest mass quarkonia is small, which is a result similar to the one obtained in the 0^{--} and 0^{++} channels⁶⁾. Equation (10) could be improved by including the effects of high-dimension condensates and possible final state interactions. As usual, these effects would not change the result obtained to leading order but, in turn, can provide a stability of the predictions versus the t_c -change.

2. - f - f' -GLUONIA SYSTEMS

Contamination of gluonium to the f - f' system has been studied by various authors⁷⁾ though inconclusive results have been obtained. In this paper, we will show that the value of the mixing angle in Eq. (10) will provide some progress for the understanding of the decay properties of the 2^{++} mesons. Within a two-component mixing formalism and assuming $SU(3)_F$ symmetry for the couplings, it is easy to obtain the couplings of the tensor mesons to pseudoscalar pairs:

$$g_{f\pi\pi}^2 \simeq \left(\frac{9}{10}\right) g^2 \left\{ 1 + 2 \sin \theta r + \mathcal{O}(\sin^2 \theta) \right\}, \quad (11a)$$

$$g_{fK\bar{K}}^2 \simeq \left(\frac{3}{10}\right) g^2 \left\{ 1 + 4 \sin \theta r + \mathcal{O}(\sin^2 \theta) \right\}, \quad (11b)$$

$$g_{f'K\bar{K}}^2 \simeq \left(\frac{3}{5}\right) g^2 \left\{ 1 + 2\sqrt{2} \sin \theta r + \dots \right\}, \quad (11c)$$

where $r \equiv g_{G\pi\pi}/g_{f\pi\pi}$ is the ratio of the gluonium over the f -coupling to $\pi\pi$. g is the uncontaminated coupling normalized in such a way that:

$$\Gamma(T \rightarrow PP') \simeq g_{TPP'}^2 |\vec{P}'|^5 / (30 M_T^2 \pi) : T \equiv A_2, K^{**} \quad (12)$$

Using⁸⁾ $\Gamma(A_2 \rightarrow K\bar{K}) \simeq (5.4 \pm 0.9) \text{ MeV}$ and $\Gamma(K^{**} \rightarrow K\pi) \simeq (45 \pm 3) \text{ MeV}$, one can deduce the average value:

$$g^2 \approx (205 \pm 10) \text{ GeV}^{-2}, \quad (13)$$

which supports the assumption of the smallness of the SU(3) breaking effects. Using⁸⁾ $\Gamma(f \rightarrow \pi\pi) = (148 \pm 17) \text{ MeV}$ and $\Gamma(f \rightarrow K\bar{K}) \approx (5.3 \pm 0.7) \text{ MeV}$, one can deduce using Eq. (10):

$$\lambda \approx -(0.68 \pm 0.23). \quad (14)$$

Considering that Eq. (14) provides the maximal effects of gluonia to the f - f' system, one can deduce:

$$\Gamma(G \rightarrow \pi\pi + K\bar{K} + \eta\eta) \lesssim (200 \pm 60) \text{ MeV}. \quad (15)$$

We have used a gluonium mass of 1.7 GeV ³⁾ and assumed universal couplings of gluonium to pseudoscalar boson pairs, which can be justified from a sum rule analysis of the vertex in Fig. 2^{*)}. The value obtained in Eq. (15) might be compared with the gluonium candidates $\theta(1.7)$ or $g_T(2.1)$ experimental widths. The same range of values for the g_T -width has been obtained from FESR⁹⁾ but using the θ -experimental width as input for the lowest mass gluonium. A definite conclusion on the lowest mass gluonium width in order to decide if it is narrow (say, 30 MeV) or large (say, 150 MeV) can be answered from the estimate of the GPP coupling from, e.g., Fig. 2. Another phenomenologically useful quantity is the gluonium decay width into $\gamma\gamma$. That can be estimated by using the 0^{++} gluonium width obtained within the context of effective Lagrangian applied to the energy momentum tensor¹⁰⁾. In this way, we have:

$$\Gamma(G \rightarrow \gamma\gamma) \approx \left(\frac{M_G}{M_{0^+}}\right)^3 \left(\frac{4}{15}\right) \Gamma(0^{++} \rightarrow \gamma\gamma) \approx 0.26 \text{ keV}, \quad (16)$$

where we have used $M_{0^+} \approx 1 \text{ GeV}$ from various theoretical expectations. Equations (15) and (16) will serve as a basis for the following attempts to interpret the $\theta(1.7)$.

*) We plan to carefully study this technically cumbersome problem in a future publication.

3. - ON THE $\theta(1.7)$ PUZZLE

The smallness of the f , f' mixing with the gluonium cannot explain the anomalous flavour dependence of the $\theta(1.7)$ branching ratios. Therefore, some additional contributions beyond the f - f' ones are needed. We present in the following two possibilities.

A) One could assume that the physical state $\theta(1.7)$ results from a large mixing between the gluonium $\theta(1.7)$, and the radial excitation of the f -meson. The smallness of the $\theta \rightarrow \pi\pi$ width might be due to some destructive interference. The possible danger of this interpretation could come from the $\theta \rightarrow \gamma\gamma$ width which could be enhanced by the one of the radial excitation of the f [recall that $\Gamma(f \rightarrow \gamma\gamma) \approx 2$ keV] whilst the experimental upper limit for $\theta \rightarrow \gamma\gamma$ is one order of magnitude less.

B) One could alternatively assume that the θ is a superposition of a narrow gluonium, responsible for the $\theta \rightarrow \pi\pi$ width of the order of 27 MeV, and an $\bar{s}s$ radial excitation of the f' which is the source of the $K\bar{K}$ enhancement. This scheme can be saved from the $\theta \rightarrow \gamma\gamma$ upper bound as the $f' \rightarrow \gamma\gamma$ width is about 0.11 keV but one is now faced with the explanation of the anomalous ordering of the levels between the $\bar{s}s$ and $\bar{u}u$ states, which we are unable to answer due to the complexity of the dynamics in this energy region.

Schemes A) and B) both seem possible but one should wait for independent tests of these assumptions, from theoretical and experimental information, before reaching a definite conclusion. However, we have shown that the f - f' system is understood and that a small gluonium contamination appears to be present in it.

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FIGURE CAPTIONS

Fig. 1 QCD contributions to the off-diagonal two-point correlator $\phi_{gq}(q^2)$:
a) perturbative part;
b) diagrams induced by the mixing under renormalization (Z_{ij} are renormalization constants);
c) lowest dimension non-perturbative contributions in the chiral limit $m_q = 0$.

Fig. 2 QCD three-point functions which govern the G-decay into pseudoscalar pairs.

a)

b)

c)

- Figure 1 -

- Figure 2 -