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COUPLING THE ANTISYMMETRIC TENSOR TO THE SUPERGRAVITY-MATTER SYSTEM

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A B S T R A C T

The description of the antisymmetric tensor gauge field with Chern-Simons forms in Kähler superspace is used to derive a particular coupling of the antisymmetric tensor to the general supergravity-matter system in terms of superfields as well as component fields. The construction is performed directly in terms of the linear multiplet. The proper duality transformations are presented at the full superfield level. General couplings are shortly discussed.

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1. The coupling of the four-dimensional $N=1$ supersymmetric non-linear model¹⁾ to supergravity and to supersymmetric Yang-Mills theory leads to a general superfield action which contains the matter fields only through two arbitrary functions^{2,3)}. The matter fields are described in terms of chiral and antichiral superfields ϕ and $\bar{\phi}$, respectively,

$$\bar{D}^{\dot{\alpha}} \phi = 0, \quad D_{\alpha} \bar{\phi} = 0. \quad (1)$$

The general superfield action⁴⁾ consists of three pieces,

$$\mathcal{L} = -3 \int E + \mathcal{L}_{\text{pot}} + \mathcal{L}_{\text{YM}}. \quad (2)$$

The first term contains at the same time the usual supergravity action and the gauge invariant kinetic term for the matter fields. The second term is the superpotential,

$$\mathcal{L}_{\text{pot}} = \frac{1}{2} \int \frac{E}{R} e^{\mathcal{G}/2} + \text{h.c.} \quad (3)$$

In the first two terms the matter superfields appear only through the particular combination

$$\mathcal{G}(\phi, \bar{\phi}) = K(\phi, \bar{\phi}) + \log W(\phi) + \log \bar{W}(\bar{\phi}) \quad (4)$$

of the Kähler potential $K(\phi, \bar{\phi})$ and the superpotential superfields $W(\phi)$ and $\bar{W}(\bar{\phi})$. The Yang-Mills piece,

$$\mathcal{L}_{\text{YM}} = \frac{1}{8g^2} \int \frac{E}{R} f_{(ab)}(\phi) W^{\alpha(a)} W_{\alpha}^{(b)} + \text{h.c.} \quad (5)$$

contains the chiral matter superfields also through the superfields $f_{(ab)}(\phi)$ and their complex conjugates.

The crucial point in this construction is the proper generalization of Kähler transformations¹⁾ to curved (flexible) superspace⁴⁾: Kähler transformations appear as field dependent chiral rotations in $U(1)$ superspace and the Kähler potential is identified as the corresponding superspace prepotential. This allows one to obtain directly the superfield Kähler transformations of any superfield as well as the Kähler covariant derivative³⁾.

The expansion of the complete action in terms of component fields is then obtained directly without performing any rescaling nor using the Wess-Zumino gauge for the Yang-Mills prepotential superfield. This simplification is due to the geometrical construction in superspace and to the use of Yang-Mills covariant

derivatives, both in the chirality conditions and in the definition of component fields. Hence the coupling of chiral matter superfields to $N=1$ four-dimensional supergravity and Yang-Mills theory is quite well understood.

On the other hand, compactifications⁵⁾ of ten-dimensional supergravity⁶⁾ lead to four-dimensional theories which contain an antisymmetric tensor gauge field⁷⁾ in relation with Yang-Mills and Lorentz Chern-Simons forms. In four-dimensional $N=1$ supersymmetry an antisymmetric tensor gauge field appears in the linear multiplet⁸⁾, together with a real scalar field, a Majorana spinor and no auxiliary fields. Antisymmetric tensor gauge fields can be used to compensate for the change under gauge transformations of Chern-Simons forms and to construct gauge invariant objects^{9,10)}.

A linear multiplet can be related to a chiral multiplet by a duality transformation¹¹⁾, in much the same way as an antisymmetric tensor gauge field is, in four dimensions, dual to a real scalar field⁷⁾. This duality still holds in the presence of Chern-Simons forms in the field strength of the antisymmetric tensor^{12,13)}. One possibility to include an antisymmetric tensor and Chern-Simons forms in the theory described in eq.(2) is to take advantage of this duality : specify a gauge singlet chiral superfield S and subject it, under a few further assumptions, to a duality transformation. One thus recovers a supersymmetric component action which indeed contains an antisymmetric tensor as well as Chern-Simons forms¹³⁾.

In this paper, we take the alternate path and directly couple the antisymmetric tensor gauge field in the presence of Chern-Simons forms to the general supergravity Yang-Mills-matter system. Our construction is genuinely a superspace formulation of this system and relies heavily on the (super)geometric description of Chern-Simons forms and two-form gauge potentials, presented in detail in Refs 9,10,14). Although the superspace geometry for both Yang-Mills and Lorentz Chern-Simons forms has been constructed, we shall limit ourselves here to the case of Yang-Mills Chern-Simons forms. The inclusion of Lorentz Chern-Simons forms is technically more involved and will be investigated elsewhere¹⁵⁾.

2. A possibility to couple the linear multiplet is to promote the function $\mathcal{G}(\phi, \bar{\phi})$ of eq.(4) to a more general function $\Gamma(\phi, \bar{\phi}, L)$ which depends on the superfield L for the linear multiplet as well. Motivated by superstring compactifications we will treat in detail the case

$$\Gamma(\phi, \bar{\phi}, L) = \mathcal{G}(\phi, \bar{\phi}) + 4n \log L. \quad (6)$$

The case of arbitrary $\Gamma(\phi, \bar{\phi}, L)$ will be shortly discussed below. In the presence of Chern-Simons forms the superfield L obeys

$$(\mathcal{D}^\alpha \mathcal{D}_\alpha - 8R^+) L = 2k \operatorname{tr} (\bar{W}_\alpha \bar{W}^{\dot{\alpha}}) , \quad (7)$$

$$(\mathcal{D}_\alpha \mathcal{D}^{\dot{\alpha}} - 8R) L = 2k \operatorname{tr} (W^\alpha W_\alpha) .$$

These equations follow from the superspace Bianchi identities $dH = k \operatorname{tr}(\mathcal{F}\mathcal{F})$ for the three-form field strength $H = \frac{1}{3!} E^A E^B E^C H_{CBA}$, defined as

$$H = dB + k \operatorname{tr} \left(\mathcal{A}\mathcal{F} - \frac{1}{3} \mathcal{A}\mathcal{A}\mathcal{A} \right) . \quad (8)$$

Here $\mathcal{A}$ is the Yang-Mills potential, a one-form in superspace with field strength $\mathcal{F} = d\mathcal{A} + \mathcal{A}\mathcal{A}$, B is the two-form gauge potential in superspace and k is a normalization constant. The field strengths $\mathcal{F}$ and H are subject to certain constraints in superspace, compatible with the Bianchi identities. The superfields W^α , $\bar{W}_{\dot{\alpha}}$ and L are identified as follows:

$$\mathcal{F}_{\beta\alpha} = i(\sigma_a \varepsilon)_\beta{}^{\dot{\beta}} \bar{W}_{\dot{\beta}} , \quad \mathcal{F}^{\dot{\beta}}{}_\alpha = -i(\bar{\sigma}_a \varepsilon)^{\dot{\beta}}{}_\beta W^\beta , \quad (9)$$

$$H_{\gamma}{}^{\dot{\beta}}{}_\alpha = -2i(\sigma_a \varepsilon)_\gamma{}^{\dot{\beta}} L . \quad (10)$$

The gaugino superfields W^α , $\bar{W}_{\dot{\alpha}}$ have the usual properties:

$$\mathcal{D}^{\dot{\alpha}} W_\alpha = \mathcal{D}_\alpha \bar{W}^{\dot{\alpha}} = 0 , \quad \mathcal{D}^{\dot{\alpha}} W_\alpha - \mathcal{D}_\alpha \bar{W}^{\dot{\alpha}} = 0 . \quad (11)$$

Of course, in the absence of Chern-Simons forms, eqs.(7) reduce to the usual linearity conditions. Furthermore, the field strength of the antisymmetric tensor $b_{mn}(x)$ is located in the $\theta\bar{\theta}$ component of the superfield L :

$$[\mathcal{D}_\alpha, \mathcal{D}_{\dot{\alpha}}] L = 4L G_{\alpha\dot{\alpha}} - 4k \operatorname{tr} (W_\alpha \bar{W}_{\dot{\alpha}}) - \frac{1}{3} (\sigma_a)_{\alpha\dot{\alpha}} \varepsilon^{dcba} H_{cba} . \quad (12)$$

The whole system is described in curved (flexible) $U(1)$ superspace as defined in ref. 4) except that now $\mathcal{G}(\phi, \bar{\phi})$ is replaced by $\Gamma(\phi, \bar{\phi}, L)$, e.g.:

$$A_\alpha = \frac{1}{4} \mathcal{D}_\alpha \Gamma(\phi, \bar{\phi}, L) , \quad A^{\dot{\alpha}} = -\frac{1}{4} \mathcal{D}^{\dot{\alpha}} \Gamma(\phi, \bar{\phi}, L) . \quad (13)$$

The geometric structures discussed here have been exposed in refs 14,10,4) and we refer the reader to these references for more details.

To proceed, we first observe that, in the absence of matter, eq.(6) just gives the coupling of 16-16 supergravity to supersymmetric Yang-Mills theory as described in ref. 14). On the other hand, if one takes $n=0$ in (6), one gets back the supergravity-matter coupling of ref. 4).

In the replacement of $\mathcal{G}(\phi, \bar{\phi})$ by $\Gamma(\phi, \bar{\phi}, L)$ defined in (6), the form of the complete action remains the same, except for a change in normalization of the first term (which is adjusted in such a way that the Einstein curvature term in the component field expression has the usual normalization):

$$\mathcal{L} = (4n-3) \int E + \mathcal{L}_{\text{pot}} + \mathcal{L}_{\text{YM}} , \quad (14)$$

$$\mathcal{L}_{\text{pot}} = \frac{1}{2} \int \frac{E}{R} e^{\Gamma/2} + \text{h.c.} , \quad (15)$$

$$\mathcal{L}_{\text{YM}} = \frac{1}{8g^2} \int \frac{E}{R} f_{(ab)}(\phi) W^{\alpha(a)} W_{\alpha}^{(b)} + \text{h.c.} . \quad (16)$$

Although eq.(14) looks similar to eq.(2), its content is rather different. In addition to the kinetic terms of the supergravity and supermatter sectors the first term now contains also the kinetic terms for the antisymmetric tensor gauge field as well as a contribution to the kinetic term of the super Yang-Mills sector.

It is remarkable that even if one takes $f_{(ab)}$ to vanish in eq.(16) there is still a kinetic term for the Yang-Mills fields generated by the Chern-Simons forms. The gauge kinetic function derives from $\Gamma(\phi, \bar{\phi}, L)$ and is determined to be proportional to L^{-1} .

Furthermore $\log L$ can be naturally identified as a dilaton field. Due to eq.(10), the Weyl weight of the linear superfield L is determined from those of the vielbein, $\sigma(E^a) = -1/2$, $\sigma(E^a) = -1$, which gives $\sigma(E) = -2$ and $\sigma(L) = 2$. Taking the Weyl weights $\sigma(\phi)$ of the chiral matter fields to be zero, it is easy to see that the weights of the first two terms of the action are equal if $n = 1/4$. The explicit Yang-Mills kinetic term, eq.(16), has weight zero because $\sigma(R) = 1$ and $\sigma(W_{\alpha}) = 3/2$.

Conversely, if one starts with an arbitrary function $\Gamma(\phi, \bar{\phi}, L)$ and requires the Einstein term in the action to have the usual normalization and the Weyl weights of the first two terms in (14) to be equal one arrives at the form (6) with $n = 1/4$. This value $n = 1/4$ actually corresponds to truncations of certain $N=4$ and $N=2$ extended supergravity theories to $N=1$ ¹⁶⁾.

3. The component field action is obtained in the standard way¹⁷⁾ using chiral superspace coordinates and constructing the chiral density multiplet. Component fields are defined as lowest components of superfields. For the supergravity sector

one identifies the vierbein field and the gravitino together with their supersymmetry partner as follows:

$$\begin{aligned} e_m^a(x) &= E_m^a|, \quad \frac{1}{2} \psi_m^\alpha(x) = E_m^\alpha|, \\ -\frac{1}{6} M(x) &= R|, \quad -\frac{1}{6} \bar{M}(x) = R^+|, \quad -\frac{1}{3} b_a(x) = G_a|. \end{aligned} \quad (17)$$

The chiral density multiplet $\mathcal{E}$ in $U(1)$ superspace¹⁴⁾,

$$\mathcal{E} = e - ie \bar{\psi}_m \bar{\theta}^m \Theta - e \Theta^2 (\bar{M} + \bar{\psi}_m \bar{\theta}^{mn} \bar{\psi}_n) \quad (18)$$

is then used to write down the expression for the component field version of (14),

$$\begin{aligned} \mathcal{L} &= (4n-3) \int d^2\Theta \mathcal{E} R + \int d^2\Theta \mathcal{E} e^{r/2} \\ &+ \frac{1}{4g^2} \int d^2\Theta \mathcal{E} f_{(ab)}(\phi) W^{\alpha(a)} W_\alpha^{(b)} + h.c.. \end{aligned} \quad (19)$$

The details of the component field action are still hidden in the superfield expansion of the chiral superfields (of chiral $U(1)$ weight two) which multiply the chiral density.

The construction goes along the same lines as in the book by Wess and Bagger¹⁷⁾. The application to $U(1)$ superspace has been carried out in ref. 14). We merely give here the identification of the remaining component fields. The antisymmetric tensor gauge field, $b_{mn}(x) = B_{mn}|$, has supersymmetry partners

$$C(x) = \log L|, \quad \varphi_\alpha(x) = \mathcal{D}_\alpha \log L|, \quad (20)$$

and the Lie algebra valued component fields of the Yang-Mills multiplet accompanying the Yang-Mills gauge potential $ia_m(x) = \mathcal{A}_m|$ are:

$$-i \tilde{\chi}_\alpha(x) = W_\alpha|, \quad i \tilde{\chi}^\alpha(x) = \bar{W}^\alpha|, \quad -2 \tilde{D}(x) = \mathcal{D}^\alpha W_\alpha|. \quad (21)$$

Finally, the chiral matter superfields have components

$$A^i(x) = \phi^i|, \quad \chi_\alpha^i(x) = \frac{1}{\sqrt{2}} \mathcal{D}_\alpha \phi^i|, \quad F^i(x) = -\frac{1}{4} \mathcal{D}^\alpha \mathcal{D}_\alpha \phi^i|. \quad (22)$$

We now write down the component field action. As already indicated above, the first term in (19),

$$\mathcal{L}_{\text{kin}} = (4n-3) \int d^2\Theta \mathcal{E} R + \text{h.c.} , \quad (23)$$

contains all the kinetic terms. We write it as

$$\mathcal{L}_{\text{kin}} = \mathcal{L}_{16-16} + \mathcal{L}_{\text{YM}}^0 + \mathcal{L}_{\text{Matter}} . \quad (24)$$

The three contributions read:

$$\begin{aligned} e^{-1} \mathcal{L}_{16-16} = & -\frac{1}{2} \mathcal{R} + \frac{1}{2} \varepsilon^{mnkl} (\bar{\psi}_m \bar{\sigma}_n \mathcal{D}_k \psi_l - \psi_m \sigma_n \mathcal{D}_k \bar{\psi}_l) \\ & + \frac{4n-3}{9} M \bar{M} - \frac{4n-3}{9} (b^a - \frac{3n}{2} \varphi \sigma^a \bar{\varphi}) (b_a - \frac{3n}{2} \varphi \sigma_a \bar{\varphi}) \\ & - n (\partial_m C) \partial^m C + n e^{-2C} \mathcal{H}_m \mathcal{H}^m \\ & - i n \bar{\varphi}^{\dot{\alpha}} \sigma_{\alpha\dot{\alpha}}^m \mathcal{D}_m \varphi^\alpha - i n \varphi^\alpha \sigma_{\alpha\dot{\alpha}}^m \mathcal{D}_m \bar{\varphi}^{\dot{\alpha}} \\ & + i n e^{-C} (\bar{\psi}_{m\dot{\alpha}} \bar{\varphi}^{\dot{\alpha}} - \psi_m^\alpha \varphi_\alpha) \mathcal{H}^m - 2n e^{-C} (\varphi \sigma_m \bar{\varphi}) \mathcal{H}^m \\ & + i n e^{-C} \varepsilon^{mnkl} (\psi_m \sigma_l \bar{\psi}_n) \mathcal{H}_k - n (\psi_m \sigma^n \bar{\sigma}^m \varphi + \bar{\psi}_m \bar{\sigma}^n \sigma^m \bar{\varphi}) \partial_n C \\ & + \frac{n}{2} \varepsilon^{k\ell mn} (\psi_k \sigma_\ell \bar{\psi}_m) (\psi_n^\alpha \varphi_\alpha - \bar{\psi}_{n\dot{\alpha}} \bar{\varphi}^{\dot{\alpha}}) \\ & + \frac{i n}{2} (\bar{\varphi} \bar{\sigma}^m \psi_m) \varphi^\alpha \varphi_\alpha + \frac{i n}{2} (\varphi \sigma^m \bar{\psi}_m) \bar{\varphi}_{\dot{\alpha}} \bar{\varphi}^{\dot{\alpha}} \\ & + \frac{n}{8} \psi_m^\beta \psi^\beta_m \varphi^\alpha \varphi_\alpha + \frac{n}{8} \bar{\psi}_{m\dot{\beta}} \bar{\psi}^{\dot{\beta}}_m \bar{\varphi}_{\dot{\alpha}} \bar{\varphi}^{\dot{\alpha}} \\ & + \frac{n}{4} (\psi_m \sigma^k \bar{\psi}^m) (\varphi \sigma_k \bar{\varphi}) - \frac{1}{2} (3n + n^2 (4n-3)) \varphi^\alpha \varphi_\alpha \bar{\varphi}_{\dot{\alpha}} \bar{\varphi}^{\dot{\alpha}} \end{aligned} \quad (25)$$

$$\begin{aligned}
 \mathcal{L}_{YM}^0 = & n k e \cdot e^{-C} \text{tr} \left[f_{mn} f^{mn} + 2i \sigma_{\alpha\dot{\alpha}}^m (\tilde{\lambda}^\alpha \mathcal{D}_m \tilde{\lambda}^{\dot{\alpha}} + \tilde{\lambda}^{\dot{\alpha}} \mathcal{D}_m \tilde{\lambda}^\alpha) \right. \\
 & + \frac{2}{3} i (4n-3) (\tilde{\lambda} \sigma^a \tilde{\lambda}) b_a - 2 \tilde{D} \tilde{D} + 2i \tilde{D} (\varphi \tilde{\lambda} - \bar{\varphi} \tilde{\lambda}) \\
 & + \varepsilon^{\kappa\ell mn} f_{\kappa\ell} (\psi_m \sigma_n \tilde{\lambda} - \bar{\psi}_m \sigma_n \tilde{\lambda}) - 2i f^{mn} (\psi_m \sigma_n \tilde{\lambda} + \bar{\psi}_m \bar{\sigma}_n \tilde{\lambda} \\
 & - i \varphi \sigma_{mn} \tilde{\lambda} - i \bar{\varphi} \bar{\sigma}_{mn} \tilde{\lambda}) - \frac{1}{2} (\tilde{\lambda} \sigma_\kappa \tilde{\lambda}) (g^{\kappa m} g^{\ell n} + g^{\kappa\ell} g^{mn} + g^{\kappa n} g^{m\ell} \\
 & - i \varepsilon^{\kappa\ell mn}) (\bar{\psi}_\ell \bar{\sigma}_m \psi_n) - i (\tilde{\lambda} \sigma_m \tilde{\lambda}) (\varphi \sigma^{mn} \psi_n + \frac{3}{2} g^{mn} \varphi \psi_n - \bar{\varphi} \bar{\sigma}^{mn} \bar{\psi}_n \\
 & - \frac{3}{2} g^{mn} \bar{\varphi} \bar{\psi}_n) + \frac{1}{4} \tilde{\lambda}^\alpha \tilde{\lambda}_\alpha (3 \bar{\psi}_m \bar{\psi}^m + 2 \bar{\psi}_m \bar{\sigma}^{mn} \bar{\psi}_n - 2i \varphi \sigma^m \bar{\psi}_m - 8 \varphi \varphi) \\
 & + \frac{1}{4} \tilde{\lambda}_{\dot{\alpha}} \tilde{\lambda}^{\dot{\alpha}} (3 \psi_m \psi^m + 2 \psi_m \sigma^{mn} \psi_n - 2i \bar{\varphi} \bar{\sigma}^m \psi_m - 8 \bar{\varphi} \bar{\varphi}) \\
 & \left. - 4 \tilde{\lambda}^\alpha \tilde{\lambda}_\alpha e^{-C} k \text{tr} (\tilde{\lambda}_{\dot{\alpha}} \tilde{\lambda}^{\dot{\alpha}}) \right] ,
 \end{aligned}
 \tag{26}$$

$$\begin{aligned}
 e^{-1} \mathcal{L}_{\text{Matter}} = & -g_{i\bar{k}} (\mathcal{D}_m A^i) \mathcal{D}^m \bar{A}^{\bar{k}} - \frac{i}{2} g_{i\bar{k}} (\bar{\chi}^{\bar{k}} \bar{\sigma}^m \hat{D}_m \chi^i \\
 & + \chi^i \sigma^m \hat{D}_m \bar{\chi}^{\bar{k}}) + \frac{1}{6} (4n-3) g_{i\bar{k}} (\chi^i \sigma^a \bar{\chi}^{\bar{k}}) b_a + g_{i\bar{k}} \hat{F}^i \hat{F}^{\bar{k}} \\
 & + \frac{1}{4} R_{i\bar{k}j\bar{\ell}} (\chi^i \chi^j) (\bar{\chi}^{\bar{k}} \bar{\chi}^{\bar{\ell}}) - \frac{1}{2} \sqrt{2} g_{i\bar{k}} (\bar{\psi}_n \bar{\sigma}^m \sigma^n \bar{\chi}^{\bar{k}}) \mathcal{D}_m A^i \\
 & - \frac{1}{2} \sqrt{2} g_{i\bar{k}} (\psi_n \sigma^m \bar{\sigma}^n \chi^i) \mathcal{D}_m \bar{A}^{\bar{k}} - \frac{1}{2} g_{i\bar{k}} (\psi_m \chi^i) (\bar{\psi}^m \bar{\chi}^{\bar{k}}) \\
 & - g_{i\bar{k}} (\psi_m \sigma^{mn} \chi^i) (\bar{\psi}_n \bar{\chi}^{\bar{k}}) - g_{i\bar{k}} (\bar{\psi}_m \bar{\sigma}^{mn} \bar{\chi}^{\bar{k}}) (\psi_n \chi^i) \\
 & + \sqrt{2} i g_{i\bar{k}} \chi^{\alpha i} (\tilde{\lambda}_\alpha \bar{A})^{\bar{k}} - \sqrt{2} i g_{i\bar{k}} \bar{\chi}_{\dot{\alpha}}^{\bar{k}} (A \tilde{\lambda}^{\dot{\alpha}})^i + (A \tilde{D})^i \partial_i \mathcal{G} \\
 & + \frac{1}{2} (\bar{\psi}_m \bar{\sigma}^m)^{\alpha} (A \tilde{\lambda}_\alpha)^i \partial_i \mathcal{G} - \frac{1}{2} (\psi_m \sigma^m)_{\dot{\alpha}} (A \tilde{\lambda}^{\dot{\alpha}})^i \partial_i \mathcal{G} ,
 \end{aligned}
 \tag{27}$$

where we have used the definitions

$$\begin{aligned}
 \hat{D}_m \chi_\alpha^i &= \mathcal{D}_m \chi_\alpha^i + \Gamma_{jk}^i \chi_\alpha^j \mathcal{D}_m A^k , \\
 \hat{F}^i &= F^i - \frac{1}{2} \Gamma_{jk}^i \chi^j \chi^k .
 \end{aligned}
 \tag{27'}$$

In these expressions, the covariant derivative of a field $\gamma(x)$ reads:

$$\mathcal{D}_m \gamma(x) = \mathcal{D}_m^{L,YM} \gamma(x) + w(\gamma) A_m^0(x) \gamma(x), \quad (28)$$

where $w(\gamma)$ is the chiral $U(1)$ weight of γ , $\mathcal{D}_m^{L,Y.M.}$ is covariant with respect to Lorentz and Yang-Mills transformations^{*}) and $A_m^0(x)$ is given by:

$$\begin{aligned} A_m^0(x) = & \frac{1}{4} (\partial_i \mathcal{G}) \mathcal{D}_m A^i - \frac{1}{4} (\partial_{\bar{k}} \mathcal{G}) \mathcal{D}_m \bar{A}^{\bar{k}} + \frac{i}{4} g_{i\bar{k}} (\chi^i \sigma_m \bar{\chi}^{\bar{k}}) \\ & + i n e^{-C} \mathcal{H}_m - \frac{i n}{2} (\varphi \sigma_m \bar{\varphi}) + \frac{n}{2} (\psi_m^\alpha \varphi_\alpha - \bar{\psi}_{m\dot{\alpha}} \bar{\varphi}^{\dot{\alpha}}). \end{aligned} \quad (29)$$

It is this combination (up to a term proportional to $e_m^a b_a$, which we have decided to write explicitly) that occurs naturally in our present generalisation of Kähler superspace as a consequence of eq.(13) and the superspace constraints^{4,10)}

$$F_{\beta}^{\dot{\alpha}} = 3(\sigma^a \epsilon)_{\beta}^{\dot{\alpha}} G_a.$$

Furthermore, $\mathcal{H}^m$ denotes the generalisation of the field strength of the anti-symmetric tensor gauge field which contains the Yang-Mills Chern-Simons form and certain quadratic spinor terms:

$$\begin{aligned} \mathcal{H}^k(x) = & \frac{1}{2} \varepsilon^{k\ell mn} \partial_\ell b_{mn} + \varepsilon^{k\ell mn} k \operatorname{tr} (a_\ell \partial_m a_n - \frac{2i}{3} a_\ell a_m a_n) \\ & - i e^C (\psi_m \sigma^{mk} \varphi - \bar{\psi}_m \bar{\sigma}^{mk} \bar{\varphi}) + \frac{i}{2} e^C \varepsilon^{k\ell mn} (\psi_\ell \sigma_m \bar{\psi}_n) \\ & - k \operatorname{tr} (\tilde{\chi} \sigma^k \tilde{\chi}). \end{aligned} \quad (30)$$

Finally, and as usual, the Lorentz-connection $\varphi_{mb}^a(x) = \phi_{mb}^a$ is determined from the superspace constraint $T_{cb}^a = 0$.

The second term in (19),

$$\mathcal{L}_{\text{pot}} = \int d^2\Theta \, \mathcal{E} e^{r/2} + \text{h.c.}, \quad (31)$$

has the following component field expression

^{*}) The field strength of the Yang-Mills potential is given by

$$f_{mn} = \partial_m a_n - \partial_n a_m - i[a_m, a_n].$$

$$\begin{aligned}
\mathcal{L}_{\text{pot}} = & e \cdot e^{g/2} e^{4nC} \left[\frac{1}{3} (4n-3) (M + \bar{M}) \right. \\
& - \frac{1}{2} (\chi^i \chi^j) (\partial_i \partial_j \mathcal{G} - \Gamma_{ij}^k \partial_k \mathcal{G} + (\partial_i \mathcal{G}) \partial_j \mathcal{G}) + \hat{F}^i \partial_i \mathcal{G} \\
& - \frac{1}{2} (\bar{\chi}^i \bar{\chi}^j) (\partial_{\bar{i}} \partial_{\bar{j}} \mathcal{G} - \Gamma_{\bar{i}\bar{j}}^{\bar{k}} \partial_{\bar{k}} \mathcal{G} + (\partial_{\bar{i}} \mathcal{G}) \partial_{\bar{j}} \mathcal{G}) + \hat{\bar{F}}^{\bar{k}} \partial_{\bar{k}} \mathcal{G} \\
& - \psi_m \sigma^{mn} \psi_n - \bar{\psi}_m \bar{\sigma}^{mn} \bar{\psi}_n + \frac{i}{\sqrt{2}} (\bar{\psi}_m \bar{\sigma}^m \chi^i) \partial_i \mathcal{G} + \frac{i}{\sqrt{2}} (\psi_m \sigma^m \bar{\chi}^{\bar{k}}) \partial_{\bar{k}} \mathcal{G} \\
& + 2in \bar{\psi}_m \bar{\sigma}^m \varphi + 2in \psi_m \sigma^m \bar{\varphi} - n(4n-1) (\varphi \varphi + \bar{\varphi} \bar{\varphi}) \\
& \left. - 2n\sqrt{2} (\varphi \chi^i) \partial_i \mathcal{G} - 2n\sqrt{2} (\bar{\varphi} \bar{\chi}^{\bar{k}}) \partial_{\bar{k}} \mathcal{G} + 2nk e^{-C} \text{tr} (\tilde{\lambda}^\alpha \tilde{\lambda}_\alpha + \tilde{\lambda}_{\dot{\alpha}} \tilde{\lambda}^{\dot{\alpha}}) \right].
\end{aligned} \tag{32}$$

As for the explicit Yang-Mills kinetic term,

$$\mathcal{L}_{YM} = \frac{1}{4g^2} \int d^2\Theta \, \mathcal{E} f_{(ab)}(\Phi) W^{\alpha(a)} W_{\alpha}^{(b)} + h.c. , \tag{33}$$

it is identical to the one given in Ref. 4), provided one uses $A_m^0(x)$ as defined in eq.(29) in the definition of covariant derivatives and replaces $-3b_a$ by $(4n-3)b_a$.

This concludes our discussion of the component field action. The elimination of the fields $M, \bar{M}, b_a, F, \bar{F}$ and D via their algebraic equations of motion is straightforward, and will not be given here.

In the absence of matter, the action displayed in eqs.(25) and (26) is the one given earlier in ref. 14). It describes the coupling of 16-16 supergravity and supersymmetric Yang-Mills theory via Chern-Simons forms. If one wants to include matter fields one simply has to add the matter part of the Lagrangian, eq.(27). In eqs.(25) and (26) the matter fields appear then only through the covariant derivatives as defined in (28) and (29). On the other hand, taking $n=0$ brings us back to the situation described in ref. 4).

4. Finally, it is instructive to perform explicitly the duality transformation that relates our approach to the formulation using a chiral superfield¹³⁾. We first observe that, since the right-hand side of eq.(7) is (anti)chiral, we may replace it by

$$\begin{aligned}
2k \, \text{tr} (\bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}}) &= (\mathcal{D}^{\alpha} \mathcal{D}_{\alpha} - 8R^+) \Omega , \\
2k \, \text{tr} (W^{\alpha} W_{\alpha}) &= (\mathcal{D}_{\dot{\alpha}} \mathcal{D}^{\dot{\alpha}} - 8R) \Omega ,
\end{aligned} \tag{34}$$

and define a superfield $L' = L - \Omega$ which satisfies the linearity conditions

$$(\mathcal{D}^\alpha \mathcal{D}_\alpha - 8R^+) L' = 0, \quad (\mathcal{D}_\alpha \mathcal{D}^\alpha - 8R) L' = 0. \quad (35)$$

It is then easy to see that this constraint and hence the theory described in this paper is obtained from varying the first order action

$$\mathcal{L}^{(1)} = (4n-3) \int E \left(1 - \frac{4n}{4n-3} (S+\bar{S}) L'\right) + \mathcal{L}_{\text{pot}} + \mathcal{L}_{\text{YM}} \quad (36)$$

with respect to the chiral and antichiral superfields S and $\bar{S}$. On the other hand, properly varying the action (36) with respect to L^* (observe that $\mathcal{L}_{\text{pot}}$ and $\mathcal{L}_{\text{YM}}$ as given in (15), (16) are inert under this variation), one obtains

$$L = \frac{1}{S+\bar{S}}. \quad (37)$$

Hence, the function Γ defined in eq.(6) takes now the form

$$\Gamma = \mathcal{G}(\phi, \bar{\phi}) - 4n \log(S+\bar{S}). \quad (38)$$

Substituting (37) back into (36) gives

$$\mathcal{L} = -3 \int E + 4n \int E (S+\bar{S}) \Omega + \mathcal{L}_{\text{pot}} + \mathcal{L}_{\text{YM}}. \quad (39)$$

Upon integration by parts in superspace the second term in (39) becomes, using eqs (34)

$$\begin{aligned} 4n \int E (S+\bar{S}) \Omega &= \\ &= -n \int \frac{E}{R} S k \text{tr}(W^\alpha W_\alpha) - n \int \frac{E}{R^+} \bar{S} k \text{tr}(\bar{W}_\alpha \bar{W}^\alpha). \end{aligned} \quad (40)$$

This shows that the action given in eq.(14) is related by a duality transformation to a special case of the superfield action presented in ref. 4). The Kähler potential is $K(\phi, \bar{\phi}, S, \bar{S}) = K(\phi, \bar{\phi}) - 4n \log(S+\bar{S})$, the superpotential $W(\phi)$ does not depend on S and the gauge kinetic superfield $f_{ab}(\phi, S)$ splits as $f_{ab}(\phi, S) = f_{ab}(\phi) + f_{ab}(S)$, such that the ϕ dependence is still arbitrary but the second term is proportional to $\delta_{ab} S$.

5. In this paper we have extended our geometric superspace construction of the general supergravity-matter-Yang-Mills system to include an antisymmetric tensor gauge field in the presence of Yang-Mills Chern-Simons forms. This can be achieved

*) We use in particular the following variation laws: $\delta E = -\frac{4n}{3} L^{-1} \delta L E$, $\delta \Omega = \frac{4n}{3} L^{-1} \delta L \Omega$.

by allowing the general function $\mathcal{G}$ to depend on the superfield L of the anti-symmetric tensor as well. For illustrative purpose we have treated in detail the case $\Gamma(\phi, \bar{\phi}, L) = \mathcal{G}(\phi, \bar{\phi}) + 4n \log L$. A more general coupling is given by the Lagrangian

$$\mathcal{L}_{\text{kin}} = -3 \int E f(\phi^i, \bar{\phi}^{\bar{i}}, L^A) \quad (41)$$

where $f(\phi^i, \bar{\phi}^{\bar{i}}, L^A)$ and the Kähler potential $K(\phi^i, \bar{\phi}^{\bar{i}}, L^A)$ are arbitrary functions of several chiral and linear superfields. From the component form of (41), which we do not specify here, we find, however, that f and K have to satisfy the condition

$$f - L^A \frac{\partial f}{\partial L^A} = 1 - \frac{1}{3} L^A \frac{\partial K}{\partial L^A} \neq 0 \quad (42)$$

in order to give the correct normalization of the curvature scalar. The special Lagrangian ($n = 1/4$) dealt with in this paper is obtained from (41) by the simple restrictions that (i) $\mathcal{L}$ contains only one linear multiplet and (ii) $\mathcal{L}$ is globally scale covariant. These limitations may turn up in the study of low-energy effective theories of ten-dimensional supergravity or superstring theories^{5,18}.

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