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SEMI-HARD QCD: MINIJETS AND ELASTIC SCATTERING

J. Dias de Deus^{*)} and J. Kwiecinski⁺⁾

CERN - Geneva

A B S T R A C T

Starting from the QCD calculation of the amplitude for semi-hard interactions we establish a quantitative correlation between the growth of the cross-section for minijet production and the changes in the $p\bar{p}$ elastic differential cross-section between ISR and Sp $\bar{p}$ S collider energies. The fact that the semi-hard amplitude is central plays an essential rôle. The predicted behaviour for σ_{tot} , $\sigma_{\text{el}}/\sigma_{\text{tot}}$, ρ and $d\sigma/dt$ at higher energies is presented.

^{*)} On leave from CFMC/INIC, Lisbon.

⁺⁾ On leave from the Institute of Nuclear Physics, Krakow, Poland.

The experimental results from the CERN SppS collider, in the energy range $200 < \sqrt{s} < 900$ GeV/c, on inelastic production and elastic scattering¹⁾⁻³⁾ may force us to change some of our views on soft hadron physics. In fact, the possible increasing rôle of semi-hard physics, with the observation of minijets associated with larger multiplicities and larger values of p_T , is becoming very transparent. It is then important to implement semi-hard QCD calculations and to include a consistent treatment of semi-hard effects in studies of conventional soft hadron physics, namely, inelastic production and elastic scattering.

Inelastic production and elastic scattering are related at a very fundamental level by unitarity. The basic and obvious question is then the following: what to expect in elastic scattering from the appearance of semi-hard physics and minijet cross-sections? Our purpose, with this note, is to give a possible answer to that question.

More specifically, our aim will be to show that the following two phenomena are intimately related: the rise of minijet cross-sections, on one side, and various changes in elastic scattering between ISR and collider energies³⁾ corresponding to violations of geometrical scaling⁴⁾⁻⁶⁾. We will be concerned not only with changes in energy dependence of global quantities like σ_{tot} , σ_{el} and ρ but with the discussion of the local behaviour of the differential cross-section, as well. We would like to emphasize the possible relation between the onset of semi-hard interactions and the very strong increase with energy of the height of the diffractive shoulder in $\bar{p}p$ elastic scattering³⁾.

At a qualitative level our argument is fairly simple. The semi-hard interactions, due to the perturbative QCD increase of gluon and sea distributions at very small x , become very important above ISR energies and, as a result, the minijet cross-section has a very rapid growth with energy. On the other hand, in the impact parameter, while the purely soft amplitude (soft Pomeron) is characterized by varying with energy essentially by increasing the radius (shrinkage is a feature of soft ladders), the semi-hard amplitude arising from QCD diagrams with large virtualities has essentially a constant radius related to the proton form factor. The semi-hard contribution makes then the elastic amplitude more central and fastly changing with energy. Naturally, σ_{el}/σ_{tot} will increase rapidly with energy. The imaginary part of the (absorbed) semi-hard amplitude in t space interferes with the soft amplitude, displaces the dip of $d\sigma/dt$ to smaller $|t|$ and dominates the large $|t|$ region. The rapid increase of the semi-hard amplitude gives the rapid increase with energy of the shoulder and second maximum of $d\sigma/dt$. The real part of the semi-hard amplitude, again because of the rapid

increase of the minijet cross-section and because of analyticity, makes ρ to increase up to very high energies.

We shall now present our model. We use the multiple scattering formalism in the eikonal approximation. We consider two driving contributions corresponding to purely soft and to semi-hard interactions and write the eikonal function as the sum of two "bare" terms: the soft bare term χ_S and the semi-hard bare term χ_{SH} . As our model is unitarized, at the amplitude level these two terms will be mixed.

The soft bare term will be assumed to correspond to a geometrical scaling amplitude and will be obtained by a straightforward extrapolation from a parametrization used at ISR energies.

The input bare term for semi-hard interactions can be computed within QCD^{7),8)}. In the forward direction it gives the bare cross-section $\sigma_{SH}^{(0)}$ for semi-hard events. Physically, it corresponds to semi-hard interactions of partons, predominantly gluons, since at very high energy, collisions of partons carrying very small momentum fractions ($x \lesssim 10^{-2}$) contribute appreciably. The perturbative QCD indefinite increase of the gluon distribution in the $x \rightarrow 0$ limit causes the very strong increase with the energy of the bare cross-section $\sigma_{SH}^{(0)}$. The calculation of the detailed energy dependence of $\sigma_{SH}^{(0)}$ can be quite involved, due to the complicated j-plane singularity structure of the bare semi-hard Pomeron. However, it turns out that the results of the QCD calculations can be parametrized in terms of an effective contribution⁹⁾:

$$\sigma_{SH}^{(0)} = C * (s - s_0)^{0.35} \quad (1)$$

with $s_0 = 2450 \text{ GeV}^2$, over a very wide energy range, from the top ISR energy up to $\sqrt{s} \approx 2 \text{ TeV}$. At much higher energy, i.e., for $\sqrt{s} = 40 \text{ TeV}$, the parametrization (1) overestimates the results of the QCD calculation by about 30% or so. The "threshold" at s_0 should of course be regarded as artificial, since the quantity $\sigma_{SH}^{(0)}$ calculated from QCD, albeit very small, is different from zero down to energies much smaller than $\sqrt{s_0}$. The parametrization (1) is therefore not valid for $\sqrt{s} \lesssim \sqrt{s_0}$.

The strong energy dependence of the bare cross-section $\sigma_{SH}^{(0)}$ with a bare intercept of 1.35 reflects, as previously mentioned, the singular behaviour of gluon distributions in the $x \rightarrow 0$ limit in perturbative QCD. The relatively large value of the effective intercept of this bare semi-hard Pomeron is an essential feature of perturbative QCD¹⁰⁾.

The constant C in (1) controls the overall normalization of $\sigma_{SH}^{(0)}$ and is model-dependent, being sensitive to the assumed value of $p_T^{\min}$ defining hard scattering, to the input (non-perturbative) gluon distribution, etc. Following Ref. 9) we fix $C = 0.15$ mb in order to reproduce the UA1 minijet cross-section.

We next discuss the t dependence of the semi-hard Pomeron: The bare term is obtained from the contribution of the perturbative QCD ladder diagram with two gluon coupling (non-perturbatively) to the proton (anti-proton) at each end of the ladder. It can be convincingly argued¹¹⁾ that, since the QCD ladder diagram involves exchange of large transverse momenta, the t dependence comes essentially from the non-perturbative coupling of the gluons to the proton and can thus be factored out. The imaginary part of the semi-hard bare term is then

$$\text{Im } A_{SH}^{(0)}(s, t) = \sigma_{SH}^{(0)}(s) F(t) \quad (2)$$

with $F(0) = 1$. It can be further argued that $F(t)$ should be related to the proton form factor⁸⁾ or to the energy independent part of the slope of the diffractive peak.

One should note that the slope parameter of the semi-hard scattering amplitude, because of the large momentum transfers along the QCD ladder, is independent of energy. The contribution of the semi-hard term to the impact parameter amplitude is then clearly central. This is in agreement with the observed "blackening" of the proton at small b in going from ISR to collider energies⁶⁾.

We write next the elastic amplitude $A(s, t)$, with normalization

$$\text{Im } A(s, t=0) = \sigma_{tot} \quad (3)$$

in the impact parameter representation

$$A(s, t) = \pi \int_0^\infty db^2 J_0(b\sqrt{-t}) f(b, s) \quad (4)$$

with

$$f(b,s) = 2i (1 - e^{i\chi(b,s)}) \quad (5)$$

The eikonal $\chi(b,s)$ assuming additivity is given by

$$\chi(b,s) = \chi_S(b,s) + \chi_{SH}(b,s) \quad (6)$$

where the subscript S stands for "soft" and SH for "semi-hard". For convenience we shall define

$$f_{S,SH}(b,s) = 2i (1 - e^{i\chi_{S,SH}(b,s)}) \quad (7)$$

and write

$$f(b,s) = f_S(b,s) + f_{SH}(b,s) + \frac{i}{2} f_S(b,s) f_{SH}(b,s) \quad (8)$$

We discuss now the impact parameter amplitudes to be used in (8).

The imaginary part of the semi-hard eikonal, $\text{Im}\chi_{SH}(b,s)$, is the Fourier-Bessel transform of the bare semi-hard Pomeron (2). The form factor $F(t)$ of (2) will be approximated by two exponentials:

$$F(t) = c e^{b_1 t} + (1-c) e^{b_2 t} \quad (9)$$

with the parameters c , b_1 and b_2 adjusted to reproduce the small t behaviour of the dipole formula of electromagnetic form factor:

$$F(t) = \left(\frac{1}{1 - t/0.71 \text{ GeV}^2} \right)^4 \quad (10)$$

The real part $\text{Re}A_{SH}^{(0)}(s,t)$ of the semi-hard bare term is obtained from $\text{Im}A_{SH}^{(0)}(s,t)$ by analyticity, for $s \gg s_0$,

$$\text{Re} A_{SH}^{(0)}(s,t) = \tan\left(\frac{0.35\pi}{2}\right) \text{Im} A_{SH}^{(0)}(s,t) \quad (11)$$

For the soft amplitude $A_s(s,t)$ we assume the geometrical scaling type parametrization of Amaldi and Schubert¹²⁾ with Gaussian functions.

The energy dependence of σ_s is obtained from a simultaneous fit compatible with geometrical scaling to the slope parameter $B^{13)}$ and to the total cross-sections through ISR energies:

$$\sigma_s(s) = (6.79 + 0.77 \ln s) / 0.31 \text{ mb} \quad (12)$$

The real part of the soft amplitude at $t = 0$, the only quantity needed to fix the real part at any t , is, by analyticity

$$\text{Re } A_s(s, t=0) = \frac{\pi}{2} \frac{0.77}{0.31} \quad (13)$$

We have now all the quantities required in (8). The Fourier-Bessel transforms are very easy to perform because of our Gaussian approximations.

We start by checking the validity of Eqs. (1) and (2), required to compute $\chi_{SH}(b, s)$, and calculate the inelastic cross-section for minijet events, with unitarity effects taken into account^{7)-9), 14)}

$$\sigma_{SH} = \pi \int_0^\infty db^2 (1 - e^{-2 \text{Im } \chi_{SH}(b, s)}) \quad (14)$$

The cross-section σ_{SH} is plotted in Fig. 1 and compared with the UA1 minijet data. The results concerning σ_{SH} are similar to those obtained in Ref. 9), despite the slightly different assumptions used to compute the b dependence of $\chi_{SH}(b, s)$.

In Fig. 2, we show the predicted energy dependence of $\sigma_{tot}(s)$, σ_{el}/σ_{tot} and the ratio $\rho \equiv \text{Real } A(s, 0) / \text{Im}(s, 0)$ in the energy range $0.1 < \sqrt{s} < 40$ TeV. For the Sp̄pS energy $\sqrt{s} = 546$ GeV we have $\sigma_{tot} = 60.7$ mb (exp: 61.9 ± 1.5 mb¹⁵⁾), $\sigma_{el}/\sigma_{tot} = 0.216$ mb (exp: 0.215 ± 0.005)¹⁵⁾, and $\rho = 0.129$. At very high energy (> 10 TeV) our predicted total cross-sections agree with cosmic ray measurements¹⁶⁾. For the ratio of the inelastic cross-sections between $\sqrt{s} = 200$ and 900 GeV, determined by the UA5 collaboration, we obtain 1.22, to be compared with the experimental value, $1.20 \pm 0.02 \pm 0.03$ ¹⁷⁾.

In Fig. 3, we show the elastic differential cross-section at collider energies. In Fig. 3a, data in small $|t|$ and large $|t|$ region at $\sqrt{s} = 546$ GeV³⁾ are presented and compared to our curves. The agreement is very good up to $|t| \approx 0.45$ GeV². From $|t| \approx 0.45$ to $|t| \approx 0.8$ data points are above the curve and for $|t| > 0.8$ they are below. The dip shoulder behaviour is, however,

qualitatively well reproduced. In Fig. 3b, the large $|t|$ data at $\sqrt{s} = 546$ and 630 GeV^3 are shown in comparison with our curves.

In Fig. 4, we present $d\sigma/dt$, for $0 < |t| < 2.5 \text{ GeV}^2$, from ISR ($\sqrt{s}=52.5 \text{ GeV}$) up to the SSC collider energy ($\sqrt{s}=40 \text{ TeV}$).

The main point we want to emphasize is the dominant rôle played by the semi-hard amplitude at the shoulder and second maximum of $d\sigma/dt$. It correctly reproduces the increase of the differential cross-section in these regions by more than one order of magnitude between ISR and SppS collider energies. This results as a consequence of centrality of the bare semi-hard amplitude together with the observed rise of the cross-section for semi-hard interactions. The increase of the cross-section with energy alone would not have been sufficient to reproduce this effect. The failure of geometrical scaling in the dip and second maximum regions, even with the experimental value of the collider cross-section put in, is a manifestation of that fact.

We have not attempted to adjust some of the parameters within the model to fit data. In the dip and second maximum regions, the model tends to be below the data points before the dip and above the data points after the dip. We believe that this discrepancy may be partially due to the use of the eikonal approximation for the calculation of multiple scattering. We have noticed, in particular, that if the term corresponding to soft/semi-hard absorption, $f_S(b,s)f_{SH}(b,s)$ in (9), is multiplied by a reducing factor (i.e., ≈ 0.75) the agreement with data is achieved for the whole range in t . The fact that the eikonal prescription gives too strong double scattering terms has been noticed in other occasions too¹⁸⁾.

One should mention that the model, in its present form, is not accurate enough in the ISR region, giving too high a value for the minijet cross-section (1.88 mb at $\sqrt{s} = 62.5 \text{ GeV}$) and for the differential cross-section at large $|t|$. This is related to the behaviour of parametrization (1) near s_0 . In the future, this parametrization should be improved and better adapted to the low energy region.

There are several approaches to the high energy behaviour of pp and $\bar{p}p$ elastic scattering. All the existing models, as modern versions of the models of Good-Walker¹⁹⁾, Chou-Yang²⁰⁾ or Cheng-Wu²¹⁾, the models of Reggeon calculus type^{18),22),23)} and models assuming essential dominant differences between pp and $\bar{p}p$ scattering^{24),25)} give reasonable descriptions of $\bar{p}p$ $d\sigma/dt$ at present

energies. At higher energies there will be significant differences which may help to discriminate the dynamics. Concerning the relation of our work to other models we would like to make the following comments:

- i) none of the models mentioned above has included, at least in a transparent manner, the effect of semi-hard physics in the differential elastic cross-section;
- ii) our semi-hard amplitude, originated in the interaction of small x partons, gives clearly a contribution to the Pomeron and so we differ from odderon models;
- iii) due to the strong energy dependence of the semi-hard Pomeron, we predict a continuous growth of ρ with energy up to $\sqrt{s} \approx 20$ TeV while in Ref. 23) ρ has a maximum at $\sqrt{s} \approx 2$ TeV and in Ref. 21) at $\sqrt{s} \approx 1$ TeV;
- iv) due to the centrality and increase with energy of the semi-hard Pomeron, we predict a fast rise with energy of the ratio σ_{el}/σ_{tot} [at $\sqrt{s} = 1.6$ TeV we have $\sigma_{el}/\sigma_{tot} = 0.24$, while in Refs. 23) and 21) $\sigma_{el}/\sigma_{tot} \approx 0.23$, and at $\sqrt{s} = 40$ TeV we have $\sigma_{el}/\sigma_{tot} = 0.38$, Ref. 23) 0.33 and Ref. 21) 0.29].

To conclude, we stress again that the dramatic changes observed in $d\sigma/dt$ between ISR and SppS collider energies can be thought as coming from the onset of semi-hard interactions. Taking that point of view we were then able to correlate quantitatively two independent pieces of experimental information, the increase of the cross-section for minijet events and the increase with energy of $d\sigma/dt$ in the shoulder region.

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FIGURE CAPTIONS

- Fig. 1 The semi-hard inelastic cross-section, Eqs. (14), (2) and (1). Data from Ref. 1).
- Fig. 2 Predicted energy dependence of σ_{e1}/σ_{tot} , ρ and σ_{tot} . Data points from Ref. 15).
- Fig. 3 The $p\bar{p}$ differential elastic cross-section at SppS collider energies:
a) small $|t|$ and large $|t|$ $d\sigma/dt$ at $\sqrt{s} = 546$ GeV;
b) large $|t|$ $d\sigma/dt$ at $\sqrt{s} = 546$ and 630 GeV.
Data are from Ref. 3).
- Fig. 4 The $p\bar{p}$ differential elastic cross-section from ISR up to SSC energies. The rise of the cross-section at the dip and second maximum is a consequence of semi-hard interactions.

- Figure 1 -

- Figure 2 -

- Figure 3a -

- Figure 3b -

- Figure 4 -