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NEUTRAL CURRENTS WITHIN AND BEYOND THE STANDARD MODEL

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ABSTRACT

We present new, model-independent analyses of the observables in low-energy neutral-current experiments in the neutrino-hadron, neutrino-electron, electron/muon-hadron and electron-muon/tau sectors. We combine them with the latest experimental measurements of the W and Z masses to make global fits in the context of the Standard Model and of superstring-inspired models. We find in the Standard Model that $\sin^2\theta_w = 0.228 \pm 0.004$ if ρ is fixed to be 1, whilst a two-parameter fit yields $\sin^2\theta_w = 0.229 \pm 0.006$ and $\rho = 1.001 \pm 0.007$. This new value of $\sin^2\theta_w$ for $\rho = 1$ lies uncomfortably outside the predicted region of the minimal SU(5) model, but is consistent with supersymmetric SU(5) models. In the minimal Standard Model with $\rho = 1$ and equal Higgs and Z masses we find that $m_t < 168$ GeV at the 90% confidence level. We establish lower bounds on the masses of additional neutral gauge bosons Z' in three superstring-inspired models, and upper limits on their mixing with the Standard Model Z^0 . In particular, we find that $m_{Z'} > 129$ GeV at the 90% confidence level in a minimal rank-5 superstring-inspired model with canonical Higgs structure.

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1. INTRODUCTION

The central dilemma of particle physics today is that the Standard Model (SM) is consistent with all confirmed experimental data, but is theoretically very unsatisfactory. This means that there is no shred of concrete evidence for any theoretical attempt to go beyond the SM. There are two experimental frontiers where such evidence might be found. One is the energy frontier: go to higher energies and hope to find a new particle threshold. The other is the precision frontier: measure the parameters of the SM as accurately as you can, and hope to find a small discrepancy or subtle new effect which cannot be interpreted within it. One sector of the SM in which can be made very precise theoretical predictions and very precise experimental measurements is that of the neutral currents (NCs). Therefore this sector is a good place to measure the parameters of the SM and to look for its breakdown. Thus we make no apology for presenting yet another analysis of the available NC data.

Interest in NCs has been rekindled by two recent developments—one experimental and one theoretical—and one anticipated development. The experimental development has been the completion of the first generation of W and Z mass measurements [1, 2] at the CERN $p\bar{p}$ Collider. These enable low-energy and high-energy data to be confronted and electroweak radiative corrections to be probed experimentally for the first time. The theoretical development has been the advent of superstring-inspired models which suggest the possible existence of additional NCs mediated by an extra neutral boson Z' that may mix with the SM Z^0 *). The anticipated development is the advent of a new generation of precision measurements on the Z peak at the SLAC Linear Collider (SLC) and at the CERN Large Electron-Positron Storage Ring (LEP). Between them, these developments mean that now is a good moment to take stock of our knowledge of NCs, and give us some new small reason for hope that a discrepancy might be found.

There have been several other recent analyses [4] of the NC data with which ours can be compared. Our principal claims to originality are the following. i) We combine a new analysis [5] of the most recent neutrino-hadron scattering data, including an improved analysis of radiative corrections, with other NC data. ii) We analyse $e^+e^- \rightarrow \ell^+\ell^-$ ($\ell = \mu, \tau$) neutral-current data, using a convenient model-independent parametrization of the interference effects which can also be used outside the SM framework**). iii) We include the latest pre-ACOL, pre-Tevatron values of m_W and m_Z as measured by UA1 [2] and UA2 [1], and combine them with low-energy NC data. There are two principal outputs of our analysis. 1) An updated fit to the NC parameters, especially ρ and $\sin^2\theta_w$ in the SM. 2) Optimized constraints on non-minimal superstring models [3] with an extra Z' , especially lower limits on its mass and upper limits on its mixing with the Z^0 .

The outline of our paper is as follows. In Section 2 we introduce the different pieces of NC data that we use, and describe our analysis procedures. Subsections 2.1 and 2.2 are devoted to neutrino-quark and neutrino-electron scattering respectively, and include the re-analysis of radiative corrections mentioned above. Subsection 2.3 is concerned with charged lepton-quark reactions: parity violation effects in atoms, asymmetries in deep-inelastic eD and μ C scattering, and NC measurements in $e^+e^- \rightarrow q\bar{q}$. Subsection 2.4 contains a discussion of the information on neutral currents in the lepton-lepton sector which is obtained from measurements of $e^+e^- \rightarrow \ell^+\ell^-$ ($\ell = \mu, \tau$). In each of these subsections we give best-fit values of model-independent parameters which characterize the effective low-energy NC interactions. Finally, subsection 2.5 includes the latest information [1, 2] on the electroweak boson masses from the CERN $p\bar{p}$ Collider.

Section 3 combines the above pieces of information in a global fit to the parameters of the SM. We give fits to $\sin^2\theta_w$ with $\rho = 1$, and fits with both ρ and $\sin^2\theta_w$ left free. We also establish an important upper bound on m_i in the minimal SM.

*) Complete references can be traced from ref. [3].

**) This is not the case for the most commonly used parametrization [6].

Section 4 uses the same pieces of information to constrain superstring-inspired models [3] with an additional neutral gauge boson Z' . We focus on three particular models with just one extra Z' : either in $SU(6)/SU(5)$ or in $SO(10)/SU(5)$ or an alternative neutral generator in $E_6/SU(5)$.

Finally, Section 5 summarizes the results of our fits, discusses their implications for models, and reviews the prospects for future improvements.

2. MODEL-INDEPENDENT ANALYSIS OF THE DATA

In this section we collect the experimental data from low-energy NC experiments and on the W and Z masses, and give the results of a model-independent fit. Specifically, we consider the following sets of experiments:

- i) deep-inelastic neutrino-hadron scattering;
- ii) neutrino-electron scattering;
- iii) a) parity violation in atoms;
b) asymmetries in $e q$ and μq interactions;
c) forward-backward asymmetries in $e^+e^- \rightarrow q\bar{q}$;
- iv) forward-backward asymmetries in $e^+e^- \rightarrow \mu^+\mu^-, \tau^+\tau^-$;
- v) electroweak boson masses.

Our aim is to obtain a model-independent determination of the coefficients which enter in the expression of the effective NC Lagrangian, whose relevant terms are

$$\mathcal{L}_{\text{eff}}^{(\nu f)} = -\frac{G_F}{\sqrt{2}} \left[\bar{\nu} \gamma_\mu (1-\gamma_5) \nu \right] \times \left[u_L \bar{u} \gamma^\mu (1-\gamma_5) u + u_R \bar{u} \gamma^\mu (1+\gamma_5) u + d_L \bar{d} \gamma^\mu (1-\gamma_5) d + d_R \bar{d} \gamma^\mu (1+\gamma_5) d + g_L \bar{e} \gamma^\mu (1-\gamma_5) e + g_R \bar{e} \gamma^\mu (1+\gamma_5) e \right] \quad (2.1)$$

for the neutrino-quark and neutrino-electron sectors, and

$$\mathcal{L}_{\text{eff}}^{(eq)} = \frac{G_F}{\sqrt{2}} \left[(\bar{e} \gamma_\mu \gamma_5 e) (c_{1u} \bar{u} \gamma^\mu u + c_{1d} \bar{d} \gamma^\mu d) + (\bar{e} \gamma_\mu e) (c_{2u} \bar{u} \gamma^\mu \gamma_5 u + c_{2d} \bar{d} \gamma^\mu \gamma_5 d) + (\bar{e} \gamma_\mu \gamma_5 e) (h_{AA}^u \bar{u} \gamma^\mu \gamma_5 u + h_{AA}^d \bar{d} \gamma^\mu \gamma_5 d) \right] \quad (2.2)$$

for the charged lepton-quark sector. One has to go beyond this approximation for point (iv), as specified later on. We discuss separately the data for the neutrino-quark, neutrino-electron, charged lepton-quark, electron-muon/tau, and (W, Z) sectors, specifying in each case the experiments, the approximations we use, and estimates of the theoretical uncertainties.

Radiative corrections have to be applied to the model-independent solutions, and we shall briefly discuss the approximation which has been adopted for each set of data. Here we anticipate the general features of these approximations. Radiative corrections have been evaluated in the framework of the SM, even when considering models with an extra neutral gauge boson Z' . This is because we know that the SM is consistent with all present experimental data and anticipate that the effects of the extra Z' on measurable quantities are small perturbations of the SM predictions. Thus the effects of radiative corrections involving Z' itself are expected to be a second-order perturbation, and will be neglected altogether.

For a consistent analysis of the radiative corrections, we adopt, in general, the same renormalization prescription in the different sectors. However, we shall relax this requirement in

those cases for which the data are not accurate enough. We assume, as input parameters [7], the gauge boson masses m_W , m_{Z^0} (where m_{Z^0} is the physical neutral boson mass in the SM and the corresponding unmixed mass parameter in extensions of the SM), the Higgs mass m_H , the fine structure constant α , and the masses and mixing angles of three generations. One can define the parameters $s_W \equiv \sin^2 \theta_W$ and ρ by

$$s_W \equiv A_c^2 / m_W^2 : A_c^2 \equiv \left(\frac{\pi \alpha}{\sqrt{2} G_F} \right) / (1 - \Delta r) \quad (2.3)$$

and

$$\rho \equiv m_W^2 / m_{Z^0}^2 \cos^2 \theta_W \quad (2.4)$$

where $(\pi \alpha / \sqrt{2} G_F)^{1/2} = (37.2810 \pm 0.0003) \text{ GeV}$. Taking $m_u = m_d = 0.1 \text{ GeV}$, $m_s = 0.3 \text{ GeV}$, $m_c = 1.5 \text{ GeV}$, $m_b = 4.5 \text{ GeV}$, $m_t = 36 \text{ GeV}$ and $\xi = m_H / m_{Z^0} = 1$, one finds $\Delta r = 0.0711 \pm 0.0013$ [8] for $s_W = 0.227$ and $\rho = 1$. In the minimal version of the SM one has $\rho = 1$ and can directly define s_W as

$$s_W \equiv 1 - m_W^2 / m_{Z^0}^2 \quad (2.5)$$

as in Ref. [9].

2.1 Neutrino-quark sector

Several processes have been experimentally investigated which involve the NC structure of the neutrino-quark interaction, including deep-inelastic scattering (on isoscalar, proton, and neutron targets), exclusive and semi-inclusive pion production, elastic scattering, and, more recently, coherent π^0 production.

In principle, each of these processes contributes to a better knowledge of the four couplings in terms of which the neutrino-quark NC interaction can be described in a model-independent way. However, deep-inelastic neutrino scattering is by far the most important of all these processes—on the one hand, because of the impressive amount of experimental data, some of them coming from recent, high-precision experiments; on the other hand, because the theoretical description of this process, based on the quark parton model (QPM) assuming a QCD parametrization of the nucleon structure functions, appears quite accurate although it has a rather large number of parameters, and allows one to control the theoretical uncertainty typical of all processes involving hadrons.

The estimate of the NC neutrino-quark couplings reported in this paper [see (2.1) for the convention adopted in the effective Lagrangian] is based on a recent model-independent analysis of the structure of the hadronic NC starting from deep-inelastic neutrino scattering [5]. The analysis includes all the experiments on isoscalar [10–22] and on neutron and proton targets [23–28] which are reported in Tables 1 and 2, respectively (except the old Gargamelle experiment, reported only for the sake of completeness) corresponding to a total of 36 independent experimental measurements.

The applicability of the QPM is assumed in the analysis of the deep-inelastic scattering data. The scaling violations related to the Q^2 -dependence of the parton distributions are parametrized in a manner (approximately) consistent with QCD along the lines suggested by Buras and Gaemers [29], by introducing an approximate functional form of the Q^2 -dependence satisfying the most important

moment equations. The parametrization adopted takes into account the most recent results coming from the study of the structure functions in the charged-current (CC) sector, in particular those obtained by CHARM [30], CCFRR [31], WA25 [32] and CDHSW [33].

The valence-quark distributions are parametrized in the form

$$x u_v(x, Q^2) + x d_v(x, Q^2) = \frac{3}{B(\gamma_1(S), 1 + \gamma_2(S))} x^{\gamma_1(S)} (1-x)^{\gamma_2(S)} \quad (2.6a)$$

$$x d_v(x, Q^2) = \frac{1}{B(\gamma_3(S), 1 + \gamma_4(S))} x^{\gamma_3(S)} (1-x)^{\gamma_4(S)} \quad (2.6b)$$

where $B(x, y)$ is the usual Euler function and

$$S \equiv \ln \left(\frac{\ln Q^2 / \Lambda^2}{\ln Q_0^2 / \Lambda^2} \right) \quad (2.7)$$

with Q_0^2 being some fixed value. Taking $Q_0^2 = 5 \text{ GeV}^2$, the numerical values adopted in the analysis are given by

$$\left. \begin{aligned} \gamma_1(0) &= 0.46 \pm 0.02 \\ \gamma_2(0) &= 2.32 \pm 0.06 \\ \gamma_3(0) &= 0.41 \pm 0.02 \\ \gamma_4(0) &= 2.52 \pm 0.12 \end{aligned} \right\} \quad (2.8)$$

with, as far as the QCD parameter Λ is concerned,

$$\Lambda = 0.25 \pm 0.18 \text{ GeV} \quad (2.9)$$

The large error in (2.9) takes into account the large uncertainty still affecting this parameter.

The sea-quark and gluon distributions are parametrized in the forms

$$x \bar{u}(x, Q^2) = x \bar{d}(x, Q^2) = C_u(S) (1-x)^{\gamma_u(S)} \quad (2.10a)$$

$$x s(x, Q^2) = x \bar{s}(x, Q^2) = C_s(S) (1-x)^{\gamma_s(S)} \quad (2.10b)$$

$$x c(x, Q^2) = x \bar{c}(x, Q^2) = C_c(S) (1-x)^{\gamma_c(S)} \quad (2.10c)$$

$$x G(x, Q^2) = C_g(S) (1-x)^{\gamma_g(S)} \quad (2.10d)$$

In equations (2.6) and (2.10) the S -dependence of the functions $C_i(S)$ and $\eta_i(S)$ is provided by QCD [29], once their values at $Q^2 = Q_0^2$ are introduced in agreement with the experimental data. On the basis of the recent results mentioned above [30–33], the following initial values are assumed at $Q_0^2 = 5 \text{ GeV}^2$:

$$\gamma_u(0) = \gamma_s(0) = \gamma_c(0) = 6.5 \pm 1.2 \quad (2.11)$$

with $\eta_G(0) = \eta_s(0) + 2$, the coefficients $C_i(0)$ appearing in (2.10) being estimated starting from the initial values of the parameters:

$$r_{sea} = R_{sea}(Q_0^2) = \int_0^1 dx x \bar{q}(x, Q_0^2) / \int_0^1 dx x [q(x, Q_0^2) + \bar{q}(x, Q_0^2)] \quad (2.12a)$$

$$a_s = A_s(Q_0^2) = \int_0^1 dx x s(x, Q_0^2) / \int_0^1 dx x \bar{u}(x, Q_0^2) \quad (2.12b)$$

$$a_c = A_c(Q_0^2) = \int_0^1 dx x c(x, Q_0^2) / \int_0^1 dx x \bar{u}(x, Q_0^2) \quad (2.12c)$$

where $q(x, Q^2)$ and $\bar{q}(x, Q^2)$ are the total amounts of quarks and antiquarks, respectively. In the analysis, the usual threshold effect for c-quark production is included through $a_c(Q^2 = 1.8 \text{ GeV}^2) = 0$, and starting from the CC analysis and the existing determinations [34, 35] it is assumed that

$$r_{sea} = 0.165 \pm 0.016, \quad a_s = 0.42 \pm 0.06 \quad (2.13)$$

Finally, the gluon distribution is obtained from the well-known momentum sum rule involving gluons together with quarks and antiquarks:

$$\int_0^1 dx [x q(x, Q^2) + x \bar{q}(x, Q^2) + x G(x, Q^2)] = 1 \quad (2.14)$$

It is seen from (2.8), (2.9), (2.11), and (2.13) that, in estimating the different parameters entering the quark and gluon distribution functions, their initial values, and their QCD evolution, a certain error is introduced. This error represents the uncertainty affecting our knowledge of the different parameters, our uncertainty estimated within the scheme provided by three generations of quarks and including our present knowledge of the generalized Cabibbo mixing [35].

The uncertainties considered above contribute to the “theoretical” error effecting the estimates of the coupling constants in the neutrino-quark sector. Other sources of uncertainty come from

- i) the Callan-Gross relation [36]: it will be assumed, consistently with the most recent data [37, 33], that

$$R \equiv \frac{F_2 - 2xF_1}{2xF_1} = R_{QCD} \quad (2.15)$$

with an uncertainty $\Delta R = \pm 0.1$;

- ii) deviation from isoscalarity: a 2% uncertainty is assumed, except when the n/p ratio of the target is known with extreme accuracy (as in the case of deuterium and hydrogen targets, for instance).

The program of implementing the electroweak radiative corrections in neutrino-induced deep-inelastic scattering has been carried out by several authors [38–42], in particular in a systematic way by Marciano and Sirlin [41]. Radiative corrections are expected to contribute in an appreciable way in deep-inelastic scattering. This is indeed the case: a comparison of neutrino-induced deep-inelastic scattering with the direct measurements of the vector-boson masses is able to give a first, rather convincing, indication that radiative corrections are at work in the electroweak interactions [43].

The way suggested by Marciano and Sirlin [41] for estimating the different effects in NCs and CCs is followed. However, we introduce their dependences on the scaling variables x and y , which are not fully integrated in the experiments because of their kinematical cuts.

The resulting determination of the four squared chiral couplings, inclusive of radiative correction effects is [5]

$$u_L^2 = 0.1269 \pm 0.0136 (\pm 0.0008) \quad (2.16a)$$

$$d_L^2 = 0.1732 \pm 0.0140 (\pm 0.0031) \quad (2.16b)$$

$$u_R^2 = 0.0261 \pm 0.0084 (\pm 0.0023) \quad (2.16c)$$

$$d_R^2 = 0.0041 \pm 0.0078 (\pm 0.0063) \quad (2.16d)$$

Of the two errors, the former is of experimental origin and corresponds to the effects of the statistical and systematic experimental errors added in quadrature. The latter error, however, is of theoretical origin, and reflects the cumulative effect of all the uncertainties described above. In the following the two errors will be added in quadrature, taking into account the correlation coefficients of the resulting total errors.

From the squared chiral couplings given in (2.16), the linear couplings are then derived, the signs being dictated by a comparison with other processes sensitive to these choices (for a recent approach to this problem, see Ref. [44]). It follows that

$$u_L = 0.3562 \pm 0.0192 \quad (2.17a)$$

$$d_L = -0.4162 \pm 0.0172 \quad (2.17b)$$

$$u_R = -0.1617 \pm 0.0271 \quad (2.17c)$$

$$d_R = 0.0639 \pm 0.0551 \quad (2.17d)$$

with off-diagonal correlation coefficients

$$\begin{pmatrix} 0.965 & 0.381 & 0.376 \\ & 0.362 & 0.418 \\ & & 0.903 \end{pmatrix} \quad (2.18)$$

A further comment is in order. The above estimate of the chiral couplings is rather sensitive to the c-quark mass m_c , as discussed in Ref. [5]. Here the value $m_c = 1.5$ GeV is fixed, thereby avoiding the introduction of a further uncertainty into the chiral couplings. The effect of relaxing this assumption in the analysis is discussed in Ref. [5].

2.2 Neutrino-electron sector

The scattering of neutrinos off electrons, which was the first NC phenomenon observed ($\bar{\nu}_\mu e$ scattering at CERN by the Gargamelle Collaboration [45] in 1973), since it is a purely leptonic process, avoids the difficulties that are typical of hadronic targets and hence is characterized by a greater simplicity from the theoretical point of view. Strong limitations, however, come from the experimental problems: very low cross-sections, due to the smallness of the electron mass, and problems inherent to ν_e and $\bar{\nu}_e$ beams. It follows that even if the two coupling constants which describe the weak neutral electron current in a model-independent way are estimated with negligible

theoretical uncertainties, errors are large because of the relatively small number of events observed in the different experiments.

In the present approach we use the results of a recent analysis [46] of neutrino–electron scattering, in which the most up-to-date experimental results for both $\langle \bar{\nu}_\mu e \rangle$ and $\langle \bar{\nu}_e e \rangle$ sectors are considered and combined. Moreover, for the first time, radiative corrections are explicitly applied following the scheme proposed in Ref. [47], even though their effects are very small compared with the errors affecting the experimental determination.

In Table 3 the updated results from the $\langle \bar{\nu}_\mu e \rangle$ experiments [48–53] are reported: the most important experiments are the most recent ones, namely CHARM [52] and BBKOPST [53]. However, the others are also of relevance in constraining the two squared coupling constants g_L^2 and g_R^2 . It is also important to note the correlations between the errors affecting $\sigma(\nu_\mu e)$ and $\sigma(\bar{\nu}_\mu e)$.

In Table 4 we give the cross-sections for $\langle \bar{\nu}_\mu e \rangle$ scattering. The experiments include the old data of Reines et al. [54], with the results coming from the two different electron-energy regions taken as two independent measurements, and the very recent result obtained at LAMPF [55]. Together they allow a determination of the two chiral couplings in the combinations $(1 + g_L)^2$, g_R^2 typical of $\langle \bar{\nu}_\mu e \rangle$ interactions.

The results coming from $\langle \bar{\nu}_\mu e \rangle$ and $\langle \bar{\nu}_e e \rangle$ can be combined and g_L , g_R can be estimated, making use of e^+e^- data to remove the residual ambiguity in the sign of g_R . One finds [46]

$$g_L = -0.273 \pm 0.018 \quad (2.19a)$$

$$g_R = 0.228 \pm 0.022 \quad (2.19b)$$

$$\rho_{LR} = 0.042 \quad (2.19c)$$

corresponding, in terms of the usual vector and axial couplings g_V and g_A , to

$$g_V = -0.046 \pm 0.029 \quad (2.20a)$$

$$g_A = -0.501 \pm 0.028 \quad (2.20b)$$

$$\rho_{AV} = -0.165 \quad (2.20c)$$

which we use in the subsequent combined fit.

2.3 Lepton–quark sector

This is probed in the three classes of experiments described in the following sub-subsections.

2.3.1 Parity violation in atoms

Different kinds of experiments have been performed on heavy atoms. They are characterized by different experimental methods, atomic elements, and transitions observed. From these experiments information can be obtained regarding the coefficients C_{1u} and C_{1d} defined in (2.1).

The first class of experiments searched for optical rotation of light in atomic vapour. The relevant transition is, in general, an allowed M1 magnetic transition; parity violation induces mixing with an E1 component. The quantity which is measured by each experiment is

$$R \equiv \text{Im } E_1^{PV} / M_1 \quad (2.21)$$

A rough estimate gives $R = O(10^{-7})$ as the expected order of magnitude. Different theoretical calculations have been performed, which allow us to determine from the data the so-called weak charge Q_w :

$$Q_w(Z, A) = -2 \left[(Z+A)C_{1u} + (2A-Z)C_{1d} \right] \quad (2.22)$$

Following Ref. [56] we adopt the theoretical estimates which are based on calculations that go beyond the central field independent particle model (IPM) approximation. In Table 5 we collect the experimental values and theoretical estimates of the quantity R for bismuth (two transition lines) [57–61] and lead [62].

A similar experiment was performed on thallium [63]. In this case, the analysed transition is a forbidden magnetic one, and its expected value is of the order 10^{-3} . The measured values and theoretical estimates of R are also given in Table 5.

The most accurate class of experiments is based on the so-called parity-violating Stark interference. One deals with an M1 forbidden transition; the parity-violating amplitude is observed by interference with an external field E . The experimental conditions are such that $\beta E \gg M_1$, where β is the vector polarizability. Experiments on caesium have been performed at the Ecole Normale Supérieure in Paris [64–66] and at the University of Colorado, Boulder [67]. The measured quantity is expressed in terms of

$$\text{Im } E_1^{PV} / \beta \quad (2.23)$$

where β is given (for Cs) by [66]

$$\beta = (26.8 \pm 0.8) a_0^3 \quad (2.24)$$

where a_0 is the Bohr radius. An accurate theoretical estimate has been carried out in Ref. [68]:

$$\text{Im } E_1^{PV} = (0.91 \pm 0.04) \frac{Q_w}{(A-Z)} \times 10^{-11} |e| a_0 \quad (2.25)$$

The experimental values obtained for $\text{Im } E_1^{PV} / \beta$, and the theoretical estimates, are shown in Table 6.

Similar experiments were performed on thallium at Berkeley [69, 70]. The experimental value is again expressed in terms of the quantity (2.23), but the vector polarizability is now given by [71]

$$\beta = 198 a_0^3 \quad (2.26)$$

A theoretical estimate has been reported in Ref. [57]:

$$\text{Im } E_1^{PV} = (0.68 \pm 0.03) \frac{Q_w}{(A-Z)} \times 10^{-10} |e| a_0 \quad (2.27)$$

Again, the experimental results and theoretical estimates are shown in Table 6.

In Table 7 we collect the values obtained for Q_w from the different experiments. In the same table we give the values for the combination

$$C_{1u}^{(0)} + \zeta C_{1d}^{(0)} \quad ; \quad \zeta = \frac{2A-Z}{A+Z} \quad (2.28)$$

found from Q_w neglecting radiative corrections. The corresponding values obtained by including radiative corrections are given in the last column of Table 7, and are denoted by $C_{1u} + \zeta C_{1d}$.

Electroweak radiative corrections in atomic parity violation have been analysed by several authors [41, 72–74]. For our purposes, it is convenient to use the estimates given in Ref. [41]. The coefficients $C_{1u}^{(0)}$ and $C_{1d}^{(0)}$ are identified, to order α , with

$$C_{1u}^{(0)} = -\frac{1}{2} \rho'_{PV} \left(1 - \frac{8}{3} \sin^2 \theta_w(0)_{\text{eff}}\right) \quad (2.29a)$$

$$C_{1d}^{(0)} = \frac{1}{2} \rho'_{PV} \left(1 - \frac{4}{3} \sin^2 \theta_w(0)_{\text{eff}}\right) \quad (2.29b)$$

where an expression for ρ'_{PV} is given explicitly in Ref. [41]. The parameter $\sin^2 \theta_w(0)_{\text{eff}}$ can be expressed in terms of s_w (2.3), making use of the relation given in Refs. [41] and [47]. Assuming the values specified above for the input parameters, we get: $\rho'_{PV} = 0.981$ and $\sin^2 \theta_w(0)_{\text{eff}} = 0.996 s_w$. This allows us to extract from (2.29a,b) the radiatively corrected coefficients C_{1u} and C_{1d} , which in the framework of the SM would be specified by $C_{1u} = -\frac{1}{2} + \frac{4}{3} s_w$ and $C_{1d} = \frac{1}{2} - \frac{2}{3} s_w$.

2.3.2 Asymmetries in electron–quark and muon–quark interactions

A parity-violating asymmetry was measured in the inelastic scattering of longitudinally polarized electrons by deuterium in the SLAC experiment [75]. The reaction is the following:

$$e_{L,R}^-(E_0) + D \rightarrow e^-(E') + X \quad (2.30)$$

and the relevant asymmetry is defined by

$$A_D = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} \quad (2.31)$$

where $\sigma_{R,L}$ is the inelastic cross-section $d^2\sigma_{R,L}/d\Omega dE'$. The asymmetry can be fitted by the formula (see Ref. [75])

$$A_D / Q^2 = a_1 + a_2 F(y) \quad (2.32)$$

where

$$y \equiv \frac{E_0 - E'}{E_0} \quad (\text{energy transfer}) \quad (2.33a)$$

$$Q^2 \equiv -q^2 > 0 \quad (\text{momentum transfer}) \quad (2.33b)$$

and

$$F(y) = \frac{1 - (1-y)^2}{1 + (1-y)^2} \quad (2.33c)$$

For isoscalar targets, a_1 and a_2 are expected to be constants, i.e. independent of the kinematical parameters. The experimental value given for A_D in Ref. [75] is

$$\frac{A_D}{Q^2} = \{(-9.7 \pm 2.6) + (4.9 \pm 8.1) F(y)\} \times 10^{-5} \quad (2.34)$$

In terms of the theoretical coefficients in the PV terms of the effective Lagrangian (2.2), we can write

$$\frac{A_D}{Q^2} = K \left[(C_{1u} - \frac{1}{2} C_{1d}) + F(y) (C_{2u} - \frac{1}{2} C_{2d}) \right] \quad (2.35)$$

with

$$K = \frac{3G_F}{5\sqrt{2}\pi\alpha} = 2.158 \times 10^{-4} \text{ GeV}^{-2} \quad (2.36)$$

In order to determine the two combinations $(C_{1u} - \frac{1}{2} C_{1d})$ and $(C_{2u} - \frac{1}{2} C_{2d})$ and their correlation ρ , we have performed a fit to the published data (Table 1 of Ref. [75]). We obtain

$$C_{1u} - \frac{1}{2} C_{1d} = -0.45 \pm 0.12 \quad (2.37a)$$

$$C_{2u} - \frac{1}{2} C_{2d} = 0.23 \pm 0.38 \quad (2.37b)$$

$$\rho = -0.95 \quad (2.37c)$$

including only pure QED radiative corrections.

Concerning the radiative corrections [45, 76-79], those coming from bremsstrahlung and pure QED [76], which give the main effects, were already taken into account in the presentation of the data. Complete one-loop electroweak corrections have been evaluated in Refs. [42] and [79]. These additional corrections depend on y and Q^2 , and range from 4‰ to 4%. Therefore they can be safely neglected when compared with the experimental errors which range from 15% to 50%. Thus the values given in (2.37a-c) represent the final results for our purposes.

A different kind of asymmetry can be measured in the deep-inelastic scattering of longitudinally polarized muons off nuclear targets. Because of the interference between the weak neutral and electromagnetic currents, there is a non-vanishing asymmetry parameter defined by

$$B = \frac{\sigma^+(-|P|) - \sigma^- (+|P|)}{\sigma^+(-|P|) + \sigma^- (+|P|)} \quad (2.38)$$

where $\sigma^*(P)$ is the cross-section with polarization P for $\mu^* N \rightarrow \mu^* X$.

An experiment was performed at the CERN Super Proton Synchrotron (SPS) with muons of 120 and 200 GeV incoming energy on an isoscalar carbon target [80]. The data are fitted with the expression

$$B = a + b F(y) Q^2 \quad (2.39)$$

where a and b are constant parameters, y and Q^2 are the energy and momentum transfer, respectively, and $F(y)$ is the function given in (2.33c). The published results of the fit are [80]

$$a = (0.06 \pm 0.17) \times 10^{-2} \quad (2.40a)$$

$$b = (-0.174 \pm 0.075) \times 10^{-3} \text{ GeV}^{-2} \quad (2.40b)$$

$$(\text{at } 120 \text{ GeV}, |P| = 0.66 \pm 0.05)$$

$$a = (0.15 \pm 0.17) \times 10^{-2} \quad (2.41a)$$

$$b = (-0.147 \pm 0.037) \times 10^{-3} \text{ GeV}^{-2} \quad (2.41b)$$

$$(\text{at } 200 \text{ GeV}, |P| = 0.81 \pm 0.04)$$

including just the statistical errors. Assuming $a = 0$, we get, in the frame of the quark parton model,

$$b = \frac{B}{F(y)Q^2} = - \frac{3G_F}{5\sqrt{2}\pi\alpha} \left[\left(h_{AA}^u - \frac{1}{2} h_{AA}^d \right) + P \left(C_{2u} - \frac{1}{2} C_{2d} \right) \right] \quad (2.42)$$

where, besides the PV coefficients C_{2u} and C_{2d} , the axial-axial couplings h_{AA}^u and h_{AA}^d defined in (2.2) also appear. It should be noted that in the SM with $s_w = 0.23$ we get $C_{2u} - \frac{1}{2} C_{2d} \approx 0$, so that the asymmetry coefficient B is almost independent of the polarization and is essentially parity-conserving [80].

We should point out that before extracting the values of a and b from the data, the effects of one-loop electroweak corrections, as evaluated in Ref. [81], have been included. In fact, these corrections are not negligible, even though the data have large errors. From (2.42) and the quoted data, including in quadrature the systematic errors as quoted in ref. [80], we obtain the following values:

$$\left(h_{AA}^u - \frac{1}{2} h_{AA}^d \right) + P \left(C_{2u} - \frac{1}{2} C_{2d} \right) = \begin{cases} 0.81 \pm 0.37 & (|P| = 0.66) \\ 0.68 \pm 0.19 & (|P| = 0.81) \end{cases} \quad (2.43)$$

after including these radiative corrections.

2.3.3 Forward-backward asymmetry in $e^+e^- \rightarrow q\bar{q}$. The forward-backward asymmetry has been measured in $e^+e^- \rightarrow q\bar{q}$ reactions at PETRA and PEP. The main problems in the experiments are flavour identification and the distinction between quark and antiquark, and the fact that no reliable method has been found for the light quarks u, d, s . In the case of the c -quark, two methods have been used:

- i) direct observation of $D^{0,*}, D^{*+}$,
- ii) observation of direct leptons in decays $c \rightarrow \ell \nu X$ ($\ell = e, \mu$).

In the case of the b -quark, only direct leptons in decays $b \rightarrow \ell \nu X$ have been observed. Method (i) for the c -quark relies on the identification of exclusive D -decay channels, whilst in method (ii) one has to analyse all the possible sources of direct leptons, such as $c \rightarrow e^+$, $b \rightarrow e^-$, $b \rightarrow c \rightarrow e^+$. Unfortunately, all the experiments have the common features of low yields and high backgrounds. Different selection criteria have been adopted in the various experiments. The values obtained for the asymmetries involve multiparameter fits, specific assumptions on the models, etc. Here we do not discuss the specific features of the fits; details can be found in Refs. [82–89].

The quantity which is obtained from the experiments is the differential cross-section

$$\frac{d\sigma_{c,b}}{d\cos\theta} = \frac{3s}{32\pi} \left[S_{c,b} (1 + \cos^2\theta) + A_{c,b} 2\cos\theta \right] \quad (2.44)$$

for $e^+e^- \rightarrow c\bar{c}$ and $e^+e^- \rightarrow b\bar{b}$. The forward-backward asymmetry is given by the coefficient

$$A_{FB}^{(c,b)} = \frac{3}{4} \frac{A_{c,b}}{S_{c,b}} \quad (2.45)$$

The values obtained for this coefficient in the two reactions are collected in Table 8.

For our model-independent fit, we adopt the following approximations:

- i) we neglect the widths of the neutral gauge bosons, which seems to be justified at PETRA and PEP energies;

- ii) we neglect electroweak radiative corrections, and mass effects in the final states [90], which can be justified by the poor experimental determination of the asymmetries;
 - iii) moreover, for the sake of simplicity in the fit, we expand the asymmetry coefficient A_{FB} in powers of s (squared c.m. energy) and keep only the first term.
- It follows from the last assumption that $A_{FB}^{c,b}$ can be expressed in a simple form in terms of the coefficients $h_{AA}^c = h_{AA}^u$ and $h_{AA}^b = h_{AA}^d$ which appear in the effective Lagrangian (2.2):

$$A_{FB}^{(c)} = -\frac{9}{32} \frac{1}{A_c^2} h_{AA}^u s = a^{(c)} s \quad (2.46a)$$

$$A_{FB}^{(b)} = \frac{9}{16} \frac{1}{A_c^2} h_{AA}^d s = a^{(b)} s \quad (2.46b)$$

with A_c defined by (2.4).

The approximation we adopt does not seem so bad. We have estimated the theoretical errors made by neglecting the terms $O(s^2)$: they depend on s and are of order $\delta A_{FB}^{th}(s) \approx -(1-2)\%$, i.e. rather small in comparison with the experimental errors, while their model-dependence is negligible in the region of parameter space relevant to our analysis. Nevertheless, in order to improve the approximation we have corrected the data of Table 8 according to:

$$A_{FB} \rightarrow A_{FB} - \delta A_{FB}^{th}(s)$$

The correction term δA_{FB}^{th} is the difference between the full tree-level expression and the approximate form (2.46). We have calculated this term in the SM and also in models with an extra Z' allowing the parameters to vary around the SM values. The correction depends strongly on the value of s , but it is nearly model-independent in a reasonable range of variation of the parameters of the model. After these corrections, best fits to these data with (2.46a,b) give

$$a^{(c)} = (-1.28 \pm 0.32) \times 10^{-9} \text{ GeV}^{-2} \quad (2.47a)$$

$$a^{(b)} = (-1.98 \pm 0.40) \times 10^{-9} \text{ GeV}^{-2} \quad (2.47b)$$

for the $c\bar{c}$ and $b\bar{b}$ measurements, respectively.

2.3.4 Summary of the data in the lepton-quark sector

The coefficients obtained from the experiments on parity violation, asymmetries in electron-quark and muon-quark, and forward-backward asymmetries in $e^+e^- \rightarrow q\bar{q}$ are all correlated. From a best fit of the different pieces of information, we obtain the following values of the coefficients appearing in equation (2.2):

$$C_{1u} = -0.181 \pm 0.056 \quad (2.48a)$$

$$C_{1d} = 0.327 \pm 0.050 \quad (2.48b)$$

$$C_{2u} - \frac{1}{2} C_{2d} = -0.101 \pm 0.242 \quad (2.48c)$$

$$h_{AA}^u = 0.605 \pm 0.152 \quad (2.48d)$$

$$h_{AA}^d = -0.513 \pm 0.105 \quad (2.48e)$$

and the off-diagonal elements of the correlation matrix ρ_{ij} are

$$\begin{pmatrix} -0.980 & -0.895 & 0.530 & -0.150 \\ & 0.888 & -0.526 & 0.149 \\ & & -0.592 & 0.167 \\ & & & 0.070 \end{pmatrix} \quad (2.49)$$

These results will be combined with those of subsections 2.1, 2.2, 2.4, and 2.5 in the fits of Sections 3 and 4.

2.4 Lepton-lepton sector

This is probed in measurements of $e^+e^- \rightarrow \ell^+\ell^-$ ($\ell = \mu, \tau$). The forward-backward asymmetries have been measured at PEP and PETRA in the processes

$$e^+e^- \rightarrow \mu^+\mu^-$$

and

$$e^+e^- \rightarrow \tau^+\tau^-$$

These final states represent idealized situations, whilst experiments must take into account the fact that in the detectors there is an unavoidable contamination of bremsstrahlung events of the type $e^+e^- \rightarrow \ell^+\ell^-\gamma$ ($\ell = \mu, \tau$). Forgetting for the moment these bremsstrahlung contributions, the differential cross-sections can be written in the form

$$\frac{d\sigma}{d\cos\theta} = \frac{s}{32\pi} [S_L \cdot (1 + \cos^2\theta) + A_L \cdot 2\cos\theta] \quad (2.50)$$

and the forward-backward asymmetry is then given by

$$A_{FB} = \frac{3}{4} \frac{A_L}{S_L} \quad (2.51)$$

For the sake of illustration, we give the tree-level expression for A_{FB} in the SM with $q = 1$:

$$A_{FB} = \frac{3}{2} \frac{\chi + a\chi^2}{1 + a\chi + b\chi^2} \quad (2.52)$$

where

$$a = 2(1 - 4s_w)^2 \quad (2.53a)$$

$$b = 2(1 - 4s_w + 8s_w^2) \quad (2.53b)$$

The effect of the finite width of Z^0 can be neglected at the PEP and PETRA energies, so that we can take [6]

$$\chi = \frac{1}{16s_w(1-s_w)} \frac{s}{s - m_{Z^0}^2} \quad (2.54)$$

Note also that, for $s \ll m_{Z^0}^2$, making use of the relation $m_{Z^0}^2 = A_c^2/q \sin^2\theta_w \cos^2\theta_w$ one gets $A_{FB} = -3/32 (s/A_c^2) [1 + O(s/m_{Z^0}^2)]$, which implies a very weak dependence of A_{FB} on $\sin^2\theta_w$, since it appears only at second order in s . Therefore this sector cannot be used to measure $\sin^2\theta_w$ in the SM with $q = 1$ at PEP and PETRA. However, it is useful for testing possible departures from the SM,

such as the possible existence of extra Z' bosons. Equation (2.52) can easily be generalized to the case of an extra Z' .

However, in order to discuss the experimental data, we have to take into account the radiative corrections [91–94]. In the SM at order α^2 we have to include the one-loop electroweak corrections to γ and Z^0 exchange diagrams, the γ and Z^0 self-energy insertions, and the γ – Z^0 mixing diagrams. To order α^3 , we get for the differential cross-section [92],

$$\frac{d\sigma}{d\cos\theta} = \frac{e^2(s)}{32\pi} s \left[S_\ell(s) (1 + \cos^2\theta) + A_\ell(s) \cdot 2\cos\theta \right] \quad (2.55)$$

We have factorized out the running electromagnetic coupling constant $e^2(s)$, which is given by

$$\frac{e^2(s)}{e^2} = \left[1 - \text{Re} \Pi_{\gamma\gamma}(s) \right]^{-1} \quad (2.56)$$

with

$$\text{Re} \Pi_{\gamma\gamma}(s) \simeq \text{Re} \Pi_{\gamma\gamma}(m_Z^2) + \frac{20}{9} \frac{\alpha}{\pi} \ln \left(\frac{s}{m_Z^2} \right) \quad (2.57)$$

The coefficients A_ℓ and S_ℓ , besides the trivial s -dependence from the propagators, have a dependence on s due to the inclusion of the leading radiative corrections.

Equation (2.51) can be applied to models with an extra Z' by including all the other diagrams with internal Z' lines in the evaluation of the quantities A_ℓ and S_ℓ . However, as already pointed out in the Introduction, it seems reasonable to keep only the tree-level contributions of the Z' , neglecting the one-loop radiative corrections involving the Z' itself.

For our purposes, it is important to have a reasonable way of confronting the experimental ($A_{\text{FB}}^{\text{ex}}$) and theoretical ($A_{\text{FB}}^{\text{th}}$) determinations of the asymmetry. It appears convenient to make use of an approximate expression for $A_{\text{FB}}^{\text{th}}(s)$, which consists in keeping only the first two terms of the power expansion in s :

$$A_{\text{FB}}^{\text{th}}(s) \simeq \frac{A}{10^4 \text{ GeV}^2} s + \frac{B}{10^8 \text{ GeV}^4} s^2 \quad (2.58)$$

where s is measured in GeV^2 . The purpose of this approximation is to condense all the information coming from $e^+e^- \rightarrow \mu^+\mu^-$, $\tau^+\tau^-$ asymmetries into the model-independent parameters A and B , which can be used as inputs in the fits to the models.

Experiments measure a quantity $(d\sigma/d\cos\theta)^{\text{raw}}$ which corresponds to certain cuts in $\cos\theta$, in the magnitudes of the $\ell^\pm$ momenta $q_\pm$, and in the acollinearity angle ζ : typical values are $|\cos\theta| < 0.8$, $q_\pm > E_B/2$, $\zeta < 10^\circ$. Correcting the raw quantity for detection efficiency, experiments provide a quantity $(d\sigma/d\cos\theta)^{\text{meas}}$ which describes the processes $e^+e^- \rightarrow \ell^+\ell^-$ and $e^+e^- \rightarrow \ell^+\ell^-\gamma$ within the given cuts. In order to extract from the latter quantity the asymmetry $A_{\text{FB}}^{\text{ex}}$, experimentalists follow two main lines. The more popular of the two consists in subtracting, from $(d\sigma/d\cos\theta)_{\text{meas}}$, the contributions of the ‘reduced QED corrections’ [91, 94], which correspond to photon-exchange bremsstrahlung, one-loop QED corrections to photon exchange, and fermion contributions to the photon self-energy diagrams. The other consists in subtracting the ‘full QED corrections’ [91, 94], coming from the other one-loop diagrams, which include Z^0 -exchange bremsstrahlung and one-loop

QED in Z^0 -exchange diagrams. The values obtained for A_{FB}^{ex} in the different experiments [6, 84, 95–102] are collected in Table 9.

However, such experimental values of A_{FB}^{ex} must be further corrected, before making a comparison with A_{FB}^{th} given by (2.58) according to

$$A_{FB}^{ex} \text{ (fully corrected)} = A_{FB}^{ex} - \delta A_{FB}^{rc} - \delta A_{FB}^{th} \quad (2.59)$$

where δA_{FB}^{rc} includes the residual radiative corrections and δA_{FB}^{th} corrects for the approximation of eq. (2.58). Concerning the term δA_{FB}^{rc} , it has been explicitly computed in Ref. [91] for the cases of the most accurate experimental determinations. Here we can use their results: the corrections are generally small apart from the cases of TASSO and PLUTO, where the experimental data for A_{FB}^{ex} are presented after ‘full QED corrections’. In fact, as noted already by different authors [6, 94], if A_{FB}^{ex} is given after ‘reduced QED corrections’, the remaining corrections tend to cancel for realistic detectors and available energies. When detailed calculations are not to hand (in those cases where A_{FB}^{ex} is given after reduced QED corrections), we will assume $\delta A_{FB}^{rc} = 0.0 \pm 0.5\%$.

The correction term δA_{FB}^{th} is the difference between the full tree-level expression and the approximate form (2.56). We have calculated this term in the SM and also in models with an extra Z' allowing the parameters to vary around the SM values. The correction depends strongly on the value of s , but it is nearly model-independent in a reasonable range of variation of the parameters of the model. The corrected values A_{FB}^{corr} obtained from (2.59) are listed in the last column of Table 9.

Finally, we have fitted the 28 independent pieces of data, obtaining the results:

$$A = -0.56 \pm 0.13 \quad (2.60a)$$

$$B = -1.34 \pm 0.96 \quad (2.60b)$$

with the correlation $\rho = -0.947$.

2.5 Vector boson masses

We report here the most recent measurements of the vector boson masses at the CERN $p\bar{p}$ collider reported by the UA2 [1] and UA1 [2] collaborations. The final analyses of the present UA2 data samples for the processes $p\bar{p} \rightarrow (W \rightarrow e^+ \bar{\nu}) + \text{anything}$ and $p\bar{p} \rightarrow (Z \rightarrow e^+ e^-) + \text{anything}$ yield [1]:

$$m_W = 80.2 \pm 0.6(\text{stat.}) \pm 0.5(\text{syst.}_1) \pm 1.3(\text{syst.}_2) \text{ GeV} \quad (2.61a)$$

$$m_Z = 91.5 \pm 1.2(\text{stat.}) \pm 1.7(\text{syst.}) \text{ GeV} \quad (2.61b)$$

In the measurement (2.61a) of m_W , (syst.₁) is the systematic uncertainty arising from possible biases in the determination of p_T^2 , and (syst.₂) is the systematic uncertainty arising from the overall energy scale uncertainty. This latter uncertainty is also responsible for (syst.) in the measurement (2.61b) of m_Z . In the case of the UA1 experiment, the most recent results [2] from $(W \rightarrow e\nu)$ and $(Z \rightarrow e^+ e^-)$ are

$$m_W = 82.7 \pm 1.0(\text{stat.}) \pm 2.7(\text{syst.}) \text{ GeV} \quad (2.62a)$$

$$m_Z = 93.1 \pm 1.0(\text{stat.}) \pm 3.1(\text{syst.}) \text{ GeV} \quad (2.62b)$$

In the measurements (2.62), the systematic errors (syst.) are essentially due to the overall energy scale uncertainty. Since the $\mu^\pm$ (and *a fortiori* the $\tau^\pm$) decays are less precise, they are not included in the fits.

It is clear that the measurements (2.61) of UA2 are at present somewhat more precise than those of UA1. However, we have chosen to combine the two independent estimates (2.61, 2.62). We weight these results as follows: (i) total errors are used as weights, (ii) systematic errors are added linearly, and (iii) the correlations between systematic errors of the same origin are included, leading to

$$m_W = 80.76 \pm 1.72 \text{ GeV} \quad (2.63a)$$

$$m_Z = 91.59 \pm 2.14 \text{ GeV} \quad (2.63b)$$

with a correlation $\rho = 0.879$. These values are used in the following as further constraints in the analyses of the SM (section 3) and of the superstring-inspired models (section 4).

3. FITS WITHIN THE STANDARD MODEL

We now fit to the SM the 13 model-independent low-energy coefficients which have been extracted from NC data as discussed in the previous section. We have made two types of fits: i) one-parameter fits imposing the minimal model assumption $\rho = 1$, and ii) two-parameter fits in which both $\sin^2\theta_w$ and ρ are left free.

We have compared the relevance of the different sectors to the global fits, by making a separate fit for each sector. Moreover, we have made fits including two more pieces of experimental information, namely m_W and m_Z , and compared these fits with those obtained from the low-energy quantities alone. The results obtained are shown in Tables 10 and 11. It should be remembered from the discussion of subsection 2.4 that the (e-L) sector alone is not sufficient to give a determination of $\sin^2\theta_w$. For this reason, we have not made fits using the (e-L) asymmetry alone, but only fits combining the (e-L) asymmetry data with other sectors, specifically with the (e-q) sector and with the (m_W, m_Z) sector, as shown in the Tables. In these cases, we have checked that the approximate relation for A_{FB} given in eq. (2.58) gives essentially the same fits of the exact expression (2.52).

We see in Tables 10,11 that all the data sets are consistent. The central values of $\sin^2\theta_w$ obtained from the separate sectors in both types of fits (with one and two parameters) are very close to the global fit values, while the errors vary from sector to sector by factors in the range 1/2.5. In particular, the fits of the whole set of 'low energy' (LE) data are perfectly consistent with the fits of the (m_W, m_Z) data from UA1 and UA2. The situation is summarized in Fig. 1, where we compare the values of $\sin^2\theta_w$ obtained in case (a) where ρ is fixed equal to 1, and in Fig. 2, where we show the solutions obtained in case (b) with both ρ and $\sin^2\theta_w$ left free. The results of our fits are:

$$(a) \quad \rho=1 \text{ (fixed)} : \quad \sin^2\theta_w = 0.228 \pm 0.004 \quad (3.1a)$$

$$(b) \quad \begin{cases} \sin^2\theta_w = 0.229 \pm 0.006 \\ \rho = 1.001 \pm 0.007 \end{cases} \quad (3.1b)$$

In both cases the χ^2/DOF is very low. Note that in the fit (a) with $\rho = 1$ the other data do not greatly improve the results obtained from the neutrino-quark sector alone, whereas they do make a significant improvement in fit (b) with variable ρ .

The reader might also find useful the corresponding 90% C.L. ranges of variation:

$$(a) \quad \rho = 1 \text{ (fixed)} : \quad 0.222 < \sin^2 \theta_w < 0.234 \quad (3.2a)$$

$$(b) \quad \begin{cases} 0.220 < \sin^2 \theta_w < 0.238 \\ 0.989 < \rho < 1.012 \end{cases} \quad (3.2b)$$

The reader should be aware when comparing eq. (3.2b) with fig. 2 that the limits given in eq. (3.2b) are given for each of the two parameters independently, irrespective of the value assumed by the other parameter.

The above value of $\sin^2 \theta_w$ for $\rho = 1$ was obtained assuming (arbitrarily) that $m_t = 36$ GeV. It is altered by less than one standard deviation for $m_t < 80$ GeV. We find a 90% confidence level upper limit

$$m_t < 168 \text{ GeV} \quad (3.3)$$

in the minimal SM from a joint fit to the neutrino-quark sector and to the intermediate vector boson masses, or $m_t < 120$ GeV at the $1-\sigma$ level.

We conclude this section by noting that care should be taken when interpreting the χ^2/DOF in Tables 10, 11. They are not the χ^2 coming from direct comparisons with the experiments, but are a measure of the agreement between the model-independent fit and the SM parametrization with or without the constraint $\rho = 1$. The small values of χ^2 describe the impressive agreement with the SM of each individual sector. More surprising is the lack of dispersion exhibited by the different sectors in figs. 1, 2. Such behaviour, which seems anomalous from a statistical point of view, cannot be ascribed to the present analysis, but has its origin elsewhere.

4. BOUNDS ON SUPERSTRING Z' BOSONS

In previous sections we have made a general analysis of the available NC data, made a model-independent fit to them, shown that the result of the fit agrees with the parametrization of the SM, and used it to constrain the parameters of the SM. Clearly, one can also use the result of the model-independent fit to probe and constrain models containing NCs beyond those in the SM. Indeed, this has been the main driving force behind this study. Models of unification use larger gauge groups which often contain additional neutral gauge bosons, one or more of which may be accessible to experiment. This week's fashion in unification is string theory [103]. It is possible that the low-energy four-dimensional field theory obtained from the ultimate variant of string theory contains no additional neutral Z' boson, but there are some interesting suggestions for some possible superstring Z' 's.

One approach to superstring phenomenology starts from the $E_8 \times E_8$ heterotic superstring in $D = 10$ dimensions, and then compactifies six of the dimensions at the Planck scale. In this approach, the compactification space may be either a manifold [104] or an orbifold [105]. In the former case, Calabi-Yau manifolds have been prominent among those considered [104]. They would give a four-dimensional gauge group that is some subgroup of E_6 [106]. Other, related manifolds could give gauge groups that are subgroups of $SO(10)$ or even $SU(5)$ [107], themselves subgroups of E_6 . Orbifold compactifications [105], on the other hand, could in principle give some more general subgroup of E_8 not contained in E_6 . However, it has been argued [108] that even in this case the only phenomenologically interesting additional $U(1)'$ gauge factor has hypercharge assignments for the known matter particles that are identical with the assignments for one of the E_6 subgroups suggested by Calabi-Yau compactification. This result was based on the following phenomenological

requirements [108]: i) chiral fermion representations, ii) a trilinear superpotential including fermion mass and Higgs mixing terms, and iii) perturbative gauge couplings at energies below m_P without invoking an intermediate scale. The unique possible $U(1)'$ was that contained in $SU(6)/SU(5)$. We recall that E_6 contains a maximal subgroup $SU(6) \times SU(2)$, so all these phenomenological scenarios based on the $D = 10$ heterotic superstring lead to some $U(1)'$ subgroup of E_6 .

An alternative approach to string phenomenology is based on string theories formulated directly in $d = 4$ dimensions [109]. A large class of such models based on fermions instead of internal bosonic coordinates gives gauge groups which are subgroups of $O(16)$. Arguments similar to those made in the case of orbifold compactifications lead to the same uniquely preferred $U(1)'$.

In view of the above arguments, we concentrate on the neutral gauge bosons associated with $U(1)$ subgroups of E_6 . Reference [3] contains a general parametrization of their properties which we summarize here. Table 12 contains the hypercharge assignments for the SM $U(1)_Y$ and the two other possible $U(1)$'s in E_6 of the conventional quarks and leptons, and of unseen particles completing a fundamental representation of E_6 . An arbitrary flavour-conserving additional E_6 NC can be characterized by two parameters: θ_1 , which is a mixing angle between the $U(1)_Y$ current and the additional NC $\hat{Y}$; and θ_2 , which fixes $\hat{Y}$ as a combination of Y' and Y'' : $\hat{Y} = \cos\theta_2 Y' - \sin\theta_2 Y''$. In addition, the ratio of the $\hat{Y}$ and Y gauge couplings is an unknown parameter λ in the effective low-energy theory. Finally, the masses of the two neutral gauge bosons m_Z and $m_{Z'}$ are also unknown.

A general exploration of the constraints imposed by the NC analysis of the previous sections on the full five-parameter space $(\theta_1, \theta_2, \lambda, m_Z, m_{Z'})$ would be tedious and not very illuminating. Therefore we have made two simplifications. One is to assume that $\lambda = 1$, which is true if the low-energy ($m \lesssim 1$ TeV) theory contains full multiplets of E_6 , and all the gauge couplings are equal at some grand unification scale [3]. Note, however, that this assumption is not necessarily valid either in models based on manifold compactification, or in orbifold models. After fixing $\lambda = 1$, physical quantities do not depend on θ_1 , so we are reduced to three parameters. Our second simplification is to consider only three discrete values of θ_2 which have appeared in previously proposed models. These are A: $\theta_2 = 0$ as found in models [110] based on E_6 broken by the Hosotani mechanism [111] down to a rank-5 subgroup; B: $\cos\theta_2 = \sqrt{3}/8, \sin\theta_2 = \sqrt{5}/8$ as found in one class of models [112] with E_6 broken to rank 6 by the Hosotani mechanism [111], followed by Higgs breaking to rank 5; and C: $\cos\theta_2 = \sqrt{3}/8, \sin\theta_2 = -\sqrt{5}/8$ as found in another class of such hybrid Hosotani/Higgs models [113].

After these assumptions, we are left with two parameters to characterize each of the models A, B, and C. These can be taken as $(m_Z, m_{Z'})$, the two neutral gauge boson masses, or as $(m_{Z'}, \theta_3)$, where θ_3 is an angle mixing the Z' with the conventional Z^0 . Arbitrary values of $(m_{Z'}, \theta_3)$ are not possible in models based on Calabi-Yau compactification, because of their restricted Higgs content.

We show in figs. 3, 4, and 5 the domains of the $(m_{Z'}, \theta_3)$ plane which are allowed in each model with the two Higgs doublet v.e.v.'s: $v \equiv \langle 0|H|0 \rangle$ and $\bar{v} \equiv \langle 0|\bar{H}|0 \rangle$, and the singlet v.e.v.: $x \equiv \langle 0|H|0 \rangle$. We have used MINUIT to perform 3 fits for each of the models A, B, C above both with and without the assumption that $\varrho = 1$, and, in the case $\varrho = 1$, both with and without the additional Higgs constraints. The results of the fits obtained for the three models are shown separately in Tables 13, 14, and 15. The values obtained for $\sin^2\theta_w$ are very similar in the different fits of each of the three models and are perfectly compatible with those obtained in the SM fits. The values of the mass m_1 of the ordinary Z^0 , even in the case of arbitrary ϱ , agree very well with the SM value. On the other hand, the regions allowed at 90% C.L. in the $(m_{Z'}, \theta_3)$ plane are rather different in the three models. In model A as shown in Table 13, the lower limits on $m_{Z'}$ are rather weak and the $\varrho = 1$ constraint does not appear to tighten these limits. In fact, values as low as 129 GeV are allowed for $m_{Z'}$. The mixing angle is in general small: $|\theta_3| \lesssim 0.1$ rad, but it can reach $0.2 \sim 0.3$ rad for the smallest values of $m_{Z'}$. If one imposes the additional Higgs constraints, $m_{Z'}$ is pushed to higher values. We have considered two forms of the Higgs constraints in this model. One (a) is with the range $0 \leq \bar{v}/v \leq 1$

which is expected in any model with radiative symmetry breaking, if $m_t > m_b$ and there are no heavier quarks, and so is (almost) model-independent. The other (b) is the more restricted range $0.2 \leq \bar{v}/v \leq 0.6$ which was found in model studies assuming $m_t \sim 40$ GeV and that the dominant seed of supersymmetry breaking in the observable sector was a universal gaugino mass. As is shown in Table 13, we find the 90% confidence level lower limits

$$(a) \quad m_{Z'} > 161 \text{ GeV} \quad (0 < \frac{\bar{v}}{v} < 1) \quad (4.1a)$$

$$(b) \quad m_{Z'} > 256 \text{ GeV} \quad (0.2 < \frac{\bar{v}}{v} < 0.6) \quad (4.1b)$$

In model B as shown in Table 14, one gets in general somewhat stronger limits for $m_{Z'}$ and θ_3 . The value of 287 GeV, obtained for the lower limit of m_2 in the case of arbitrary ϱ , is increased to 352 GeV by imposing the condition $\varrho = 1$, which becomes 350 GeV when the additional Higgs constraint is imposed. The mixing angle is generally very small: $|\theta_3| < 0.05$ rad. Finally, in model C as shown in Table 15, neither the condition $\varrho = 1$ nor the additional Higgs constraints appear to modify the lower bound of 180 GeV obtained in the case of arbitrary ϱ . The mixing angle is again rather small in this model: $|\theta_3| < 0.05$. The situation is represented graphically in figs. 3, 4 and 5 for the three models. Both in the tables and in the figures, we present only the results of the global fits of the whole set of data. We have also performed fits to different subsets of the data, but the changes are rather small in going from one sector to another. It should be noted that the $(e-q)$ and (m_w, m_Z) sectors contribute significantly to the constraints in the $(m_{Z'}, \theta_3)$ plane, especially in the case of model A.

Constraints on extra Z 's coming from neutral current data have been analysed previously in several papers [4]. Electron-positron annihilation cross-sections and asymmetries are analysed in particular in ref. [114]. Recently, more indirect constraints have been obtained from electroweak radiative corrections to charged current data [115]. The analysis performed by Durkin and Langacker [4] is the most similar to ours. The models A and B correspond to their models Z_η and Z_χ , respectively. The set of neutral current data used in our analysis includes results from new experiments, in particular in the $\nu-q$ and $\nu-e$ sectors, and our fits give a more accurate determination of the parameters $\sin^2\theta_w$ and m_1 . On the other hand, the limits we obtain on the parameters m_2 and θ_2 are rather similar to theirs in the case of model A, both for arbitrary ϱ and for $\varrho = 1$ without additional Higgs constraints. In the case of model B, we obtain consistently somewhat stronger lower bounds on m_2 . Model C was not considered by Durkin and Langacker [4].

A natural question to ask is how these indirect lower bounds on the masses of possible Z' bosons compare with those obtainable from direct searches at the CERN $p\bar{p}$ collider [3]. Before answering this question, one has to confront the model-dependent issue of possible novel Z and Z' decay modes. If all the spin- $1/2$ and spin-0 particles in 3 generations of E_6 27 representations are kinematically accessible to Z' decay, the branching ratios $B(Z' \rightarrow \ell^+\ell^-)$ may be reduced by factors up to ~ 6 in the models under study. In the case of a Z' much heavier than the Z , a conservative estimate of $\sigma B(Z' \rightarrow \ell^+\ell^-)$ and hence a conservative lower bound on $m_{Z'}$ must allow for the possibility that all these decay modes are indeed available. However, this assumption is clearly too conservative for a Z' close in mass to the Z , whose observed $\sigma B(Z \rightarrow e^+e^-)$ agrees with the Standard Model prediction, and whose natural width has an experimental upper limit close to the value expected in the SM. In the cases of models B and C this is a moot point, since the 90% confidence level lower limits on $m_{Z'}$ obtained from our NC analysis ($m_{Z'} > 287$ GeV and 180 GeV, respectively) are much larger than any possible lower limit from the CERN $p\bar{p}$ collider irrespective of the mass spectra of the models. However, the collider lower limit on $m_{Z'}$ in model A could conceivably be competitive with our low-energy NC limits. If the model A Z' could only decay into conventional

particles (i.e. no Higgses, no sparticles and no extra E_6 states) then we estimate that the CERN $p\bar{p}$ collider could establish a 90% confidence level lower limit $m_{Z'} \gtrsim 150$ GeV. This would be stronger than our NC bound of 129 GeV at the 90% confidence level obtained without applying the Higgs constraints, but is weaker than our bound of 161 GeV at the 90% confidence level obtained with the Higgs constraints. If the Z' in model A could decay into 3 full generations of E_6 particles and sparticles the CERN $p\bar{p}$ collider could not establish any useful limit. This extreme case is, however, impossible if the Z' weighs only slightly more than the Z . However, we estimate that even if one takes into account the experimental constraints on $\sigma_B(Z \rightarrow \ell^+ \ell^-)$ and $\Gamma(Z \rightarrow \text{all})$, the CERN $p\bar{p}$ collider limit would still be much below our NC limits. This situation is likely to change, however, with the advent of the Tevatron collider and ACOL at CERN.

5. SUMMARY, CONCLUSIONS AND PROSPECTS

Motivated by the end of the pre-SLC and pre-LEP era of NC studies, and by the advent of superstring-inspired models that suggest the possible existence of additional NC mediated by an extra Z' boson, we have made in this paper a new analysis of the available NC data. The results of this analysis can be stated in three forms: model-independent determinations of the low-energy NC parameters in section 2, fits within the Standard Model in section 3, and constraints on models containing an additional $U(1)$ in section 4.

As has been mentioned in Section 3 and can be seen in fig. 1, the overall level of consistency between the Standard Model and the available data on neutrino-quark scattering, neutrino-electron scattering, charged lepton-quark interactions, e^+e^- annihilation into leptons, m_W and m_Z is surprisingly high. The low values of χ^2/DOF indicate that the experimental and theoretical errors have not been underestimated. The values of $\sin^2\theta_w$ quoted in equations (3.2) lie in the range suggested by various Grand Unified Theories (GUTs) [117]. They favour non-minimal models such as supersymmetric $SU(5)$, and the minimal non-supersymmetric $SU(5)$ GUT can be almost completely excluded, since $0.222 < \sin^2\theta_w < 0.234$ at the 90% confidence level in the one-parameter fit. The 90% confidence level limits on q are $0.989 < q < 1.012$. A combined fit to the neutrino-quark sector and to m_W , m_Z in the one-parameter SM with $q = 1$ yields $m_t < 168$ GeV at the 90% confidence level.

As for the superstring-inspired models discussed in section 4, we find 90% c.l. lower limits on the extra neutral boson masses in the range 129 to 352 GeV, depending on the model. The existence of such an extra neutral boson is not (yet) a definite prediction of the superstring, but there are reasons to think that if such a boson exists, its mass should be $\lesssim 1$ TeV. Therefore such bounds begin to be significant constraints on model-building.

In the foreseeable future, the principal improvements in low-energy NC measurements are expected to be in neutrino-electron scattering [118] and in atomic physics parity-violation experiments. We expect that the errors in neutrino-electron scattering will be reduced to levels comparable to those now reached in neutrino-hadron scattering. In the long term it may be possible to reduce the errors in atomic physics parity-violation experiments even further. However, we expect that the next large factor of improvement in NC measurements will come from measurements at the Z^0 peak in e^+e^- annihilation [119]. Measurements of the Z^0 mass and of the left-right asymmetry on the peak should be able to reduce the errors on $\sin^2\theta_w$ by about an order of magnitude. They would improve [114, 119] the lower bounds on the mass of an extra neutral boson up to about 0.5 or 1 TeV, if its mixing with the Z^0 is of the form discussed in section 4. Direct searches for a Z' in hadron-hadron collisions at the Tevatron $p\bar{p}$ collider could reach masses of order 350 to 400 GeV, depending on the model [3, 116]. In the more distant future, searches at the SSC or LHC could reach up to several TeV [3, 116, 120], thereby confirming or excluding the suggestion of an extra light neutral boson.

Note: In the final stages of preparing this paper we received ref. [121], which reports similar NC analyses with similar conclusions.

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Table 1

Neutrino-induced deep-inelastic scattering experiments on isoscalar targets.
(The quoted experimental errors include both statistical and systematic errors, added quadratically.)

Experiment	Ref.	Target	$R_\nu = \sigma_{\text{NC}}^\nu / \sigma_{\text{CC}}^\nu$	$R_{\bar{\nu}} = \sigma_{\text{NC}}^{\bar{\nu}} / \sigma_{\text{CC}}^{\bar{\nu}}$
Gargamelle	10	Freon	0.25 ± 0.04	0.56 ± 0.08
CDHS I	11	Iron	0.293 ± 0.010	0.35 ± 0.03
HPWF	12	Propane	0.30 ± 0.04	0.33 ± 0.09
CITF	13	Iron	0.28 ± 0.03	0.35 ± 0.11
CDHS II	14	Iron	0.301 ± 0.007	0.363 ± 0.015
CHARM (1981)	15	Marble	0.320 ± 0.010	0.377 ± 0.020
ABCDLOS	16	H + neon	0.345 ± 0.018	0.364 ± 0.030
BEBC D	17	Deuterium	0.33 ± 0.03	0.35 ± 0.05
BM-ITEP	18	Neon + H	0.32 ± 0.03	0.406 ± 0.028
FMM	19	Steel + sand	0.307 ± 0.011	0.384 ± 0.018
CDHSW	20	Iron	0.3072 ± 0.0032	0.382 ± 0.016
CCCFRR	21	Iron	0.292 ± 0.009	0.388 ± 0.020
CHARM (1986)	22	Marble	0.3093 ± 0.0031	-

Table 2

Neutrino-induced deep-inelastic scattering experiments on neutron and proton targets.
(The quoted experimental errors include both statistical and systematic errors, added quadratically.)

Experiment	Ref.	Target	ν data	$\bar{\nu}$ data
BEBC TST	23	Hydrogen	$R_p^p = 0.47 \pm 0.04$	$R_p^p = 0.33 \pm 0.04$
BM-ITEP	18	Neon + H	$R_{n/p}^{n/p} = 1.08 \pm 0.19$	$R_{n/p}^{n/p} = 0.88 \pm 0.17$
SIMTT	24	Deuterium	$R_{n/p}^{n/p} = 1.01 \pm 0.14$	-
FNAL	25	Hydrogen	$R_p^p = 0.40 \pm 0.14$	-
ABCMO	26	Hydrogen	$R_p^p = 0.51 \pm 0.04$	-
SIMTT	24	Deuterium	$R_p^p = 0.49 \pm 0.06$	-
FNAL	27	Hydrogen	-	$R_p^p = 0.35 \pm 0.05$
BEBC D	17	Deuterium	$r_p = 0.06 \pm 0.06$	$r_p = 0.02 \pm 0.09$
BBCLMO	28	Hydrogen	$R_p^p = 0.384 \pm 0.028$	$R_p^p = 0.338 \pm 0.021$

Table 3 $(\bar{\nu}_\mu)e$ scattering experiments

Experiment	Ref.	No. of ν events	σ_ν/E_ν (10^{-42} cm ² /GeV)	No. of $\bar{\nu}$ events	$\sigma_{\bar{\nu}}/E_{\bar{\nu}}$ (10^{-42} cm ² /GeV)
Gargamelle (PS, SPS)	48	9	$2.4^{+1.2}_{-0.9}$	3	$1.0^{+1.3}_{-0.6}$
AP	49	11.5	1.1 ± 0.6	9.6	2.2 ± 1.0
FNAL	50	20.5	1.6 ± 0.4	–	–
VPI	51	34	1.4 ± 0.4	–	–
CHARM	52	83	$1.8 \pm 0.3 \pm 0.4$	112	$1.5 \pm 0.3 \pm 0.3$
BBKOPST (E734)	53	107	$1.85 \pm 0.25 \pm 0.27$	59	$1.16 \pm 0.20 \pm 0.14$

Table 4 $(\bar{\nu}_e)e$ scattering experiments

Experiment	Ref.	σ_ν/E_ν (10^{-42} cm ² /GeV)	$\sigma_{\bar{\nu}}/E_{\bar{\nu}}$ (10^{-42} cm ² /GeV)	
UCI	54	–	$1.5 \leq E_e \leq 3.0$ MeV	0.76 ± 0.22
			$3.0 \leq E_e \leq 4.5$ MeV	0.186 ± 0.048
LAMPF (E225)	55	$9.8 \pm 2.7 \pm 1.6$	–	–

Table 5

Experimental values and theoretical estimates of $\text{Im } E_1^{\text{PV}}/M_1$

Experiment	Ref.	Element (line)	$10^8 \text{ Im } E_1^{\text{PV}}/M_1$	
			Exp. value	Th. estimate/ Q_w
Novosibirsk	58	Bi(648)	$-20.2 \pm 2.7^{\text{a)}}$	} 0.1093 ± 0.0336
Oxford	59	Bi(648)	$-9.3 \pm 1.5^{\text{b)}}$	
Moscow	60	Bi(648)	-7.8 ± 1.8	
Seattle (1981)	61	Bi(876)	$-10.4 \pm 1.7^{\text{b)}}$	
Seattle (1983)	62	Pb(1280)	$-9.9 \pm 2.5^{\text{b)}}$	
			$10^3 \text{ Im } E_1^{\text{PV}}/M_1$	
Berkeley (1981)	63	Tl	$-1.40 \pm 0.37^{\text{d)}}$	$(0.829 \pm 0.118) \times 10^{-2}$

a) Quoted error is statistical only.

b) Quoted error is predominantly systematic.

c) Quoted errors include systematic and statistical ones.

d) Statistical and systematic errors are added in quadrature.

Table 6

Experimental values and theoretical estimates of $\text{Im } E_1^{\text{PV}}/\beta$

(The quoted experimental errors include both statistical and systematic errors, added in quadrature.)

Experiment	Ref.	Element	$\text{Im } E_1^{\text{PV}}/\beta \text{ (mV/cm)}$	
			Exp. value	Th. estimate/ Q_w
Berkeley (1981)	69	Tl	-1.80 ± 0.48	} 1.436 ± 0.063
Berkeley (1984)	70	Tl	-1.73 ± 0.27	
Paris (1982)	64	Cs	-1.33 ± 0.25	} 2.24 ± 0.012
Paris (1984)	65,66	Cs	-1.75 ± 0.27	
Boulder (1985)	67	Cs	-1.65 ± 0.13	

Table 7

Coefficients C_{1u} and C_{1d} from parity violation in atoms

Experiment	$\frac{Z}{A}$ Element	$\zeta = \frac{(2A - Z)}{(A + Z)}$	Q_w	$C_{1u}^{(0)} + \zeta C_{1d}^{(0)}$	$C_{1u} + \zeta C_{1d}$
Novosibirsk	$^{83}_{209}\text{Bi}$	1.147	-185 ± 62	0.317 ± 0.106	0.323 ± 0.108
Oxford	$^{83}_{209}\text{Bi}$	1.147	-85 ± 29	0.146 ± 0.050	0.149 ± 0.051
Moscow	$^{83}_{209}\text{Bi}$	1.147	-71 ± 27	0.122 ± 0.046	0.124 ± 0.047
Seattle (1981)	$^{83}_{209}\text{Bi}$	1.147	-115 ± 37	0.197 ± 0.063	0.201 ± 0.064
Seattle (1983)	$^{82}_{207}\text{Pb}$	1.148	-76 ± 22	0.131 ± 0.038	0.134 ± 0.039
Berkeley (1981)	$^{81}_{204}\text{Tl}$	1.147	-169 ± 51	0.269 ± 0.089	0.274 ± 0.091
Berkeley (1981)	$^{81}_{204}\text{Tl}$	1.147	-125 ± 34	0.219 ± 0.060	0.223 ± 0.061
Berkeley (1984)	$^{81}_{204}\text{Tl}$	1.147	-120 ± 16	0.211 ± 0.028	0.215 ± 0.029
Paris (1982)	$^{55}_{133}\text{Cs}$	1.122	-59.3 ± 11.5	0.158 ± 0.031	0.161 ± 0.032
Paris (1984)	$^{55}_{133}\text{Cs}$	1.122	-78.2 ± 12.7	0.208 ± 0.034	0.212 ± 0.035
Boulder (1985)	$^{55}_{133}\text{Cs}$	1.122	-73.7 ± 7.0	0.196 ± 0.019	0.200 ± 0.019

Table 8

Experimental data on A_{FB} in $e^+e^- \rightarrow c\bar{c}$ and $e^+e^- \rightarrow b\bar{b}$

a) $e^+e^- \rightarrow c\bar{c}$				
Experiment	Ref.	$\sqrt{s}$ (GeV)	$\int L dt$ (pb^{-1})	A_{FB} (%)
HRS	82	29	255	-14 ± 5
MAC	82, 84	29	(a)	-14 ± 32
TPC	85	29	77	$-17 \pm 9 \pm 4$
JADE	86	34.4	74.7	-14 ± 9
MARK J	87	35.3	76	-16 ± 9
PLUTO	82, 84	34	(a)	-10 ± 10
TASSO	88	34.6	72.1	$+5 \pm 24$
TASSO	88	34.4	(a)	-28 ± 13
b) $e^+e^- \rightarrow b\bar{b}$				
MAC	82, 84	29	≈ 30	-11 ± 12
MARK II	82, 84	29	≈ 30	-27 ± 12
TPC	85	29	77	$-20 \pm 16 \pm 5$
CELLO	89	34	7.9	-40 ± 17
JADE	86	34.6	76	$-22.8 \pm 6.0 \pm 2.5$
MARK J	87	35.3	76	-21 ± 19
PLUTO	82	34	44	-36 ± 25
TASSO	88	34.6	72.1	-30 ± 18

(a) For details of the luminosity, see the original publication.

Table 9

Experimental data on A_{FB} in $e^+e^- \rightarrow \mu^+\mu^-$ and $e^+e^- \rightarrow \tau^+\tau^-$.
 (Note: Either the statistical and systematic errors are shown separately,
 or the combined error is given after they are added in quadrature.)

a) $e^+e^- \rightarrow \mu^+\mu^-$					
Experiment	Ref.	$\sqrt{s}$ (GeV)	$\int L dt$ (pb^{-1})	A_{FB}^{exp} (%)	A_{FB}^{corr} (%)
HRS	95	29	106	$-4.9 \pm 1.5 \pm 0.5$	-4.9 ± 1.6
MAC	96	29	222	$-5.9 \pm 0.8 \pm 0.2$	-5.9 ± 1.0
MARK II	97	29	100	-7.1 ± 1.7	-7.5 ± 1.7
CELLO	98	34.2	11.2	-6.4 ± 6.4	-6.3 ± 6.4
JADE	99	34.4	71.2	$-11.1 \pm 1.8 \pm 1.0$	-10.7 ± 5.2
MARK J	100	34.6	76.3	$-11.7 \pm 1.7 \pm 0.5$	-11.6 ± 1.8
PLUTO	101	34.7	44.0	$-13.2 \pm 2.8 \pm 1.0$	-12.1 ± 3.0
TASSO	102	34.5	74.7	$-9.8 \pm 2.3 \pm 0.5$	-8.6 ± 2.4
CELLO	98	39	(a)	$-4.8 \pm 6.5 \pm 1.0$	-4.5 ± 6.6
JADE	6	38.0	11.9	-9.7 ± 5.2	-9.4 ± 5.2
MARK J	6	39.1	16.5	-10.0 ± 4.1	-9.7 ± 4.1
TASSO	84	38.3	(a)	$+2.4 \pm 8.6 \pm 0.5$	$+3.7 \pm 8.6$
CELLO	98	44	34.4	$-18.8 \pm 4.5 \pm 1.0$	-18.1 ± 4.6
JADE	6	43.7	43.1	-19.1 ± 3.1	-18.5 ± 3.1
MARK J	6	44.1	43.0	-16.0 ± 3.0	-15.3 ± 3.0
TASSO	84	43.6	(a)	$-17.3 \pm 4.4 \pm 0.5$	-15.7 ± 4.5
b) $e^+e^- \rightarrow \tau^+\tau^-$					
HRS	95	29	256	$-4.4 \pm 1.4 \pm 0.5$	-4.4 ± 1.6
MAC	96	29	210	$-5.5 \pm 1.2 \pm 0.5$	-5.5 ± 1.4
MARK II	97	29	100	-4.2 ± 2.0	-4.2 ± 2.1
CELLO	98	34.2	11.3	-10.3 ± 5.2	-10.2 ± 5.2
JADE	99	34.6	62.4	$-6.0 \pm 2.5 \pm 1.0$	-5.8 ± 2.7
MARK J	100	35.0	(a)	$-8.6 \pm 3.7 \pm 1.5$	-8.4 ± 4.0
PLUTO	101, 106	34.6	42.3	$-5.9 \pm 6.8^{+0}_{-2.5}$	$-4.7^{+6.8}_{-7.3}$
TASSO	102	34.5	69.4	$-4.9 \pm 5.3^{+1.3}_{-1.2}$	-3.8 ± 5.5
JADE	6	38.0	11.8	$+7.5 \pm 6.3 \pm 1.0$	$+7.8 \pm 6.4$
CELLO	6	43.9	21.0	$-14.2 \pm 4.8 \pm 1.0$	-13.6 ± 4.9
JADE	6	43.7	43.1	$-17.0 \pm 3.6 \pm 1.0$	-16.4 ± 3.8
MARK J	100	43.9	(a)	$-12.8 \pm 7.0 \pm 1.5$	-12.2 ± 7.2

(a) For details of the luminosity, see the original publication.

Table 10

Case (a): One-parameter fits within the Standard Model

	(ν -q)	(ν -e)	(e-q)	(e-q) + (e-L)
$\sin^2 \theta_w$	$0.2283^{+0.0048}_{-0.0048}$	$0.2271^{+0.0143}_{-0.0143}$	$0.2272^{+0.0112}_{-0.0112}$	$0.2277^{+0.0111}_{-0.0111}$
m_{Z^0}	$92.15^{+0.68}_{-0.67}$	$92.33^{+2.18}_{-1.94}$	$92.31^{+1.67}_{-1.53}$	$92.24^{+1.65}_{-1.51}$
χ^2	0.47/3	$0.6 \times 10^{-3}/1$	1.48/4	1.99/6
	Low-energy	(m_w, m_z)	(e-L) + (m_w, m_z)	Everything
$\sin^2 \theta_w$	$0.2281^{+0.0042}_{-0.0042}$	$0.2279^{+0.0089}_{-0.0084}$	$0.2282^{+0.0089}_{-0.0084}$	$0.2281^{+0.0038}_{-0.0038}$
m_{Z^0}	$92.18^{+0.59}_{-0.58}$	$92.21^{+1.24}_{-1.22}$	$92.17^{+1.24}_{-1.22}$	$92.18^{+0.53}_{-0.52}$
χ^2	2.54/12	0.13/1	0.64/3	2.65/14

Note that the qualities of the fits in this and the subsequent Tables, as measured by the χ^2/DOF values, are always very good. This may indicate that the errors have been treated very conservatively.

Table 11

Case (b): Two-parameter fits within the Standard Model

	$(\nu\text{-}q)$	$(\nu\text{-}e)$	$(e\text{-}q)$	$(e\text{-}q) + (e\text{-}L)$
$\sin^2 \theta_w$	$0.2310^{+0.0113}_{-0.0116}$	$0.2272^{+0.0146}_{-0.0150}$	$0.2278^{+0.0116}_{-0.0116}$	$0.2280^{+0.0114}_{-0.0113}$
q	$1.0031^{+0.0121}_{-0.0121}$	$1.0014^{+0.0557}_{-0.0557}$	$0.9695^{+0.0463}_{-0.0452}$	$0.9917^{+0.0346}_{-0.0347}$
m_{Z^0}	$91.64^{+2.22}_{-2.00}$	$92.25^{+3.98}_{-3.47}$	$93.66^{+2.81}_{-2.55}$	$92.59^{+2.24}_{-2.07}$
χ^2	0.47/2	0	1.05/3	1.93/5
	Low-energy	$(m_W + m_Z)$	$(e\text{-}L) + (m_W + m_Z)$	Everything
$\sin^2 \theta_w$	$0.2284^{+0.0068}_{-0.0069}$	$0.2294^{+0.0101}_{-0.0095}$	$0.2299^{+0.0098}_{-0.0092}$	$0.2287^{+0.0056}_{-0.0056}$
q	$1.0004^{+0.0080}_{-0.0080}$	$1.0090^{+0.0258}_{-0.0244}$	$1.0110^{+0.0228}_{-0.0220}$	$1.0010^{+0.0071}_{-0.0071}$
m_{Z^0}	$92.12^{+1.32}_{-1.24}$	$91.59^{+2.14}_{-2.14}$	$91.43^{+1.97}_{-1.94}$	$92.06^{+1.07}_{-1.03}$
χ^2	2.53/11	0	0.39/2	2.65/13

Table 12

Hypercharge (Y, Y', Y'') assignments
for the fundamental representation **27**

	$\sqrt{5}/3 Y$	$\sqrt{5}/3 Y'$	Y''
$\begin{pmatrix} u \\ d_1 \end{pmatrix}_L$	1/6	1/3	0
d_{2L}	-1/3	-2/3	0
u_L^c	-2/3	1/3	0
d_{1L}^c	1/3	-1/6	1/2
d_{2L}^c	1/3	-1/6	-1/2
$\begin{pmatrix} \nu_1 \\ e_1 \end{pmatrix}_L$	-1/2	-1/6	1/2
$\begin{pmatrix} \nu_2 \\ e_2 \end{pmatrix}_L$	-1/2	-1/6	-1/2
$\begin{pmatrix} e_2^c \\ \nu_3 \end{pmatrix}_L$	1/2	-2/3	0
e_{1L}^c	1	1/3	0
ν_{4L}	0	5/6	-1/2
ν_{5L}	0	5/6	1/2

Table 13

Fits for Model A

	Arbitrary ϱ	$\varrho = 1$	Higgs constraints	
			(a)	(b)
$\sin^2 \theta_w$	$0.2299^{+0.0059}_{-0.0056}$	$0.2313^{+0.0057}_{-0.0057}$	$0.2313^{+0.0056}_{-0.0057}$	$0.2311^{+0.0058}_{-0.0058}$
θ_3 (rad)	$-0.038^{+0.032}_{-0.076}$	$-0.038^{+0.065}_{-0.090}$	$-0.040^{+0.047}_{-0.056}$	$-0.019^{+0.025}_{-0.027}$
m_Z (GeV)	$91.50^{+1.16}_{-1.38}$	$91.22^{+1.33}_{-1.42}$	$91.21^{+1.35}_{-1.40}$	$91.40^{+1.25}_{-1.24}$
$m_{Z'}$ (GeV)	159 (1σ)	151 (1σ)	183 (1σ)	291 (1σ)
lower bound	132 (90% c.l.)	129 (90% c.l.)	161 (90% c.l.)	256 (90% c.l.)
θ_3 (rad) (90% c.l.)	$-0.183 \leq \theta_3 \leq 0.012$	$-0.204 \leq \theta_3 \leq 0.031$	$-0.137 \leq \theta_3 \leq 0.034$	$-0.064 \leq \theta_3 \leq 0.022$
χ^2/DOF	1.20/11	1.91/12	1.88/13	2.04/13

Table 14

Fits for Model B

	Arbitrary ϱ	$\varrho = 1$	Higgs constraints
$\sin^2 \theta_w$	$0.2301^{+0.0061}_{-0.0058}$	$0.2305^{+0.0057}_{-0.0055}$	$0.2273^{+0.0038}_{-0.0038}$
θ_3 (rad)	$-0.020^{+0.022}_{-0.020}$	$-0.018^{+0.017}_{-0.018}$	$-0.008^{+0.012}_{-0.013}$
m_Z (GeV)	$91.27^{+1.29}_{-1.29}$	$91.22^{+1.25}_{-1.26}$	$92.15^{+0.55}_{-0.54}$
$m_{Z'}$ (GeV)	349 (1σ)	407 (1σ)	410 (1σ)
lower bound	287 (90% c.l.)	352 (90% c.l.)	350 (90% c.l.)
θ_3 (rad) (90% c.l.)	$-0.052 \leq \theta_3 \leq 0.015$	$-0.047 \leq \theta_3 \leq 0.011$	$-0.029 \leq \theta_3 \leq 0.011$
χ^2/DOF	1.43/11	1.46/12	2.12/13

Table 15

Fits for Model C

	Arbitrary g	$g = 1$	Higgs constraints
$\sin^2 \theta_w$	$0.2289^{+0.0056}_{-0.0056}$	$0.2285^{+0.0054}_{-0.0050}$	$0.2286^{+0.0039}_{-0.0038}$
θ_3 (rad)	$0.000^{+0.029}_{-0.031}$	$-0.005^{+0.033}_{-0.015}$	$-0.005^{+0.008}_{-0.012}$
m_Z (GeV)	$91.95^{+1.15}_{-1.11}$	$92.11^{+0.71}_{-0.84}$	$92.11^{+0.58}_{-0.56}$
$m_{Z'}$ (GeV)	209 (1σ)	209 (1σ)	210 (1σ)
lower bound	180 (90% c.l.)	180 (90% c.l.)	181 (90% c.l.)
θ_3 (rad) (90% c.l.)	$-0.052 \leq \theta_3 \leq 0.046$	$-0.029 \leq \theta_3 \leq 0.046$	$-0.017 \leq \theta_3 \leq 0.004$
χ^2/DOF	1.77/11	1.80/12	1.80/12

APPENDIX A

PARAMETRIZATION OF THE THEORETICAL COEFFICIENTS

In this Appendix we give the expressions of the coefficients which appear in the effective Lagrangian (2.1) and (2.2) in terms of the parameters which specify the model. Following the notation of Ref. [3], we start from the neutral-current Lagrangian,

$$\mathcal{L}_{NC} = \sum_{\alpha\mu} \bar{\psi}_\alpha Z_\mu^\alpha = e \sum_{\substack{\alpha=1 \\ R=u,d,e,\dots}}^3 \bar{\psi}^R \gamma_\mu (v_\alpha^R + a_\alpha^R \gamma_5) \psi^R Z_\mu^\alpha \quad (A1)$$

where e is the electromagnetic gauge coupling constant and the Z_α^μ are the gauge boson mass eigenstates: the photon $Z_1 \equiv \gamma$, and $Z_2 \equiv Z$ and $Z_3 \equiv Z'$ with masses $M_2 \equiv m_Z$ and $M_3 \equiv m_{Z'}$. In the case of the Standard Model, $M_2 = m_{Z0}$. A comparison between (A1) and (2.1), (2.2) gives

$$\begin{aligned} (u_{L,R}, d_{L,R}, g_{L,R}) &= 2 \sum_{i=2}^3 \left(\frac{A_c}{M_i} \right)^2 (v_i^{(v)} - a_i^{(v)}) (v_i^{(u,d,e)} - a_i^{(u,d,e)}) \\ (h_{AA}^u, h_{AA}^d) &= -8 \sum_{i=2}^3 \left(\frac{A_c}{M_i} \right)^2 (a_i^{(e)} a_i^{(u,d)}) \\ (C_{2u}, C_{2d}) &= -8 \sum_{i=2}^3 \left(\frac{A_c}{M_i} \right)^2 (v_i^{(e)} a_i^{(u,d)}) \end{aligned} \quad (A2)$$

where A_c is the constant defined in (2.3). Concerning the processes $e^+e^- \rightarrow \mu^+\mu^-$, $\tau^+\tau^-$, we are going beyond the effective Lagrangian approximation, and the coefficients A and B , which give a model-independent fit of the asymmetry A_{FB} [Eq. (2.58)], have to be evaluated directly starting from (A1). One gets explicitly

$$\begin{aligned} \frac{A}{10^4 \text{GeV}^2} &= -\frac{3}{2} \left\{ \left[\frac{a_2^{(l)}}{M_2} \right]^2 + \left[\frac{a_3^{(l)}}{M_3} \right]^2 \right\} \\ \frac{B}{10^8 \text{GeV}^4} &= -\frac{3}{2} \left\{ \frac{(a_2^{(l)})^2}{M_2^4} + \frac{(a_3^{(l)})^2}{M_3^4} - \frac{(v_2^{(l)} a_3^{(l)} + v_3^{(l)} a_2^{(l)})^2}{M_2^2 M_3^2} - 2 \left[(a_2^{(l)} v_3^{(l)})^2 + (a_3^{(l)} v_2^{(l)})^2 \right] \right\} \end{aligned} \quad (A3)$$

where M_2 and M_3 are expressed in GeV. In the case of the SM, the r.h.s. of (A2) and (A3) contain only the terms $i = 2$, which correspond to the Z^0 . In this case, the coefficients $v^k \equiv v_2^k$ and $a^k \equiv a_2^k$ are shown in Table A1. They depend only on the Weinberg angle θ_w . If one relaxes the SM assumption $\varrho = 1$ [$\varrho = A_c^2/m_{Z0}^2 s_w(1-s_w)$], the complete expressions (A2) and (A3) depend on the two parameters θ_w and m_{Z0} ; one can take, equivalently, θ_w and ϱ .

In the models we have considered beyond the SM, the coefficients v^k and a^k depend only on the parameters θ_w and θ_3 (mixing angle), besides the given value of θ_2 which is fixed for a specific model. Their expressions are given explicitly in Table A2. Then, the complete expressions (A2) and (A3) depend on the four-independent parameters m_Z , $m_{Z'}$, θ_3 , and θ_w , which reduce to only three, e.g. $m_{Z'}$, θ_3 , and θ_w , with the assumption $\varrho = 1$.

Table A1

Vector and axial-vector couplings for the Standard Model

$$v^u = \left(\frac{1}{4} - \frac{2}{3} \sin^2 \theta_w \right) \frac{1}{\sin \theta_w \cos \theta_w}$$

$$v^d = \left(-\frac{1}{4} + \frac{1}{3} \sin^2 \theta_w \right) \frac{1}{\sin \theta_w \cos \theta_w}$$

$$v^e = \left(-\frac{1}{4} + \sin^2 \theta_w \right) \frac{1}{\sin \theta_w \cos \theta_w}$$

$$v^\nu = -a^\nu = \frac{1}{4} \frac{1}{\sin \theta_w \cos \theta_w}$$

$$a^u = -\frac{1}{4} \frac{1}{\sin \theta_w \cos \theta_w}$$

$$a^d = a^e = \frac{1}{4} \frac{1}{\sin \theta_w \cos \theta_w}$$

Table A2

Vector and axial-vector couplings for models beyond the SM

$$V_2^u = \frac{1}{\sin\theta_w \cos\theta_w} \left[C_3 \left(\frac{1}{4} - \frac{2}{3} \sin^2\theta_w \right) \right]$$

$$V_3^u = \frac{1}{\sin\theta_w \cos\theta_w} \left[S_3 \left(\frac{1}{4} - \frac{2}{3} \sin^2\theta_w \right) \right]$$

$$V_2^d = \frac{1}{\sin\theta_w \cos\theta_w} \left[C_3 \left(-\frac{1}{4} + \frac{1}{3} \sin^2\theta_w \right) + \frac{1}{4} \sin\theta_w S_3 (C_2 + \sqrt{\frac{5}{3}} S_2) \right]$$

$$V_3^d = \frac{1}{\sin\theta_w \cos\theta_w} \left[S_3 \left(-\frac{1}{4} + \frac{1}{3} \sin^2\theta_w \right) - \frac{1}{4} \sin\theta_w C_3 (C_2 + \sqrt{\frac{5}{3}} S_2) \right]$$

$$a_2^u = \frac{1}{\sin\theta_w \cos\theta_w} \left[-\frac{1}{4} C_3 - \frac{1}{3} \sin\theta_w S_3 C_2 \right]$$

$$a_3^u = \frac{1}{\sin\theta_w \cos\theta_w} \left[-\frac{1}{4} S_3 + \frac{1}{3} \sin\theta_w C_3 C_2 \right]$$

$$a_2^d = \frac{1}{\sin\theta_w \cos\theta_w} \left[\frac{1}{4} C_3 + \frac{1}{4} \sin\theta_w S_3 \left(-\frac{1}{3} C_2 + \sqrt{\frac{5}{3}} S_2 \right) \right]$$

$$a_3^d = \frac{1}{\sin\theta_w \cos\theta_w} \left[\frac{1}{4} S_3 - \frac{1}{4} \sin\theta_w C_3 \left(-\frac{1}{3} C_2 + \sqrt{\frac{5}{3}} S_2 \right) \right]$$

$$V_\alpha^e = -2V_\alpha^u - V_\alpha^d \quad (\alpha = 2, 3)$$

$$a_\alpha^e = a_\alpha^d \quad (\alpha = 2, 3)$$

$$V_\alpha^{\nu} = -a_\alpha^{\nu} = -V_\alpha^d - \frac{1}{2} V_\alpha^u - \frac{1}{2} a_\alpha^u \quad (\alpha = 2, 3)$$

$$(C_i \equiv \cos\theta_i, S_i \equiv \sin\theta_i, i=2,3)$$

PARAMETERS OF MODELS BEYOND THE SM

The superstring-inspired models considered in the text belong to three classes (A, B, C) characterized by fixed values of θ_2 (when $\lambda = 1$, the parameter θ_1 becomes irrelevant for the physical quantities, as described in Section 4).

If one takes into account the constraints coming from the restricted Higgs content of the models, one obtains bounds for the domain of the parameters $m_{Z'}$ and θ_3 . These constraints can be specified in terms of $\xi \equiv \bar{v}^2/(v^2 + \bar{v}^2)$, where v and $\bar{v}$ are the v.e.v.'s of $H \equiv \nu_{2L}$ and $\bar{H} \equiv \nu_{3L}$ generally favoured by radiative electroweak symmetry breaking for each of the three models, and corresponding to $0 < |\bar{v}/v| \leq 1$.

In order to specify the bounds in the $(m_{Z'}, \theta_3)$ plane, we recall the usual mass matrix for the two gauge bosons Z and Z' :

$$\begin{pmatrix} m_Z^2 & 0 \\ 0 & m_{Z'}^2 \end{pmatrix} = \begin{pmatrix} c_3 & -s_3 \\ s_3 & c_3 \end{pmatrix} \begin{pmatrix} 1 & b \\ b & a \end{pmatrix} \begin{pmatrix} c_3 & s_3 \\ -s_3 & c_3 \end{pmatrix} m_{Z^0}^2 \quad (B1)$$

where [3]:

$$a = (m_Z^2 + m_{Z'}^2)/m_{Z^0}^2 - 1 \quad (B2)$$

$$b = \frac{m_{Z'}^2 - m_Z^2}{m_{Z^0}^2} c_3 s_3 = -\left[\frac{1}{3}c_2(4-5\sqrt{3}) + \sqrt{\frac{5}{3}}s_2\right] \sin\theta_w \quad (B3)$$

The situation is summarized in Table B1. The Higgs sector contains only $SU(2)_L$ doublets and singlets, so that $q = 1$. Then one can use the relation

$$m_Z^2 c_3^2 + m_{Z'}^2 s_3^2 = m_{Z^0}^2 \quad (B4)$$

to eliminate m_Z from (B3), getting

$$m_{Z'}^2/m_{Z^0}^2 = 1 + b \sin\theta_w \cot\theta_3 \quad (B5)$$

Table B1

Values of the parameters for the models A, B, C

Model	(c_2, s_2)	ξ	b
A	(1, 0)	[0, 0.5]	[− 1.33, − 0.90]
B	$(\sqrt{3}/8, \sqrt{5}/8)$	–	– 0.82
C	$(\sqrt{3}/8, -\sqrt{5}/8)$	[0, 0.5]	[− 0.82, 0.20]

Figure captions

- Fig. 1** Values of $\sin^2\theta_w$ obtained in one-parameter fits within the Standard Model, with fixed $q = 1$. The central values and $1-\sigma$ error bars for different combinations of the data are shown. Note the high degree of consistency of the different fits.
- Fig. 2** Values of $(q, \sin^2\theta_w)$ obtained in two-parameter fits within the Standard Model. The ellipses are 90% confidence-level contours. Note again the high degree of consistency of the different fits.
- Fig. 3** Fits to Model A, both with and without the Higgs constraints motivated by the Calabi–Yau models described in the text.
- Fig. 4** Fits to Model B (as for Fig. 3).
- Fig. 5** Fits to Model C (as for Fig. 3).

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5