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ANOMALY FREE SUPERGRAVITY AND SUPER YANG-MILLS THEORIES IN TEN DIMENSIONS

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A B S T R A C T

We formulate in ten dimensions a supergravity theory coupled to a super Yang-Mills theory through the Green-Schwarz ansatz, which ensures chiral anomaly cancellation. The formulation is obtained by fixing suitable constraints in the superspace and solving the Bianchi identities. In this way we obtain the whole set of equations of motion. They are characterized by highly non-polynomial expressions, when expressed only in terms of the physical superfields. We discuss the problem of the massive modes appearing in some of these equations.

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1. - INTRODUCTION

The massless spectrum of type I and heterotic superstring theory coincides with the field content of the models of $N = 1$ supergravity (SUGRA) coupled to super Yang-Mills (SYM) theory in ten dimensions. These fields are: a graviton, a chiral Weyl spinor (gravitino), a scalar (dilaton), an antichiral Weyl spinor (gravitello), a two-index antisymmetric tensor (graviphoton), plus the gauge bosons and chiral Weyl gauginos in the adjoint representation of the gauge group G [$SO(32)$ or $E_8 \times E_8$]. This correspondence justifies the interest in SUGRA + SYM models in ten dimensions. If one is able to write down a Lagrangian or at least the equations of motion for such models, one may hope that they are in some way related to the low energy limit of the superstring theories, given by the effective theory (for the massless vector). By the latter we mean the action obtained (so far partially) by representing by means of an effective Lagrangian the results of computations of string amplitudes with zero modes on the external legs^{1),2)}. We recall that this should also be compared with the equations of motion obtained from the vanishing of the β functions in σ models representing the superstrings propagating in the background given by the relevant zero mode fields^{3),4)}.

Superstrings, of course, contain an infinite number of massive states and the effective actions for the massless sector are obtained by integrating out the massive modes. As a consequence, these actions are highly non-polynomial. In practice, only the first few local terms in the string tension expansion are obtained.

Expecting that the actions of SUGRA-SYM models reproduce exactly the above effective actions would be clearly excessive. However, an analysis of the former is useful, in our opinion, to understand the supersymmetric structure of the latter. Moreover, consistent formulations of SUGRA-SYM models are interesting in themselves, independently of their relation to superstrings.

Pure SUGRA⁵⁾ and pure SYM⁶⁾ in ten dimensions have been known for a long time. The coupling of SYM to SUGRA was obtained by Chapline and Manton⁷⁾ who extended previous works^{8),9)} dealing with the Abelian case. The main ingredient consists in modifying H , the graviphoton field strength, through the addition of the Chern-Simons (CS) three-form for the gauge connection. It is interesting that this term is also required by the group manifold approach to pure SYM^{6a)}.

The next important step was accomplished by Green and Schwarz¹⁰⁾ (GS). They showed that by modifying H through the addition of a CS three-form for the Lorentz connection, one can define a local counterterm which cancels all the anomalies when the gauge group is either $SO(32)$ or $E_8 \times E_8$. Since the corresponding superstring theories are expected to be anomaly free^{10),11)}, the GS modification appears to be a necessary ingredient for SUGRA-SYM models related to superstrings. However, the GS modification naïvely introduced in the Chapline-Manton Lagrangian breaks its supersymmetry and the problem arises of getting the supersymmetric completion.

A way to address such a problem is to work in a superspace. One assumes suitable constraints on the torsion and the two-form graviphoton and Yang-Mills curvatures, defined in the superspace. Then the supersymmetry transformation rules and the field equations for the physical fields follow from the Bianchi identities¹²⁾⁻¹⁶⁾. A related approach is based on the group manifold¹⁷⁾. A dual formulation based on a six-form graviphoton is that of Refs. 18). In this way several authors¹²⁾⁻¹⁸⁾ have succeeded in reproducing the Chapline-Manton model. But the straightforward application of this approach in the presence of the GS modification meets with a difficulty. Indeed, in this case, the Bianchi identities for H contain in addition to $\text{Tr}(FF)$ the term $\text{Tr}(RR)$, which comes from the differential of the Lorentz CS three-form (R and F are the Lorentz and gauge curvatures, respectively). Now, the superspace structure of $\text{Tr}(RR)$ is not as simple as the structure of $\text{Tr}(FF)$ and prevents a straightforward extension of the solution of the Bianchi identities for H , from the Chapline-Manton to the GS case.

The attempts at obtaining the supersymmetric completion of the GS term have yielded only partial results. In particular the first terms of a perturbative expansion in c_2 (see below for the definition) have been obtained in Refs. 19) and 18) working along the lines of Refs. 12)-19) or using the Noether procedure. Off-shell completion of the linearized level has been studied in Ref. 20) by means of superconformal calculus. A full supersymmetric completion for the $D = 4$ reduced theory has been obtained in Ref. 21). These attempts and their relation to superstrings have been reviewed and discussed in Ref. 22).

A way to bypass the above-mentioned difficulty has been presented in Ref. 23) [see also 24)]. There a theorem was proved which allows us to decompose $\text{Tr}(RR)$ in such a way that, after a suitable redefinition of H , the superspace structure of the Bianchi identities for the new H is the same as in the absence of the Lorentz CS form. Therefore the new Bianchi identities can be solved along

the lines of Refs. 12)-14). In Ref. 20) the superforms which appear in the decomposition of $\text{Tr}(\text{RR})$ and the relevant field equations were not explicitly calculated. In the present paper we give details of the proof of the above-mentioned theorem, calculate the explicit formulas and obtain all the field equations. Parallel results have been obtained in Ref. 25) by means of group manifold techniques.

Our field equations are given in terms of polynomial expressions which, however, involve an auxiliary superfield T_{abc} , belonging to the 120 irreducible representation (IR) of $\text{SO}(1,9)$. T_{abc} in turn satisfies an equation involving in a non-linear way T_{abc} itself. Therefore once this equation is solved recursively and T_{abc} removed, the equations of motion become infinite series in the physical superfields, in agreement with what one expects from previous contiguous results^{25),12),13),17),20)}.

A striking feature of the field equation we find is that, in addition to the massless SUGRA-SYM spectrum, they contain massive modes, which propagate at the Planck mass. These modes are present in the field equations for the graviton and the gravitino, as well as in the equation for the auxiliary superfield T_{abc} . Altogether there are 128+128 massive modes. Unfortunately most of them are poltergeist (poltergeists²⁶⁾ are physical ghosts, i.e., negative norm states which do not decouple from the physical states). Since we do not consider the theory under investigation as a fundamental theory but rather as an approximation to an effective theory of the massless sector, ghosts propagating at the Planck mass do not represent a disaster. Moreover they can be avoided by superfield redefinitions to any finite order in an expansion in the inverse of the Planck mass. Anyhow, see the detailed discussion at the end of the paper.

At this point we should add that the Green-Schwarz anomaly cancellation mechanism would deserve a closer examination concerning the geometrical aspects. The latter are treated extensively in Ref. 27), so we drop them here. We only notice that here we are interested with the purely algebraic aspects of the problem and leave aside any geometrical consideration.

The paper is organized as follows. In Section 2 we introduce our notations, present the constraints and solve the Bianchi identities for the torsion and the gauge curvature. These results can be found in the literature¹²⁾⁻¹⁸⁾ and are reported here in order to render the paper self-contained. In Section 3 we state and prove again the theorem of Ref. 23) and give the complete expressions of the superforms involved. In Section 4 we solve the Bianchi identities for H and

obtain the relevant field equations. In the last section we discuss the freedom present in our solution, which can be expressed in terms of suitable transformations of the supervielbeins and the superconnection, with special emphasis on the ghost problem.

2. - CONSTRAINTS AND SOLUTIONS OF THE T- AND F-BIANCHI IDENTITIES

A superspace in ten dimensions is spanned by the co-ordinates $z^M \equiv (x^m, \theta^\mu)$, where x^m ($m=0,1,\dots,9$) are the ordinary space-time co-ordinates and θ^μ ($\mu=1,\dots,16$) are Grassmann variables. A superform ψ can be written in the base $\{dz^\mu\}$ as

$$\psi(z) = \frac{1}{n!} dz^{M_1} \dots dz^{M_n} \psi_{M_1 \dots M_n}(z)$$

Here the dx^m ($d\theta^\mu$) anticommute (commute) and each dx^m anticommutes with each $d\theta^\mu$. One mimics ordinary differential geometry and introduces the supervielbeins $e_M^A(z)$ and the corresponding superforms $e^A(z) = dz^M e_M^A(z)$. Here $A = \{a, \alpha\}$ ($a=0,\dots,9$, $\alpha=1,\dots,16$) is a flat index (letters from the beginning of the alphabet represent flat indices, letters from the middle of the alphabet are used for world indices, with obvious meaning of the capital, small Latin and small Greek letters). We can express any superform in the basis $\{e^A(z)\}$

$$\psi = \frac{1}{n!} e^{A_1} \dots e^{A_n} \psi_{A_1 \dots A_n}$$

and $\psi_{A_1 \dots A_n}$ are related to the $\psi_{M_1 \dots M_n}$ by means of the $e_M^A(z)$. We will often use the following decomposition of ψ :

$$\psi = \sum_{p+q=n} \frac{1}{n!} \psi_{p,q}$$

where the (p,q) -superform $\psi_{p,q}$ is defined by

$$\psi_{p,q} = \frac{n!}{p!q!} e^{\alpha_1} \dots e^{\alpha_q} e^{a_1} \dots e^{a_p} \psi_{a_1 \dots a_p, \alpha_1 \dots \alpha_q}$$

Any equation where an n -superform is equated to zero will split into $n+1$ independent equations, one for each $(p,n-p)$ -sector, with $p = 0, \dots, n$.

Next, one introduces the Lorentz connection 1-superform Ω_A^B and the $\mathcal{G}$ -valued gauge connection 1-superform A , where $\mathcal{G}$ is the Lie algebra of the relevant gauge group G , as well as the 2-superform B . Ω_A^B is Lorentz-valued: a superfield ϕ_A^B is Lorentz-valued if $\phi^{ab} = -\phi^{ba}$, $\phi_\alpha^\beta = 1/4(\Gamma^{[a}\Gamma^{b]})_\alpha^\beta \phi_{ab}$ and vanishes otherwise. The symbol $[]$ represents antisymmetrization and we have introduced two sets of Γ -matrices with upper and lower indices respectively, satisfying

$$\Gamma_{\alpha\beta}^a \Gamma^{b\beta\gamma} + \Gamma_{\alpha\beta}^b \Gamma^{a\beta\gamma} = 2 \eta^{ab} \delta_\alpha^\gamma$$

η_{ab} is the Minkowski metric with signature -8 . $\Gamma^{a_1 \dots a_n}$ will denote completely antisymmetrized products of n such Γ matrices, where antisymmetrization is understood to unit weight. Let us now introduce the total differential $d = dz^M (\partial/\partial z^M)$ in the superspace, while Δ will represent the covariant differential w.r.t. both gauge and Lorentz transformations. Then the torsion is defined by:

$$T^A = \Delta e^A = \frac{1}{2} e^B e^C T_{CB}^A \quad (2.1)$$

The Lorentz and gauge supercurvatures are

$$R_A^B = d \Omega_A^B + \Omega_A^C \Omega_C^B = \frac{1}{2} e^C e^D R_{DC}{}^B \quad (2.2)$$

$$F = dA + AA = \frac{1}{2} e^C e^B F_{BC} \quad (2.3)$$

respectively. We also set

$$H = dB + c_1 \omega_{3YM}(A) - c_2 \omega_{3L}(\Omega) = \frac{1}{6} e^C e^B e^A H_{ABC} \quad (2.4)$$

where

$$\omega_{3YM} = \text{Tr}(A dA + \frac{2}{3} A^3) \quad , \quad \omega_{3L}(\Omega) = \text{Tr}(\Omega d\Omega + \frac{2}{3} \Omega^3)$$

are the gauge and Lorentz CS superforms, and c_1 and c_2 are constants. H is assumed to be gauge and Lorentz invariant, therefore, due to the transformation properties of ω_{3YM} and ω_{3L} one can assume that

$$\delta B = c_1 \text{Tr}(c dA) - c_2 \text{Tr}(u d\Omega) \quad (2.5)$$

where δ represents the total gauge + Lorentz variation and $c(u)$ represents the corresponding $\mathcal{J}$ -valued (Lorentz-valued) infinitesimal parameter. Equation (2.5) is basic for the cancellation scheme of Green and Schwarz, which, however, works only, for specific values of c_1 and c_2 for $SO(32)$ and $E_8 \times E_8$. Once we have suitably normalized the generators of $\mathcal{J}$ and of the Lie algebra of the Lorentz group [if $\text{Tr}(RR) = R_{ab} R^{ba}$, then $\text{Tr}(\tau_i \tau_j) = -\delta_{ij}$, in the defining vector representation of the orthogonal groups], then anomalies cancel if $c_1 = c_2$ in the $SO(32)$ case and $c_1/c_2 = 30$ in the $E_8 \times E_8$ case. We remark that what we are going to do in this paper holds independently of the values of c_1 and c_2 .

By differentiating (2.1)-(2.4) we obtain the Bianchi identities

$$\Delta T^A = e^B R_B^A \quad (2.6)$$

$$\Delta R_A^B = 0 \quad (2.7)$$

$$\Delta F = 0 \quad (2.8)$$

$$dH = c_1 \text{Tr}(FF) - c_2 \text{Tr}(RR) \quad (2.9)$$

Let us deal now with the constraints. The basic superfields (vielbeins, gauge and Lorentz connections as well as B) contain a huge number of superfields. In order to render the above general "geometric" setting fit for describing dynamics, the strategy consists in getting rid of a large amount of such superfields by imposing suitable constraints on the intrinsic components of T^A , F and H . When these constraints are introduced into the Bianchi identities, the latter are no longer identically satisfied and their consistency implies physical consequences. In general the Bianchi identities allow us to express R_A^B , F and H in terms of the surviving (basic) superfields and determine the supersymmetry transformations of the latter. In some cases they imply in addition field equations for the set of basic fields, i.e., they put the theory on shell. This is what occurs for SUGRA-SYM theories in ten dimensions. Let us then write down the constraints we have chosen for such theories:

$$T_{\alpha\beta}^c = 2(\Gamma^c)_{\alpha\beta} \quad (2.10)$$

$$T_{\alpha\beta}{}^\gamma = 0 = T_{\alpha b}{}^c \quad (2.11)$$

$$T_{a\beta}{}^\gamma = (\Gamma_a)_{\beta\epsilon} \psi^{\epsilon\gamma}(z) \quad , \quad \psi^{\epsilon\gamma}(z) = -\psi^{\gamma\epsilon}(z) \quad (2.12)$$

$$F_{\alpha\beta} = 0 \quad (2.13)$$

$$H_{\alpha\beta\gamma} = 0 + O(c_2) \quad (2.14)$$

$$H_{a\beta\gamma} = \Gamma_{a\beta\gamma} \phi + O(c_2) \quad (2.15)$$

ϕ is a scalar superfield, $O(c_2)$ in Eqs. (2.14) and (2.15) represent terms which vanish as $c_2 \rightarrow 0$ (see the next section). The above constraints are taken from Refs. 13) and 15). Another set of constraints can be found in the literature^{12),14)}: they can be shown to be equivalent to the above up to redefinitions of the basic fields. The question of uniqueness of the constraints in ten dimensional SUGRA-SYM theories will be discussed elsewhere. The relation between constraints and consistency of the superstring propagating in curved superspace background has been discussed in Refs. 28) and 29).

The T-Bianchi identities, Eq. (2.6), under the constraints (2.10)-(2.12) (or equivalent ones) have been solved by several groups¹³⁾⁻¹⁸⁾. Here we simply record the results.

a) $T_{ab}{}^c$ and $T_{a\beta}{}^\gamma$ contain only a 120 IR of $SO(1,9)$ which turns out to be the same for both:

$$T_{a\beta}{}^\gamma = -\frac{1}{24} (\Gamma_a \Gamma^{b_1 b_2 b_3})_{\beta}{}^\gamma T_{b_1 b_2 b_3} \quad (2.16)$$

where $T_{b_1 b_2 b_3}$ is a completely antisymmetric superfield, which is divergenceless

$$D^a T_{a b_1 b_2} = 0 \quad (2.17)$$

b) T_{ab}^γ can be expressed in terms of $D_\alpha T_{b_1 b_2 b_3}$ as follows

$$T_{ab}^\gamma = -\frac{1}{14} (\Gamma^{c_1 c_2}{}_{[a})^\gamma{}^\beta D_\beta T_{b] c_1 c_2} + \frac{3}{28} (\Gamma^c)^\gamma{}^\beta D_\beta T_c{}^{ab} \quad (2.18)$$

where D_A denotes the covariant derivative. When expressed in terms of $SO(1,9)$ IR's it can be written as

$$T_{ab}^\alpha = \frac{3}{2} \sigma_{ab}^\alpha + \frac{1}{4} \hat{\sigma}_{ab}^\alpha$$

where σ_{ab}^α is the 560 IR, i.e., $\Gamma_{\alpha\beta}^c \sigma_{cb}^\beta = 0$, and $\sigma_{ab}^\alpha = (\Gamma_{[a})^{\alpha\beta} L_{\beta c]}$ with $L_{\alpha\beta} = \Gamma_{\alpha\beta}^c T_{cb}^\beta$ being the 144 IR. In terms of $D_\alpha T_{abc}$, $L_{\alpha\beta}$ is given

$$L_{\alpha\beta} = \frac{1}{28} (\Gamma^{c_1 c_2})_{\alpha}{}^\gamma D_\gamma T_{c_1 c_2 \beta} \quad (2.19)$$

The 16 IR in T_{ab}^α vanishes, so that $\Gamma_a^{\alpha\beta} L_{\beta a} = 0$. Moreover

$$D_\alpha T_{abc} = 2 \left[(\Gamma_{[a})_{\alpha\beta} T_{b c]}^\beta - (\Gamma_{ab})_{\alpha}{}^\beta L_{\beta c]} \right] \quad (2.20)$$

From the vanishing of the 16 IR it follows $(\Gamma^{c_1 c_2 c_3})^{\alpha\gamma} D_\gamma T_{c_1 c_2 c_3} = 0$. In addition

$$D_\gamma T_{ab}^\gamma = 0 \quad (2.21)$$

c) The components of the Lorentz curvature are given by

$$R_{\alpha\beta}{}^{c_1 c_2} = \frac{1}{6} (\Gamma^{c_1 c_2 d_1 d_2 d_3})_{\alpha\beta} T_{d_1 d_2 d_3} + 3 (\Gamma^d)_{\alpha\beta} T_{d c_1 c_2} \quad (2.22)$$

$$R_{\alpha a c_1 c_2} = \frac{1}{2} D_\alpha T_{a c_1 c_2} + 2 (\Gamma_{[c_1})_{\alpha\beta} T_{c_2] a}^\beta - (\Gamma_a)_{\alpha\beta} T_{c_1 c_2}^\beta \quad (2.23)$$

$$\begin{aligned} R_{ab c_1 c_2} = & -\frac{1}{8} D_\beta T_{ab}^\beta (\Gamma_{c_1 c_2})_\gamma{}^\beta - D_{[a} T_{b] c_1 c_2} - \frac{1}{2} T_{ab}^d T_{d c_1 c_2} \\ & + \frac{1}{12} \eta_{c_1 [a} \eta_{b] c_2} T^2 + \frac{1}{4} \eta_{c_1 [b} T_{a] c_2}^2 - \frac{1}{4} \eta_{c_1 [b} T_{a] c_2}^2 \end{aligned} \quad (2.24)$$

where $T_{\alpha\beta}^2 = T_a{}^{c_1 c_2} T_{c_1 c_2 \beta}$, $T^2 = T^{c_1 c_2 c_3} T_{c_1 c_2 c_3}$. The Ricci tensor R_{ab} is the symmetric part of $R_{ac b}$, and is given by

$$R_{ab} = -\frac{1}{8} D_\beta L \gamma_a (\Gamma_b)^{\gamma\beta} + \frac{1}{4} \eta_{ab} T^2 - \frac{3}{2} T_{ab}^2 \quad (2.25)$$

The vanishing of the antisymmetric part of R^c_{acb} as a consequence of the constraints takes the form:

$$\Gamma_{[a}^{\gamma\beta} D_\beta L \gamma_{b]} = 0$$

Finally

$$R \equiv R_a^a = T^2 \quad (2.26)$$

It is convenient to assign a dimension N to the components of the superforms involved: we first fix the dimension of dx^m to -1 and of $d\theta^\mu$ to $-\frac{1}{2}$, and the dimensions of the components in such a way that the scalar superforms be dimensionless. The flat indexed components are then assigned a dimension by setting $N(e^a_m) = 0$ and $N(e^a_\mu) = -\frac{1}{2}$. In this way the different sectors of the Bianchi identities can be classified according to their dimensions. They are solved step by step starting from those with lower dimensions. One can see that the components of T^A and R_A^B are expressed in terms of a single completely antisymmetric superfield T_{abc} and its derivatives. T_{abc} is unconstrained except for Eq. (2.17) and the constraint ensuing from Eq. (2.21). According to Dragon's theorem³⁰⁾, once the T-Bianchi identities are satisfied, the R-Bianchi identities, Eq. (2.7), are also satisfied automatically and do not provide any further information. Therefore the T- and R-Bianchi identities and the constraints (2.10)-(2.12) leave the SUGRA sector largely off shell. As we will see in Section 4, also the H-Bianchi identities, Eq. (2.9), and the H constraints, Eqs. (2.14) and (2.15), are needed in order to put this sector on shell.

The situation is different for the F-Bianchi identities which, under the constraints (2.13), are sufficient to put the SYM sector on shell. In fact they imply:

$$F_{a\alpha} = (\Gamma_a)_{\alpha\beta} \chi^\beta \quad (2.27)$$

$$D_\alpha \chi^\beta = \frac{1}{2} (\Gamma^{c_1 c_2})_\alpha{}^\beta F_{c_1 c_2} \quad (2.28)$$

$$D_{\alpha} F_{ab} = 2 (\Gamma_a D_b)_{\alpha\beta} \chi^{\beta} - T_{abc} (\Gamma^c)_{\alpha\beta} \chi^{\beta} - 2 T_{ca\gamma} (\Gamma_b)_{\gamma\beta} \chi^{\beta} \quad (2.29)$$

together with the field equations

$$(D^a \Gamma_a)_{\alpha\beta} \chi^{\beta} = 0 \quad (2.30)$$

$$D^b F_{ba} = - (\Gamma_a)_{\alpha\beta} \chi^{\alpha} \chi^{\beta} - T_a{}^{bc} F_{bc} - \chi^{\gamma} L_{\gamma a} \quad (2.31)$$

We end this section by collecting some useful relations which follow from Eqs. (2.13), (2.27)-(2.29);

$$\text{Tr}(FF)_{0,4} = 0 = \text{Tr}(FF)_{1,3} \quad (2.32)$$

$$\text{Tr}(FF)_{2,2} = e^a e^b e^{\gamma} e^{\beta} [- (\Gamma_b \Gamma^{c_1 c_2 c_3} \Gamma_a)_{\alpha\beta} \chi_{c_1 c_2 c_3}] \quad (2.33)$$

$$\text{Tr}(FF)_{3,1} = e^a e^b e^c e^{\gamma} [- (\Gamma_c)_{\alpha\beta} F_{ba}{}^{\beta}] \quad (2.34)$$

where

$$\chi_{c_1 c_2 c_3} \equiv \frac{1}{96} (\Gamma_{c_1 c_2 c_3})_{\alpha\beta} \text{Tr}(\chi^{\alpha} \chi^{\beta}) \quad (2.35)$$

$$F_{ab}{}^{\beta} \equiv \text{Tr}(F_{ab} \chi^{\beta}) \quad (2.36)$$

Moreover

$$\frac{1}{2} D_{\alpha} \chi_{c_1 c_2 c_3} = (\Gamma_{c_1})_{\alpha\gamma} Z^{\gamma}{}_{c_2 c_3} \quad (2.37)$$

and

$$F_{ab}{}^{\gamma} = -8 Z_{ab}{}^{\gamma} - 12 (\Gamma_a)_{\alpha\gamma} Z_{b\gamma}{}^{\gamma} + 2 (\Gamma_{ab})^{\alpha}{}_{\beta} Z^{\beta} \quad (2.38)$$

where

$$Z_{a\alpha} \equiv -\frac{1}{3} (\Gamma^c)_{\alpha\beta} Z^{\beta}_{ca} \quad , \quad Z^{\alpha} \equiv (\Gamma^c)^{\alpha\beta} Z_{c\beta} \quad (2.39)$$

Before ending this section it is perhaps worth recalling that the physical fields are identified with specific superfields at $\theta = 0$; the graviton with e_m^a , the gravitino with e_m^{α} , the dilaton with ϕ and the gravitello with $D\phi$, the graviphoton with B_{mn} , the gauge field with A_m and the gaugino with χ^{α} .

3. - THE MAIN IDENTITY FOR $\text{Tr}(\text{RR})$

The H-Bianchi identities, Eq. (8), subject to the constraints (2.14) and (2.15) have been solved for $c_2 = 0$ ¹²⁾⁻¹⁴⁾. They lead to field equations for the SUGRA sector which, together with Eqs. (2.30) and (2.31) reproduce the Chapline-Manton model, modulo a redefinition of the fields involved. The crucial point in deriving this solution is the vanishing of the right-hand side of Eq. (2.9) for $c_2 = 0$ in the sectors (0,4) and (1,3). For $c_2 \neq 0$ the right-hand side of Eq. (2.9) in the same sectors no longer vanishes. This cannot be cured by means of a field redefinition so as to obtain a new curvature R' with $R'^b_{\alpha\beta a} = 0$, which was shown to be possible in Ref. 31) for pure SUGRA. In the present case, it is not possible.

However, in Ref. 23) it was proved that Eq. (2.9) can be rewritten in a form which has the identical structure as the same equation with $c_2 = 0$. This is based on the following theorem.

Theorem (3.1)

Under the constraints (2.10) and (2.11), the T-Bianchi identities imply:

$$\text{Tr}(\text{RR}) = dX + K \quad (3.2)$$

where $X = 1/6 e^A e^B e^C X_{CBA}$ and $K = 1/24 e^A e^B e^C e^D K_{DCBA}$ are gauge and Lorentz invariant superforms and

$$K_{0,4} = 0 = K_{1,3} \quad (3.3)$$

Moreover $K_{2,2}$ and $K_{3,1}$ have the structure

$$K_{2,2} = e^a e^b e^c e^d [- (\Gamma_b \Gamma^{c_1 c_2 c_3} \Gamma_a)_{\alpha\beta} L_{c_1 c_2 c_3}] \quad (3.4)$$

$$K_{3,1} = e^a e^b e^c e^d [- (\Gamma_c)_{\alpha\beta} K^{\beta}_{ba}] \quad (3.5)$$

and there exists a tensor V^{γ}_{ab} such that

$$\frac{1}{2} D_{\alpha} L_{a_1 a_2 a_3} = (\Gamma_{[c_1})_{\alpha\gamma} V^{\gamma}_{c_2 c_3]} \quad (3.6)$$

$$K^{\alpha}_{ab} = -8 V^{\alpha}_{ab} - 12 (\Gamma_{[a})^{\alpha\beta} V_{b]\beta} + 2 (\Gamma_{ab})^{\alpha\beta} V_{\beta} \quad (3.7)$$

where

$$V_{a\alpha} \equiv -\frac{1}{3} (\Gamma^c)_{\alpha\gamma} V^{\gamma}_{ca} \quad , \quad V^{\alpha} \equiv (\Gamma^c)^{\alpha\gamma} V_{c\gamma}$$

Before proving the theorem, let us remark that Eqs. (3.3)-(3.7) are in one-to-one correspondence with Eqs. (2.32)-(2.34) and (2.37)-(2.39). It is apparent that $K_{2,2}$ and $K_{3,1}$ have the same content in terms of $SO(1,9)$ IR's as $\text{Tr}(F,F)_{2,2}$ and $\text{Tr}(FF)_{3,1}$, respectively. Therefore, if we define

$$\hat{H} = H + c_2 X \quad (3.8)$$

Equation (2.9) becomes

$$d\hat{H} = c_1 \text{Tr}(FF) - c_2 K \quad (3.9)$$

where now the right-hand side has the same structure as Eq. (2.9) with $c_2 = 0$. As a consequence it is natural to impose the constraints

$$\hat{H}_{\alpha\beta\gamma} = 0 \quad (3.10)$$

$$\hat{H}_{a\alpha\beta} = (\Gamma_a)_{\alpha\beta} \phi \quad (3.11)$$

Once X is known, they just amount to fixing the unspecified terms $O(c_2)$ in the constraints (2.14) and (2.15). As we will see $X_{\alpha\beta\gamma}$ vanishes, so that Eq. (2.14) is left unchanged.

Equation (3.9) subject to the constraints (3.10) and (3.11) can be solved consistently in the same way as Eq. (2.9) when $c_2 = 0$. Now let us pass to the detailed proof of Theorem (3.1) and to the explicit calculation of X and K . To this end we enrich our notation as follows³¹⁾. We split the exterior operator d in the superspace according to the splitting of superforms introduced at the beginning of Section 2; we simply regard d as a sum of operators each of which sends a (p,q) -superform into a (p',q') -superform and will be thus assigned a degree $(r,s) = (p'-p, q'-q)$. Therefore, if d is applied to scalar superforms, we write

$$d = \Delta = \bar{d} + D + T + \tau \quad (3.12)$$

where $\bar{d}$, D , T , τ have degree $(1,0)$, $(0,1)$, $(-1,2)$ and $(2,1)$, respectively. More specifically $\bar{d} = e^a D_a + T_{(1,0)}$, $D = e^\alpha D_\alpha + T_{(0,1)}$, $T = T_{(-1,2)}$, $\tau = T_{(2,-1)}$; D_a and D_α are the ordinary flat space-time and spinor covariant derivatives acting only on the components, while $T_{(r,s)}$ acts only on the vielbein basis as follows:

$$\begin{aligned} T_{(-1,2)} e^a &= (\Gamma^a)_{\alpha\beta} e^\alpha e^\beta & T_{(-1,2)} e^\alpha &= 0 \\ T_{(2,-1)} e^a &= 0 & T_{(2,-1)} e^\alpha &= \frac{1}{2} e^b e^c T_{cb}{}^\alpha \\ T_{(1,0)} e^a &= \frac{1}{2} e^b e^c T_{cb}{}^a & T_{(1,0)} e^\alpha &= e^b e^\gamma T_{b\gamma}{}^\alpha \\ T_{(0,1)} e^a &= e^b e^\gamma T_{\gamma b}{}^a = 0 & T_{(0,1)} e^\alpha &= \frac{1}{2} e^\beta e^\gamma T_{\beta\gamma}{}^\alpha \end{aligned} \quad (3.13)$$

From the nilpotency of d , $d^2 = 0$, a set of useful identities follows

$$\tau^2 = 0 \quad (3.14a)$$

$$\bar{d}\tau + \tau\bar{d} = 0 \quad (3.14b)$$

$$\bar{d}^2 + D\tau + \tau D = 0 \quad (3.14c)$$

$$\bar{d}D + D\bar{d} + T\tau + \tau T = 0 \quad (3.14d)$$

$$D^2 + \bar{d}T + T\bar{d} = 0 \quad (3.14e)$$

$$DT + TD = 0 \quad (3.14f)$$

$$T^2 = 0 \quad (3.14g)$$

In particular T is a coboundary operator.

Let us set $Q \equiv \text{Tr}(RR)$ and consider the identity

$$dQ = 0 \quad (3.15)$$

which follows from the Bianchi identity (2.7). We can project it onto the sectors $(p, 5-p)$ and obtain

$$TQ_{0,4} = 0 \quad (3.16a)$$

$$TQ_{1,3} + DQ_{0,4} = 0 \quad (3.16b)$$

$$TQ_{2,2} + DQ_{1,3} + \bar{d}Q_{0,4} = 0 \quad (3.16c)$$

$$TQ_{3,1} + DQ_{2,2} + \bar{d}Q_{1,3} + \tau Q_{0,4} = 0 \quad (3.16d)$$

$$TQ_{4,0} + DQ_{3,1} + \bar{d}Q_{2,2} + \tau Q_{1,3} = 0 \quad (3.16e)$$

$$DQ_{4,0} + \bar{d}Q_{3,1} + \tau Q_{2,2} = 0 \quad (3.16f)$$

$$\bar{d}Q_{4,0} + \tau Q_{3,1} = 0 \quad (3.16g)$$

$$\tau Q_{4,0} = 0 \quad (3.16h)$$

Let us denote by $S_{p,q}^N$ the space of (p,q) -superforms with dimension N which are functions of the components of torsion, curvatures and their derivatives. Then T maps $S_{p,q}^N$ into $S_{p-1,q+2}^N$.

Using Eq. (2.22) and the formulas of Appendix A, one can verify that $Q_{0,4}$ is indeed a coboundary of T

$$Q_{0,4} = T X_{1,2} \quad (3.17)$$

and $X_{1,2} \in S_{1,2}^2$. Actually one easily gets

$$Q_{0,4} = T \bar{X}_{1,2} \quad (3.18)$$

where

$$\bar{X}_{1,2} = e^a e^\alpha e^\beta (-R_{\alpha\beta}{}^{c_1 c_2} T_{c_1 c_2} a) \quad (3.19)$$

However, $X_{1,2}$ is still ambiguous since Eq. (3.18) is also satisfied by

$$X_{1,2} = \bar{X}_{1,2} + Y_{1,2} \quad (3.20)$$

provided $Y_{1,2}$ is a cocycle of T belonging to $S_{1,2}^2$. The T -coboundaries in Eq. (3.20) can be disregarded. They simply express the fact that X in Eq. (3.2) is defined modulo trivial cocycles of d and can be absorbed in a redefinition of the two-superform B in Eq. (3.8).

There are three independent non-trivial cocycles of T in $S_{1,2}^2$. They are:

$$Y_{1,2}^{(0)} = e^a e^\alpha e^\beta (\Gamma_a)_{\alpha\beta} T^2 \quad (3.21)$$

$$Y_{1,2}^{(1)} = e^a e^\alpha e^\beta D_c R_{\alpha\beta} a^c \quad (3.22)$$

$$Y_{1,2}^{(2)} = e^a e^\alpha e^\beta (\Gamma_a{}^{c_1 c_2 c_3 c_4})_{\alpha\beta} T_{c_1 c_2}{}^d T_{c_3 c_4} \quad (3.23)$$

$Y_{1,2}^{(0)}$ will be omitted for the time being. It amounts to a redefinition of the superfield ϕ in Eq. (3.11). Then $Y_{1,2}$ in Eq. (3.20) is, a priori, a linear combination of $Y_{1,2}^{(i)}$ ($i=1,2$):

$$Y_{1,2} = \kappa_{(1)} Y_{1,2}^{(1)} + \kappa_{(2)} Y_{1,2}^{(2)} \quad (3.24)$$

$Y_{1,2}^{(i)}$ will play an important rôle in the following. Substituting Eq. (3.17) into Eq. (3.16b) and using Eq. (3.14b), one obtains

$$T(Q_{1,3} - DX_{1,2}) = 0 \quad (3.25)$$

so that $Q_{1,3} - DX_{1,2} \in S_{1,3}^{5/2}$ is a cocycle of T . If $X_{1,2}$ is given by (3.20), it is in general non-trivial. However, and this is the crucial point of the theorem, it becomes a coboundary if the numerical coefficients in Eq. (3.24) are suitably chosen: precisely, when $\kappa_{(1)} = -1$ and $\kappa_{(2)} = 0$. In such case

$$X_{1,2} = e^a e^\alpha e^\beta (-R_{\alpha\beta}{}^{\epsilon_1\epsilon_2} T_{\epsilon_1\epsilon_2} a - D_\epsilon R_{\alpha\beta} a^\epsilon) \quad (3.26)$$

an equivalent form for $X_{1,2}$ is

$$\begin{aligned} X_{1,2} = e^a e^\alpha e^\beta \left[\frac{1}{4} D_\gamma L_{\beta\alpha} + \frac{1}{4} (\Gamma_a{}^{\epsilon_1\epsilon_2\epsilon_3\epsilon_4})_{\alpha\beta} T^{\epsilon_1\epsilon_2\epsilon_3\epsilon_4} + \right. \\ \left. - 2 \Gamma_{\alpha\beta}^\epsilon \left(\frac{1}{4} \eta_{ac} T^{\epsilon\epsilon} - R_{ac} \right) \right] \end{aligned} \quad (3.27)$$

With this choice of $X_{1,2}$ we have

$$Q_{1,3} = TX_{2,1} + DX_{1,2} \quad (3.28)$$

A lengthy calculation gives for $X_{2,1}$ the explicit form

$$\begin{aligned} X_{2,1} = e^a e^b e^\gamma \left[-D^\epsilon R_{\gamma cab} - T_{c\gamma}{}^\delta R_{scab} - 2 T_{ab}{}^\epsilon L_{\gamma\epsilon} + \right. \\ \left. + \frac{1}{4} D_{[b} L_{a]\gamma} + \frac{1}{4} T_{\alpha\gamma}{}^\delta L_{\delta b} - \frac{1}{4} T_a{}^{\epsilon_1\epsilon_2} (\Gamma_{\epsilon_1\epsilon_2}{}^\delta)_{\gamma}{}^\delta L_{\delta c\epsilon_2} + \right. \\ \left. + \frac{1}{3} (\Gamma_{ab})_{\gamma}{}^\delta (D_\delta T_{\epsilon_1\epsilon_2\epsilon_3}) T^{\epsilon_1\epsilon_2\epsilon_3} \right] \end{aligned} \quad (3.29)$$

There is no arbitrariness in $X_{2,1}$ since there are no cocycles of T in the sector (2.1) (see Appendix B). Inserting Eqs. (3.17) and (3.28) into Eq. (3.16c) and using (3.14b,c) we obtain

$$T W_{2,2} = 0 \quad (3.30)$$

where

$$W_{2,2} = Q_{2,2} - D X_{2,1} - \bar{d} X_{1,2}$$

The non-trivial cocycles of T in the sector $(2,2)$ have, modulo coboundaries, the general structure

$$e^a e^b e^q e^p (\Gamma_b \Gamma^{c_1 c_2 c_3} \Gamma_a)_{q\beta} L_{c_1 c_2 c_3} \quad (3.31)$$

so that in full generality one can write

$$W_{2,2} = T X_{3,0} - e^a e^b e^q e^p (\Gamma_b \Gamma^{c_1 c_2 c_3} \Gamma_a)_{q\beta} L_{c_1 c_2 c_3} \quad (3.32)$$

An explicit (and lengthy) calculation gives

$$\begin{aligned} X_{3,0} = & e^{c_3} e^{c_2} e^{c_1} \left[-\frac{1}{6} \square T_{c_1 c_2 c_3} - \frac{1}{3} \left(\frac{4}{2} R_{b_1 b_2 c_1 c_3} - 2 D_{b_1} T_{b_2 c_1 c_3} \right) T^{b_1 b_2 c_1} + \right. \\ & -\frac{1}{3} T_{c_1 c_2}^b \left(\frac{1}{2} R_{b c_3} - 3 T_{b c_3}^2 \right) - \frac{1}{12} T^2 T_{c_1 c_2 c_3} + \frac{2}{3} T^3 c_1 c_2 c_3 + \\ & -\frac{1}{48} \varepsilon_{c_1 c_2 c_3}^{b_1 \dots b_7} \left(T_{b_1 b_2 b_3} D_{b_4} T_{b_5 b_6 b_7} + \frac{3}{2} T_{b_1 b_2 b_3} T_{b_4 b_5 b_6 b_7}^2 \right) + \\ & \left. -2 T_{c_1 c_2}^{s\gamma} (\Gamma_{c_3})_{s\gamma} + T_{c_1 c_2}^q L_{q c_3} - \frac{1}{12} (\Gamma_{c_1 c_2 c_3})^{\gamma\delta} L_{\gamma\delta}^2 - \frac{1}{48} (\Gamma_{c_1 c_2 c_3})^{\gamma\delta} D_{\gamma} T^{b_1 b_2 b_3} D_{\delta} T_{b_1 b_2 b_3} \right] \quad (3.33) \\ L_{c_1 c_2 c_3} = & -\frac{1}{12} \square T_{c_1 c_2 c_3} - T_{c_1}^{b_1 b_2} \left(-\frac{1}{6} R_{b_1 b_2 c_2 c_3} + \frac{5}{8} D_{c_2} T_{c_3 b_1 b_2} + \frac{19}{24} D_{b_1} T_{b_2 c_2 c_3} \right) + \\ & -T_{c_1 c_2}^b \left(\frac{4}{8} T_{b c_3}^2 + \frac{1}{3} R_{b c_3} \right) - \frac{1}{288} \varepsilon_{c_1 c_2 c_3}^{b_1 \dots b_7} \left(T_{b_1 b_2 b_3} \Delta_{b_4} T_{b_5 b_6 b_7} + \frac{3}{2} T_{b_1 b_2 b_3} T_{b_4 b_5 b_6 b_7}^2 \right) + \\ & -\frac{13}{12} T_{c_1 c_2 c_3}^3 + \frac{1}{16} T^2 T_{c_1 c_2 c_3} - \frac{1}{24} (\Gamma_{c_1 c_2 c_3})_{s\gamma} T^{s\gamma} - \frac{1}{48} (\Gamma_{c_1 c_2 c_3})^{\gamma\delta} L_{\gamma\delta}^2 \\ & + \frac{1}{2} L_{\gamma c_1} (\Gamma_{c_2})^{\gamma\delta} L_{\delta c_3} + \frac{1}{288} (\Gamma_{c_1 c_2 c_3})^{\gamma\delta} (D_{\gamma} T^{b_1 b_2 b_3}) D_{\delta} T_{b_1 b_2 b_3} \quad (3.34) \end{aligned}$$

where in the right-hand side of Eq. (3.34) antisymmetrization of c_1 c_2 and c_3 is understood and

$$T_{a_1 a_2 a_3 a_4}^2 \equiv T_{a_1 a_2}^c T_{c a_3 a_4} \quad , \quad T_{a_1 a_2 a_3}^3 \equiv T_{b a_1}^c T_{c a_2}^d T_{d a_3}^b$$

$$L_{\gamma\delta}^2 \equiv L_{\gamma b} L_{\delta}^b \quad , \quad T^{2\delta\gamma} \equiv T_{c_1 c_2}^{\delta} T^{c_1 c_2 \gamma} \quad , \quad T_{c_1 c_2}^{\delta\gamma} \equiv T_{c_1 b}^{\delta} T_{c_2}^{b\gamma} \quad , \quad \square = D^a D_a$$

From Eqs. (3.17), (3.28) and (3.32), by defining

$$X = X_{1,2} + X_{2,1} + X_{3,0} \quad (3.35)$$

one obtains Eq. (3.2), where K satisfies Eq. (3.3) and both X and K are invariant superforms.

In order to prove Eqs. (3.4)-(3.7) let us consider the obvious identity

$$dK = 0 \quad (3.36)$$

In the sector (2,3) this identity takes the form

$$TK_{3,1} + DK_{2,2} = 0 \quad (3.37)$$

The content of both $K_{3,1}$ and $DK_{2,2}$ in terms of $SO(1,9)$ IR's is

$$\overline{16} \times 120 = \overline{1200} + 560 + \overline{144} + 16$$

as shown in Appendix C the 1200 IR cannot fulfil Eq. (3.37) so that Eqs. (3.5) and (3.6) follow. Inserting Eq. (3.5) into Eq. (3.37), due to the invertibility of T in the (4,1) sector, one obtains Eq. (3.7).

Again we have explicitly checked Eq. (3.6) and calculated $V_{c_1 c_2}^{\alpha}$, $V_{\alpha a}$ and v^{α} . The result is given in Appendix D.

This concludes the proof of Theorem (3.1).

Finally, $K_{4,0}$ can be obtained from its definitions, i.e.,

$$K_{4,0} = Q_{4,0} - (dX)_{4,0} \quad (3.38)$$

An alternative method consists in projecting Eq. (3.36) into the sector (3,2)

$$T K_{4,0} + \bar{d} K_{2,2} + D K_{3,1} = 0$$

Inverting the T operator in this equation one gets

$$K_{a_1 a_2 a_3 a_4} = -\frac{3}{8} D_\beta K_{[a_1 a_2} \gamma^{\beta} (\Gamma_{a_3 a_4])_\gamma} - 36 D_{[a_1} L_{a_2 a_3 a_4]} + 36 T_{[a_1}^b L_{b a_2 a_3 a_4]}$$

This concludes the explicit calculation of the superforms X and K.

4. - SOLUTION OF THE H-BIANCHI IDENTITIES

Let us go back to Eq. (3.9). Projected onto the various sectors it becomes

$$(d\hat{H})_{0,4} = 0 \quad (4.1a)$$

$$(d\hat{H})_{1,3} = 0 \quad (4.1b)$$

$$(d\hat{H})_{2,2} = c_1 \text{Tr}(FF)_{2,2} - c_2 K_{2,2} \quad (4.1c)$$

$$(d\hat{H})_{3,1} = c_1 \text{Tr}(FF)_{3,1} - c_2 K_{3,1} \quad (4.1d)$$

$$(d\hat{H})_{3,2} = c_1 \text{Tr}(FF)_{4,0} - c_2 K_{4,0} \quad (4.1e)$$

We remark that once Eqs. (4.1a,b,c) are satisfied, Eqs. (4.1d,e) automatically hold. Indeed let us trivially observe that

$$d[d\hat{H} - c_1 \text{Tr}(FF) + c_2 K] = 0 \quad (4.2)$$

This equation projected into the sector (2,3) becomes

$$T [(d\hat{H})_{3,1} - c_1 (Tr(FF))_{3,1} + c_2 K_{3,1}] = 0 \quad (4.3)$$

if Eqs. (4.1a,b,c) hold. Equation (4.3) implies Eq. (4.1d) since T applied to (3,1)-superforms is invertible. In the same way Eq. (4.2) in the (3,2)-sector, tells us that if Eqs. (4.1a,b,c,d) are true then Eq. (4.1e) holds.

We want now to find a solution of Eqs. (4.1) under the restrictions (3.10) and (3.11). The above remark tells us that we may limit ourselves to solving Eqs. (4.1a,b,c). We will not find any additional information in Eqs. (4.1d,e).

Let us define

$$\begin{aligned} D_\alpha \phi &\equiv \lambda_\alpha \quad , \quad \phi_{abc} \equiv \frac{1}{96} (\Gamma_{abc})^{\tau\beta} D_\tau D_\beta \phi \\ \lambda_{abc} &\equiv \frac{1}{96} \lambda_\alpha (\Gamma_{abc})^{\tau\beta} \lambda_\beta \end{aligned} \quad (4.4)$$

Equation (4.1a) is identically satisfied. Equation (4.1b) implies

$$\hat{H}_{ab\alpha} = -\frac{1}{2} (\Gamma_{ab})_\alpha{}^\beta \lambda_\beta \quad (4.5)$$

If Eqs. (4.1a,b) hold, both sides of Eq. (4.1c) are cocycles of T. In the sector (2,2) there are trivial and non-trivial cocycles: they all belong to the 120 IR of SO(1,9). The non-trivial ones have the general structure given in Eq. (3.31). Therefore, Eq. (4.1c) gives rise to the following two conditions:

$$\hat{H}_{abc} = -\phi T_{abc} + 3 \phi_{abc} + 12 c_1 \chi_{abc} - 12 c_2 L_{abc} \quad (4.6)$$

$$\phi_{abc} = -\frac{1}{6} T_{abc} + 4 c_1 \chi_{abc} - 4 c_2 L_{abc} \quad (4.7)$$

Eliminating ϕ_{abc} from these two equations one obtains

$$T_{abc} = -\frac{2}{3\phi} \hat{H}_{abc} + \frac{16}{\phi} (c_1 \chi_{abc} - c_2 L_{abc}) \quad (4.8)$$

Inserting Eq. (4.1d) into the spinor-like derivative of Eq. (4.8) and remembering Eq. (2.20), we derive the following expression for $L_{a\alpha}$:

$$\begin{aligned} L_{a\alpha} = & -\frac{1}{2\phi} \left[D_a \lambda_\alpha + T_{a\alpha}{}^\beta \lambda_\beta + \frac{1}{2} (\Gamma^{c_1 c_2})_\alpha{}^\beta T_{c_1 c_2 a} \lambda_\beta + \right. \\ & \left. + 8 (\Gamma_a)_{\alpha\gamma} (c_1 Z^\gamma - c_2 V^\gamma) - 48 (c_1 Z_a - c_2 V_a \alpha) \right] \quad (4.9) \end{aligned}$$

Recalling the definition of $L_{a\alpha}$ and the absence in it of the 16 IR, we obtain from Eq. (4.9) the equations of motion of the gravitino, gravitello and dilaton, respectively:

$$(\Gamma^{ab})_{\alpha\beta} T_{ab}{}^\beta = -2 L_{c\alpha} \quad (4.10)$$

$$(\Gamma^a)^\alpha{}^\beta D_a \lambda_\beta = 2 \psi^\alpha{}^\beta \lambda_\beta - 32 (c_1 Z^\alpha - c_2 V^\alpha) \quad (4.11)$$

$$\begin{aligned} \square \phi = & -\frac{\phi}{2} T^2 + 12 T^{a_1 a_2 a_3} (c_1 \chi_{a_1 a_2 a_3} - c_2 L_{a_1 a_2 a_3}) + \\ & + 2 D_\alpha (c_1 Z^\alpha - c_2 V^\alpha) \quad (4.12) \end{aligned}$$

Equation (4.8) together with Eq. (2.17) implies the equation of motion for the graviphoton

$$D^a \hat{H}_{abc} = -\frac{3}{2} (D^a \phi) T_{abc} + 24 (c_1 D^a \chi_{abc} - c_2 D^a L_{abc}) \quad (4.13)$$

Equation (2.25) provides the equation of motion for the graviton when Eq. (4.9) is taken into account:

$$\begin{aligned} R_{ab} = & -\frac{1}{2\phi^2} \lambda_\alpha (\Gamma_a)^\alpha{}^\beta D_b \lambda_\beta - \frac{1}{\phi} D_a D_b \phi - 2 T_{ab}{}^2 + \frac{1}{4} \eta_{ab} T^2 + \\ & -\frac{1}{4\phi^2} \lambda_\alpha (\Gamma_{d_1 d_2 a})^\alpha{}^\beta \lambda_\beta T_b{}^{d_1 d_2} + \frac{1}{48\phi^2} \eta_{ab} \lambda_\alpha (\Gamma_{d_1 d_2 d_3})^\alpha{}^\beta \lambda_\beta T^{d_1 d_2 d_3} \\ & + \frac{1}{\phi^2} \left[4 (c_1 Z^\beta - c_2 V^\beta) \eta_{ab} - 24 (\Gamma_b)^\alpha{}^\beta (c_1 Z_a \alpha - c_2 V_a \alpha) \right] \lambda_\beta \\ & + \frac{1}{2\phi} \left[24 T_{d_1 d_2 a} (c_1 \chi^{d_1 d_2}{}_b - c_2 L^{d_1 d_2}{}_b) + \eta_{ab} D_\beta (c_1 Z^\beta - c_2 V^\beta) \right. \\ & \left. - 6 (\Gamma_b)^\alpha{}^\beta D_\beta (c_1 Z_a \alpha - c_2 V_a \alpha) \right] \quad (4.14) \end{aligned}$$

where () denotes symmetrization.

No further independent conditions follow from Eqs. (4.1). Finally we recall that $\chi_{a_1 a_2 a_3}$ and $L_{a_1 a_2 a_3}$ are given by Eqs. (2.35) and (3.34) respectively, Z^α and $Z_{a\alpha}$ by Eq. (2.39), while V^α and $V_{a\alpha}$ can be derived from the expression of V_{ab}^γ which is given in Appendix D.

Let us now come to the auxiliary superfield T_{abc} . It must satisfy Eq. (4.8), where

$$\hat{H}_{abc} = H_{abc} + c_2 X_{abc}$$

where $X_{3,0} = e^c e^b e^a 1/6 X_{abc}$. So, from Eqs. (3.33) and (3.34) one sees that both L_{abc} and X_{abc} depend on T_{abc} . Therefore if we remove T_{abc} from the field equations by a recursive procedure, they formally become equations for infinite series. This is what the approximate solutions existing in the literature suggested.

However, these equations also present a perhaps unexpected feature. For it is evident from Eqs. (3.33) and (3.34) and from the formulas of Appendix D, that the superfields X_{abc} , L_{abc} , $V_{a\alpha}$ and $\Gamma_{(a}^{\alpha\beta} D_{\alpha} V_{\beta b)}$ contain linear terms which involve $\square$.

$$\begin{aligned} X_{abc} &= -\square T_{abc} + \hat{X}_{abc} \\ L_{abc} &= -\frac{1}{12} \square T_{abc} + \hat{L}_{abc} \\ V_{a\alpha} &= -\frac{1}{12} \square L_{\alpha a} + \hat{V}_{a\alpha} \\ (\Gamma_{(a}^{\alpha\beta} D_{\alpha} V_{\beta b)}) &= \frac{2}{3} \square R_{ab} + \hat{W}_{(ab)} \end{aligned} \quad (4.15)$$

where $\hat{X}_{abc}$, $\hat{L}_{abc}$, $\hat{V}_{a\alpha}$ and $\hat{W}_{(ab)}$ represent genuine currents (i.e. terms non-linear in the superfields). Let us suppose that ϕ acquire a v.e.v., i.e.,

$$\phi = \kappa + \hat{\phi}$$

where κ is a constant related to the Planck mass and the v.e.v. of ϕ vanishes. Therefore the graviton and the gravitino equations contain modes which propagate at the Planck mass. Also the torsion T_{abc} propagates at the Planck mass as can be seen by rewriting Eq. (4.8) in the form

$$\begin{aligned} -2c_2 \square T_{abc} + \kappa T_{abc} = & -\frac{2}{3} H_{abc} - \hat{\phi} T_{abc} - \frac{2}{3} c_2 \hat{X}_{abc} + \\ & + 16 (c_1 X_{abc} - c_2 \hat{L}_{abc}) \end{aligned} \quad (4.16)$$

We remark that, from Eqs. (4.15) the same linear combination $\kappa T_{abc} - 2c_2 T_{abc}$ appears in Eqs. (4.6) and (4.7). The equations of motion of the graviphoton, gravitello and dilaton and those of the gauge sector do not contain propagating massive modes.

Unfortunately some of the above massive modes are poltergeist ghosts and the problems connected with their presence will be discussed in the next section. In order to prepare such a discussion it is convenient to rewrite Eqs. (4.9), (4.14) and (4.16) in a different form.

From Eq. (4.11) one gets

$$\square \lambda_\alpha = (\Gamma^\alpha)_{\alpha\gamma} D_\alpha [2 \psi^{\gamma\beta} \lambda_\beta - 32 (c_1 Z^\gamma - c_2 V^\gamma)] - (\Gamma_{ab})_\alpha{}^\beta D^a D^b \lambda_\beta \quad (4.16)$$

From the (4,0) sector of Eq. (2.9) one obtains

$$\begin{aligned} \square H_{a_1 a_2 a_3} = & 3 D^b D_{[a_1} H_{a_2 a_3] b} - 6 D^b [T_{[a_1 a_2}{}^\gamma (\frac{1}{2} (\Gamma_{a_3 b])}{}^\delta \lambda_\delta + c_2 X_{(a_1) a_3 b}]^\gamma + \\ & - T_{[a_1 a_2}{}^d H_{d a_3 b]} + c_1 \text{Tr} (F_{[a_1 a_2} F_{a_3 b]}) + c_2 R_{[a_1 a_2}{}^{d_1 d_2} R_{a_3 b] d_1 d_2} \end{aligned} \quad (4.17)$$

Therefore with the definitions:

$$P_{abc} = T_{abc} + \frac{2}{3\kappa} H_{abc} \quad (4.18a)$$

$$P_{a\alpha} = L_{a\alpha} + \frac{1}{2\kappa} D_\alpha \lambda_\alpha \quad (4.18b)$$

$$P_{ab} = R_{ab} + \frac{1}{\kappa} D_{(a} D_{b)} \phi \quad (4.18c)$$

and taking into account Eqs. (4.12), (4.16) and (4.17), Eqs. (4.8), (4.9) and (4.14) become

$$(\square - \frac{\kappa}{2c_2}) P_{abc} = J_{abc} \quad (4.19a)$$

$$\left(\square - \frac{\kappa}{2\zeta_2}\right) P_{a\alpha} = J_{a\alpha} \quad (4.19b)$$

$$\left(\square - \frac{\kappa}{2\zeta_2}\right) P_{ab} = J_{ab} \quad (4.19c)$$

J_{abc} , $J_{a\alpha}$ and J_{ab} are genuine currents and do not contain terms linear in the fields. Their actual expressions are not needed in our discussion and will not be given here.

It is important to verify that the field equations obtained by applying the space-like covariant derivative D_α to Eqs. (4.19) do not involve new massive modes. When we apply D_α to Eq. (4.19a), the new field T_{ab}^α pops up in the left-hand side; however, the contribution from the 560 IR vanishes at the linear level, while that of the 144 IR just gives Eq. (4.19b). No problem arises with Eqs. (4.19b,c) since $D_\alpha L_{a\beta}$ involves only R_{ab} and $D_\alpha R_{ab}$ does not involve new superfields.

It is easy to count the number of massive modes in Eq. (4.19): at the linear level, $\Gamma^{\alpha\beta} P_{a\alpha}$, $D^a P_{a\alpha}$, $D^a P_{ab}$ and $D^a P_{abc}$ vanish; then by taking into account the condition $D^a P_{abc} = 0$ at the linear level, one has $(120-45+10-1) + (54-10) = 128$ bosonic modes and $160-16-16 = 128$ fermionic modes.

5. - DISCUSSION

At this point a major question concerns the relation between the field equations obtained in the previous section and the effective actions derived from superstring models. In particular we should clarify the rôle of the ghosts we have found.

First we examine the consequences on Eqs. (4.9)-(4.14) of superfield redefinitions which preserve the transformation properties under Lorentz and gauge transformations. The supervielbeins can transform as follows

$$e'^A = e^B Q_B^A, \quad e^A = e'^B \bar{Q}_B^A \quad (5.1)$$

where

$$Q_A^B = \begin{pmatrix} Q_a^b & Q_\alpha^{\beta} \\ Q_r^b & Q_\gamma^{\beta} \end{pmatrix} \quad \bar{Q}_A^B = \begin{pmatrix} \bar{Q}_a^b & \bar{Q}_\alpha^{\beta} \\ \bar{Q}_r^b & \bar{Q}_\gamma^{\beta} \end{pmatrix}$$

and

$$Q_A^C \bar{Q}_C^B = \delta_A^B$$

Q_A^B are Lorentz-covariant and (gauge-invariant) superfields.

The superconnections, the two-form B and the dilaton superfield can be modified as follows:

$$\Omega_A'^B = \Omega_A^B - Y_A^B = \Omega_A^B - e^C Y_{CA}^B \quad (5.2)$$

$$A' = A - C = A - e^B C_B \quad (5.3)$$

$$B' = B - \Sigma + c_1 \text{Tr}(CA) - c_2 Y_A^B \Omega_B^A \quad (5.4)$$

$$\phi' = \phi - \psi \quad (5.5)$$

$Y_A^B(C)$ is a covariant Lorentz-valued ($\mathcal{H}$ -valued) one-superform, Σ is an invariant two-superform and ϕ is a scalar superfield. All these superfields of course depend on the superfields which characterize our model. The new torsion, curvature and H field are

$$\begin{aligned} T_{B_1 B_2}^A = & (-1)^{B_2 E_2} \bar{Q}_{B_1}^{E_1} \bar{Q}_{B_2}^{E_2} (-T_{E_2 E_1}^D Q_D^A - 2 D_{E_2} Q_{E_1}^A) + \\ & + 2 \bar{Q}_{[B_1}^E Y_{E B_2]}^A \end{aligned} \quad (5.6)$$

$$R_A'^B \equiv \frac{1}{2} e^{E_1 E_2} e^{E_3 E_4} R_{E_2 E_1 A}^B = -D Y_A^B + Y_A^E Y_E^B + R_A^B \quad (5.7)$$

$$H' \equiv \frac{1}{6} e^{iA} e^{iB} e^{iC} H_{CBA} = H + c_1 \text{Tr}(C D C) - c_2 \text{Tr}(Y D Y) + \\ - 2 c_1 \text{Tr}(F C) + 2 c_2 \text{Tr}(R Y) - \frac{2}{3} c_1 \text{Tr} C^3 + \frac{2}{3} c_2 \text{Tr} Y^3 - d \Sigma \quad (5.8)$$

Since

$$\text{Tr}(R R') = \text{Tr}(R R) + d \text{Tr}(Y D Y - \frac{2}{3} Y^3 - 2 R Y)$$

and

$$\text{Tr}(F' F') = \text{Tr}(F F) + d \text{Tr}(C d C - \frac{2}{3} C^3 - 2 F C)$$

it follows that

$$\hat{H}' = \hat{H} - d \Sigma, \quad K' = K \quad (5.9)$$

The new field equations which follow from Eqs. (4.9)-(4.14) by performing the above transformations involve the superfields Q_A^B , Y_{Ab}^B , etc. When comparing our results with the effective superstring actions, we must take into account the arbitrariness arising from the possibility of redefining the superfields as above. However, even allowing for such an arbitrariness, it is not clear whether our field equations reproduce those deriving from an effective superstring action to all orders. Actually it is still unclear whether we can reproduce in this way the term quartic in $R_{a_1 a_2 a_3 a_4}$ which arises in superstring models^{2),4)}. The supersymmetric completion of such terms has been discussed by Kallosh²⁹⁾.

Let us now turn to the ghost problem and consider the transformations (5.1) with

$$Q_a^b = \delta_a^b + \frac{2c_2}{K} R_a^b - \frac{1}{8K} \delta_a^b \phi \\ Q_a^b = \frac{2c_2}{K} (\Gamma^b)^{\beta\gamma} D_b L_{\gamma a} - \frac{1}{4K} (\Gamma_a)^{\beta\gamma} \lambda_{\gamma}, \quad \bar{Q}_a^{\beta} = -Q_a^{\beta} \\ Y_{abc} = \frac{4c_2}{K} D_{[b} R_{c]a} + \frac{1}{4K} \eta_{a[b} D_{c]} \phi \\ Q_{\alpha}^{\beta} = \delta_{\alpha}^{\beta} \\ Q_{\alpha}^a = Y_{\alpha ab} = C = \Sigma = 0 \quad (5.10)$$

Then in the field equations for the transformed gravitino and graviton, the linear terms corresponding to the ghost propagation are absent. We obtain, for example:

$$(5.11)$$

where R'_{ab} is given by Eqs. (5.7) and (5.10) and j'_{ab} is a true current. Equation (5.11) represents the equation of propagation for a zero mass mode only. However, the massive modes do not decouple from the theory since in j'_{ab} their fields, i.e., P_{abc} , $P_{a\alpha}$ and P_{ab} , are still present and keep propagating according to Eqs. (4.19). In other words, what we have achieved with the above field redefinitions is to formally separate the propagation equations of the zero modes (the primed fields) from those of the massive modes (the P fields), without, however, completely decoupling the dynamics of the two sets of modes. The P superfields can be eliminated from the field equations of the massless sector by solving Eqs. (4.19) recursively through an expansion in the parameter $1/K$. In such a way, the massive modes decouple from the massless sector to any finite order in $1/K$. Needless to say this ignores locality.

What we have just done corresponds to a well-known procedure employed in the case of gravitational actions quadratic in the curvature. These contain poltergeists unless the quadratic terms are in the Gauss-Bonnet combination³²⁾. To any finite order in an expansion into the inverse of the Planck mass these ghosts can be avoided through a suitable redefinition³³⁾ of the metric tensor which restores the Gauss-Bonnet combination [see Refs. 2), 33), 34) for the connection with effective string actions].

We interpret the presence of poltergeists in our solution as the signal of a conflict between our requirements of absence of anomalies, supersymmetry and unitarity, and our insisting on a local field theoretical model with only a finite number of propagating modes. We finally add that poltergeists propagate at an energy at which the perturbative derivation of effective string actions is unreliable.

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APPENDIX A

In this appendix we list a few remarkable properties of the Γ matrices, which have been used extensively for our calculations.

$$(\Gamma_{a_1 \dots a_K})_{\alpha\beta} = -(-1)^{\frac{1}{2}K(K+1)} \frac{1}{(10-K)!} \varepsilon_{a_1 \dots a_K a_{K+1} \dots a_{10}} (\Gamma^{a_{K+1} \dots a_{10}})_{\alpha\beta} \quad (A.1)$$

$$(\Gamma_{a_1 \dots a_K})^{\alpha\beta} = (-1)^{\frac{1}{2}K(K+1)} \frac{1}{(10-K)!} \varepsilon_{a_1 \dots a_K a_{K+1} \dots a_{10}} (\Gamma^{a_{K+1} \dots a_{10}})^{\alpha\beta} \quad (A.2)$$

$$(\Gamma^{a_1 \dots a_K} \Gamma_{b_1 \dots b_K})_{\alpha}^{\gamma} = (-1)^{\frac{1}{2}K(K-1)} 16 \cdot K! \delta_{b_1 \dots b_K}^{[a_1 \dots a_K]} \delta_{\alpha}^{\gamma}, \quad K \leq 4 \quad (A.3)$$

$$(\Gamma^{a_1 \dots a_5} \Gamma_{b_1 \dots b_5})_{\alpha}^{\gamma} = 16 \cdot 5! \delta_{b_1 \dots b_5}^{[a_1 \dots a_5]} \delta_{\alpha}^{\gamma} - 16 \varepsilon^{a_1 \dots a_5}_{b_1 \dots b_5} \delta_{\alpha}^{\gamma} \quad (A.4)$$

$$2 \cdot 5! (\Gamma_a)_{\alpha\beta} (\Gamma^a)^{\gamma\delta} + (\Gamma_{a_1 \dots a_5})_{\alpha\beta} (\Gamma^{a_1 \dots a_5})^{\gamma\delta} = 2 \cdot 5! 16 \delta_{\alpha}^{\gamma} \delta_{\beta}^{\delta} \quad (A.5)$$

$$(\Gamma_a)_{\gamma\delta} (\Gamma^a)_{\alpha\beta} = 0 \quad (A.6)$$

$$(\Gamma^{[a_1 \dots a_5]})_{(\alpha\beta} (\Gamma^{a_6]})_{\gamma\delta)} = 0 \quad (A.7)$$

Equations (A.6) and (A.7) follow from the fact that $(\Gamma_a)_{\alpha\beta}$ and $(\Gamma_a)^{\alpha\beta}$ are $SO(1,9)$ scalars.

APPENDIX B

In this Appendix we show that $TX = 0$ implies $X = 0$, if the superform X belongs either to the sector $(2,1)$ or to the sector $(3,1)$. If X is a $(2,1)$ -superform the components $X_{ab\alpha}$ contain the 16, 144 and 560 IR's of $SO(1,9)$. TX is a $(3,1)$ -superform $Y_{1,3}$ whose components also contain the 16, 144 and 560 IR's. This means that the action of T on the superform X does not project out any IR. Then $TX = 0$ implies $X = 0$.

Similarly if X is a $(3,1)$ -superform, the flat components $X_{abc\alpha}$ contain the 16, 144, 560 and 1200 IR of $SO(1,9)$. TX is a $(2,3)$ -superform Y and again the components of Y contain the same IR's, so the same result follows.

APPENDIX C

Using Eqs. (3.4) and the definition $K_{3,1} = 1/6 e^a e^b e^c e^\gamma K_{\gamma cba}$, Eq. (3.37) becomes

$$(\Gamma_{ab}{}^{c_1 c_2 c_3})_{(\alpha\beta} D_\gamma) L_{c_1 c_2 c_3} + (\Gamma_a{}^{c_1 c_2 c_3})_{(\alpha\beta} [-\frac{1}{2} K_\gamma)_{cab} + 6 D_\gamma L_{cba}] = 0 \quad (C.1)$$

The first term contains the IR's present in $(144 \times 45 + 672 \times 45) \cap (16 \times 120)$, while the second term only contains the IR's present in $(144 \times 45) \cap (16 \times 120)$. Then the contribution of the first term to the $(672 \times 45) \cap (16 \times 120)$ which only contains the 1200 must vanish. Then $(D_\gamma L_{c_1 c_2 c_3})^{(1200)} = 0$ and therefore also $K_{\gamma cab}^{(1200)} = 0$.

APPENDIX D

Here we write down the explicit expression for V_{ab}^Y . Let us remark before that our being able to obtain $D_{\alpha} L_{a_1 a_2 a_3}$ in the form $D_{\alpha} L_{a_1 a_2 a_3} = (\Gamma_{[a_1})_{\alpha \gamma} V_{a_2 a_3]}^Y$, is a check of the correctness of Eq. (3.34).

$$\begin{aligned}
 V_{ab}^Y = & (\Gamma_{ab})^{\delta\beta} \left\{ -\frac{1}{576} (\Gamma^{c_1 \dots c_5})^{\beta\delta} L_{\delta}^{c_6} T_{c_1 c_2 c_3} T_{c_4 c_5 c_6} + \frac{1}{96} (\Gamma^{c_1 \dots c_4})^{\beta\delta} T^{\delta c_5 c_6} T_{c_1 c_2 c_3} T_{c_4 c_5 c_6} + \right. \\
 & + (\Gamma^{c_1 c_2 c_3})^{\beta\delta} \left[-\frac{1}{24} T_{c_1 c_2}^d D_{c_3} L_{\delta}^d + L_{\delta}^{c_4} \left(\frac{19}{96} T_{c_1 c_2 c_3}^2 c_4 + \frac{1}{64} D_{c_4} T_{c_3 c_2 c_1} + \right. \right. \\
 & + \frac{7}{192} D_{c_1} T_{c_2 c_3 c_4} \left. \right] + (\Gamma^{c_1 c_2})^{\beta\delta} \left[T^{\delta d_1 d_2} \left(\frac{1}{24} T_{d_1 c_1 c_2}^2 d_2 + \frac{1}{32} T_{d_1 d_2 c_1 c_2}^2 + \right. \right. \\
 & + \frac{1}{24} D_{c_1} T_{c_2 d_1 d_2} - \frac{1}{48} R_{d_1 d_2 c_1 c_2} \left. \right] + (\Gamma^{c_1})^{\beta\delta} \left[-\frac{13}{288} L_{\delta}^{c_2} T_{c_1 c_2}^2 - \frac{7}{144} L_{\delta}^{c_2} R_{c_1 c_2} \right] \left. \right\} + \\
 & + (\Gamma_a)^{\delta\beta} \left\{ -\frac{5}{288} (\Gamma^{c_1 \dots c_5})_{\beta\delta} T^{\delta c_6} T_{c_1 c_2 c_3} T_{c_4 c_5 c_6} + (\Gamma^{c_1 \dots c_4})_{\beta\delta} \left[L_{\delta}^{c_5} \left(-\frac{1}{16} T_{c_1 c_2} T_{c_3 c_4 c_5} + \right. \right. \right. \\
 & - \frac{5}{36} T_{c_1 c_2 c_3} T_{c_4 c_5 c_6} \left. \right) + L_{\delta}^b \left(\frac{5}{288} D_{c_1} T_{c_2 c_3 c_4} + \frac{97}{576} T_{c_1 c_2 c_3}^2 c_4 + \frac{1}{24} T_{c_1 c_2 c_3} D_{c_4} L_{\delta}^b \right] + \\
 & + (\Gamma^{c_1 c_2 c_3})_{\beta\delta} \left[T^{\delta c_4 c_5} \left(-\frac{1}{8} T_{b c_1 c_2} T_{c_3 c_4 c_5} + \frac{5}{288} T_{c_1 c_2 c_3} T_{c_4 c_5 c_6} \right) + \frac{1}{8} T^{\delta d_1 d_2} T_{b c_1 d_1} T_{d_2 c_2 c_3} + \right. \\
 & + T^{\delta c_4} b \left(-\frac{1}{16} T_{c_1 c_2 c_3}^2 c_4 + \frac{1}{12} D_{c_1} T_{c_2 c_3 c_4} + \frac{1}{72} D_{c_4} T_{c_3 c_2 c_1} \right) + \frac{1}{72} T_{c_1 c_2 c_3} \not{D}^{\delta\lambda} \lambda_b \left. \right] + \\
 & + (\Gamma^{c_1 c_2})_{\beta\delta} \left[L_{\delta}^{c_3} \left(\frac{13}{18} T_{c_1 c_2 c_3}^2 b - \frac{47}{36} T_{b c_1 c_2 c_3}^2 + \frac{7}{16} D_{c_1} T_{c_2 c_3 b} - \frac{55}{288} D_b T_{c_1 c_2 c_3} + \right. \right. \\
 & - \frac{43}{288} D_{c_3} T_{c_2 c_1 b} - \frac{1}{36} R_{c_1 c_2 c_3 b} \left. \right) - \frac{1}{12} T_{c_1 c_2}^d D_{\delta}^d L_{\delta}^b + \frac{1}{8} T_{c_1 c_2}^d D_b L_{\delta}^d + \frac{1}{4} T_{b c_1}^d D_{c_2} L_{\delta}^d \left. \right] + \\
 & + (\Gamma^{c_1})_{\beta\delta} \left[T^{\delta c_2 c_3} \left(\frac{3}{4} T_{c_1 c_2 c_3}^2 b - \frac{1}{4} T_{b c_1 c_2 c_3}^2 + \frac{1}{8} D_{c_1} T_{c_2 c_3 b} - \frac{1}{8} D_b T_{c_1 c_2 c_3} + \right. \right. \\
 & + \frac{1}{4} D_{c_3} T_{c_2 c_1 b} + \frac{1}{4} R_{b c_1 c_2 c_3} \left. \right) + T^{\delta c_2} b \left(\frac{31}{144} T_{c_1 c_2}^2 + \frac{1}{18} R_{c_1 c_2} \right) \left. \right] + \\
 & + L_{\beta}^d \left(\frac{47}{72} T_{d b}^2 + \frac{37}{72} R_{d b} \right) - \frac{7}{36} T^2 L_{\beta}^b + \frac{5}{12} T_b^{d_1 d_2} D_{d_1} L_{\beta}^{d_2} + \frac{1}{6} \square L_{\beta}^b \left. \right\} +
 \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{16} (\Gamma^{c_1 \dots c_5})^{\delta\delta} L_{\delta a} T_{b c_1 c_2} T_{c_3 c_4 c_5} + (\Gamma^{c_1 \dots c_4})^{\delta\delta} \left[T^{\delta c_5} a \left(-\frac{5}{24} T_{b c_1 c_2} T_{c_3 c_4 c_5} + \right. \right. \\
& \left. \left. - \frac{1}{24} T_{c_1 c_2 c_3} T_{c_4 c_5 b} \right) + T^{\delta a} b \left(\frac{1}{48} T^2_{c_1 c_2 c_3 c_4} + \frac{7}{144} D_{c_1} T_{c_2 c_3 c_4} \right) \right] + \\
& + (\Gamma^{c_1 c_2 c_3})^{\delta\delta} \left[L_{\delta}{}^{c_4} \left(-\frac{1}{4} T_{a b c_1} T_{c_2 c_3 c_4} + \frac{1}{6} T_{c_1 c_2 c_3} T_{c_4 a b} + \frac{1}{6} T_{a c_1 c_2} T_{c_3 c_4 b} \right) + \right. \\
& \left. + L_{\delta a} \left(\frac{31}{24} T^2_{b c_1 c_2 c_3} + \frac{1}{9} D_b T_{c_1 c_2 c_3} - \frac{1}{4} D_{c_1} T_{c_2 c_3 b} \right) + \frac{1}{9} T_{c_1 c_2 c_3} D_a L_{\delta b} - \frac{1}{4} T_{a c_1 c_2} D_{c_3} L_{\delta b} \right] + \\
& + (\Gamma^{c_1 c_2})^{\delta\delta} \left[T^{\delta c_3 c_4} \left(-\frac{1}{4} T_{a b c_1} T_{c_2 c_3 c_4} - \frac{17}{48} T_{a c_1 c_2} T_{c_3 c_4 b} - \frac{1}{6} T_{c_1 c_2 c_3} T_{c_4 a b} \right) + \right. \\
& \left. + T^{\delta c_3} a \left(-\frac{5}{36} T^2_{c_1 c_2 c_3 b} + \frac{1}{18} T^2_{b c_1 c_2 c_3} - \frac{1}{6} D_{c_1} T_{c_2 c_3 b} - \frac{1}{36} D_b T_{c_1 c_2 c_3} - \frac{7}{36} D_{c_3} T_{c_2 c_1 b} + \right. \right. \\
& \left. \left. + \frac{1}{9} R_{c_1 c_2 c_3 b} \right) + \frac{1}{24} T_{c_1 c_2 d} D^d T^{\delta a} b - \frac{1}{6} T_{c_1 c_2 a} \not{D}^{\delta\lambda} L_{\lambda b} \right] + (\Gamma^{c_1})^{\delta\delta} \left[L_{\delta}{}^{c_2} \left(-\frac{8}{9} T^2_{c_1 a b c_2} + \right. \right. \\
& \left. \left. - \frac{7}{9} T^2_{a b c_1 c_2} + \frac{5}{24} D_{c_2} T_{c_1 a b} - \frac{13}{42} D_{c_1} T_{c_2 a b} - \frac{25}{36} D_a T_{b c_1 c_2} + \frac{2}{9} R_{c_1 a b c_2} + \right. \right. \\
& \left. \left. - \frac{1}{9} R_{c_1 c_2 a b} \right) + L_{\delta a} \left(-\frac{59}{42} T^2_{b c_1} - \frac{7}{18} R_{b c_1} \right) + \frac{1}{4} T_{a b}{}^{c_2} D_{c_1} L_{\delta c_2} + \frac{1}{3} T_{c_1 c_2 a} D_b L_{\delta}{}^{c_2} + \right. \\
& \left. - \frac{1}{6} T_{a c_1 c_2} D^{c_2} L_{\delta a_3} \right] + T^{\delta c_1 c_2} \left(-\frac{43}{18} T^2_{a c_1 c_2 b} - \frac{29}{36} T^2_{c_1 c_2 a b} + \frac{11}{18} R_{c_1 c_2 a b} - \right. \\
& \left. - \frac{1}{9} D_{c_1} T_{c_2 a b} - \frac{2}{3} D_{a_2} T_{a_3 c_1 c_2} \right) + T^{\delta d} a \left(\frac{7}{6} T^2_{b d} + \frac{2}{3} R_{d b} \right) - \frac{1}{6} T_a{}^{c_1 c_2} D_{c_1} T^{\delta}{}_{c_2 b} + \\
& - \frac{1}{36} T^d{}_{a b} \not{D}^{\delta\delta} L_{\delta d} - \frac{1}{6} \square T^{\delta}{}_{a b} + \frac{11}{42} T^2 T^{\delta}{}_{a b} - \frac{5}{12} T_a{}^{c_1 c_2} D_b T^{\delta}{}_{c_1 c_2} .
\end{aligned}$$

where the antisymmetrization over a and b is understood and $\not{D}^{\alpha\beta} = (\Gamma^a)^{\alpha\beta} D_a$.

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