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PRECISION MEASUREMENTS OF FINAL STATE
 WEAK COUPLING
 FROM POLARIZED ELECTRON-POSITRON
 ANNIHILATION

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Abstract: We introduce and analyse the longitudinally polarized forward-backward asymmetry A_{FB}^{Pol} for electron-positron annihilation on Z^0 resonance. We show that a specific combination of data taken with two opposite electron polarizations is independent of the initial state details including one loop electroweak and bremsstrahlung effects. This remarkable property would allow the weak coupling of the produced final state ($\mu, \tau, b, c, \dots$) to be measured in a clean and very promising way at LEP/SLC.

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1. INTRODUCTION

One of the most promising aspects of the new LEP and SLC colliders is their ability to perform high precision investigations of the Z_0 coupling to fermions (and may be scalars?). This is due to several reasons, including :

- i) the ability to tune the energy of the e^+e^- annihilation to precisely the Z_0 mass, thus generating high statistics ;
- ii) the absence of hadrons in the initial state which greatly reduces the theoretical uncertainty due to strong interactions relative to $p\bar{p}$, pp or even ep colliders ;
- iii) the possibility of measuring asymmetries (ratios of cross sections), which allows the cancellation of a large fraction of experimental sources of error such as the uncertainty in the luminosity, etc.

It has been shown^{(1),(2)} that the capacity of electron beam polarization vastly improves the ability of these machines to measure the electron- Z_0 coupling. The left-right electron beam polarization asymmetry

$$(1) \quad A_{LR} = \frac{\sigma(e^+e_L^- \rightarrow \text{hadrons}) - \sigma(e^+e_R^- \rightarrow \text{hadrons})}{\sigma(e^+e_L^- \rightarrow \text{hadrons}) + \sigma(e^+e_R^- \rightarrow \text{hadrons})}$$

at $q^2 \equiv (p_{e^+} + p_{e^-})^2 = M_Z^2$ depends only on the initial state e coupling

to the Z_0 and is extremely sensitive to interesting new physics such as the one-loop renormalization of coupling constants in the Standard Model of Glashow, Salam and Weinberg (GSW), the existence of new fermions or scalars⁽³⁾ or new Z 's beyonds GSW⁽⁴⁾⁽⁵⁾. Further, it is insensitive to one photon initial state bremsstrahlung⁽⁶⁾ because it is a very flat function of $\sqrt{s}$ near the Z_0 resonance, thus eliminating a source of theoretical uncertainty due to many photon bremsstrahlung from the initial state.

The quantitative consequences of all these nice properties have been discussed elsewhere. In particular, it has been shown^{(2),(7)} that a measurement of A_{LR} on Z_0 would provide the value of $\sin^2 \theta_W$, the

weak mixing angle, with an experimental error⁽⁷⁾ of ± 0.0005 and a theoretical uncertainty⁽²⁾ of ± 0.0005 .

In this paper we wish to show that polarization is also an important tool for high precision measurements of the couplings of Z_0 to the final state fermion f in $e^+e^- \rightarrow f\bar{f}$ (with $f \neq e, \nu_e$). This is due to the possibility of forming a very specific polarized forward-backward asymmetry $A_{FB}^{pol(f)}$ to be defined below, which depends at $q^2 = M_Z^2$ only on the f - Z_0 coupling and not on the e - Z_0 coupling. Further, it is also insensitive to initial state photon bremsstrahlung and so QED effects are more reliably calculated.

This paper is organized as follows :

Section II gives a description and discussion of $A_{FB}^{pol(f)}$ at the tree level. Section III examines the one-loop $O(\alpha_{em})$ corrections to A_{FB}^{pol} . The numerical results for the one-loop complete electroweak corrections in GSW are obtained using the Monte Carlo computer program BREMMUS. In Section IV the experimental accuracy is discussed, while section V gives an analysis of the interesting physics issues. Finally in Section VI we give some conclusions.

2. TREE LEVEL RESULTS

Consider the process $e^+e^- \rightarrow f\bar{f}$ with f a fermion ($f \neq e, \nu_e$), with only s channel graphs at tree level in terms of the helicity matrix elements which include both photon and Z^0 exchange ($i, j = L, R$)

$$(2) \quad M_{ij} = g_{ie}^\gamma g_{jf}^\gamma G_\gamma + g_{ie}^Z g_{jf}^Z G_Z \quad .$$

Here we define the pure helicity coupling constants of left-handed electrons to photons g_{Le}^γ and to Z^0 's g_{Le}^Z or right-handed electrons to photons g_{Re}^γ and to Z^0 's g_{Re}^Z . Similarly we define the pure helicity couplings of the final state fermion to γ and Z by g_{jf}^γ, g_{jf}^Z , $j=L, R$. G_γ and G_Z are respectively the photon and Z propagators.

In terms of these matrix elements the differential cross section for $e^+e^- \rightarrow f\bar{f}$ with massless f , electron polarisation P_{e^-} and positron polarisation P_{e^+} reads :

$$(3) \quad \frac{d\sigma}{d\Omega} \simeq (1-P_{e^-})(1+P_{e^+}) \left[|M_{LL}|^2 \frac{u^2}{s} + |M_{LR}|^2 \frac{t^2}{s} \right] \\ + (1-P_{e^+})(1+P_{e^-}) \left[|M_{RR}|^2 \frac{u^2}{s} + |M_{RL}|^2 \frac{t^2}{s} \right]$$

where u, t, s are the usual Mandelstam variables.

From this expression one can derive the forward backward asymmetry for a given polarization degree defined as

$$(4) \quad A_{FB}^{(P)(f)} = \frac{(N_{PF} - N_{PB})}{N_{PF} + N_{PB}}$$

where $P = \frac{P_{e^+} - P_{e^-}}{1 - P_{e^+} P_{e^-}}$ is the polarization degree of the e^+e^- system,

$N_{PF,B}$ is the number of events with polarization P in the forward hemisphere ($0 \leq \cos \theta \leq 1$) or backward hemisphere ($-1 \leq \cos \theta \leq 0$) and θ is the angle between e^- and f .

On Z_0 resonance we may drop the photon exchange and write :

$$(5) \quad A_{FB}^{P(f)} \simeq \frac{3}{4} \frac{(g_{Lf}^Z)^2 - (g_{Rf}^Z)^2}{(g_{Lf}^Z)^2 + (g_{Rf}^Z)^2} \left[\frac{g_{Le}^2(1+P) - g_{Re}^2(1-P)}{g_{Le}^2(1+P) + g_{Re}^2(1-P)} \right]$$

where $G_{L,Re}$ are the initial state couplings to Z_0 . Thus $A_{FB}^{P(f)}$ depends *both* on the initial state and on the final state couplings to Z_0 (and on the polarization degree as well).

We can construct a much simpler quantity on Z_0 resonance defining a "polarized forward backward" asymmetry in the following way :

$$(6) \quad A_{FB}^{pol(f)} = \frac{1}{\bar{P}} \frac{(N_{PF} - N_{\neg PF}) - (N_{PB} - N_{\neg PB})}{(N_{PF} + N_{\neg PF} + N_{PB} + N_{\neg PB})}$$

On Z_0 resonance one may easily verify that

$$(7) \quad A_{FB}^{pol(f)} \simeq \frac{3}{4} \frac{(g_{Lf}^Z)^2 - (g_{Rf}^Z)^2}{(g_{Lf}^Z)^2 + (g_{Rf}^Z)^2}$$

which is completely independent of the *initial* state electron coupling to Z_0 . This latter property has to be compared with that which characterises the longitudinal polarization asymmetry A_{LR} eq.(1) which, in the same kinematical configuration, turns out to be independent of the *final* state coupling. Defining

$$(8) \quad A_f = \frac{(g_{Lf}^Z)^2 - (g_{Rf}^Z)^2}{(g_{Lf}^Z)^2 + (g_{Rf}^Z)^2} = \frac{2 g_{vf} g_{af}}{(g_{vf})^2 + (g_{af})^2}$$

we have, at this level :

$$(9) \quad A_{FB}^{pol(f)} = \frac{3}{4} A_f$$

$$(10) \quad A_{LR} = A_e .$$

These expressions become more meaningful if one remembers that the corresponding value of the unpolarized forward-backward asymmetry would be :

$$(11) \quad A_{FB}^{(f)} = \frac{3}{4} A_e A_f .$$

The interest of longitudinally polarized beams is clearly brought to line by these formulae : whereas unpolarized beams invariably lead to a measurement of a product of initial and final state couplings, polarized asymmetries allow to disentangle the different couplings, thus providing a number of unambiguous tests of several basic assumptions of the Standard Model.

Before closing this section we would like to stress an important property of $A_{FB}^{pol(f)}$ around Z_0 resonance. Actually, the combination of the initial couplings which appeared in the numerator and in the denominator before cancellation is the sum $(g_{Le}^Z)^2 + (g_{Re}^Z)^2$. This is *not* suppressed by a factor of $1-4 s_\theta^2$ (θ is the weak mixing angle) and consequently $A_{FB}^{pol(f)}$ will be a slowly varying function of $\sqrt{s}$ near (i.e. within a few GeV) the Z_0 pole, when the photon terms has to be included as well.

3. RESULTS INCLUDING ONE LOOP ELECTROWEAK CORRECTIONS

We now discuss the $O(\alpha_{em})$ corrections to A_{FB}^{pol} near the Z^0 resonance. These can be divided into four groups :

- i) oblique corrections : vector boson self-energies. As shown elsewhere ^{(1),(8)} they can be included as renormalizations of the various coupling constants $g_{ie}^\gamma, g_{ie}^Z, g_{jf}^\gamma, g_{jf}^Z$ and to the effective γ and Z propagators G_γ and G_Z and so they do not change the previous results.
 - ii) vertex corrections involving virtual heavy weak $W^\pm$ and Z bosons (and even Higgs H) in internal loops effectively renormalize the coupling constants g and again the previous discussion is unaffected.
 - iii) box diagrams involving only heavy $W^\pm$ and Z 's do not factorize as in the previous discussion but are very small.
 - iv) QED corrections : initial and final state hard photon bremsstrahlung do not factorize ; neither do box diagrams involving a virtual photon and a Z^0 . In particular, initial state radiation poses a real threat to the unpolarized forward-backward asymmetry in $e^+e^- \rightarrow \mu^+\mu^-$; its steepness as a function of $\sqrt{s}$, due to suppression factors of $4\sin^2\theta_w - 1$, makes one photon QED corrections very large and renders necessary the inclusion of many-photon initial state radiation. As stressed in Section 2, A_{FB}^{pol} on the contrary is a very flat function of $\sqrt{s}$ near the Z^0 resonance and we expect from this a strong suppression of QED effects.
- Actually, all these electroweak one loop effects have been investigated in detail using the Monte Carlo computer program BREMMUS ⁽⁹⁾, and we shall devote the next part of this section to an illustration of the numerical results. This program calculates the differential cross section of the process $e^+e^-_{pol} \rightarrow f\bar{f}(\gamma)$ completely to $O(\alpha_m^3)$ for arbitrary electron longitudinal polarization. Many $O(\alpha_{em}^4)$ and higher order effects are also included such as those coming from the running of the coupling constants in the $e^+e^- \rightarrow f\bar{f}\gamma$ cross section for hard and soft bremsstrahlung.

NUMERICAL CALCULATIONS

The results of the numerical simulations are displayed in Fig. 1. and 2. The acceptance was defined as :

- . both outgoing fermions at an angle of more than 20° with respect to the beam axis
- . both outgoing fermions having more than 15 GeV energy .

In addition, the effect of a further acolinearity cut of 2° between the two outgoing fermions was studied (dashed line). One can see clearly from Fig.1 and 2 that the unpolarized forward-backward asymmetry and the forward-backward asymmetries with fixed $\mathbb{P}$ (= polarization degree) both show a very strong dependence on $\sqrt{s}$, as well as a substantial dependence on the further acolinearity cut. This is due to the dependence of the asymmetries on the initial e^+e^-Z vertex and thus on the γ -Z interference and initial state bremsstrahlung.

The left-right asymmetry and the polarized forward-backward asymmetry defined above are on the contrary practically energy independent and cut-independent. Taking in fact the difference of cross-sections with opposite initial state helicity isolates the parity violating Z^0 exchange and eliminates the disturbing photon exchange, making moreover $A_{FB}^{pol}(f)$ insensitive on cuts as a consequence of its energy independence.

The conclusion of this numerical analysis summarized in Fig. 1 and 2 is that $A_{FB}^{pol}(f)$ is, from the theoretical point of view, a very appealing quantity. We shall now devote the next section IV to a short discussion of the related experimental accuracy.

4. EXPERIMENTAL ACCURACIES

In this section we will give a rough estimate of the experimental accuracies achievable in measurements of the individual couplings A in a standard SLC/LEP detector capable of charge and momentum measurements and of electron and muon identification. We assume that the polarization degree is $\mathbb{P} = .5$ and that the accumulated luminosity is of 100 pb^{-1} . The data taking mode is that requested for the left-right asymmetry measurement, namely one with an equal amount of luminosity for polarization $\mathbb{P} = +P$ and $\mathbb{P} = -P$. This is obtained by reversing the electron

polarization for SLC, where the positrons are not polarized, or by depolarizing an equal number of electron and positron bunches in LEP.

The statistical error in measuring A_{FB}^{pol} eq (6) is

$$(12) \quad \Delta A_{FB}^{pol} = \frac{1}{\mathcal{P}} \sqrt{\frac{1}{N}}$$

resulting in $\Delta A_f \approx \frac{4}{3\mathcal{P} \sqrt{N}}$

where $N = N_{PF} + N_{-PF} + N_{PB} + N_{-PB}$

to be compared with the precision obtained from the unpolarized asymmetry $\Delta A_f (unpol) = \frac{4}{3 A_e \sqrt{N}}$.

The gain in statistical precision for a given luminosity is thus $\mathcal{P}/A_e \approx 3$ for $\sin^2 \theta_W \approx .23$ and $\mathcal{P} = .5$.

It goes beyond the purposes of this paper to analyze all the systematic errors of all possible final states. They can be divided in general in three classes :

a) systematic errors resulting in a systematic shift in the forward-backward asymmetry. These would be typically forward-backward and charge dependent asymmetries in detection efficiency, uncertainties in the center of mass energy or in the initial state radiative correction. Such uncertainties will vanish for A_{FB}^{pol} , owing to its specific definition eq (6).

b) systematic errors resulting in a systematic relative increase or decrease of the asymmetry. These include errors coming from contamination of the sample by unwanted processes, i.e., two photon physics, cosmics, interchannel contamination such as c contamination in the b sample, B- $\bar{B}$ mixing, experimental smearing ... etc. These errors enter proportionally to the asymmetries and will affect in general the determination of both polarized and unpolarized asymmetries. Note however that the error due to the uncertainty on A_e , which enters in the unfolding of the A_f from the unpolarized asymmetries, does not affect A_{FB}^{pol} .

Consider, as an example, the precision that one can obtain without polarization with 100 pb^{-1} and $10^5 e^+e^- \rightarrow \mu^+\mu^-$ events, the precision on A_e is limited to $\Delta A_e \approx 0.012$ ($\Delta \sin^2 \theta_W = 0.0015$) and limits the extraction of A_f to $\Delta A_f/A_f \geq \Delta A_e/A_e = 7.5\%$ for $\sin^2 \theta_W = 0.23$.

c) errors entering specifically in measurements with polarized beams come from the polarization measurement and the relative normalization of the samples taken with opposite polarization. If the normalization is good enough for the measurement of A_{LR} it turns out that this error is negligible in the measurement of A_{FB}^{pol} . The polarization measurement error ΔP enters as $\Delta A_f = A_f \frac{\Delta P}{P}$. A precision of $\frac{\Delta P}{P} = 1\%$ would limit the knowledge of the A_f 's to 1% of their value ; this is always smaller than the statistical errors, at least for $P = 0.5$, but not quite negligible when the value of A_f is large, i.e. for d-type quarks.

Let us consider now, as an example, the precision that one can obtain from measuring A_{FB}^{pol} for $e^+e^- \rightarrow b\bar{b}$ at the Z^0 pole. The unpolarized forward-backward asymmetry for b's has been measured at PEP and PETRA⁽¹⁰⁾. $b\bar{b}$ events are selected via their decay into muons and electrons having a large $P_{\perp}$ with respect to, say, the thrust axis. An overall efficiency of typically 10% can be achieved. The only important contribution to the systematic error was the 20% contamination from semi-leptonic charm decay leading to a 10% relative error on A_{FB} . This error will be greatly reduced when running at the Z pole, since the relative contribution of the $b\bar{b}$ to $c\bar{c}$ final state in Z decays will be enhanced by a factor $[(a_b^2 + v_b^2)/(a_c^2 + v_c^2)]/[(1/9)/(4/9)] \approx 5$, which reduces the error to $\pm 2\%$. These numbers have been recently confirmed by detailed calculations⁽¹¹⁾, leading to a precision in the measurement of

$A_{FB}^{b\bar{b}}$ of ± 0.006 (stat) ± 0.002 (syst), leading to a measurement of A_b to ± 0.08 , taking into account the error on A_e obtained with unpolarized beams.

From the previous discussion we expect thus an error on the polarized forward-backward asymmetry $A_{FB}^{pol(b)}$ of ± 0.012 (stat) ± 0.012 (syst), leading to a measurement of A_b to ± 0.02 with a gain in precision with respect to the unpolarized case of a factor $\gtrsim 4$.

Having discussed in detail the predictable experimental accuracy on a measurement of A_{FB}^{pol} we are now in a position to discuss in the next section V the relevant physics implications.

5. PHYSICS ISSUES

The fact that $A_{FB}^{pol}(M_Z^2)$ is independent of the initial electron coupling to Z_0 and of disturbing QED effects provides us with a rather powerful tool for investigating the physics issues of the produced final state. We shall illustrate this appealing possibility with three particularly simple examples.

Consider first μ and τ pair production. A measurement of $A_{FB}^{pol(\mu,\tau)}(M_Z^2)$ represents then, to our knowledge, one of the most unbiased tests of $e-\mu$, $e-\tau$ universality. Note that the information contained in $A_{FB}^{pol(\tau)}$ is *exactly* the same which would be provided by a measurement of the τ polarization (assuming the standard picture for $\tau \rightarrow \pi \nu$ decay), but with the *important* increase in statistics coming from the possibility of using *all* the produced τ pairs.

As a second example, consider $b\bar{b}$ or $c\bar{c}$ production. If the effect of the strong interactions of the final state were washed out (as in the case of the longitudinal asymmetry A_{LR} illustrated in a previous paper⁽²⁾), the same kind of possibility of measuring the b and c coupling to Z_0 would exist as in the μ, τ pair case.

The corresponding ideal situation is summarized in Table I and Fig.2 where we show the numerical values of the various A_f coefficients eqs. (8), (9), together with their sensitivity on the assumed value of $\sin^2 \theta_W$. One can see from this table and this figure that for d-type quarks A_f is large and very little sensitive on $\sin^2 \theta_W$, thus providing a different and independent information from that obtained by measuring the smaller and more $\sin^2 \theta_W$ dependent u-quarks and charged leptons A factors.

As a matter of fact however, strong interaction effects do not cancel, in general, for a quantity like $A_{FB}^{pol(b,c)}$, as we will show below. From the point of view of a measurement of the weak quark charges, this would be a difficulty if the effect were large, owing to the intrinsic theoretical uncertainty which affects strong interactions computations.

In this philosophy, one would thus look for that specific set of $b\bar{b}$ ($c\bar{c}$) events for which the strong interaction effect is minimized. On the other hand, if for a certain different set of $b\bar{b}$ ($c\bar{c}$) events the strong interaction effect were particularly enhanced, one could turn the argument around and use such events as a possible interesting way of measuring, say, α_s . We consider both possibilities as equally valuable and mainly determined by the actual technical characteristics of the specific experiment. Thus, we will give here a formula, which is a simple generalization of that given in a previous paper⁽¹²⁾, and which allows to compute the strong interaction effect in any practical case. To do this, we assume the validity of a perturbative QCD expansion on Z_0 pole. This is certainly justified as far as $b\bar{b}$, $c\bar{c}$ production is concerned ($t\bar{t}$ production will be considered separately below). Then, to first order in α_s , we write :

$$(13) \quad A_{FB}^{\text{pol}(b,c)}(M_Z^2) = A_{FB}^{\text{pol}(b,c)}(M_Z^2) (\alpha_s=0) \cdot \left[1 + \frac{\alpha_s (M_Z^2)}{\pi} \Delta \right]$$

where

$$(14) \quad \Delta = \int dx \, dx' [W_3^{(s)}(x, x') - W_u^{(s)}(x, x')] \rho_{\text{exp}}(x, x') .$$

The meaning of the quantities entering eq.(14) is the following :

$W_{3,u}^{(s)}(x, x')$ are structure functions densities in the process

$e^+e^- \rightarrow q\bar{q} + q\bar{q}g$, where we have defined x, x' as the usual reduced quark, antiquark energies. They are related to the conventional structure functions $\tilde{W}_{1,2,3}(x, s = (p_{e^-} + p_{e^+})^2)$ defined in reference (12) in the following way :

$$(15) \quad \frac{sX^2}{2} [\tilde{W}_3(x, s) - \tilde{W}_3(x, s) (\alpha_s = 0)] = \frac{\alpha_s}{\pi} (s) \int dx' W_3^{(s)}(x, x', s)$$

$$(16) \quad 2x [(\tilde{W}_1(x, s) + \frac{sX^2}{12} \tilde{W}_2(x, s)) - (\tilde{W}_1(x, s) + \frac{sX^2}{12} \tilde{W}_2(x, s))(\alpha_s=0)] = \\ = \frac{\alpha_s}{\pi} (s) \int dx' W_u^{(s)}(x, x', s) .$$

Using this definition and eqs. (A1-A3) of ref.(12) one can easily work out the explicit expression of $W_{3,u}^{(s)}(x,x',s)$. Starting from such expressions, one can then build the quantity Δ which is of practical relevance by specifying the kind of events which are particularly convenient from the experimental point of view. This corresponds to giving a certain prescription for the weight function $\rho_{\text{exp}}(x,x')$ which depends on the experimental choice for identifying the produced quark (leptonic modes, jet identification, etc.etc.). We have already given some results in two extreme ideal configurations in ref (12), where we found :

$$(17) \quad \Delta = -\frac{1}{3} \quad (2 \text{ jets events})$$

$$(18) \quad \Delta = -\frac{4}{3} \quad (\text{full 2 jet} + 3 \text{ jet contribution})$$

assuming $m_q = 0$. Although these numbers are qualitative, they fix the scale of the effect which has to be expected for this particular asymmetry. It would be a relatively easy job to compute the effect in a more realistic configuration⁽¹³⁾ and to decide how to optimize it for the two alternative purposes that we mentioned before. Note that this relatively simple procedure which exploits $W_{3,u}^{(s)}(x,x')$ is only allowed by the complete cancellation in this specific asymmetry of unwanted QED effects coming in particular from $Z\gamma$ boxes interference, which would make the previous formulation with structure functions not applicable. To make the previous statements more quantitative, suppose Δ , for certain set of events, to be of order one. Then, from the previous discussion of Section IV, one would be able to obtain a *measurement* of α_s from $A_{\text{FB}}^{\text{pol}(b)}$ to a precision of

$$(19) \quad \delta \left[\frac{\alpha_s}{\pi} (M_Z^2) \right]_{\Delta=1} \simeq \pm 0.02.$$

This measurement would be of a precision comparable to that coming from other available measurements, but is *free of any extra ambiguity*.

As a third example we mention a simple possible test of New Physics beyond the Standard Model coming from the fact that any deviation of the final state coupling to Z^0 due, for instance, to one heavy Z' with essential mixing with Z^0 might in principle be observed

in A_{FB}^{pol} . In particular, if the final state were of hadronic kind, this deviation would be *different* from that observed in A_{LR} , thus allowing in principle to disentangle the source.

Finally, we would like to spend a few words about the possibility of using $A_{FB}^{pol(f)}$ to measure the couplings of new, still undiscovered, fermions f (for scalars, A_{FB} would obviously be zero). This requires some caution since, in this case, the final state mass can no longer be ignored. One can easily be persuaded, though, that in the specific case of A_{FB}^{pol} taking the final mass into account will not destroy the remarkable independence of the initial state coupling, at least as far as the most dangerous initial state radiation effects are concerned. Thus, the main effect will be to modify the structure of final state coupling which will be measured. To lowest order, this will actually become :

$$(20) \quad A_{FB}^{pol(f)0}(M_Z^2) = \frac{3\beta v_f a_f}{v_f^2[3-\beta^2] + 2 a_f^2 \beta^2} \quad (\beta = \sqrt{1 - 4m_f^2/M_Z^2}) .$$

To compute QCD corrections to eq (20) is lengthy but straightforward if the top quark mass is less than, say 35 GeV (the formalism to be used has been given elsewhere^{(14),(15)}). For higher masses other deeper problems, including the validity of a lowest order QCD expansion itself, arise which have to be very carefully studied.

6. CONCLUSIONS

We have introduced and analyzed the main properties of the longitudinal forward backward asymmetry A_{FB}^{pol} for e^+e^- collisions on Z_0 resonance. We have shown that it is independent of the details of the *initial* state, thus allowing a clean and unambiguous test of the physics issues of the final state. This fact, combined with the corresponding improvement of statistical precision, makes this specific asymmetry rather special and, we believe, worth to be analyzed in the next series of LEP/SLC experiments.

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TABLE CAPTIONS

Table I. Value of fermion A_f coefficients and their sensitivity to $\sin^2 \theta_W$ (for $\sin^2 \theta_W = 0.23$)

Table II. Values computed by BREMMUS for $A_{FB}^{\text{pol}(\mu, c, d)}$ for $M_Z = 92$ and 94 GeV ($m_H = 100$ GeV, $m_t = 60$ GeV).

TABLE I

Fermion type	A_f	$\partial A_f / \partial \sin^2 \theta_W$
e, μ , τ	0.16	- 7.85
u, c, t	0.67	- 3.45
d, s, b	0.94	- 0.63

TABLE II

M_Z	μ	b	c
92	0.121	0.680	0.491
94	0.191	0.686	0.521

FIGURE CAPTIONS

Fig. 1(a) Ecm-dependence of :

(top) the forward-backward asymmetry A_{FB} for muons with longitudinal beam polarization $P = (P^+ - P^-)/(1 - P^+ P^-)$, $P = -0.5, 0., +0.5$.

(middle) The polarization asymmetry A_{LR} .

(bottom) The polarized forward-backward asymmetry A_{FB}^{pol} as defined in the text.

Full lines are calculated with BREMMUS⁽⁹⁾ for $p_\mu > 15$ GeV, $20^\circ < \theta_\mu < 160^\circ$. Dashed lines indicate the effect of an additional acolinearity cut of 2° .

- (b) same as (a) for b quarks
- (c) same as (a) for c quarks

Fig.2(a) Ecm variation of the unpolarized forward-backward asymmetry A_{FB} , the polarization asymmetry A_{LR} , the polarized forward-backward asymmetry A_{FB}^{pol} for $M_Z = 92$ GeV (full lines) and $M_Z = 94$ GeV (dashed lines). Results are obtained with BREMMUS⁽⁹⁾.

- (b) same as (a) for b quarks
- (c) same as (a) for c quarks.

FIG. 1

FIG. 2