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ABSTRACT

We show that one-loop string effects do not alter the form of the leading terms in the low-energy effective 10-dimensional $N=1$ supergravity theory derived from the heterotic string. We extend this result to the 4-dimensional $N=1$ supergravity theory obtained after compactification, showing in particular that its Kähler potential is not renormalized by string loop effects or by the exchange of Kaluza-Klein modes. We also demonstrate that the Kähler potential and the gauge kinetic function are not renormalized by one-loop string effects in a wide class of 4-dimensional formulations of superstring theories.

NO RENORMALIZATION OF THE $N = 1$ SUPERGRAVITY THEORY
DERIVED FROM THE HETEROTIC STRING

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Much superstring phenomenology is dependent on the form of the effective low-energy field theory derived from the superstring. To obtain this from the heterotic superstring in 10 dimensions, one usually starts by deriving the $D=10$, $N=1$ supergravity field theory describing the interactions of the massless degrees of freedom, and then compactifies it to a $d=4$, $N=1$ supergravity theory [1]. Alternatively, one may work directly from a four-dimensional formulation of superstring theory, taking its low-energy limit to obtain a $d=4$, $N=1$ supergravity field theory [2]. In both cases, analyses have to date concentrated on the lowest genus term in the string topological expansion ("tree level"), and have in the case of the $D=10$ heterotic superstring [3-8] included some higher order terms in the string slope parameter α' . However, neither in the $D=10$ theory nor in the $d=4$ formulations has a detailed analysis of higher genus terms in the string topological expansion ("loop effects") been carried out.

One has some general ideas about the possible forms of such terms. For example, in $D=10$ the dilaton field $\phi = e^{D/\sqrt{8}/s}$ is the loop expansion parameter [9], so higher order terms in the string topological expansion will have some power of ϕ as a prefactor. In manifold compactifications of the $D=10$ theory to $d=4$ the role of the dilaton field is taken by the quantity T/S , where T and S are the gauge-singlet chiral superfields introduced by Witten [1]. In direct 4-dimensional formulations of the superstring, the loop expansion parameter is $1/z$, where the z field has couplings analogous to those of the S field introduced previously [2].

Moreover, the local $N=1$ supersymmetry of the effective low-energy field theory imposes important restrictions on its form. Thus, one knows that all terms in an effective $d=4$, $N=1$ supergravity theory which are at most bilinear in field derivatives can be characterized by a Kähler potential K , which is a real function of the chiral superfields, together with a chiral function f_{ab} which specifies the kinetic terms of the gauge fields (a, b are gauge group indices). The tree-level forms of these functions are well-known to be of a generalized no-scale form [10]. In the case of manifold compactifications from $D=10$ to $d=4$ we have [1]:

$$K = -\ln (S + S^\dagger) - 3 \ln (T + T^\dagger) + \dots \quad (1a)$$

$$f_{ab} = S \delta_{ab} \quad (1b)$$

where the dots include terms involving gauge non-singlet fields, whilst in the direct $d=4$

formulations [2]

$$K = -\ln (z + z^\dagger) + \dots \quad (2a)$$

$$f_{ab} = z \delta_{ab} \quad (2b)$$

where the dots again represent gauge non-singlet fields. The algebraic forms of higher-order terms that might be generated by string loop effects or by the exchange of Kaluza-Klein modes are also well-known [11]. However, it is not completely known which terms are in fact generated, although there are arguments that the superpotential terms in the Kähler potential (1a) are not renormalized [12], nor are there corrections to f_{ab} (1b) [13].

In this paper we study the one-loop string corrections to the functions (1,2) characterizing the effective $d=4$, $N=1$ supergravity action, and show in particular that the Kähler structures ((1a),(2a)) are not renormalized. In the case of the $D=10$ theory compactified to $d=4$, this proof proceeds by first demonstrating that loop corrections to the completed Chamseddine-Chapline-Manton [14] $D=10$ supergravity theory vanish when $\alpha' \rightarrow 0$. Simple counting of fermionic zero modes tells us that corrections to the bilinear and trilinear terms vanish, while corrections to quadrilinear terms vanish because of the periodicity of a Green function integrated over the fundamental domain. In the case of $d=4$ formulations, we demonstrate by explicit calculation that one-loop corrections to the two-point functions determining the Kähler potential (2a) and the chiral function f_{ab} (2b) both vanish. Finally, we comment on the possibility of extending these no-renormalization results to higher-order loop corrections.

We start with the computation of one-loop corrections to the effective $D=10$ supergravity action. The tree-level effective low-energy theory of the heterotic string was first derived in Ref. [3] where the tree-level two- and three-point bosonic amplitudes were studied. This analysis has been further developed for tree-level four-point couplings in Refs. [4] up to [8]. As a result of these studies the Chamseddine-Chapline-Manton [14] action is found with the addition of terms containing up to six derivative couplings of the gravitational field:

$$I = \int d^4x e \left[-\frac{R}{2k^2} - \frac{1}{8g_{10}^2} e^{-KD/\sqrt{2}} (tr F F^{\mu\nu} - tr R_{\mu\nu} R^{\mu\nu}) \right] \quad (3)$$

$$-\frac{1}{2} \partial_\mu \partial^\mu D - \frac{3k^2}{2g_{10}^4} e^{-KD/\sqrt{2}} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{gk^2}{g_{10}^2} R^{\mu\nu\rho\sigma} \bar{H}_{\nu\rho\alpha} \bar{H}^\alpha_{\sigma\mu}$$

where

$$H_{\mu\nu\rho} = \frac{1}{3} \left[\partial_\mu B_{\nu\rho} - \frac{K}{4} tr(A_\mu F_{\nu\rho} - \frac{2g_{10} A_\mu A_\nu A_\rho)}{3} + \frac{K}{4g_{10}^2} tr(\omega_\mu R_{\nu\rho} - \frac{2}{3} \omega_\mu \omega_\nu \omega_\rho) + c_{ycl} \right] \quad (4a)$$

$$\equiv \frac{1}{3} \left[\partial_\mu B_{\nu\rho} - \frac{K(\omega_\mu^2)}{4} \gamma_{\mu\nu\rho} + \frac{K}{4g_{10}^2} (\omega_{3L}^2)_{\mu\nu\rho} + c_{ycl} \right]$$

and

$$\bar{H}_{\mu\nu\rho} = \frac{1}{3} [\partial_\mu \bar{B}_{\nu\rho}] \quad (4b)$$

The vertex operators for the emission of gravitons - including the dilaton and the antisymmetric tensor field $B_{\mu\nu}$ - and gauge bosons in the heterotic string involve, as well as the usual exponential factor, the left-moving space-time and internal momenta and the right-moving supersymmetrized space-time momentum. This supersymmetrized momentum can be written in the Green-Schwarz formalism [15] as

$$B^i = P^i + \frac{1}{2} k_i R^{ij} \quad (5)$$

where P^i is the momentum operator in the right-moving sector, and R^{ij} is the contribution of the fermionic fields S^a . In calculating the amplitudes, traces over both bosonic and fermionic degrees of freedom must be carried out. In particular, the trace over the fermionic zero modes S_a^i has to be calculated. It is well-known that

$$Tr(R_o^n) = 0 \quad \text{for } n \leq 3 \quad (6)$$

where

$$R_o^{ij} \equiv \frac{1}{8} \bar{\gamma}_o^{ij} \gamma^{ij} \int_0$$

(7)

and γ^{ij} is the antisymmetric product of γ^i, γ^j and $\gamma^- \equiv (\gamma^0 - \gamma^3)/\sqrt{2}$. Therefore, there are no contributions to one-, two- or three-point functions at the one-loop level, and contributions to higher-point functions must contain at least four powers of the momenta k , resulting from the $Tr(\frac{1}{2} k R)^4$ factor implied by Eq. (5). It follows immediately that there can be no renormalization of the coefficients of $R, \partial_\mu D \partial^\mu D, tr F_{\mu\nu} F^{\mu\nu}, \partial_\mu B_{\nu\rho} [\partial^\mu B^{\nu\rho} - \omega_{3Y}^{\mu\nu\rho}]$ or $\omega_{3Y} \omega_{3Y}$. We would therefore expect that there is no renormalization of any term in the Chamseddine-Chapline-Manton action (3) as we now check explicitly.

To analyze one-loop effects on $R_{\mu\nu} R^{\mu\nu} e^{-KD/\sqrt{2}}, \partial_\mu B_{\nu\rho} \omega_{3L}^{\mu\nu\rho} e^{-KD/\sqrt{2}}$ and $\omega_{3L} \omega_{3L} e^{-KD/\sqrt{2}}$ we need a more detailed analysis of one-loop corrections to the 4-graviton interaction. These are in fact of order k^6 in the limit $\alpha' \rightarrow 0$ as we discuss below, whilst $R_{\mu\nu} R^{\mu\nu} \sim k^4, \partial_\mu B_{\nu\rho} \omega_{3L}^{\mu\nu\rho} \sim k^4$ and $\omega_{3L} \omega_{3L} \sim k^6$. Therefore neither are any of these interactions renormalized at the one-loop level. Finally, the $\omega_{3Y} \omega_{3L}$ interaction does not get renormalized because it is $\sim k^4$, whilst the one-loop interaction between 2 gravitons and two gauge bosons $\sim k^6$ when $\alpha' \rightarrow 0$.

To see how the potential one-loop contributions to the 4-graviton and 2-graviton/2-gauge boson interactions which appear to be of lower order in the momenta k in fact vanish, we discuss now the four-graviton amplitude in some detail. The one-loop contribution to this amplitude in the heterotic string was calculated in Ref. [16] to be:

$$A_{4g} = (2\pi g)^4 \int d^2\tau \prod_{a=1}^3 dV_a^2 [f(\bar{\tau})]^{-24} e^{\frac{2\pi i \bar{\tau}}{1 \leq b < c \leq 4}} \prod_{1 \leq b < c \leq 4} \left[\chi(\nu_{bc}, \tau) \right]^{k_b k_c k_d} \times \left(\frac{-2}{Im \bar{\tau}} \right)^5 \sum_{L \in \Lambda} \exp(-\pi i L^2 \bar{\tau}) K_{\delta_1 \delta_2 \delta_3 \delta_4} \prod_{\delta_1 \delta_2 \delta_3 \delta_4} \quad (8)$$

Here

$$f(\bar{\tau}) = \prod_{n=1}^{\infty} (1 - e^{-2\pi i n \bar{\tau}}) \quad (9)$$

is the partition function, $\ln X(\nu_{ab}, \tau) \equiv G_{ab}$ is the Green function, $\nu_{ab} \equiv \nu_a - \nu_b$, and $K_{j_1 j_2 j_3 j_4}$ is a kinematical factor which is of fourth order in the external momenta k_a ($a = 1, 2, 3, 4$):

$$K_{j_1 j_2 j_3 j_4} = \prod_{a=1}^4 k_a \int_{\alpha_a}^{\alpha_a} Tr (P_0^{i_1 i_2} P_0^{i_3 i_4} R_0^{i_1 i_3} R_0^{i_2 i_4}) \quad (10)$$

with $\rho_{i_1 \dots i_4}$ ($a = 1, \dots, 4$) the polarization tensor of the graviton. In Eq. (8) we sum over the internal loop momenta $L \in \Lambda$, where Λ is the root lattice of $E_8 \times E_8$ or $SO(32)$, and $T_{j_1 j_2 j_3 j_4}$ is given by

$$\begin{aligned} T_{j_1 j_2 j_3 j_4} &= A_{j_1 j_2 j_3 j_4} A_{j_1} + \frac{1}{2\alpha'} A_{j_1} A_{j_2} R_{j_3 j_4} \delta_{j_1 j_4} \\ &+ \left(\frac{1}{2\alpha'}\right)^2 R_{j_1 j_2 j_3 j_4} \delta_{j_1 j_2} \delta_{j_3 j_4} + \text{permutations} \end{aligned} \quad (11)$$

where

$$A_a = \frac{1}{2} \sum_{b=1}^4 k_b \left[\tilde{B}_{ab} + \frac{I_m \bar{\nu}_{ab}}{I_m \bar{\tau}} \right] \quad (12a)$$

$$R_{ab} = D_{ab} - \frac{1}{4\pi I_m \bar{\tau}} \quad (12b)$$

and explicit expressions for the functions $\tilde{B}_{ab}(\nu_{ab}, \bar{\tau})$ and $D_{ab}(\nu_{ab}, \bar{\tau})$ can be found in Ref. [16]. Note that we have included explicitly in Eqs. (8) and (11) the dependence on α' , as it is the limit $\alpha' \rightarrow 0$ which we intend to study. Some overall factors of α' which appear in Eq. (8) are ignored as they do not enter the argument we now develop.

In order to take the limit $\alpha' \rightarrow 0$, we first perform an expansion of

$$\prod_{a < b} [X(\nu_{ba}, \tau)]^{\alpha' k_a \cdot k_b} = 1 + \alpha' \sum_{a < b} k_a \cdot k_b G_{ba} + \dots$$

Multiplying now by $T_{j_1 j_2 j_3 j_4}$ in Eq. (11) we find that the coefficient of $(\frac{1}{2\alpha'})^2$ in Eq. (8) has typically the form $R_{12} R_{34} \delta_{j_1 j_2} \delta_{j_3 j_4}$, whereas the coefficient of $\frac{1}{2\alpha'}$ has typically the form $A_1 A_2 R_{34} \delta_{j_3 j_4}$ or $R_{12} R_{34} \delta_{j_1 j_2} \delta_{j_3 j_4} \sum_{a < b} k_a \cdot k_b G_{ba}$. We integrate now over the variables ν_a in the region

$$T = \left\{ -\frac{1}{2} \leq R_e \nu_a \leq \frac{1}{2} \right\}, \quad 0 \leq I_m \nu_a \leq I_m \tau \quad (13)$$

and we find that all these coefficients vanish as results of the following identities [17]:

$$\int d^2 \nu \left[\tilde{B}_{ab} + \frac{I_m \bar{\nu}}{I_m \bar{\tau}} \right] = 0 \quad (14a)$$

$$\int d^2 \nu \left[D_{ab} - \frac{1}{4\pi I_m \bar{\tau}} \right] = 0 \quad (14b)$$

Explicit proofs of these identities were not exhibited in Ref. [17], so we constructed our own independent proofs †, which are of a powerful nature susceptible to generalization. We observe that the integrands of (14a) and (14b) can be rewritten in terms of derivatives of the Green function G_{ab} :

$$\frac{1}{2\pi} \int d^2 \nu \partial_{\bar{\nu}} G_{ab} = 0 \quad (15a)$$

$$\frac{1}{2\pi^2} \int d^2 \nu \partial_{\bar{\nu}}^2 G_{ab} = 0 \quad (15b)$$

† We thank N. Sakai and Y. Tanii for correspondence on these identities, which we received during this work.

Both integrals vanish due to the periodicity of the Green function on the torus

$$G(\nu+1, \tau) = G(\nu, \tau) = G(\nu, \tau) \quad (16)$$

This periodicity is not immediately sufficient to prove (15) because there are poles both in $\partial_\nu G_{ab}$ and $\partial_\tau^2 G_{ab}$. However, a careful study of the pole contribution to the integrals in (15) shows that it is zero. We therefore find that the leading contribution to Eq. (8) when $\alpha' \rightarrow 0$ is of order 1. Its coefficients are higher order in the momentum and they have typically the forms: $A_{ij} A_{jk} A_{kl} A_{li}$, $A_{ij} A_{jk} R_{4j3j}$, $\sum_{a<b} k_a \cdot k_b G_{ab}$ and $R_{12} R_{34} \delta_{j1j2} \delta_{j3j4} (\sum_{a<b} k_a \cdot k_b G_{ab})^2$. This means in particular that the contribution of A_{4g} to the effective Lagrangian in the limit $\alpha' \rightarrow 0$ contains at least 8 derivatives and hence cannot renormalize the term $e^{-\alpha D/\sqrt{2}} \epsilon_{\mu\nu} R^{\mu\nu}$ in Eq. (3). For the same reason, $e^{-\alpha D/\sqrt{2}} \omega_{3L} \omega_{3L} \sim k^0$ cannot be renormalized by the 4-graviton interaction derived from the heterotic string one-loop amplitude.

A similar study of the 2-graviton/2-gauge-boson amplitude shows that it contains at least 6 powers of momenta, as the leading term which contains 4 powers of momenta (resulting from the kinematical factor (10)) vanishes by use of the identities (15). Thus we also find that $\omega_{3L} \omega_{3L} e^{-\alpha D/\sqrt{2}}$ in Eq. (3) does not get renormalized by one-loop contributions to the string amplitudes.

To complete our non-renormalization theorem we need to study the limit $\alpha' \rightarrow 0$ in higher-point functions. We have verified by arguments similar to the above that the term containing 4 powers of momenta in the n-graviton amplitude vanishes and it seems that the term $\sim k^0$ vanishes as well, though we have not confirmed this in all details. Thus we conclude that none of the terms in the Chamseddine-Chapline-Manton action are renormalized by one-loop string corrections. As to higher loop corrections, the counting of fermionic zero modes and the vanishing of integrations analogous to (15) may enable one to extend this non-renormalization theorem [18]. We hope to return to this subject in a future publication.

We now consider what happens when the $D=10$, $N=1$ effective supergravity field theory (3) is compactified to $d=4$. We are only interested in corrections to terms that are at most bilinear in the field derivatives, which are determined by the Kähler potential K and the chiral kinetic function f_{ab} , that can be read off from the forms of the kinetic energy terms

$$\mathcal{L}_{KE} = K_i^j \partial_\mu \phi_i^+ \partial^\mu \phi_j + \text{Re} \int_{ab} f_{ab}^{\mu\nu} F_\mu^a F_\nu^b \quad (17)$$

for arbitrary values of the background fields appearing in K and f_{ab} , where $K_j^i \equiv \partial^2 K / \partial \phi_i^+ \partial \phi_j$. Using the tree-level forms (1), L_{KE} (17) becomes

$$\mathcal{L}_{KE} = \frac{2S\partial^\mu S^+}{(S+S^+)^2} + \frac{3\partial_\mu T \partial^\mu T^+}{(T+T^+)^2} + (\text{Re} S) F_{\mu\nu}^+ F^{\mu\nu} + \dots \quad (18)$$

The absence of any one-loop renormalization to the 10-dimensional action (3) means that there is no one-loop renormalization to the coefficients in (18) for any value of the background fields S and T , and hence

$$\delta_{\text{one-loop}} K_i^j = 0 \quad (19a)$$

$$\delta_{\text{one-loop}} f_{ab} = 0 \quad (19b)$$

We can integrate (19) to deduce that

$$\delta_{\text{one-loop}} K = g(\phi) + g^*(\phi^+) \quad (20)$$

where $g(\phi)$ is a holomorphic function of the chiral superfields, corresponding to a renormalization of the conventional superpotential. But it has been argued previously [12] that the superpotential is not renormalized by string loop effects, so we conclude that

$$\delta_{\text{one-loop}} K = 0 \quad (21)$$

and hence all one-loop corrections to the effective $d=4$, $N=1$ supergravity theory vanish.

In principle, there could be corrections in $d=4$ arising from terms of higher order in the fields, which may after compactification renormalize kinetic energy terms. Typical

is the term $(\text{tr} F_{MN} F^{MN})^2$ that appears in the effective Lagrangian of the D=10 theory already at the tree level. When compactified it could yield a term $\sim |C|^2 \text{tr} F_{\mu\nu} F^{\mu\nu}$ (C are the gauge non-singlet fields) which would renormalize f_{ab} . This of course would depend very strongly on the manifold of compactification, and may vanish for certain types of compactification. Indeed, we show later that they are absent in a wide class of superstring theories formulated directly in four dimensions. Moreover, these do not appear in the 10-dimensional theory as one-loop corrections, rather as higher order corrections in α' already at the tree level (see Refs. [4] up to [8]).

It seems appropriate to comment here on the possibility that exchanges of Kaluza-Klein modes could renormalize the Kähler potential K or the chiral function f_{ab} . To consider this possibility, as commented previously, we need only look at the two-point functions in an arbitrary background field. However, to the extent that we have identified correctly the massless zero modes, there is no kinetic energy mixing between them and the Kaluza-Klein modes. Hence

$$\delta_{KK} K_{ij}^i = 0 \quad (22a)$$

$$\delta_{KK} f_{ab} = 0 \quad (22b)$$

and after integration

$$\delta_{KK} K = g_{KK}(\phi) + g_{KK}^*(\phi^\dagger) \quad (23)$$

In this case, there is no theorem to prevent the Kaluza-Klein modes from changing the superpotential, but as seen in (22) they do not affect the Kähler metric or the chiral kinetic function f_{ab} .

We conclude with some comments on the renormalization of the Kähler potential (2a) and the chiral kinetic function f_{ab} (2b) in 4-dimensional superstring theories. In this case, it is already known [19] that a $U(1)$ gauge field $F_{\mu\nu}^{(1)}$ can mix with the antisymmetric tensor field $B_{\mu\nu}$ if $\text{tr} Y \neq 0$ for the low-mass fields:

$$\sum_{\partial K - \partial_{\text{top}}} \mathcal{L} \ni c \varepsilon_{\mu\nu\sigma} B^{\mu\nu} F^{(\sigma)} F^{(\sigma)} \text{Tr} Y \quad (24)$$

but this term is absent for non-Abelian gauge fields and is in any case irrelevant to the renormalization of K or f_{ab} . To determine the possible one-loop corrections to the Kähler potential and to the gauge kinetic function we must calculate the two-point functions with two external dilatons (denoted here by z) and with two external gauge bosons. Both of these vanish because of θ -function identities which cause cancellations in the sum over all spin structures in theories where modular invariance is imposed on the left- and right-moving sectors independently.

The parity-conserving amplitudes we calculate are [20]:

$$A_{PC} = \text{Tr} \left(\frac{1 + G}{2} \prod_{a=1}^2 \Delta_{NS}^{(a)} V_{NS}^{(a,o)} \right) r \frac{1}{2} \text{Tr} \left(\prod_{a=1}^2 \Delta_R^{(a)} V_R^{(a,o)} \right) \quad (25)$$

where $G = (-1)^{N_{NS}-1}$ is the G-parity operator, and Δ_{NS} , V_{NS} (Δ_R , V_R) are the propagator and vertex operators in the Neveu-Schwarz (Ramond) sector:

$$\Delta_B = \frac{i}{4\pi} \int d\bar{z}_a z_a^{L_B-1} \bar{z}_a^{\bar{L}_B-1} \quad (26)$$

$$V_B = g \int_a^{\lambda,A} \left[P_\lambda(v) + \frac{1}{2} k_a^\mu \psi_\mu(v) \psi_\lambda(v) \right] V_{(a,v)} T^A(\bar{v}) \quad (27)$$

where $B=NS1$ (NSG) for the Neveu-Schwarz sector without (with) the insertion of the G-parity operator, and $B=R$ for the Ramond sector. In the Ramond sector we use $-\frac{1}{2} \Gamma_\mu(\nu_a) \Gamma_\lambda(\nu_a)$ instead of $\frac{1}{2} \psi_\mu(\nu_a) \psi_\lambda(\nu_a)$ in the vertex operator, where the ψ_μ satisfy antiperiodic boundary conditions and the Γ_μ satisfy periodic ones. We also have

$$V_o(a, v) = e^{i k_a \cdot [\bar{X}(v) + \bar{X}(\bar{v})]} \quad (28)$$

and

$$T^A(\vec{v}) = \tilde{T}^A(\vec{v}); \quad \tilde{T}^I(\vec{v}); \quad c(k) e^{i k \cdot X^I(\vec{v})} \quad (29)$$

for graviton, neutral gauge boson and charged gauge boson respectively. The partition functions in the various sectors are given by [20]:

$$F_{NS1} = \left[2\pi \frac{\vartheta_3(o, \tau)}{\vartheta_1'(o, \tau)} \right]^4 \quad (30a)$$

$$F_{NSG} = - \left[2\pi \frac{\vartheta_4(o, \tau)}{\vartheta_1'(o, \tau)} \right]^4 \quad (30b)$$

$$F_R = \left[2\pi \frac{\vartheta_2(o, \tau)}{\vartheta_1'(o, \tau)} \right]^4 \quad (30c)$$

and the correlation functions are

$$\langle \Psi_\mu^K(\nu_1) \Psi_\nu^K(\nu_2) \rangle = - \frac{i g_{\mu\nu}}{\vartheta_1'(\nu_2, \tau)} \begin{cases} \vartheta_2(o, \tau) \vartheta_4(o, \tau) \vartheta_3(\nu_1, \tau) \vartheta_4(\nu_2, \tau) & K=NS1 \\ \vartheta_3(o, \tau) \vartheta_2(o, \tau) \vartheta_4(\nu_1, \tau) \vartheta_4(\nu_2, \tau) & K=NSG \\ -\frac{1}{2} \vartheta_3(o, \tau) \vartheta_4(o, \tau) \vartheta_4(\nu_1, \tau) \vartheta_2(\nu_2, \tau) & K=R \end{cases} \quad (31)$$

In calculating the traces in (25) we encounter the following combinations resulting from

the product of vertex operators: $(P_\mu P_\lambda); (P_\mu \frac{1}{2} k_\rho \psi_\rho(\nu_a) \psi_\lambda(\nu_a))$ and $< \frac{1}{2} k_\rho \psi_\rho(\nu_1) \psi_\lambda(\nu_1) \psi_\lambda(\nu_1) \psi_\rho(\nu_2) \psi_\lambda(\nu_2) >$ with the appropriate correlations in the Ramond sector. The first of these three correlation functions vanishes when we sum over spin structures, because the operators P_μ do not depend on the spin structure, and we have

$$F_{NS1} + F_{NSG} - F_R = 0 \quad (32)$$

as a result of θ -function identities. Note the minus sign before F_R , which is dictated by modular invariance. The second correlation function vanishes for the same reason when summed over all spin structures. In this case we have a pole contribution in (31) when $\nu_1 - \nu_2 \simeq 0$, but the pole residue is the same for all the spin structures, which when summed over will have (32) as a common factor and hence vanish. To prove that the third correlation function vanishes we use (30) combined with (31) and the following identities:

$$\vartheta_3(\nu, \tau) \vartheta_4^2(o, \tau) = \vartheta_4^2(\nu, \tau) \vartheta_3^2(o, \tau) - \vartheta_1^2(\nu, \tau) \vartheta_2^2(o, \tau) \quad (33a)$$

$$\vartheta_1^2(\nu, \tau) \vartheta_3^2(o, \tau) = \vartheta_4^2(\nu, \tau) \vartheta_2^2(o, \tau) - \vartheta_2^2(\nu, \tau) \vartheta_4^2(o, \tau) \quad (33b)$$

as well as Eq. (32).

Thus we have shown that the $O(k^2)$ kinetic energy terms in the effective Lagrangian cannot be renormalized by one-loop effects. This is true whether we use the dilaton or vector boson vertices, and the argument can be extended to all the n-point functions which could contribute to such a term in the effective Lagrangian. The effective low-energy theory found by simple truncation of the $N = 1, D = 10$ supergravity is very robust. Many possible corrections which appear to be consistent with the obvious symmetries of the theory [11] are not in fact generated by higher-order effects. This increases ones confidence in phenomenological results obtained from the tree-level truncated theory. The robustness of the tree-level results suggests that there may be more symmetries waiting to be discovered. We hope to return to these in a future publication.

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