SUPERSTRING LOOP CORRECTIONS TO σ -MODEL β -FUNCTIONS

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A B S T R A C T

We present an algorithm for string one-loop corrections to σ -model β -functions in closed superstring theories, by identifying the massless poles in string one-loop amplitudes with the ultra-violet divergences in the corresponding two-dimensional σ -model. The string one-loop corrected β -functions for the type-II superstring are given.

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1. Introduction

It has been conjectured that string propagation in the presence of massless fields (graviton, dilaton, etc.) is described by the requirement of vanishing β -functions of the corresponding two-dimensional σ -models on the string world sheet [1]. This has been demonstrated at the string tree-level up to two-loop order (in the σ -model sense) in bosonic σ -models [2] and up to four and five-loop orders in the $N = 1$ and $N = 2$ supersymmetric σ -models [3,4].

It is an important issue whether the vanishing β -functions is a useful notion at loop levels of string theories. Fischler and Susskind [5] have shown that string loop corrections to β -functions can be evaluated by cancelling the string loop divergences in scattering amplitudes $A_M(p_1, \dots, p_n)$ (of massless particles) against σ -model counter-terms. Consider a torus (T^2) correction as the simplest example to see the strategy. The string loop divergence arises from the graph of a single dilaton emitted from the S^2 and absorbed by the vacuum by coupling to the one-loop cosmological constant J . This graph has an infinity due to the pole at $p^2 = 0$, where $p = \sum_i p_i$. Roughly speaking, we have

$$A_{T^2}(p_1, \dots, p_n) = \frac{1}{\epsilon} J A_{S^2}(p_1, \dots, p_n; 0) + (\text{finite}), \quad (1.1)$$

where $1/\epsilon$ represents the infinity due to the momentum pole and $A_{S^2}(p_1, \dots, p_n; 0)$ is the n -point amplitude on S^2 with a dilaton emission

vertex inserted at zero momentum. Conformal invariance of the theory can be restored by cancelling this divergence against the counter term introduced in the σ -model lagrangian [4]:

$$\Delta \mathcal{L} = \frac{1}{4\pi} \sqrt{-g} g^{\mu\nu} (\Delta g_{ij}) \partial_\mu X^i \partial_\nu X^j, \quad (1.2)$$

$$\Delta g_{ij} = \frac{1}{\epsilon} \frac{1}{12} T g_{ij}. \quad (1.3)$$

The naive expectation that the β -functions cannot be modified by string loops [6] does not seem valid, because the ultraviolet cutoff in the σ -model breaks the conformal invariance and modifies the functional integral in regions of a small handle [5].

In this prescription string loop corrections are reduced to the insertion of local operators at points on S^2 or $\mathbb{C}$ (complex plane). Hence the argument relying on the scale invariance [7] may be extended to prove the equivalence of the vanishing β -function and the background field equations at string loop level.

The prescription of Fischler and Susskind (FS mechanism) has so far been applied to bosonic strings [5]. These theories have divergences and are not fully consistent theories of string. It is more appropriate to formulate the problem in superstring theories in which there will be no ambiguity due to the string loop divergences (at least at one-loop) unlike bosonic cases. In this letter, we present a formalism for evaluating string loop corrections to β -functions in supersymmetric σ -models and compute the one-loop corrections explicitly in the (1,1) supersymmetric σ -model which corresponds to the type-II superstring [7-9].

In superstring theories, loop corrections to amplitudes of massless particles satisfy a remarkable property, known as the non-renormalization theorem (NRT) [8]. It is interesting to see how this property of string amplitudes is reflected in the β -functions of the corresponding σ -model. This will enable us to solve the puzzle pointed out previously [5] regarding the genus-dependence of β -functions.

2. Review of Tree Level

The type-II superstring corresponds to the (1,1) supersymmetric non-linear σ -model in $D = 2$ [3]. At the string tree-level, the β -functions for the graviton g_{ij} have been evaluated up to $O(\alpha'^4)$ [3]. They are evaluated in dimensional regularization from the simple pole part $\sim 1/\epsilon$ of the σ -model loop corrections Δg_{ij} and $\Delta \Phi$ [3]. (For simplicity we neglect the effects of other massless fields, e.g., B_{ij} , A_{ijk}):

$$(\Delta S)^{(0)} = \frac{1}{4} (\alpha')^{-1} \int d^4 x \left[(\Delta g_{ij})^{(0)} D_\alpha \Phi^i D^\alpha \Phi^j + 4\alpha' (\Delta \Phi)^{(0)} R \right], \quad (2.1)$$

$$\beta_{ij}^{(0)} \equiv (\Delta g_{ij})^{(0)} \Big|_{1/\epsilon}, \quad \beta_\Phi^{(0)} \equiv (\Delta \Phi)^{(0)} \Big|_{1/\epsilon}, \quad (2.2)$$

$$\beta_{ij}^{(0)} = R_{ij} - 2 \nabla_i \nabla_j \Phi - \frac{1}{8} \alpha'^3 \zeta(3) \left[\frac{\delta \tilde{Y}}{\delta g_{ij}} - \frac{1}{4} g_{ij} \frac{\delta \tilde{Y}}{\delta \Phi} \right] + O(\alpha'^4), \quad (2.3)$$

$$\beta_\Phi^{(0)} = R + 4 (\nabla_i \Phi)^2 - 4 \nabla_i^2 \Phi + \frac{1}{16} \alpha'^3 \zeta(3) \left[\frac{\delta \tilde{Y}}{\delta \Phi} - 2 \tilde{Y} \right] + O(\alpha'^4), \quad (2.4)$$

$$\tilde{Y} \equiv t^{i_1 \dots i_8} t_{j_1 \dots j_8} \tilde{R}_{i_1 i_2}{}^{j_1 j_2} \dots \tilde{R}_{i_7 i_8}{}^{j_7 j_8}, \quad (2.5)$$

$$\tilde{R}_{ij}{}^{kl} \equiv \left[\partial_i \tilde{\omega}_j{}^{kl} + \tilde{\omega}_i{}^{km} \tilde{\omega}_j{}^{ml} \right] - (i \leftrightarrow j), \quad (2.6)$$

$$\tilde{\omega}_i{}^{jk} \equiv \omega_i{}^{jk}(e) + \delta_i{}^{[j} \nabla^{k]} \Phi. \quad (2.7)$$

It has been shown that the requirement of vanishing β -functions gives the field equations of the graviton and dilaton in $D = 10$,¹ which can be described by the (string tree-level) effective action [1-4,10]

$$\Gamma^{(0)} = -\frac{1}{2\kappa^2} \int d^{10}X e^{-2\Phi} \left[R - 4(\nabla_i \Phi)^2 - \frac{1}{8} \alpha'^3 \zeta(3) \tilde{Y} \right] + O(\alpha'^4). \quad (2.8)$$

It is easy to derive from eqs.(2.3), (2.4) and (2.8) that the β 's are related to the Γ 's as

$$\beta_{ij}^{(0)} = -2 e^{-1} \kappa^2 e^{2\Phi} \left[\frac{\delta \Gamma^{(0)}}{\delta g_{ij}} - \frac{1}{4} g_{ij} \frac{\delta \Gamma^{(0)}}{\delta \Phi} \right], \quad (2.9)$$

$$\beta_{\Phi}^{(0)} = e^{-1} \kappa^2 e^{2\Phi} \frac{\delta \Gamma^{(0)}}{\delta \Phi}. \quad (2.10)$$

By expressing the counter lagrangian (2.1) in terms of the β -functions (2.9) and (2.10), we see that it can be interpreted as the emission of a graviton or a dilaton which couples to the massless background fields. We rely on this interpretation to derive the string one-loop corrections to the β -functions in the next section.

¹ At $O(\alpha'^3)$ the Φ -dependent terms in the β -functions have not been worked out completely [3]. In eqs.(2.3) and (2.4) we give those which are determined from the present requirement.

3. One-Loop Effects.

The conformal invariance of the σ -model is broken by introducing a short-distance (in the world sheet) cutoff a or the parameter ε in the dimensional regularization. ($\log a \approx 1/\varepsilon$). Hence the string loop effects, in particular the ultraviolet behaviour of the σ -model and the resulting β -functions are in general not invariant under the conformal transformations of the world-sheet metric $g_{\mu\nu}$. The string loop corrected β -functions can be a useful notion if the condition that they vanish describes the loop corrected string propagation. We wish to suggest that the β -functions possessing this property at string loop levels may be obtained in the following way. At one-loop in the case of bosonic string, we consider a torus (on which the σ -model is defined) with the metric corresponding to the shape shown in Fig.1, with a cylinder (tube) of radius δ propagating from a sphere to a torus. The β -functions probe the short-distance property of the theory and hence for the torus of metric with $\delta \gg a$, the β -functions of the σ -model on T^2 will coincide with those on S^2 . If one chooses the radius of the cylinder to be of the cutoff size, i.e., $\delta \approx a$, the short-distance property of the theory gets modified by the presence of the handle, yielding the β -functions of the σ -model corrected by such a T^2 -effect.

In this letter we will not pursue the direct computation of the β -functions within the σ -model as described above. Instead we approach the problem in the direction put forward by Fischler and Susskind [5]. We evaluate the ultraviolet divergences of the σ -model as $\delta \approx a \rightarrow 0$ by identifying them with the infinities due to poles of massless particles (graviton and dilaton) in one-loop scattering amplitudes of the superstring theory coupled to the background massless fields.

It is important to note the difference in the divergence behaviour between the closed bosonic string and the closed superstrings. Consider the n -particle scattering amplitude in both theories. In the bosonic string, the one-loop amplitude has an infinity due to the dilaton graph of Fig.2a. This occurs through the dilaton coupling to the loop-induced cosmological term J and contributes a g_{ij} -term to the β -function β_{ij} of the σ -model. (The J itself is divergent in bosonic theories, but it is not relevant to the present problem.) In the superstring, the one-loop M -point amplitudes of massless particles vanish for $M \leq 3$ (the NRT). Hence the dilaton tadpole graph of Fig.2a is absent and no g_{ij} -term is produced in the β_{ij} . The same argument applies to the graph of a single massless particle emitted from the S^2 and coupling to one or two massless fields, so that terms linear and quadratic in the curvature tensor are not produced in the β_{ij} in superstring theory.

The superstring one-loop effects on β -functions start with cubic terms in the curvature tensor. These effects can be evaluated by considering the superstring amplitude corresponding to the graph in Fig.2b. This graph has a pole at $k^2 = 0$, where k is the momentum in the thin tube. As will be shown below, this pole due to the massless propagators corresponds to the infinity in the σ -model, which arises from taking the short-distance and thin tube limit $\delta \approx a \rightarrow 0$. By this identification of the pole at $k^2 = 0$ in the superstring amplitudes with the ultraviolet behaviour of the σ -model, the graph of massless particle emission in Fig.2b can be interpreted in the σ -model language as divergent counter-terms, and hence it contributes to the σ -model β -functions. We restrict ourselves to the terms of string one-loop and up to $O(\alpha'^4)$ in the σ -model loop.

To see the explicit existence of such a pole at $k^2 = 0$, it suffices to analyze the 5-point one-loop amplitude of Fig.3. The 5-point amplitude is worked out in Ref.[11] and expressed as an integral over the coordinates v_i of the vertex operators as well as the Teichmüller coordinate τ :

$$A(1, \dots, 5) = (2\pi)^{10} g^5 \int d^2\tau \int d^2v_1 \dots \int d^2v_4 \left(-\frac{2}{\tau_2}\right)^5 \times \left[K(z) \bar{K}(\bar{z}) + \frac{C}{\tau_2} \right] \prod_{1 \leq J} (\chi_{IJ})^{k_I \cdot k_J / 2}, \quad (3.1)$$

where we use the notation by Frampton et al. in Ref.[11]. We are interested in the pole at $k^2 = (k_1 + k_2)^2 \rightarrow 0$ (on-shell limit). This singularity arises as the two points 1 and 2 get close to each other (Fig.3). The parametrization appropriate for this kinematical region is

$$v_1 - v_2 = \xi e^{i\phi}, \quad \xi \approx 0, \\ \int d^2v_1 \dots \int d^2v_4 = i \int_0^1 d\xi \xi \int_0^{2\pi} d\phi \int d^2v_2 \dots \int d^2v_4. \quad (3.2)$$

Using the fact that $K(z)$ has the pole behaviour $\sim \tau_2/(v_1 - v_2)$ as $v_1 - v_2 \rightarrow 0$, we have

$$A(1, \dots, 5) \approx \int_F \frac{d^2\tau}{\tau_2^3} \int_0^1 d\xi \xi^{-1+k_1 \cdot k_2/2} \approx \frac{4}{(k_1 + k_2)^2} \quad (3.3)$$

One can show, as follows, that the on-shell limit $k^2 = (k_1 + k_2)^2 \rightarrow 0$ in the superstring amplitude can be interpreted as the ultraviolet limit in the σ -model, i.e., the limit $\varepsilon \rightarrow 0$ in the dimensional regularization:

$$\begin{aligned} \lim_{k^2 \rightarrow 0} \frac{4}{k^2} &= 4 \lim_{\substack{\mu \rightarrow 0 \\ k^2 \rightarrow 0}} \int_{\mu}^1 dx x^{k^2-1} = 4 \lim_{\mu \rightarrow 0} \int_{\mu}^1 \frac{dx}{x} \\ &= 2 \lim_{\mu \rightarrow 0} \left(\int_{\mu}^1 \frac{dx}{x} + \int_1^{\frac{1}{\mu}} \frac{dy}{y} \right) \quad (y = \frac{1}{x}) \\ &= \lim_{\mu \rightarrow 0} \int_0^{\frac{1}{\mu^2}} \frac{dz}{z + \mu^2} = \lim_{\substack{\Lambda \rightarrow \infty \\ \mu \rightarrow 0}} \int_0^{\Lambda} \frac{dz}{z + \mu^2} \end{aligned}$$

$$= \frac{1}{\pi} \lim_{\substack{\mu \rightarrow 0 \\ D \rightarrow 2}} \int \frac{d^D k}{k^2 + \mu^2} = \lim_{\substack{\mu \rightarrow 0 \\ \varepsilon \rightarrow 0}} \pi^{-\varepsilon} (\mu^2)^{-\varepsilon} \Gamma(\varepsilon) = \lim_{\varepsilon \rightarrow 0} \frac{1}{\varepsilon} . \quad (3.4)$$

Thus we can identify the pole at $k^2 = 0$ in the scattering amplitude with the pole at $\varepsilon = 0$ in the dimensional regularization in the β -function calculation. By this identification, the pole term of the above one-loop effect gives rise to a counter-term of $O(1/\varepsilon)$ in the σ -model lagrangian (Fig.4). (See also the papers by Callan et al. in Ref.[5].)

The finite factor which multiplies the singularity $1/\varepsilon$ corresponding to the top part of the graph in Fig.2b can be easily obtained from $M = 4$ amplitude [8,9]:

$$A(1, \dots, 4) = \frac{\alpha'^3}{32\pi^6} \kappa^2 g^2 \rho_{i_1 j_1}^1 \dots \rho_{i_4 j_4}^4 K^{i_1 \dots i_4} K^{j_1 \dots j_4} \\ \times \int_F \frac{d^2 \tau}{\tau_2^5} \int d^2 \nu_1 \dots \int d^2 \nu_3 + O(\alpha'^4), \quad (3.5)$$

where g is the closed string coupling constant. This result can be described in terms of the one-loop effective action [9]:

$$\Gamma^{(1)} = \frac{g^2}{768\pi^5 \kappa^2} \alpha'^3 \tilde{Y} + O(\alpha'^4). \quad (3.6)$$

The one-loop corrections to the σ -model lagrangian are then obtained from $\Gamma^{(1)}$ in the same way as the tree-level counter-terms (2.1) are obtained from $\Gamma^{(0)}$ through eqs.(2.9) and (2.10):

$$(\Delta g_{ij})^{(1)} = -2 \frac{1}{\varepsilon} \kappa^2 e^{-1} e^{2\Phi} \left[\frac{\delta \Gamma^{(1)}}{\delta g_{ij}} - \frac{1}{4} g_{ij} \frac{\delta \Gamma^{(1)}}{\delta \Phi} \right], \quad (3.7)$$

$$(\Delta \Phi)^{(1)} = \frac{1}{\varepsilon} \kappa^2 e^{-1} e^{2\Phi} \frac{\delta \Gamma^{(1)}}{\delta \Phi}. \quad (3.8)$$

The pole $1/\epsilon$ corresponds to the massless on-shell propagators of the thin tube, while the differentiation $\delta/\delta g_{ij}$ or $\delta/\delta \Phi$ comes from the replacement of one of the polarization tensors by the graviton or dilaton propagator of the thin tube. These counter-terms reveal the emissions of the graviton and dilaton from a σ -model lagrangian on S^2 , where the thin tube is inserted.

We have now the following β -functions with the genus-one corrections:

$$\begin{aligned} \beta_{ij}^{(0)} + \beta_{ij}^{(1)} = & R_{ij} - 2 \nabla_i \nabla_j \Phi - \frac{1}{8} \alpha'^3 \zeta(3) \left[\frac{\delta \tilde{Y}}{\delta g_{ij}} - \frac{1}{4} g_{ij} \frac{\delta \tilde{Y}}{\delta \Phi} \right] \\ & - e^{2\Phi} \frac{g^2}{384 \pi^5} \alpha'^3 \left[\frac{\delta \tilde{Y}}{\delta g_{ij}} - \frac{1}{4} g_{ij} \frac{\delta \tilde{Y}}{\delta \Phi} \right] + \frac{g^2}{768 \pi^5} e^{2\Phi} \alpha'^3 g_{ij} \tilde{Y} + O(\alpha'^4), \end{aligned} \quad (3.9)$$

$$\begin{aligned} \beta_{\Phi}^{(0)} + \beta_{\Phi}^{(1)} = & R + 4(\nabla_i \Phi)^2 - 4 \nabla_i^2 \Phi + \frac{1}{16} \alpha'^3 \zeta(3) \left[\frac{\delta \tilde{Y}}{\delta \Phi} - 2 \tilde{Y} \right] \\ & + e^{2\Phi} \frac{g^2}{768 \pi^5} \alpha'^3 \frac{\delta \tilde{Y}}{\delta \Phi} + O(\alpha'^4). \end{aligned} \quad (3.10)$$

We can also obtain the genus-one corrected effective action in $D = 10$:

$$\begin{aligned} \Gamma^{(0)} + \Gamma^{(1)} = & \frac{1}{\kappa^2} \int d^{10} X e \left\{ e^{-2\Phi} \left[-\frac{1}{2} R + 2(\nabla_i \Phi)^2 + \frac{1}{16} \zeta(3) \alpha'^3 \tilde{Y} \right] \right. \\ & \left. + \frac{g^2}{768 \pi^5} \alpha'^3 \tilde{Y} \right\} + O(\alpha'^4). \end{aligned} \quad (3.11)$$

This result agrees with the direct calculation of type-II superstring 1-loop amplitudes [8,9]. The absence of the one, two and three-point terms in $\Gamma^{(1)}$ is a reflection of the NRT.

4. Discussions and Remarks.

It is a widely accepted observation that some of the infinities appearing in the string theories are related to the (logarithmic) ultraviolet divergence in the corresponding σ -model. This relation was used to evaluate the string one-loop corrections to the β -functions in bosonic theories [5]. However, bosonic string and superstring theories have apparently different divergence properties, and it was not quite clear [5] whether the same relation holds for superstring, satisfying the NRT. We have demonstrated that the string one-loop corrections to the β -functions of supersymmetric σ -models can be computed exploiting this relation in the superstring. It is not the string loop divergence but the pole due to the massless propagators in the superstring amplitudes that is related to the ultraviolet divergences in the σ -model. The pole corresponds to the thin tube of Fig.2b, and we have to choose the limit $\delta \approx a \rightarrow 0$ to translate the pole singularity into the ultraviolet divergence in the σ -model.

By picking up only the massless pole terms, the one-loop (T^2) amplitudes are reduced to the tree (S^2) amplitudes with zero momentum insertion of local operators with divergent coefficients. This means in the σ -model language that the string loop effects produce genus corrections to the β -functions of the σ -model on S^2 . The equivalence between the vanishing genus-corrected β -functions and $D = 10$ field equations [7] is now valid, because our σ -model is defined on S^2 .

Superstring loop amplitudes have a remarkable property, i.e., they vanish for $M \leq 3$ and are finite for $M \geq 4$, therefore the FS mechanism can be tested more definitely in superstring theories. We have shown that in the type-II case the β -functions evaluated by the FS mechanism indeed possesses

this property. The present method can be easily extended so that the effects of other massless fields, e.g. antisymmetric tensors, can be taken into account. Our prescription is applicable to the heterotic string as well as other types of closed strings which are one-loop finite. The computation of the β -functions of the corresponding σ -models will be straightforward.

We have suggested that the string one-loop corrections to the σ -model β -functions arise as a consequence of the breaking of the conformal invariance by short-distance cutoff, and that one has to consider a torus with the thin tube of the size $\delta \approx a$. We suspect that the dependence of the theory on δ will disappear when the genus corrected β -functions are required to vanish. It is an interesting question whether the β -functions computed within the σ -model considering the extra effect from the limit $\delta \approx a \rightarrow 0$ will coincide with those we have evaluated based on the FS mechanism.

There are several directions in which the present scheme of β -function computation may be extended. To be able to extend the scheme to string loops of higher-genus one needs superstring theories which are finite beyond one-loop. In some sense the conformal invariance of the σ -model is related to the finiteness of the superstring. Presently a rigorous and general proof of the equivalence between the vanishing β -functions and string equation of motion is lacking, and hence to extend the β -function calculation to higher (σ -model) loops is also important.

We are grateful to M. Freeman and G. Veneziano for valuable discussions and reading the manuscript. We also thank C. Pope for useful comments. We are indebted to M. Jacob for kind hospitality at TH-Division, CERN.

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Figure Captions

Fig.1 A torus on which the σ -model is defined. The δ 's denotes the radius of the tube propagating from a sphere to a torus.

Fig.2(a) Bosonic string one-loop amplitude of n massless particles. The dilaton emitted from the sphere is absorbed by the vacuum by coupling to the one-loop cosmological constant.

Fig.2(b) Superstring one-loop amplitude of n massless particles (solid lines) in the presence of massless background fields (curly lines).

Fig.3 Five-point one-loop amplitude of massless particles. The pole at $(k_1+k_2)^2 \rightarrow 0$ arises as $v_1-v_2 \rightarrow 0$.

Fig.4 The massless pole terms of superstring one-loop amplitudes interpreted as the local counter-terms in the limit $\delta \approx a \rightarrow 0$ in the σ -model defined on the sphere.

FIG. 1

FIG. 2

FIG. 3

FIG. 4