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HEAVY QUARK MASS IN THE $\overline{MS}$ -SCHEME: REVISITED

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ABSTRACT

We re-examine the relation between the heavy quark mass extracted from QCD duality sum rules and the ones used in the $\overline{MS}$ -scheme. Then, we deduce the values of the pole and running masses in the $\overline{MS}$ -scheme and briefly explain how to use these masses in some processes.

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In our present understanding of strong interactions, QCD perturbation theory provides a satisfactory description of various hard processes of quark and gluon jet phenomena in terms of the falling coupling $\bar{\alpha}_s(Q^2/\Lambda^2)$. However, due to the approximate solution of the theory, one has to choose a regularization and renormalization framework for the evaluation of the Green's function. Among others, the dimensional regularization and renormalization scheme, the so-called $\overline{\text{MS}}$ -scheme¹⁾ appears to be the most convenient one in QCD. In a massless theory, the scheme dependence is reflected by the value of the QCD scale Λ . In a theory with massive quarks, the scheme dependence appears both in Λ and in the value of the quark mass where the latter can be treated like a coupling constant when solving the renormalization group equation²⁾. The values of the running light quark masses have been studied using QCD duality sum rules³⁾ within the $\overline{\text{MS}}$ -scheme, whilst their ratio, which is scale-independent, has been known (before the advent of QCD) from the current algebra low-energy approach⁴⁾. Despite the additional difficulty related to the non-observation of free quarks, it appears quite clearly that the mass of the light quarks to be used for a perturbative analysis is the running mass. For heavy quark systems, which are highly non-relativistic, the situation is not so. The aim of these short notes is to discuss the consistent definition of the heavy quark mass to be used for a given perturbative expression of a Green's function. The definition of the quark mass has already been attacked by various authors⁴⁾⁻⁶⁾ following the pioneer work of Georgi and Politzer⁷⁾, but it appears that the answer, though interesting remains incomplete. From a phenomenological approach, the most precise values of the charm and bottom quark masses come from the Shifman-Vainshtein-Zakharov (SVZ)-like⁸⁾ moments and Laplace sum rules of the two-point correlator $\Pi(q^2)$ associated to the quark current $\bar{\psi}\gamma_\mu\psi$. In this analysis, the expression of the spectral function $\text{Im } \Pi(t)$ obtained in QED by Källen-Sabry⁹⁾ or its interpolated form given by Schwinger¹⁰⁾ has been used. Then, it is clear that the quark mass which should be used is the "pole mass" defined at the singularity of the quark propagator $S_F(p)$ at $p^2 = M^2$. Also, as in QED, this pole mass is not affected by radiative corrections. That can be achieved by the mass renormalization constant¹¹⁾:

$$Z_m^{\text{QED}} = 1 - \left(\frac{\alpha_0}{\pi}\right) \left\{ \log \frac{\Lambda_{\text{UV}}^2}{M^2} + \frac{1}{2} \right\}, \quad (1)$$

within a Pauli-Villars regularization with the ultra-violet cut-off Λ_{UV} whilst there is no infra-red cut-off at least up to two loops as shown by Tarrach in Ref. 5).

In order to minimize the effects of radiative corrections in their analysis, SVZ⁸⁾ have chosen to work with the "Euclidean mass" $m(p^2 = -M^2)$ which is related to the pole mass $M(p^2 = M^2)$ as:

$$m(p^2) = M(p^2 = M^2) \left\{ 1 - \frac{\alpha_s}{\pi} \left(1 - \frac{M^2}{p^2} \right) \left[\log \left(1 - \frac{p^2}{M^2} \right) - \frac{a}{3} \left(1 + \frac{M^2}{p^2} \log \left(1 - \frac{p^2}{M^2} \right) \right) \right] \right\}. \quad (2)$$

where a is the covariant gauge parameter. Various fits of charmonium and bottomium data have given stable and accurate results⁸⁾:

$$\begin{aligned} m_c(p^2 = -M_c^2) &\approx (1.26 \pm 0.02) \text{ GeV}, \\ m_b(p^2 = -M_b^2) &\approx (4.23 \pm 0.05) \text{ GeV}, \end{aligned} \quad (3)$$

despite some disagreements for getting the value of the gluon condensate and quite independently of the type of sum rules used. The stability of the results can be understood because the quark mass plays the dominant role in the sum rules analysis. The minimization of the radiative corrections implies that the results in Eq. (3) are not sensitive to the actual value of Λ . Equations (2) and (3) give the pole mass in the QED-like scheme within which the expression of the spectral function has been derived. The results are shown in the first column of the Table for various Λ values. If we assume that $\Lambda \approx (180 \pm 80) \text{ MeV}$, one would get after adding the errors quadratically:

$$\begin{aligned} M_{\text{QED}}^c &\approx (1.46 \pm 0.05) \text{ GeV}, \\ M_{\text{QED}}^b &\approx (4.70 \pm 0.14) \text{ GeV}. \end{aligned} \quad (4)$$

Now, we have to convert the sum rule results in Eqs. (3) and (4) into the quark masses used in the standard $\overline{\text{MS}}$ -scheme. The renormalization scheme dependence of the pole mass has been studied previously⁵⁾ but only for a dimensionally regularized quark self-energy. Therefore, one cannot use directly this result in our case where the quantity used in the sum rule is Pauli-Villars regularized. Another way for getting indirectly the "mass definition" in the $\overline{\text{MS}}$ -scheme is to require that radiative corrections of the two-point correlators related to physical observables are the same in any regularization and renormalization schemes. The correlator $\Pi(q^2)$ of the vector-current which is related to the

total cross-section $e^+e^- \rightarrow \text{hadrons}$ should be scheme invariant. The authors of Ref. 6) have compared the coefficient of the m^2/q^2 term of $\Pi(q^2)$ in two ways. Firstly, one evaluates $\Pi(q^2)$ in the Euclidean region by using the "light" quark expansion in m^2/q^2 : it is clear in this case that after a leading-log resummation, m^2 is replaced by the "running" mass solution of the differential equation¹⁾:

$$\frac{d\bar{\alpha}}{dt} = - (1 + \gamma_m(\alpha_s)) \bar{\alpha} \quad ; \quad \bar{\alpha}(t=0) = \frac{\alpha}{v}, \quad (5a)$$

where v is the $\overline{\text{MS}}$ arbitrary scale; $t \equiv \log -q^2/v^2$ and $\gamma_m(\alpha_s) = \gamma_1(\alpha_s/\pi) + \gamma_2(\alpha_s/\pi)^2 + \dots$, is the mass anomalous dimension¹⁾: $\gamma_1 = 2$, $\gamma_2 = 1/6(101/2 - 5n/3)$ for $\text{SU}(n)_F$, which in terms of the invariant mass $\hat{m}$ reads¹⁾:

$$\bar{m}_{\overline{\text{MS}}}(\rho^2) = \frac{\hat{m}}{L^{\gamma_2/\beta_1}} \left\{ 1 - \delta_1 \frac{\beta_2}{\beta_1^3} \frac{\log^2 L}{L} + \frac{1}{\beta_1^2} \left(\gamma_2 - \delta_1 \frac{\beta_2}{\beta_1} \right) \frac{1}{L} \right\}, \quad (5b)$$

where :

$$L \equiv \log \sqrt{-\rho^2}/\Lambda \quad ; \quad \beta(\alpha_s) = \left(\frac{\alpha_s}{\pi} \right) \left(\beta_1 \equiv -\frac{1}{2} (11 - \frac{2}{3}n) + \left(\frac{\alpha_s}{\pi} \right)^2 \left(\beta_2 \equiv -\frac{1}{4} (51 - \frac{19}{3}n) \right) \right)$$

is the Callan-Symanzik beta function. Secondly, one evaluates $\Pi(q^2)$ from the dispersive integral by giving the expression of the spectral function $\text{Im } \Pi(t)$ obtained by Källén-Sabry in QED⁹⁾. One should note that in this case, the mass to be used in the spectral function is the QED pole mass having the value in Eq. (4). Requiring the same radiative corrections for $\Pi(q^2)$, one is led to the relation⁶⁾:

$$M_{\text{QED}}(\rho^2 = M^2) = \bar{m}_{\overline{\text{MS}}}(M) \left\{ 1 + \frac{4}{3} \frac{\bar{\alpha}_s}{\pi} + \mathcal{O}(\bar{\alpha}_s^2) \right\}, \quad (6)$$

which is also maintained for the analysis of other channels⁶⁾ [pseudoscalar¹²⁾ ...]. Now, let us compare Eq. (6) with the relation between the pole and running masses in the $\overline{\text{MS}}$ -scheme, which is¹⁾:

$$M_{\overline{\text{MS}}}(\rho^2 = M^2) = \bar{m}_{\overline{\text{MS}}}(M) \left\{ 1 + \sum_1^{\text{FP}}(M^2 = v^2) \right\}, \quad (7)$$

where Σ_1^{FP} represents the finite part of the quark self-energy decomposed as $\Sigma = m\Sigma_1 + (\hat{p}-m)\Sigma_2$. For a dimensionally regularized Green's function,

$\Sigma_1^{\text{FP}} = 4/3(\alpha_s/\pi)$. Therefore, Eqs. (6) and (7) state that the "pole mass" is regularization and renormalization schemes invariant.

This important result will allow us to derive the values of the invariant mass $\hat{m}$ and of the running mass evaluated at one GeV. These results are shown in the Table, which for $\Lambda_{\overline{\text{MS}}} = (180 \pm 80) \text{ MeV}$ gives^{*)}:

$$\begin{aligned} \hat{m}_c &\approx (1.81 \pm 0.33) \text{ GeV} ; \quad \bar{m}_c(1 \text{ GeV}) \approx (1.42 \pm 0.07) \text{ GeV}, \\ \hat{m}_b &\approx (7.90 \pm 1.01) \text{ GeV} ; \quad \bar{m}_b(1 \text{ GeV}) \approx (6.29 \pm 0.73) \text{ GeV}. \end{aligned} \quad (8)$$

The numerical values in Eq. (8) are slightly higher than the previous results of Gasser and Leutwyler⁴⁾ due to the fact that they have identified the running mass in the $\overline{\text{MS}}$ -scheme with the Euclidean mass of the QED-like scheme defined in Eqs. (2) and (3). Instead, one should use Eq. (6). The results in Eq. (8) can be used together with the SU(4) approximate symmetry assumption of the ψ and ϕ wave functions at the origin in order to predict the strange quark mass value. One has⁴⁾:

$$\left(\frac{\bar{m}_c - \bar{m}_u}{\bar{m}_d - \bar{m}_u} \right) \approx 9 \pm 2, \quad (9)$$

which leads to:

$$\bar{m}_d(1 \text{ GeV}) \approx (157.8 \pm 35.9) \text{ MeV}, \quad (10)$$

in good agreement with the result obtained from a direct sum rule analysis of the (pseudo) scalar strange quark two-point correlator^{3),14)}.

We are aware of the phenomenological implications of our results in the heavy quark processes where the ambiguities in the quark mass definitions and on its value induce the main errors into the physical observables [e.g., $\sin^2 \theta_W$ ¹⁵⁾, cross-sections¹⁶⁾, weak decays and lifetime¹⁷⁾...]. We are also aware of the fact that the values of the perturbative charm quark pole mass in Eq. (4) might

*) The corresponding value of $\bar{m}_c$ evaluated at the η_c mass is $(1.08 \pm 0.04) \text{ GeV}$ and might be compared with forthcoming lattice results (communication from G. Martinelli) or with the one based on a QCD effective potential¹³⁾. One should note that the Euclidean mass in Eq. (2) which is gauge dependent could not be physically interesting.

be lower than the one favoured by potential modelists¹⁸⁾. Unfortunately, in the case of the quarks studied here, there is not any easy connection between the QCD Lagrangian and the parameters of the potential models^{*)}. Non-perturbative effects which are larger for the charm than for the bottom might help to compensate for such numerical discrepancies. However, if these are parametrized à la SVZ^{8a)}, then their effects are suppressed like M^{-3} and are therefore negligible. Probably, these non-perturbative terms are related to the hadronization process where the QCD perturbative approach, even with the inclusion of the condensate effects, cannot say much.

Our analysis has shown that for a perturbative treatment of a process, one should use the running mass if a "light quark expansion" of the Green's function has been done in the Euclidean region. In the case where the process is evaluated via its spectral function in the time-like region, then one should use the pole mass in the analysis.

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*) If indeed we have started from the scheme where the bare mass of the QCD Lagrangian is invariant under a regularization and a renormalization procedure, we could have obtained a higher value of about 1.8 GeV of the pole charmed mass. However, this scheme would lead to inconsistencies for the radiative corrections of the two-point correlator.

TABLE

QCD Parameters in GeV

Λ	$M_{\text{QED}}(p^2 = M^2) = \overline{M}_{\overline{\text{MS}}}$		$\hat{m}$		$\overline{m}_{\overline{\text{MS}}} (1 \text{ GeV})$	
0.08	1.41±0.02	4.61±0.05	2.14±0.03	8.91±0.11	1.35±0.02	5.56±0.07
0.10	1.42±0.02	4.63±0.06	2.06±0.03	8.65±0.10	1.36±0.02	5.70±0.07
0.12	1.43±0.02	4.65±0.06	1.98±0.03	8.42±0.10	1.38±0.02	5.85±0.07
0.14	1.44±0.02	4.67±0.06	1.92±0.03	8.23±0.10	1.39±0.02	5.99±0.07
0.16	1.45±0.02	4.68±0.06	1.86±0.03	8.06±0.10	1.40±0.02	6.14±0.07
0.18	1.46±0.02	4.70±0.06	1.81±0.03	7.90±0.09	1.42±0.02	6.29±0.07
0.20	1.47±0.02	4.71±0.06	1.76±0.03	7.76±0.09	1.44±0.02	6.45±0.08
0.22	1.48±0.02	4.73±0.06	1.72±0.03	7.63±0.09	1.45±0.02	6.62±0.08
0.24	1.49±0.02	4.74±0.06	1.68±0.03	7.51±0.09	1.47±0.02	6.79±0.08
0.26	1.50±0.02	4.76±0.06	1.64±0.03	7.40±0.09	1.49±0.02	6.98±0.08
	Charm	Bottom	Charm	Bottom	Charm	Bottom

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