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A ONE-LOOP FINITE CONTORTED NON-LINEAR σ -MODEL WITHOUT THE WESS-ZUMINO TERM

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ABSTRACT

An explicit breaking interaction term of (1,0) supersymmetry, governed by a dimensionless coupling parameter, is added to the σ -model action in such a way to describe the torsion to which the fermions couple. Using (1,0) supergraph methods, it is shown that, under certain assumptions on the torsion-like explicit breaking term, one-loop finiteness is achieved without spoiling the Ricci-flatness of the target manifold.

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The propagation of a heterotic string in a fixed background space-time M can be described by a two-dimensional $(2,0)$ -supersymmetric non-linear σ -model whose target manifold is M . Hull and Witten [1] have presented a $(1,0)$ -superspace formulation of these models and established all the conditions that an arbitrary $(1,0)$ σ -model must satisfy in order to exhibit $(2,0)$ supersymmetry. In ref.[2], Hull has set the requirements for the two-loop finiteness of the $(2,0)$ models.

The torsion of the target space M is represented by means of the supersymmetric generalized Wess-Zumino term described by a rank-two anti-symmetric tensor defined on M [3]. In this letter, we shall show that we can explicitly break the $(1,0)$ supersymmetry by a coupling term which respects reparametrization invariance and is governed by a dimensionless coupling parameter so as to modify (or even cancel) the torsion contribution stemming from the (supersymmetric) Wess-Zumino term. Then, our main purpose here shall be the investigation of the structure of ultraviolet divergences induced by our torsion-like $(1,0)$ breaking term.

Using suitably modified superspace Feynman rules which account for the explicit breaking of the $(1,0)$ supersymmetry [4], it shall be shown in this work that, already at the one-loop level, the metric tensor β -function of the $(1,0)$ model receives contributions from the explicit breaking combined with the Wess-Zumino term. On the other hand, the additional torsion-like breaking term will be shown to require an independent renormalization and will induce, also via the Wess-Zumino term, other explicit breaking couplings that we shall determine with the help of our extended supergraph techniques.

The $(1,0)$ superspace is parametrized by the coordinates $Z^A = (x^+, x^-; \theta_-)$, where $x^\pm$ are the usual light-cone coordinates and θ_- is a left-handed (real) Majorana spinor. The corresponding supersymmetry covariant derivative is defined by

$$D_+ \equiv i \frac{\partial}{\partial \theta_-} + \theta_- \partial_+ \quad (1)$$

and is such that

$$D_+^2 = i\partial_+, \quad (2)$$

where $\partial_{\pm} \equiv \frac{\partial}{\partial x^{\mp}}$.

The basic relations to be employed in our supergraph calculations are:

$$\{D_+, \theta_-\} = i, \quad (3a)$$

$$D_{1+}\delta(\theta_{1-} - \theta_{2-})|_{\theta_1=\theta_2} = i \quad (3b)$$

and

$$D_{1+}\theta_{1-}\delta(\theta_{1-} - \theta_{2-})|_{\theta_1=\theta_2} = -i\theta_{1-}. \quad (3c)$$

Considering only real scalar superfields, $\Phi^i(x, \theta_-)$, whose θ -expansion is

$$\Phi^i(x, \theta_-) = A^i(x) - i\theta_- \psi_+^i(x), \quad (4)$$

and viewing them as the coordinates of some n -dimensional manifold M with metric tensor g_{ij} , the superspace action for an arbitrary $(1,0)$ non-linear σ -model with Wess-Zumino term is known [3] to be given by:

$$S = - \int d^2x d\theta_- [g_{ij}(\Phi) + B_{ij}(\Phi)](D_+ \Phi^i)(\partial_- \Phi^j), \quad (5)$$

where B_{ij} is an anti-symmetric tensor defined on M .

To extend this action to the case of $(2,0)$ supersymmetry, one realizes the extra supersymmetry non-linearly according to

$$\delta \Phi^i = I^i{}_j(\Phi) \epsilon_- D_+ \Phi^j, \quad (6)$$

where $I^i{}_j(\Phi)$ is some tensor defined on M and ϵ_- is the parameter of the second supersymmetry. As already known from ref.[1], the requirement of closure of the $(2,0)$ supersymmetry fixes the tensor $I^i{}_j(\Phi)$ to be a complex structure on M . Next, the requirement of invariance of the action (5) under the supersymmetry (6)

imposes that M be a complex hermitian manifold with a totally anti-symmetric torsion tensor given by

$$H_{ijk} \equiv \frac{1}{2}(B_{ij,k} + B_{jk,i} + B_{ki,j}). \quad (7)$$

In the particular case B_{ij} is taken to be zero or an exact two-form, M turns out to be a Kähler manifold [1].

In view of what has been discussed in the previous paragraph, a way of realizing an explicit breaking from $(2,0)$ to $(1,0)$ supersymmetry is by choosing the target space to be a complex Kähler manifold and the tensor $B_{ij}(\phi)$ to be a general two-form which does not necessarily satisfy the condition

$$H^i{}_{kl}(\Phi)I^l{}_j(\Phi) - H^l{}_{kj}(\Phi)I^i{}_l(\Phi) = 0. \quad (8)$$

Our next step, and this is our main goal, is to find reparametrization invariant coupling terms which explicitly break the $(1,0)$ supersymmetry. Restricting ourselves to those terms governed by a dimensionless coupling parameter, λ , the only term we can build up in such a way to affect the spinor-scalar couplings is given by:

$$\mathcal{L}_{breaking} = i\lambda \int d\theta_- \theta_- T_{ijk}(\Phi)(D_+ \Phi^i)(D_+ \Phi^j)(\partial_- \Phi^k), \quad (9)$$

where $T_{ijk}(\Phi)$ is a tensor defined over M . Since only its anti-symmetric part in ij appears in (9), there is no loss of generality in taking $T_{ijk}(\Phi)$ to be anti-symmetric in the first two indices.

The component-field expression of the breaking term (9) is readily shown to be:

$$\mathcal{L}_{breaking} = i\lambda T_{ijk}(A)\psi_+^i \psi_+^j \partial_- A^k. \quad (10)$$

This suggests to us an interesting geometrical interpretation for such a breaking term. Taking the component-field fermionic Lagrangian following from the action (5) and putting it together with (10), one can easily show that the ψ_+ -field Lagrangian reads:

$$\mathcal{L}_{fermionic} = -ig_{ij}(A)\psi_+^i (\nabla_- \psi_+)^j, \quad (11)$$

where the covariant derivative ∇_- is defined to be

$$(\nabla_- \psi_+)^j \equiv \partial_- \psi_+^j + \hat{\Gamma}^j{}_{kl}(\partial_- A^l) \psi_+^k. \quad (12)$$

$\hat{\Gamma}^j{}_{kl}$ is the connection containing the Christoffel symbol, $\Gamma^j{}_{kl}$, the torsion term induced by the Wess-Zumino action and an additional contribution generated by our breaking Lagrangian:

$$\hat{\Gamma}^j{}_{kl} = \Gamma^j{}_{kl}(A) + H^j{}_{kl}(A) - \lambda T^j{}_{kl}(A). \quad (13)$$

Therefore, one can notice that by suitably choosing the tensor T_{ijk} in our breaking Lagrangian, the effects of the torsion contributed by the Wess-Zumino action can be completely cancelled out. This would be the case if

$$T_{ijk} = \frac{1}{\lambda} H_{ijk}. \quad (14)$$

For the quantization of the model, we proceed as it is usually done: the background-field method is adopted and a normal coordinate expansion in terms of the quantum field ξ^i is performed. Denoting by ξ^a the quantum fluctuation with local frame index a , the quantum superpropagators turn out to be given by

$$\langle \xi^a(1) \xi^b(2) \rangle = \delta^{ab} \frac{i}{k^2} D_{1+}(k) \delta(\theta_{1-} - \theta_{2-}). \quad (15)$$

In order to perform diagrammatic calculations, we read the superspace Feynman rules for the vertices from the quantum-background couplings stemming from the normal coordinate expansions of (5) and (9).

We organize the presentation of the quantum-background vertices by giving separately the different kinds of quadratic terms in the quantum fluctuation ξ^i . We stop at this order in ξ^i since our analysis is restricted to the one-loop approximation.

(i) Terms of the form $\xi^{i_1} \xi^{i_2}$:

$$- \int d\theta_- [R_{ki_1i_2l} + \frac{1}{2} (D_{i_1} D_{i_2} B_{kl} + R^m{}_{i_1i_2k} B_{ml} + R^m{}_{i_1i_2l} B_{km})] (D_+ \Phi^k) (\partial_- \Phi^l) \xi^{i_1} \xi^{i_2} +$$

$$+i\lambda \int d\theta_- \theta_- \frac{1}{2} (D_{i_1} D_{i_2} T_{ijk} + R^m{}_{i_1 i_2 i} T_{mjk} + R^m{}_{i_1 i_2 j} T_{imk} + R^m{}_{i_1 i_2 k} T_{ijm}) (D_+ \Phi^i) (D_+ \Phi^j) (\partial_- \Phi^j) \xi^{i_1} \xi^{i_2}. \quad (16)$$

(ii) Terms of the form $\xi^{i_2} (\nabla_{\pm} \xi^{i_2})$:

$$\begin{aligned} & - \int d\theta_- [(D_{i_1} B_{i_2 k}) (D_+ \Phi^k) \xi^{i_1} (\nabla_- \xi^{i_2}) - (D_{i_1} B_{i_2 k}) (\partial_- \Phi^k) \xi^{i_1} (\nabla_+ \xi^{i_2})] + \\ & + i\lambda \int d\theta_- \theta_- [(D_{i_1} T_{ij i_2}) (D_+ \Phi^i) (D_+ \Phi^j) \xi^{i_1} (\nabla_- \xi^{i_2}) + \\ & + 2(D_{i_1} T_{ii_2 j}) (D_+ \Phi^i) (\partial_- \Phi^j) \xi^{i_1} (\nabla_+ \xi^{i_2})]. \end{aligned} \quad (17)$$

iii) Terms of the form $(\nabla_{\pm} \xi^{i_1}) (\nabla_{\pm} \xi^{i_2})$:

$$\begin{aligned} & - \int d\theta_- B_{i_1 i_2} (\nabla_+ \xi^{i_1}) (\nabla_- \xi^{i_2}) + i\lambda \int d\theta_- [2T_{ii_1 i_2} (D_+ \Phi^i) (\nabla_+ \xi^{i_1} (\nabla_- \xi^{i_2}) + \\ & + T_{i_1 i_2 i} (\partial_- \Phi^i) (\nabla_+ \xi^{i_1}) (\nabla_+ \xi^{i_2})]. \end{aligned} \quad (18)$$

In these expressions, Φ^i denotes the background field and the reparametrization covariant derivatives are defined as:

$$\nabla_+ \xi^i \equiv D_+ \xi^i + \Gamma^i{}_{jk} (D_+ \Phi^j) \xi^k \quad (19a)$$

and

$$\nabla_- \xi^i \equiv \partial_- \xi^i + \Gamma^i{}_{jk} (\partial_- \Phi^j) \xi^k. \quad (19b)$$

We are now ready to start presenting and discussing the results of our calculations for the one-loop corrections. The Feynman rules for the vertices can be directly read from the Lagrangian pieces (16), (17) and (18).

The tadpole graph of fig.1 gives the following contributions to the effective action:

$$\int \frac{d^2 k}{(2\pi)^2} \frac{1}{k^2 + \mu_r^2} \int d\theta_- [R_{ij} + \frac{1}{2} (D^k D_k B_{ij} + R^m{}_{i} B_{mj} + R^m{}_{j} B_{mi})] (D_+ \Phi^i) (\partial_- \Phi^j) \quad (20)$$

and

$$-\frac{1}{2}(i\lambda) \int \frac{d^2 k}{(2\pi)^2} \frac{1}{k^2 + \mu_r^2} \int d\theta_- \theta_- [D^l D_l T_{ijk} + R^m{}_i T_{mjk} + R^m{}_j T_{imk} + R^m{}_k T_{ijm}]$$

$$(D_+ \Phi^i)(D_+ \Phi^j)(\partial_- \Phi^k), \quad (21)$$

where μ_r stands for an infra-red cut-off mass.

The correction (20) contributes to the renormalization of the metric tensor, whereas the result (21) renormalizes the breaking interaction (9). Tadpoles with a $D_+ \Phi^i$, $\partial_- \Phi^i$ or $(D_+ \Phi^i)(D_+ \Phi^j)$ associated to the external leg are all vanishing due to the supersymmetry algebra and arguments of symmetric integration.

fig.1

1-loop tadpoles

Contrary to what happens for the ordinary non-linear σ -model, supergraphs of the type depicted in fig.2 may contribute further one-loop infinities [6]. We separate their contributions in three different categories: both vertices are taken from the supersymmetric quantum-background couplings; one is chosen supersymmetric and the other of the supersymmetry-breaking type (graphs linear in λ), and both vertices are taken from the supersymmetry-breaking Lagrangian (graphs of order λ^2).

fig.2

1-loop supergraphs responsible for new breaking terms when $B_{ij} \neq 0$.

We shall present now our results for the supergraphs of fig.2. At order λ^0 , the divergent contributions we find are:

$$\int \frac{d^2 p}{(2\pi)^2} \int \frac{d^2 k}{(2\pi)^2} \frac{1}{k^2 + \mu_r^2} \frac{1}{(k+p)^2 + \mu_r^2} (k+p)_+ (k+p)_- \int d\theta_- (D_k B_{li}) (D^k B^l{}_j) (D_+ \Phi^i) (\partial_- \Phi^j) \quad (22)$$

and

$$\int \frac{d^2 p}{(2\pi)^2} \int \frac{d^2 k}{(2\pi)^2} \frac{1}{k^2 + \mu_r^2} \frac{1}{(k+p)^2 + \mu_r^2} k_+ (k+p)_- \int d\theta_- (D_k B_{li}) (D^l B^k{}_j) (D_+ \Phi^i) (\partial_- \Phi^j). \quad (23)$$

They both contribute to the metric tensor renormalization.

The divergences encountered at the order λ are listed below:

$$\begin{aligned} & -2(i\lambda) \int \frac{d^2 p}{(2\pi)^2} \int \frac{d^2 k}{(2\pi)^2} \frac{1}{k^2} \frac{1}{(k+p)^2} (k+p)_+ (k+p)_- \\ & \int d\theta_- \theta_- (D_m B_{ni}) (D^m T_j{}^n{}_k) (D_+ \Phi^i) (D_+ \Phi^j) (\partial_- \Phi^k), \quad (24) \\ & -2(i\lambda) \int \frac{d^2 p}{(2\pi)^2} \int \frac{d^2 k}{(2\pi)^2} \frac{1}{k^2} \frac{1}{(k+p)^2} k_+ (k+p)_- \end{aligned}$$

$$\int d\theta_- \theta_- (D_m B_{ni}) (D^n T_j{}^m{}_k) (D_+ \Phi^i) (D_+ \Phi^j) (\partial_- \Phi^k), \quad (25)$$

$$2(i\lambda) \int \frac{d^2 p}{(2\pi)^2} \int \frac{d^2 k}{(2\pi)^2} \frac{1}{k^2} \frac{1}{(k+p)^2} (k+p)_+ k_-$$

$$\int d\theta_- (D^m B^{ni}) [iT_{jnm} (D_+ \Phi^j) - \theta_- D_p T_{jnm} (D_+ \Phi^p) (D_+ \Phi^j) +$$

$$-i\theta_- T_{jnm} (\partial_+ \Phi^j)] (\partial_- \Phi_i), \quad (26)$$

$$-2(i\lambda) \int \frac{d^2 p}{(2\pi)^2} \int \frac{d^2 k}{(2\pi)^2} \frac{1}{k^2} \frac{1}{(k+p)^2} k_+ (k+p)_-$$

$$\int d\theta_- B^{mn} [i(D_n T_{imj}) (D_+ \Phi^i) (\partial_- \Phi^j) +$$

$$-\theta_- (D_p D_n T_{imj}) (D_+ \Phi^p) (D_+ \Phi^i) (\partial_- \Phi^j) +$$

$$+\theta_- (D_n T_{imj}) (D_+ \Phi^i) (\partial_- D_+ \Phi^j) +$$

$$-i\theta_- (D_n T_{imj}) (\partial_+ \Phi^i) (\partial_- \Phi^j)], \quad (27)$$

$$\lambda \int \frac{d^2 p}{(2\pi)^2} \int \frac{d^2 k}{(2\pi)^2} \frac{1}{k^2} \frac{1}{(k+p)^2} k_+ (k+p)_- (k+p)_-$$

$$\int d\theta_- \theta_- B_{mn} (D^m T_{ij}{}^n) (D_+ \Phi^i) (D_+ \Phi^j) \quad (28)$$

and

$$2(i\lambda) \int \frac{d^2 p}{(2\pi)^2} \int \frac{d^2 k}{(2\pi)^2} \frac{1}{k^2} \frac{1}{(k+p)^2} k_+ (k+p)_- (k+p)_-$$

$$\int d\theta_- B_{mn} [iT_i{}^{mn} (D_+ \Phi^i) - \theta_- (D_p T_i{}^{mn}) (D_+ \Phi^p) (D_+ \Phi^i) +$$

$$-\theta_- T_i{}^{mn} (i\partial_+ \Phi^i)]. \quad (29)$$

It is interesting to notice the presence of supersymmetric corrections induced by the breaking of supersymmetry and their contribution to the metric tensor renormalization.

Taking now both vertices to be of the supersymmetry-breaking type, one can show that there will not be any order- λ^2 contribution to the renormalization of g_{ij} , the reason being that it is not possible for both θ_- -factors in the vertices to be absorbed by derivatives D_+ coming from the loop upon partial integrations. At most one θ_- can be eaten up by a D_+ ; the other will always remain to give a contribution which

explicitly breaks supersymmetry. Indeed, the only infinity induced by this class of graphs is given by:

$$4(i\lambda^2) \int \frac{d^2 p}{(2\pi)^2} \int \frac{d^2 k}{(2\pi)^2} \frac{1}{k^2} \frac{1}{(k+p)^2} (k+p)_+ k_- \int d\theta_- \theta_- T_{imn} (D^m T_j{}^m{}_k) (D_+ \Phi^i) (D_+ \Phi^j) (\partial_- \Phi^k). \quad (30)$$

Considering finally one-loop diagrams with more than two external (background) lines, one concludes that only supergraphs of the type drawn in fig.3 may actually lead to divergences. As a matter of fact, this could already be expected by a simple power counting. Explicit calculations show that the only infinite correction contributed by these graphs is given by:

$$2(i\lambda^3) \int \frac{d^2 p}{(2\pi)^2} \int \frac{d^2 k}{(2\pi)^2} \frac{1}{k^2} \frac{1}{(k+p)^2} \frac{1}{(k+p+q)^2} k_- k_- (k+p)_+ (k+p+q)_+ \int d\theta_- \theta_- T_i{}^m{}_n T_j{}^n{}_p T_k{}^p{}_m (D_+ \Phi^i) (D_+ \Phi^j) (\partial_- \Phi^k). \quad (31)$$

fig.3

Divergent one-loop diagrams with highest number of external legs.

In view of the results presented above, we would like to conclude with some remarks. Besides the well-known modification of the metric tensor renormalization due to the presence of the Wess-Zumino term, we can notice that, by virtue of an interference with the latter, our torsion- like breaking term modifies the metric

tensor β -function by terms of order λ . However, when $B_{ij} = 0$, our breaking term does not alter the result of β_{ij} for the exact model: it remains vanishing for Ricci-flat target spaces.

On the other hand, we should also pay attention to the fact that new divergent explicit breaking terms of the form

$$\int d\theta_- \theta_- (D_+ \Phi)(D_+ \Phi), \quad (32)$$

$$\int d\theta_- \theta_- (D_+ \Phi)(\partial_- D_+ \Phi) \quad (33)$$

and

$$\int d\theta_- \theta_- (\partial_+ \Phi)(\partial_- \Phi) \quad (34)$$

are induced as a result of the combination of the Wess-Zumino and our breaking term. If the former is not present, and this is the situation we are actually interested in, these new infinities are not generated.

Finally, we would like to state our main conclusion in this letter: switching off the Wess-Zumino term, and introducing the torsion by means of the breaking Lagrangian (9), we obtain that Ricci-flatness is still sufficient to ensure the vanishing of the metric tensor β -function. No new breaking term is induced and the breaking coupling (9) itself does not require any renormalization provided that the tensor T_{ijk} be covariantly constant and symmetric in its last two indices. In such a case, one would have a one-loop finite non-linear σ -model which includes torsion and is defined on a Ricci-flat target manifold.

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