INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

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1987 MIRAMARE-TRIESTE

INTERNATIONAL
ATOMIC ENERGY
AGENCY

UNITED NATIONS
EDUCATIONAL,
SCIENTIFIC
AND CULTURAL
ORGANIZATION

11-87/81

International Atomic Energy Agency

and

United Nations Educational Scientific and Cultural Organization

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The Born-Infeld Action From Conformal Invariance of the Open Superstring *

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ABSTRACT

We show that the one-loop approximation to sigma-model perturbation theory for the open superstring leads to the low energy effective action of Born-Infeld for the gauge field coupled to the ends of the string. Thus the bosonic part of the low-energy open superstring effective action is just that of the open bosonic string. We also show that these results are exact to all-loop orders of sigma-model perturbation theory.

MIRAMARE - TRIESTE

January 1987

To appear in Physics Letters B (1987).

Supported in part by INFN, Sezione di Trieste, Trieste, Italy.

On leave of absence from Imperial College, London, United Kingdom.

An important issue in string theory is the determination of the low energy effective action for the massless modes of the string. There are several approaches to this problem [1][2][3][4], one of which is to interpret the conditions for conformal invariance of the associated sigma model as the effective field equations [3]. In this approach the spacetime fields representing the massless modes appear as the "coupling constants" of the sigma model, and one calculates the beta-functions (strictly, the trace anomaly [5]) as a perturbation series in derivatives of these spacetime fields. For the open string there is one massless Lorentz vector mode whose spacetime field A_μ appears in the sigma model action as the boundary term $\int A_\mu(x) dx^\mu$. The beta-function for this term has been computed recently by Abouelsaood et al. [6] in the one-loop approximation (to sigma model perturbation theory), which is equivalent to the approximation of slowly varying field strength $F_{\mu\nu}$. It turns out that the condition of vanishing beta-function is the Euler-Lagrange equation for the Born-Infeld Lagrangian $\det^{1/2}(\eta_{\mu\nu} + F_{\mu\nu})$, which had been obtained previously by Fradkin and Tseytlin [7] as the low energy spacetime effective Lagrangian for A_μ by path integral methods.

In this letter we extend this result to the open superstring in the NSR formalism using worldsheet superspace methods. The massless modes are those of a ten dimensional Maxwell supermultiplet. Restricting attention to the bosonic field A_μ only, we find that the effective action is again the Born-Infeld action. A low energy spacetime effective action for the open superstring has been derived previously by Tseytlin [8], using the effective action method of [2]. He obtains an effective action for A_μ that is not the Born-Infeld action. The significance of this discrepancy is not clear to us.

Our other main result is that there are no higher sigma model loop corrections to the beta function for the bosonic string and the superstring. (It is still possible that there are either nonperturbative contributions to the beta function [9] or string loop contributions [10]). Thus the Born-Infeld action, which was derived in [7][6] using the slowly varying field approximation, is in fact valid for arbitrary A_μ .

The Euclidean open superstring action in an arbitrary background spacetime field A_μ is
(in the NSR formalism)

$$S = -4 \int_M d\tau d\bar{z} d\theta d\bar{\theta} (D \phi^\mu \bar{D} \phi^\mu) \eta_{\mu\nu} + 2i \int_{\partial M} d\tau d\psi A_\mu(\phi) D_\psi \phi^\mu \quad (1)$$

where M is the world supersheet with coordinates $\{z = \tau + i\sigma, \theta = \psi + i\chi, \bar{z}, \bar{\theta}\}$ which at zero string-loops we can choose to be the upper half superplane, $\sigma \geq 0$, with boundary ∂M at $\sigma = 0$, $\chi = 0$ and coordinates $\{\tau, \psi\}$ [11]. The two dimensional superfield ϕ^μ has the component expansion

$$\phi^\mu = x^\mu + i\theta \lambda^\mu + i\bar{\theta} \bar{\lambda}^\mu + i\theta\bar{\theta} \gamma^\mu \quad (2)$$

The spinor derivatives $D = i\partial/\partial\bar{\theta} + 1/2 \bar{\theta} \partial/\partial z$, $\bar{D} = i\partial/\partial\theta + 1/2 \theta \partial/\partial\bar{z}$ and

$$D_\psi = i\partial/\partial\psi + 1/2 \psi \partial/\partial\tau = (D + \bar{D})|_{\theta=\bar{\theta}, z=\bar{z}} \text{ satisfy}$$

$$D^2 = i/2 \partial/\partial z, \quad \bar{D}^2 = i/2 \partial/\partial\bar{z}, \quad D_\psi^2 = i/2 \partial/\partial\tau \quad (3)$$

The boundary condition on ϕ^μ that follows from (1) is

$$[(D + \bar{D}) \phi^\mu + F_{\mu\nu} (D + \bar{D}) \phi^\nu]_{\sigma=0, \chi=0} = 0 \quad (4)$$

We have obtained this result by reducing (1) to components, and then demanding that $\delta S=0$ when the field equation $D\bar{D} \phi^\mu = 0$ is satisfied, and that the action is supersymmetric. This yields the boundary conditions and the following restrictions on the variation λ^μ and $\bar{\lambda}^\mu$, and the supersymmetry parameter ε :

$$\delta(\lambda^\mu - \bar{\lambda}^\mu)|_{\sigma=0} = -F_{\mu\nu} \delta(\lambda^\nu + \bar{\lambda}^\nu)|_{\sigma=0}, \quad \varepsilon = \bar{\varepsilon} \quad (5)$$

The resulting boundary conditions are then assembled into the superfield equation (4).

The superpropagator $N_{\mu\nu}(z, \theta; z', \theta')$ satisfies

$$D\bar{D} N_{\mu\nu} = -1/16 \delta_{\mu\nu} \delta^2(z-z') \delta(\theta - \theta') \delta(\bar{\theta} - \bar{\theta}') \quad (6)$$

subject to the boundary condition

$$[(D + \bar{D}) N_{\mu\nu} + F_\mu^\lambda (D + \bar{D}) N_{\lambda\nu}]_{\sigma=0, \chi=0} = 0 \quad (7)$$

For constant field strength $F_{\mu\nu}$ the solution to (6) and (7) is

$$N_{\mu\nu} = -1/2\pi \left\{ \delta_{\mu\nu} \ln |z-z'| + i/2 \bar{\theta}\theta' + 1/2[(1-F)/(1+F)]_{\mu\nu} \ln(z\bar{z}') + i/2 \bar{\theta}\theta' \right. \\ \left. + 1/2[(1+F)/(1-F)]_{\mu\nu} \ln(\bar{z}z' + i/2 \theta\theta') \right\} \quad (8)$$

which generalizes the results of [11] and [6]. The vertices are obtained from the normal coordinate expansion of the boundary term $S_{\partial M}$ in (1). The superfield ϕ^μ is replaced by $\phi^\mu + \pi^\mu(\xi)$ where ϕ^μ is now a background superfield and the fluctuation π^μ is expanded in a power series of the quantum superfield ξ^μ . We find that

$$S_{\partial M}[\phi + \pi(\xi)] = 2i \int_{\partial M} d\tau d\psi F_{\mu\nu} \xi^\nu D_\psi \phi^\mu = \\ 2i \int_{\partial M} d\tau d\psi \left\{ 1/2 (F_{\mu\nu, \lambda} D_\psi \phi^\mu \xi^\lambda \xi^\nu + F_{\mu\nu} \xi^\nu D_\psi \phi^\mu) \right. \\ \left. + \sum_{n=2}^{\infty} 1/n! F_{\mu\nu, \lambda_1 \dots \lambda_n} D_\psi \phi^\mu \xi^\nu \xi^{\lambda_1} \dots \xi^{\lambda_n} + \sum_{n=2}^{\infty} (n-1)/n! F_{\mu \lambda_1 \lambda_2 \dots \lambda_n} D_\psi \phi^\mu \xi^{\lambda_1} \xi^{\lambda_2} \dots \xi^{\lambda_n} \right\} \quad (9)$$

boundary term in the (sigma model) effective action $\Gamma[\phi]$. Renormalizability and cancellation of the vacuum energy due to worldsheet supersymmetry implies that this has the form

$$\int_{\partial M} d\tau d\psi C_{\mu} D_{\psi} \phi^{\mu} \quad (13)$$

where C_{μ} is a function of (derivatives of) $F_{\mu\nu}$. To obtain a contribution of this form we need to consider diagrams with no more than one U-vertex because the U's each contain a $D_{\psi} \phi^{\mu}$ factor.

The general diagram with exactly one U-vertex is shown in Fig. 2. Each V-vertex contains one D_{ψ} -derivative. Each such derivative "collapses" a ξ^{μ} -propagator. This is because, when restricted to the boundary of the upper half superplane the propagator is

$$N_{\mu\nu}(\tau, \psi; \tau', \psi') = -i \delta_{\mu\nu} \int_{-\infty}^{+\infty} d\tau' \frac{1}{2\pi} \exp(i\tau'(\tau - \tau')) \tilde{D}_{\psi} \delta(\psi - \psi') / |\rho| \quad (14)$$

where $\tilde{D}_{\psi} = i\partial/\partial\psi + (i/2) \psi \cdot p$. Thus $\tilde{D}_{\psi} N_{\mu\nu}$ produces a factor of $\tilde{D}_{\psi}^2 = p^2/2$ in the integral which cancels the factor of $|\rho|$. The effect of the D_{ψ} factor at a V-vertex is therefore to reduce the diagram to one with one fewer vertices. As there are as many V-vertices as there are D_{ψ} factors all the V-vertices can be collapsed in this way onto the single U-vertex. The result is the "flower" diagram shown in Fig.3 where the factor at the single vertex is a product of (derivatives of) $F_{\mu\nu}$ times a factor of $D_{\psi} \phi^{\mu}$. There are two circumstances that require special attention in this process. When we use one factor of D_{ψ} to collapse a propagator there may be another factor of D_{ψ} at the other end. In this case we simply integrate by parts to push the second D_{ψ} either onto the other propagators or onto the background field dependent factors at that vertex. If the D_{ψ} is pushed onto another propagator we then proceed as before. If it is

At one loop we shall need only the terms quadratic in ξ^{μ} . By using the propagator (8) we automatically take into account the effect of the "vertex" $F_{\mu\nu} \xi^{\nu} D_{\psi} \xi^{\mu}$ [6]. To obtain a contribution to the effective action of the same form as $S_{\partial M}$ we need only consider a one loop

diagram with one insertion of the $F_{\mu\nu} \lambda D_{\psi} \phi^{\mu}$ "vertex". Thus as in [6] we obtain the associated beta-function β_{μ} from the one-loop diagram of Fig. 1, which is

$$\int_{\partial M} d\tau d\psi F_{\mu\nu} \lambda D_{\psi} \phi^{\mu} \lim_{\tau' \rightarrow \tau} \tau' \rightarrow \psi \rightarrow \psi N_{\partial M}^{\lambda\nu}(\tau, \psi; \tau', \psi') \quad (10)$$

where $N_{\partial M}$ is the propagator between points on the boundary;

$$N_{\partial M}^{\mu\nu} = -1/\pi \left[(1-F^2)^{-1} \right]^{\mu\nu} \ln |\tau - \tau'| + i/2 \psi \psi' \quad (11)$$

Observe that the ψ dependence of $N_{\partial M}$ drops out in the limit $\psi \rightarrow \psi'$ at which point the calculation is identical to that of [6]. The beta-function β_{μ} is therefore the same as for the open bosonic string, and the effective action is therefore again the Born-Infeld action.

We now turn to the question of higher sigma model loop corrections to the β_{μ} -function. The vertices are given in (9). They have one of the following two structures:

$$D_{\psi} \phi^{\nu} U^{(n)}_{\nu\mu_1 \dots \mu_n} \xi^{\mu_1} \dots \xi^{\mu_n}, \quad V^{(n)}_{\nu\mu_1 \dots \mu_n} D_{\psi} \phi^{\nu} \xi^{\mu_1} \dots \xi^{\mu_n} \quad (12)$$

For the argument to follow it is convenient to consider the terms quadratic in ξ^{μ} on the same footing as the terms higher order in ξ^{μ} , so that we can take the ξ^{μ} -propagator to be the usual $F_{\mu\nu}$ -independent one (i.e. (11) with $F_{\mu\nu}=0$). We need to compute the divergent part of the

pushed onto the background field dependent factors then it must produce a factor of $D_{\psi} \phi^{\mu}$. As we

already have such a factor at the U-vertex and we are looking for contributions to $\Gamma[q]$ of the form (13) we can ignore these terms. The other point to realize is that two vertices may be

connected by more than one propagator with a factor, or factors, of D_{ψ} acting on it. In this case the contraction of one of the propagators will cause the others to become simple closed loops

which "trap" factor(s) of D_{ψ} . This situation is shown in Fig. 4. Such closed loops yield one of

the two factors $\int dp$ or $\psi \int p dp$ which vanish by the usual rules of dimensional regularization, which we shall use in what follows.

We have now to consider diagrams with zero U-vertices. Proceeding as before we start to collapse propagators to reduce the effective number of vertices. If we have to integrate by parts a factor of D_{ψ} we cannot now ignore the term in which D_{ψ} acts on the background field dependent factors. However, once this happens the vertex is reduced to a U-type vertex with a factor of $D_{\psi} \phi^{\mu}$ but no factor of D_{ψ} on the propagators at that vertex. We then proceed as described above for the diagrams of the type shown in Fig. 2. Otherwise, if we do not now have to integrate a D_{ψ} factor by parts or if, when integrating by parts, we always push it onto another propagator then the diagram will eventually be reduced to the "flower" diagram of Fig. 5 in which a D_{ψ} factor is trapped in one "petal". As before, this is zero by the usual rules of dimensional regularization. Thus all diagrams of relevance to the β_{μ} -function can be reduced to one of the form of Fig. 3, which is simply a product of U and q factors of

$$I = \int_{-\infty}^{+\infty} dp / |p| \quad (15)$$

This integral is infrared divergent. This divergence is not relevant to the β_{μ} -function so we can regulate it by replacing p in (15) by $(p^2+m^2)^{1/2}$. Regularizing ultraviolet divergences by dimensional regularization we replace I by

$$I_{res.} = 2 \int_0^{\infty} d(1+\epsilon) p / (p^2 + m^2)^{1/2} = -4 \epsilon^{-1} + 2(\gamma - \ln m^2) + O(\epsilon) \quad (16)$$

The γ and $(\ln m)$ factors must cancel as usual when account is taken of counterterms, so that the net effect of the diagram of Fig. 3 is to give a contribution of

$$\epsilon^{-q} \int d\tau d\psi \hat{U}_{\mu} D_{\psi} \phi^{\mu} \quad (17)$$

to the ultraviolet divergence of $\Gamma[q]$. The β_{μ} -function is determined by the ϵ^{-1} pole only however, so that only the $q=1$ term in (17) can contribute. That is, the one loop contribution to β_{μ} is exact. The higher powers of ϵ^{-1} are required by the renormalization group pole equations if the one-loop β_{μ} -function is non-zero, but should cancel if A_{μ} is chosen to satisfy $\beta_{\mu}=0$.

It should be clear from the above that the arguments leading to the conclusion that the one-loop β_{μ} -function is exact are equally valid for the bosonic string, as well as the supersymmetric string. It would be interesting to see whether a similar result holds in the presence of nontrivial background for massless closed string modes.

An obvious question raised by our work is whether our exact result for the β_{μ} -function is consistent with the field equations obtained from the low energy limit of the open superstring amplitudes. Various points should be borne in mind here.

Although one expects that the equations of motion ($W_{\mu}=0$) derived from string scattering amplitudes should be, in some sense, equivalent to that obtained by demanding conformal

invariance of the corresponding sigma model, this does not mean that the forms of the two field equations need necessarily be the same. Presumably a configuration for which $W_\mu = 0$ will also satisfy $\beta_\mu = 0$ (whether the converse is true is not so clear). However the relation between W_μ and β_μ could be very complicated, involving derivative operators of arbitrarily high order that would render any comparison of W_μ and β_μ order by order in α' very difficult. In particular, the fact that the string scattering calculations yield amplitude involving nonlinear dependence on external momenta [1], and hence to arbitrarily many derivatives of the background $F_{\mu\nu}$ field, need not necessarily be in conflict with a β_μ -function which can be derived from the variation of the Born-Infeld action. Of course, it may also be the case that there are additional contributions to the β_μ -function from string-loop amplitudes [10], which we have not discussed in this paper.

An extension of our results to non-Abelian gauge fields may be possible using the recent results of Marcus and Sagnotti [12], who add a boundary term to the string action involving a number of anticommuting fields. We hope to report on this in a future publication.

ACKNOWLEDGMENTS

C.N.P. and P.K.T. would like to thank Professor Abdus Salam for his kind hospitality and ICTP in Trieste where this work was carried out.

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Figure Captions

Fig. 1 One-loop diagram contributing to β_{μ} . The double line indicates the

background field dependent factors and the single line represents the ξ^{μ} -propagator of eq. (8).

Fig. 2 A diagram containing one $U(p)$ vertex. The remaining q V -vertices are represented by the shaded circle.

Fig. 3 The "flower" diagram that is equivalent to that of Fig. 2 after collapse of propagators due to D_{ψ} factors at V -vertices. The factor $\hat{U}$ at the new U -type of vertex is a product of (derivatives of) $F_{\mu\nu}$ factors.

Fig. 4 Diagrams with "trapped" D_{ψ} factors in simple closed loops (indicated by dots) after the contraction of one propagator between vertices connected by more than one propagator. They vanish by the usual rules of dimensional regularization.

Fig. 5 A possible result of the collapse of a diagram with only V -vertices. The factor V at the new type V -vertex is a product of (derivatives of) $F_{\mu\nu}$ factors, without a factor of $D_{\psi} \phi^{\mu}$. This diagram is zero by the usual rules of dimensional regularization.

Fig. 1

Fig. 3

Fig. 4

Fig. 5