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CONSTRAINTS FROM STRING UNIFICATION

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A B S T R A C T

The incorporation of a standard model with n generations and softly broken $N = 1$ supersymmetry in a string theory of the heterotic type implies relations among the string scale, the compactification scale, the Planck mass and various couplings. We study these constraints, including running effects through two loops, and obtain severe limitations on possible consistent string unifications by requiring string loops not to be totally out of control.

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1. Strings and Superstrings are at present the leading candidates for a truly unified theory of all interactions. While many apparently consistent string theories have been constructed, no convincing theoretical criterium has as yet emerged for singling out any one of them.

On the other hand, all string theories that have a chance of being physically relevant share certain basic properties. On the basis of these - and of a few reasonable assumptions - we wish to discuss some constraints that arise on String Unification i. e. on the merging of gauge and gravitational interactions at a particular scale.

The claimed renormalizability, or even finiteness, of string theories should enable one to make meaningful estimates of the way coupling constants, including those of gravity, are renormalized by loops and run. This, in turn, allows one to identify the scale at which String Unification holds and to make definite low energy predictions.

Many of the basic points of our analysis are not entirely new and can be found in the literature at a more qualitative level [1].

2. We start by listing some general properties of string theories that we assume to hold. We shall then specify our assumptions.

a. Any (super) string theory contains (in units $\hbar = c = 1$) one fundamental mass scale $M_S = (\alpha')^{-1/2}$: this (or rather $2M_S$) is the scale of energy for string excitations and is also the theory's cut off in momentum space (for a more economic, geometric system of units see ref.[2])

b. All other physical parameters can be obviously expressed in terms of M_S via dimensionless numbers. A most interesting feature of string theory is that these numbers are just Vacuum Expectation Values (VEV's) of various string fields and are therefore , in principle, determined by the theory [3]. Noteworthy examples are:

1) The string loop expansion parameter:

$$(1) \quad g_{SL}^2 = \exp(-2\langle\phi_{dil.}\rangle)$$

related, as eq. (1) indicates, to the dilaton VEV.

ii) The size R_C of the compact (d-4)-dimensional manifold K_{d-4} which, together with flat Minkowski space M_4 , hopefully provides the ground state value of the d-dimensional metric. We define R_C by:

$$(2) \quad V_{d-4} = \int d^{d-4}y \ (g_{d-4})^{1/2} \equiv (R_C)^{d-4} = (M_C/\rho)^{4-d}$$

where M_C is defined to be the scale of Kaluza-Klein type excitations and ρ is a pure number to be discussed later. String dynamics should determine (M_C/M_S) [3,1].

In the compactification process the original gauge group (e.g. $E_8 \otimes E_8$) will be broken down [4] to a subgroup G_{gut} which is the true symmetry below M_C . G_{gut} will further break down to the low energy group at some scale M_{gut} .

Proton stability and other constraints discussed hereafter set M_{gut} not much more than an order of magnitude lower than M_C . Keeping M_{gut} different from M_C corresponds thus to the rather unnatural and unaesthetical possibility that gauge interactions unify first together and then, at most an order of magnitude higher, unify with gravity. This is why we shall mainly concentrate here on the case $M_C = M_{gut}$.

Below M_C the theory is effectively 4-dimensional and is controlled by gauge and gravitational couplings related at tree level by:

$$(3) \quad \alpha \equiv g^2/4\pi = 8 G_N M_S^2 = 8(M_S/M_P)^2 ; \quad M_P \approx 1.2 \cdot 10^{19} \text{ GEV}$$

where we have implicitly assumed to be working in the heterotic string type framework [5].

Above M_C the theory is d-dimensional (d>4) and we shall actually assume that d=10 between M_C and M_S (where the extra 26-10=16 dimensions produce gauge symmetries a la Halpern, Frenkel-Kac, Segal[6]). The 10-dimensional couplings are simply related to the 4-dimensional ones by[1]:

$$(4) \quad \alpha_{10} = \alpha V_6 = 8(\rho/M_c)^6 (M_s/M_p)^2$$

The fundamental (Kaluza-Klein-type) relation (3) tells us that gravity and gauge couplings are equal in string units or, in a unit independent way, that gravitational and gauge forces are identical (up to Clebsches) for a massive, non-neutral string. This is the essence of string unification.

We now specify our assumptions:

a. The string loop expansion is not totally out of control:

$$(5) \quad \alpha_{SL} = \text{LOOP/TREE} < 0(1)$$

In order to see what this relation implies we refer to some recent loop calculation in type II superstrings[7], where the corrections to R^4 terms in the effective action were computed with the result:

$$(6) \quad \text{LOOP/TREE} = \alpha_{10}/3\pi \cdot (M_s^2/4\pi)^3 \cdot \zeta(3)^{-1}$$

Even being generous we have to require:

$$\begin{aligned} (7) \quad \alpha_{10}/3\pi (M_s^2/4\pi)^3 &= 8(\rho/M_c)^6 (M_s/M_p)^2 (3\pi)^{-1} (M_s^2/4\pi)^3 \\ &= 3(\rho/2\pi)^6 \alpha (M_s/M_c)^6 = 0.0064 (\rho/2\pi)^6 \alpha^4 (M_p/M_c)^6 < 0(1) \end{aligned}$$

where eqs. (3), (4) have been used.

b. The compactification radius R_c is at least as large as the fundamental string length parameter, or, in terms of excitation energies:

$$(8) \quad M_c < M_s \text{ (or perhaps } < 2 M_s)$$

c. We shall finally assume that the relation among the $SU(3)$, $SU(2)$, and $U(1)$ couplings (at a scale to be discussed below) is the one of $SU(5)$ - E_6 . We shall briefly comment at the end on the

possibility that these relations are broken by string loop effects as advocated by Choi [8].

It is clearly important at this stage to discuss the value of the parameter ρ defined in eq.(2). For a toroidal compactification clearly $\rho=2\pi$ and eq. (7) simplifies accordingly. We shall now argue that for any compactification of size $R > 1/M_s$, ρ cannot be too different from 2π . What we really need, actually, is the relation between α_{10} , α and the scale M_c of Kaluza-Klein excitations. A 10-dimensional loop is always proportional to

$$(9) \quad \alpha_{10}/(2\pi)^{10} \int d^{10}p \longrightarrow \alpha_{10}/(2\pi)^4 \sum (\Delta p/2\pi)^6 \int d^4p \\ \equiv \alpha/(2\pi)^4 \int d^4p$$

where Δp is the spacing of the compactified momenta. Since, by our definition, $\Delta p = M_c$, eq. (9) gives $\rho=2\pi$.

Equation (3) holds at tree level [9]. It is obviously violated at low energy, where G_N coincides with the measured Newton constant and α gives the low energy gauge couplings. Clearly string loops cannot be always negligible even if eq. (7) is satisfied!

This brings us to define the scale of String Unification as one where loop corrections are under control, so that eq. (3) remains essentially valid. In turn, this leads to the problem of evaluating the running of the various coupling constants due to string loops.

We know how the gauge couplings run. What about G_N itself? In order to answer this question let us first understand [10] how a gauge coupling can run in a theory which is supposedly ultra-violet finite. There is no need to introduce a cutoff in string theory, this being M_s itself. Nonetheless (large?) logs of M_s/q can still occur if the loop is Infra-red Divergent. This is precisely the case for loops with external gauge bosons in $d=4$:

$$(10) \quad \Sigma(q) \cong \alpha b_0 \ln(M_s/q) + O(g^4) + O(\alpha_{SL})$$

Equation (10) appears to indicate that running occurs from $q=0$ to $q=O(M_s)$. This is misleading and the running is actually

negligible above M_C . The statement can be checked by direct evaluation of the loop, but its meaning is quite obvious : for $q > M_C$ the theory is effectively d -dimensional ($d > 4$) and is therefore free of infra-red singularities. Consequently, above M_C , gauge couplings cannot acquire any potentially dangerous logarithmic q -dependence. The result for the gauge boson self energy at $M_C < q < M_S$ is [10] :

$$(11) \quad \Sigma(q) = \alpha_{SL} / 2\pi (1 + O((M_S - q)^{d-4} / M_S^{d-4}))$$

which provides, by assumption (5), a small correction and negligible running.

The above argument immediately provides the answer to the question of the running of Newton's constant. Since, already at $d=4$, gravity is free of infra-red problems* (this is the counterpart to its ultra-violet problems!) G_N does not run and its renormalization is small at all values of q because of (5).

We conclude that the unifying relation (3) must hold at M_C , the compactification scale, which is thus identified with the String Unification scale. Equation (3) becomes therefore :

$$(12) \quad \alpha(M_C) = 8 (M_S / M_P)^2 (1 + O(\alpha_{SL}))$$

Although M_S is the fundamental string theory parameter it is M_P that is experimentally known. We shall thus determine M_S from M_P and the unified value of α at M_C .

3. To proceed we need to assume something on the physics below M_C . We shall introduce two intermediate scales between $M_F = 200 \text{ GeV}$ and M_C : the higher one is the scale M_X at which the group G_{gut} breaks to $SU(3) \times SU(2) \times U(1)$ (times possibly other $U(1)$'s); the lower one, M_{susy} , is the scale of supersymmetry

*) We are assuming here that, for some reason yet to be understood, the correct string theory gives a vanishing cosmological constant : in the opposite case G_N would run [10].

breaking below which the supersymmetric partners of the usual fundamental particles decouple (i.e. stop contributing to the running). This represents, of course, an average, approximate scale of SUSY breaking.

The final picture involves a priori five different scales and is usually referred to (improperly?) as the desert scenario. Note that we are implicitly assuming that no new strong coupling phenomena occur between M_F and M_C . These could involve for instance the binding of some fundamental preonic strings to yield the observed quarks and leptons. Apart from such possibilities our scenario looks rather general. As we shall argue, the extra scales that one could introduce between M_X and M_C to account for various stages of symmetry breaking would not drastically change our conclusions.

In order to visualize easily the constraints involved it is convenient to consider (fig. 1) a (double logarithmic) plot with $8(M_S/M_P)^2 = \alpha(M_C)$ in abscissae and $(M_C/M_S)^2$ in ordinates (lines of constant M_C/M_P are also indicated for convenience). The upper shaded region is excluded by (8), the lower right by (7) which implies:

$$(13) \quad 1 > (M_C/M_S)^6 > 3\alpha \text{ -----} > \alpha < 1/3$$

Finally, α cannot be too small (in order to reproduce the M_F values) and thus one must also exclude the region to the left of some vertical line (say around $\alpha = 0.02$). We thus immediately see that one is confined to a small triangle within which $M_C/M_S = 0.6 \text{---} 1$, $M_C/M_P = 0.035 \text{---} 0.2$. This is basically the statement of refs.[1] just made more explicit and quantitative.

At this point we have searched for quantitative solutions to the simplest String Unification scenario--that in which, as we said at the beginning, the standard model gauge couplings unify with one another and with gravity at the single scale $M_C=M_X$. To this purpose, we have used the program of Ref. [11] in which gauge couplings are evolved numerically according to their two-loop β -functions from the Fermi scale to M_C (including effects from perturbative Yukawa couplings).

We have found, somewhat to our surprise (see however Ref. [12]), that no such minimal scheme can be made consistent with a reasonably small string loop expansion parameter. In searching for a solution we have allowed for $O(\alpha)$ breaking of the $SU(5)$ - $E(6)$ relations among the gauge couplings at M_C . The best compromises for 3, 4 and 5 families are shown in Fig.1 and are clearly way off our target triangle. The $n=4$ case with $\alpha_{SL}=10$ is the least problematic case and still far from satisfactory. For $n>5$ the $SU(3)$ coupling blows up before M_C .

Schemes of the type proposed in Refs.[13], in which all the gauge couplings become $> O(1)$ (but otherwise arbitrary) at the same scale, do not fall in the class of perturbative schemes we have been looking for. In our framework these solutions are acceptable provided "anti-unification" occurs at $M_C \approx M_S \approx 0.3 M_P \approx 4 \cdot 10^{18}$ GeV.

A possible way out of this impasse could be to invoke a recent claim by Choi[8], according to which the $SU(5)$ - $E(6)$ relations among the gauge couplings can be violated much more than $O(\alpha)$ due to string effects not present in the field theory limit. According to Ref.8 the breaking is as large as $O(\alpha_{SL})$ i. e. $O(1)$. This would certainly allow one to avoid our difficulties with String Unification..., at least as long as string loop effects are not computed.

On the other hand Choi's claim looks quite surprising: the string effects he advocates come from a heavy non zero winding number sector of mass $O(M_S^2/M_C) \gg M_S$. If excited states of this kind are important in loops at $q=M_C$, this would imply that $SU(5)$ - $E(6)$ relations are also violated much above M_C , where compactification should become irrelevant and symmetry should be restored. Indeed the winding sector usually contributes to loops in a way proportional to:

$$\sum \exp(-l^2 R_C^2 M_S^2)$$

and, for $R_C^2 M_S^2 \gg 1$, only the $l=0$ term is relevant.

We are thus led to conjecture that these string effects, although present, will be very depressed for $R_C^2 M_S^2 \gg 1$ and thus irrelevant for our considerations.

The only two remaining possibilities for a perturbative String Unification appear to be the following ones:

a. The $SU(5)$ - $E(6)$ relations are valid at M_G and their breaking occurs at another (yet unpredicted) scale M_X lying slightly below. This is a rather unappealing way out as we have already said.

b. Since, in any case, we need $M_G = M_S$ to within 50% or so, there is no meaning in discussing an intermediate regime described by an effective 10-dimensional field theory. One should rather consider directly a 4-dimensional string theory along the lines pioneered by Narain [14] and later developed by many authors in the so-called 4-dimensional models [15].

It is possible that one of these models will be selected, either by consistency arguments or by dynamics (or, in the worst case, by hand) and that it will provide new unifying relations among the gauge couplings as to make String Unification work just right.

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FIGURE CAPTION

The small triangle where a 'good' string unification should land after evolution of the couplings. The dots (a), (b), (c) indicate how close one can get to the triangle for different numbers of families:

a) $n=3$, $M_S=4 M_C=8 \cdot 10^{17}$ GeV, $M_{\text{SUSY}}=0(1 \text{ TeV})$, $\alpha=0.035$

b) $n=4$, $M_S=2 M_C=1 \cdot 10^{18}$ GeV, $M_{\text{SUSY}}=0(1 \text{ TeV})$, $\alpha=0.08$

c) $n=5$, $M_S=3 M_C=3 \cdot 10^{18}$ GeV, $M_{\text{SUSY}}=0(10 \text{ TeV})$, $\alpha=0.5$