CONSTRAINTS FROM STRING UNIFICATION

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ABSTRACT

The incorporation of a standard model with n generations and softly broken N = 1 supersymmetry in a string theory of the heterotic type implies relations among the string scale, the compactification scale, the Planck mass and various couplings. We study these constraints, including running effects through two loops, and obtain severe limitations on possible consistent string unifications by requiring string loops not to be totally out of control.

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1. Strings and Superstrings are at present the leading candidates for a truly unified theory of all interactions. While many apparently consistent string theories have been constructed, no convincing theoretical criterion has as yet emerged for singling out any one of them.

On the other hand, all string theories that have a chance of being physically relevant share certain basic properties. On the basis of these - and of a few reasonable assumptions - we wish to discuss some constraints that arise on String Unification, i.e. on the merging of gauge and gravitational interactions at a particular scale.

The claimed renormalizability, or even finiteness, of string theories should enable one to make meaningful estimates of the way coupling constants, including those of gravity, are renormalized by loops and run. This, in turn, allows one to identify the scale at which String Unification holds and to make definite low energy predictions.

Many of the basic points of our analysis are not entirely new and can be found in the literature at a more qualitative level [1].

2. We start by listing some general properties of string theories that we assume to hold. We shall then specify our assumptions.

a. Any (super) string theory contains (in units $\hbar = c = 1$) one fundamental mass scale $M_S = (\alpha')^{-1/2}$: this (or rather $2M_S$) is the scale of energy for string excitations and is also the theory's cut off in momentum space (for a more economic, geometric system of units see ref. [2]).

b. All other physical parameters can be obviously expressed in terms of $M_S$ via dimensionless numbers. A most interesting feature of string theory is that these numbers are just Vacuum Expectation Values (VEV's) of various string fields and are therefore, in principle, determined by the theory [3]. Noteworthy examples are:

1) The string loop expansion parameter:

$$g_{SL}^2 = \exp(-2\phi_{vib})$$
related, as eq. (1) indicates, to the dilaton VEV.

ii) The size $R_c$ of the compact $(d-4)$-dimensional manifold $K_{d-4}$ which, together with flat Minkowski space $M_4$, hopefully provides the ground state value of the $d$-dimensional metric. We define $R_c$ by:

$$V_{d-4} = \int d^{d-4}y \left( g_{d-4} \right)^{1/2} \equiv (R_c)^{d-4} = (M_c/\rho)^{d-4}$$

where $M_c$ is defined to be the scale of Kaluza-Klein type excitations and $\rho$ is a pure number to be discussed later. String dynamics should determine $(M_c/M_s)$ [3,1].

In the compactification process the original gauge group (e.g. $E_6 \otimes E_6$) will be broken down [4] to a subgroup $G_{\text{gut}}$ which is the true symmetry below $M_c$. $G_{\text{gut}}$ will further break down to the low energy group at some scale $M_{\text{gut}}$.

Proton stability and other constraints discussed hereafter set $M_{\text{gut}}$ not much more than an order of magnitude lower than $M_c$. Keeping $M_{\text{gut}}$ different from $M_c$ corresponds thus to the rather unnatural and unaesthetical possibility that gauge interactions unify first together and then, at most an order of magnitude higher, unify with gravity. This is why we shall mainly concentrate here on the case $M_c = M_{\text{gut}}$.

Below $M_c$ the theory is effectively 4-dimensional and is controlled by gauge and gravitational couplings related at tree level by:

$$\alpha = g^2/4\pi = 8 G_N M_s^2 = 8 (M_s/M_p)^2 ; M_p \sim 1.2 \times 10^{19} \text{ GeV}$$

where we have implicitly assumed to be working in the heterotic string type framework [5].

Above $M_c$ the theory is $d$-dimensional ($d>4$) and we shall actually assume that $d=10$ between $M_c$ and $M_s$ (where the extra 26-10=16 dimensions produce gauge symmetries à la Halpern, Frenkel-Kac, Segal[6]). The 10-dimensional couplings are simply related to the 4-dimensional ones by [1]:

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\[(4) \quad \alpha_{10} = \alpha \sqrt{6} = 8 (\rho / M_c)^6 (M_S / M_p)^2\]

The fundamental (Kaluza-Klein-type) relation (3) tells us that gravity and gauge couplings are equal in string units or, in a unit independent way, that gravitational and gauge forces are identical (up to Clebsches) for a massive, non-neutral string. This is the essence of string unification.

We now specify our assumptions:

a. The string loop expansion is not totally out of control:

\[(5) \quad \alpha_{SL} = \frac{\text{LOOP}}{\text{TREE}} < O(1)\]

In order to see what this relation implies we refer to some recent loop calculation in type II superstrings[7], where the corrections to $R^4$ terms in the effective action were computed with the result:

\[(6) \quad \frac{\text{LOOP}}{\text{TREE}} = \frac{\alpha_{10}}{3 \pi} \cdot \left( \frac{M_S^2}{4 \pi} \right)^5 \cdot \zeta(3)^{-1}\]

Even being generous we have to require:

\[(7) \quad \frac{\alpha_{10}}{3 \pi} \cdot \left( \frac{M_S^2}{4 \pi} \right)^5 = 8 (\rho / M_c)^6 (M_S / M_p)^2 (3 \pi)^{-1} (M_S^2 / 4 \pi)^5 = 3 (\rho / 2 \pi)^6 \alpha \left( \frac{M_S}{M_c} \right)^6 \approx 0.0064 (\rho / 2 \pi)^6 \alpha^2 (M_p / M_c)^6 < O(1)\]

where eqs. (3), (4) have been used.

b. The compactification radius $R_C$ is at least as large as the fundamental string length parameter, or, in terms of excitation energies:

\[(8) \quad M_c < M_S \text{ (or perhaps } < 2 M_S)\]

c. We shall finally assume that the relation among the SU(3), SU(2), and U(1) couplings (at a scale to be discussed below) is the one of SU(5)-E_6. We shall briefly comment at the end on the
possibility that these relations are broken by string loop effects as advocated by Choi [8].

It is clearly important at this stage to discuss the value of the parameter $\rho$ defined in eq. (2). For a toroidal compactification clearly $\rho=2\pi$ and eq. (7) simplifies accordingly. We shall now argue that for any compactification of size $R > 1/M_S$, $\rho$ cannot be too different from $2\pi$. What we really need, actually, is the relation between $\alpha_{10}$, $\alpha$ and the scale $M_c$ of Kaluza-Klein excitations. A 10-dimensional loop is always proportional to

\[ (9) \quad \alpha_{10}/(2\pi)^{10} \int d^{10}p \quad \longrightarrow \quad \alpha_{10}/(2\pi)^{d} \sum (\Delta p/2\pi)^{d} \int d^{d}p \]

where $\Delta p$ is the spacing of the compactified momenta. Since, by our definition, $\Delta p = M_c$, eq. (9) gives $\rho = 2\pi$.

Equation (3) holds at tree level [9]. It is obviously violated at low energy, where $G_N$ coincides with the measured Newton constant and $\alpha$ gives the low energy gauge couplings. Clearly string loops cannot be always negligible even if eq. (7) is satisfied!

This brings us to define the scale of String Unification as one where loop corrections are under control, so that eq. (3) remains essentially valid. In turn, this leads to the problem of evaluating the running of the various coupling constants due to string loops.

We know how the gauge couplings run. What about $G_N$ itself? In order to answer this question let us first understand [10] how a gauge coupling can run in a theory which is supposedly ultra-violet finite. There is no need to introduce a cutoff in string theory, this being $M_S$ itself. Nonetheless (large?) logs of $M_S^2/q$ can still occur if the loop is Infra-red Divergent. This is precisely the case for loops with external gauge bosons in $d=4$:

\[ (10) \quad \Sigma (q) \leq \alpha b_0 \ln(M_S^2/q) + O(g^2) + O(\alpha_{SL}) \]

Equation (10) appears to indicate that running occurs from $q=0$ to $q=O(M_S)$. This is misleading and the running is actually
negligible above $M_C$. The statement can be checked by direct
evaluation of the loop, but its meaning is quite obvious: for
$q>M_C$ the theory is effectively $d$-dimensional ($d=4$) and is
therefore free of infra-red singularities. Consequently, above $M_C$,
gauge couplings cannot acquire any potentially dangerous
logarithmic $q$-dependence. The result for the gauge boson self
energy at $M_C < q < M_S$ is \cite{10}:

\begin{equation}
\Sigma(q) = \alpha_S \frac{1}{2 \pi} \left( 1 + \mathcal{O}(\alpha_S) \right)
\end{equation}

which provides, by assumption (5), a small correction and
negligible running.

The above argument immediately provides the answer to the
question of the running of Newton's constant. Since already at
d=4, gravity is free of infra-red problems\footnote{We are assuming here
that, for some reason yet to be understood, the correct string
theory gives a vanishing cosmological constant; in the opposite
case $G_N$ would run \cite{10}.} (this is the
counterpart to its ultra-violet problems!) $G_N$ does not run and
its renormalization is small at all values of $q$ because of (5).

We conclude that the unifying relation (3) must hold at $M_C$,
the compactification scale, which is thus identified with the
String Unification scale. Equation (3) becomes therefore:

\begin{equation}
\alpha(M_C) = 8 \left( \frac{M_S}{M_p} \right)^2 \left( 1 + \mathcal{O}(\alpha_S) \right)
\end{equation}

Although $M_S$ is the fundamental string theory parameter it is
$M_p$ that is experimentally known. We shall thus determine $M_S$
from $M_p$ and the unified value of $\alpha$ at $M_C$.

3. To proceed we need to assume something on the physics
below $M_C$. We shall introduce two intermediate scales between
$M_F = 200 GeV$ and $M_C$; the higher one is the scale $M_X$ at which the
group $G_{gut}$ breaks to $SU(3) \times SU(2) \times U(1)$ (times possibly other
$U(1)$'s); the lower one, $M_{susy}$, is the scale of supersymmetry.
breaking below which the supersymmetric partners of the usual fundamental particles decouple (i.e. stop contributing to the running). This represents, of course, an average, approximate scale of SUSY breaking.

The final picture involves a priori five different scales and is usually referred to (improperly?) as the desert scenario. Note that we are implicitly assuming that no new strong coupling phenomena occur between $M_F$ and $M_C$. These could involve for instance the binding of some fundamental preonic strings to yield the observed quarks and leptons. Apart from such possibilities our scenario looks rather general. As we shall argue, the extra scales that one could introduce between $M_A$ and $M_C$ to account for various stages of symmetry breaking would not drastically change our conclusions.

In order to visualize easily the constraints involved it is convenient to consider (fig. 1) a (double logarithmic) plot with $\theta (M_s / M_p) = a(M_c)$ in abscissae and $(M_c / M_s)^2$ in ordinates (lines of constant $M_c / M_p$ are also indicated for convenience). The upper shaded region is excluded by (8), the lower right by (7) which implies:

\[(13) \quad 1 > (M_c / M_s)^6 > 3a \quad \longrightarrow \quad a < 1/3\]

Finally, $\alpha$ cannot be too small (in order to reproduce the $M_F$ values) and thus one must also exclude the region to the left of some vertical line (say around $\alpha = 0.02$). We thus immediately see that one is confined to a small triangle within which $M_c / M_s = 0.6 \cdots \cdots 1, M_c / M_p = 0.035 \cdots \cdots 0.2$. This is basically the statement of refs.[1] just made more explicit and quantitative.

At this point we have searched for quantitative solutions to the simplest String Unification scenario—that in which, as we said at the beginning, the standard model gauge couplings unify with one another and with gravity at the single scale $M_c = M_X$. To this purpose, we have used the program of Ref. [11] in which gauge couplings are evolved numerically according to their two-loop $\beta$-functions from the Fermi scale to $M_c$ (including effects from perturbative Yukawa couplings).
We have found, somewhat to our surprise (see however Ref. [12]), that no such minimal scheme can be made consistent with a reasonably small string loop expansion parameter. In searching for a solution we have allowed for $O(\alpha)$ breaking of the SU(5)-E(6) relations among the gauge couplings at $M_C$. The best compromises for 3, 4 and 5 families are shown in Fig.1 and are clearly way off our target triangle. The $n=4$ case with $\alpha_{SL}=10$ is the least problematic case and still far from satisfactory. For $n=5$ the SU(3) coupling blows up before $M_C$.

Schemes of the type proposed in Refs.[13], in which all the gauge couplings become $O(1)$ (but otherwise arbitrary) at the same scale, do not fall in the class of perturbative schemes we have been looking for. In our framework these solutions are acceptable provided "anti-unification" occurs at $M_C \approx M_S \approx 0.3 M_p$, $\approx 4 \times 10^{18}$ GeV.

A possible way out of this impasse could be to invoke a recent claim by Cho[8], according to which the SU(5)-E(6) relations among the gauge couplings can be violated much more than $O(\alpha)$ due to string effects not present in the field theory limit. According to Ref.8 the breaking is as large as $O(\alpha_{SL}) \text{i.e.} O(1)$. This would certainly allow one to avoid our difficulties with String Unification... at least as long as string loop effects are not computed.

On the other hand Cho’s claim looks quite surprising: the string effects he advocates come from a heavy non zero winding number sector of mass $O(M_s^2/M_C)$ $\gg M_S$. If excited states of this kind are important in loops at $q=M_C$, this would imply that SU(5)-E(6) relations are also violated much above $M_C$, where compactification should become irrelevant and symmetry should be restored. Indeed the winding sector usually contributes to loops in a way proportional to:

$$\sum \exp(-\sqrt{2 R_C^2 M_S^2})$$

and, for $R_C^2 M_S^2 \gg 1$, only the $l=0$ term is relevant.

We are thus led to conjecture that these string effects, although present, will be very depressed for $R_C^2 M_S^2 \gg 1$ and thus irrelevant for our considerations.
The only two remaining possibilities for a perturbative String Unification appear to be the following ones:

a. The SU(5)-E(6) relations are valid at $M_c$ and their breaking occurs at another (yet unpredicted) scale $M_x$ lying slightly below. This is a rather unappealing way out as we have already said.

b. Since, in any case, we need $M_c=M_S$ to within 50% or so, there is no meaning in discussing an intermediate regime described by an effective 10-dimensional field theory. One should rather consider directly a 4-dimensional string theory along the lines pioneered by Narain [14] and later developed by many authors in the so-called 4-dimensional models [15].

It is possible that one of these models will be selected, either by consistency arguments or by dynamics (or, in the worst case, by hand) and that it will provide new unifying relations among the gauge couplings as to make String Unification work just right.

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FIGURE CAPTION

The small triangle where a 'good' string unification should land after evolution of the couplings. The dots (a), (b), (c) indicate how close one can get to the triangle for different numbers of families:

a) \( n=3, \ M_S=4 \ M_C =8 \times 10^{17} \text{ GeV}, \ M_{\text{SUSY}} = 0(1 \text{ TeV}), \ \alpha = 0.035 \)

b) \( n=4, \ M_S=2 \ M_C =1 \times 10^{18} \text{ GeV}, \ M_{\text{SUSY}} = 0(1 \text{ TeV}), \ \alpha = 0.08 \)

c) \( n=5, \ M_S=3 \ M_C =3 \times 10^{18} \text{ GeV}, \ M_{\text{SUSY}} = 0(10 \text{ TeV}), \ \alpha = 0.5 \)