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SCHERK-SCHWARZ MECHANISM AND SUPERSTRINGS

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A B S T R A C T

It is shown that a particular choice of massless classical backgrounds provides an exact solution to the equations of motion of closed superstrings while spontaneously breaking supersymmetry at an arbitrary scale.

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In any realistic superstring theory space-time supersymmetry must be broken. One could consider three possibilities. The first is an explicit breaking of the symmetry. It is an unappealing procedure which introduces a mass scale by hand and completely eliminates the content of all supersymmetric identities. Another alternative is dynamical breaking which presents big calculational obstacles although physically this is an exclusively technical and not substantial issue. Finally, spontaneous breaking retains the Ward identities of supersymmetry and allows one to calculate within some perturbation expansion as well.

We shall concentrate on the third possibility. It is well known that the field theoretic limit of closed superstrings is some extended supergravity. In the latter the general method for their spontaneous breakdown involves some gauging. A gauging of particular interest was introduced by Scherk and Schwarz¹⁾ and is based on non-semisimple, non-compact groups which are obtained by contraction of semisimple groups. Equivalently it can be thought of as a generalized (co-ordinate dependent) compactification consistent with a flat four-dimensional metric.

On the other hand, vacuum solutions of superstring theories obey the corresponding equations of motion. A convenient way to determine whether a particular ansatz is a solution involves the formulation of string perturbation theory in a general background field²⁾. By requiring the quantum string theory to maintain its conformal invariance, the background field equations are obtained. One can then check whether the ansatz in question obeys these equations.

The Scherk-Schwarz compactification. We shall use notation similar to that of Ref. 1). The total number of dimensions is $D+E$ with D space-time and E internal. World indices in $D+E$ and D and E are $\hat{\mu}, \hat{\nu}, \dots$ and $\mu, \nu, \dots$ and $\alpha, \beta, \dots$ respectively. The analogous choices for flat indices are $\hat{m}, \hat{n}, \dots$ and $m, n, \dots$ and $a, b, \dots$. Finally we decompose a complete co-ordinate $x^{\hat{\mu}}$ into a space-time part x^{μ} and an internal one y^{α} .

As a simple example consider the Einstein action in $D+E$ dimensions. We take the vielbein $\hat{e}^{\hat{\alpha}}_{\hat{\mu}}$ as our fundamental dynamical variable having in mind applications to supergravity. The theory possesses co-ordinate as well as local Lorentz invariance in $D+E$ dimensions and we have the freedom to impose $(D+E)$ as well as $(D+E)(D+E-1)/2$ gauge conditions respectively.

Let us, however, use only the internal Lorentz gauge freedom which consists of $(D+E)(D+E-1)/2 - D(D-1)/2 = D.E + E(E-1)/2$ conditions. The matrix $\hat{e}_{\mu}^{\hat{r}}$ can be conveniently decomposed as follows:

$$\hat{e}_{\mu}^{\hat{r}} = \begin{pmatrix} D \times D & D \times E \\ E \times D & E \times E \end{pmatrix} \quad (1)$$

By imposing the conditions allowed by the internal Lorentz invariance we can bring the vielbein to the form:

$$\hat{e}_{\mu}^{\hat{r}} = \begin{pmatrix} e_{\mu}^r & A_{\mu}^{\alpha} \phi_{\alpha}^a \\ 0 & \phi_{\alpha}^a \end{pmatrix} \quad (2)$$

with the understanding that out of the E^2 scalar fields ϕ_{α}^a only $E(E+1)/2$ are propagating degrees of freedom.

Scherk and Schwarz introduced a y^{α} -dependence in the vielbein in a way that it:

(i) guarantees that we obtain the well-known results in the limit in which the vielbein becomes y^{α} independent;

(ii) results in a theory with D-dimensional co-ordinate invariance and the most general possible E-dimensional gauge invariance.

The y^{α} -dependences of the various fields must take the form:

$$e_{\mu}^r(x, y) = e_{\mu}^r(x) \quad (3a)$$

$$A_{\mu}^{\alpha}(x, y) = [U^{-1}(y)]^{\alpha}_{\beta} A_{\mu}^{\beta}(x) \quad (3b)$$

$$\phi_{\alpha}^a(x, y) = [U(y)]^{\beta}_{\alpha} \phi_{\beta}^a(x) \quad (3c)$$

The non-singular matrices U define structure constants for any E-dimensional Lie group G:

$$f_{\alpha\beta}^{\gamma} = [U^{-1}(y)]^{\alpha'}_{\alpha} [U^{-1}(y)]^{\beta'}_{\beta} (\partial_{\beta'} [U(y)]^{\gamma}_{\alpha'} - \partial_{\alpha'} [U(y)]^{\gamma}_{\beta'}) \quad (4)$$

However, the resulting D-dimensional theory in general will possess, together with an E-dimensional gauge invariance, an unbounded below potential term which

must clearly be stabilized. This can be done only for the so-called flat gauge groups which are non-semisimple and non-compact.

When we consider a supergravity Lagrangian, the dimensionful parameters entering in the generators of these groups become the mass scales which characterize the supersymmetry breaking. These mass scales are undetermined a priori and no cosmological constant is generated to free order. The Scherk-Schwarz generalized dimensional reduction leads to spontaneously broken no-scale supergravities³⁾.

The closed superstring. There are two types of closed superstrings to be considered. In type II superstrings the massless background fields are the ten-dimensional metric $\hat{G}_{\mu\nu}^{\Lambda\Lambda}(X)$, antisymmetric tensor $\hat{B}_{\mu\nu}^{\Lambda\Lambda}(X)$ and dilaton $\Phi(X)$. For the heterotic string theory we must add the gauge field $\hat{A}_{\mu}^{\Lambda}(X)$.

These background fields provide an infinite collection of couplings in the two-dimensional field theory. As a result, these couplings satisfy renormalization group equations:

$$\beta_{\mu\nu}^G \equiv \frac{\partial \hat{G}_{\mu\nu}^{\Lambda\Lambda}}{\partial \ln \Lambda} \quad (4a)$$

$$\beta_{\mu\nu}^B \equiv \frac{\partial \hat{B}_{\mu\nu}^{\Lambda\Lambda}}{\partial \ln \Lambda} \quad (4b)$$

$$\beta^{\Phi} \equiv \frac{\partial \Phi}{\partial \ln \Lambda} \quad (4c)$$

$$\beta_{\mu}^A \equiv \frac{\partial \hat{A}_{\mu}^{\Lambda}}{\partial \ln \Lambda} \quad (4d)$$

The equations which impose quantum conformal invariance take the form:

$$\beta_{\mu\nu}^G = \beta_{\mu\nu}^B = \beta^{\Phi} = \beta_{\mu}^A = 0 \quad (5)$$

Let us apply the Scherk-Schwarz mechanism to the closed superstrings. The total dimensionality is $D+E = 10$. This higher-dimensional theory is first reduced to $D+1 = 5$ dimensions by assuming all the fields of the theory to be independent of the compactified dimensions. The resulting five-dimensional theory has at least an $SO(E-1) = SO(5)$ global symmetry group. The final reduction to $D = 4$ dimensions introduces dependence on the y^1 co-ordinate consistent with this global symmetry group:

$$[U(y')]^\alpha_\beta = [\exp My']^\alpha_\beta \quad (6)$$

with M a real, antisymmetric, traceless 6x6 matrix with zeros in the first row and column. The structure constants follow by inserting (6) into (4):

$$f^\gamma_{\alpha\beta} = M^\gamma_\alpha \delta'_\beta - M^\gamma_\beta \delta'_\alpha \quad (7)$$

We now write down the backgrounds of interest for the vielbein:

$$e^r_\mu(x) = \delta^r_\mu, \quad A^\alpha_\mu(x) = 0, \quad \phi^a_\alpha(x) = \delta^a_\alpha \quad (8a)$$

the antisymmetric tensor:

$$B_{\mu\nu}(x) = 0, \quad B_{\mu\alpha}(x) = 0, \quad B_{\alpha\beta}(x) = B_{\alpha\beta} \quad (8b)$$

the dilaton:

$$\Phi(x) = \Phi \quad (8c)$$

and the gauge field:

$$A_\mu(x) = 0, \quad A_\alpha(x) = 0 \quad (8d)$$

The above ansatz can be shown to solve the quantum conformal invariance equations (5) to all orders in perturbation theory of the two-dimensional action. Since this action has general co-ordinate invariance, a perturbation expansion in terms of Riemann normal co-ordinates is the most convenient²⁾. Then, one can write the beta functions of this theory purely as functions of:

- (i) the field strength tensors available and their covariant derivatives;
- (ii) covariant derivatives of the dilaton field.

The higher the order of perturbation theory, the more field strength tensors and covariant derivatives of them, or of the dilaton, will appear. In our background choice (8), it is obvious that all such derivatives of the dilaton field will vanish. The same is true for the field strength tensors of $\hat{B}^{\Lambda\Lambda}_{\mu\nu}$ and $\hat{A}^{\Lambda}_{\mu}$ since they both vanish identically.

We now have to consider the Riemann tensor $\hat{R}^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}$. For this purpose it is very convenient not to work with the vielbein, but with the metric tensor $\hat{g}^{\hat{\mu}\hat{\nu}}$. For our ansatz (8) it takes the form:

$$\hat{g}^{\mu\nu} = \begin{pmatrix} \eta^{\mu\nu} & 0 \\ 0 & -h^{\alpha\beta} \end{pmatrix} \quad (9)$$

and we only have to examine the internal metric $h_{\alpha\beta}$ defined by:

$$h_{\alpha\beta} \equiv \phi_{\alpha}^a \delta_{ab} \phi_{\beta}^b \quad (10)$$

Then,

$$\begin{aligned} h_{\alpha\beta} &= [U(y')]_{\alpha}^{\alpha'} \delta_{\alpha'}^a \delta_{ab} [U(y')]_{\beta}^{\beta'} \delta_{\beta'}^b \\ &= [U^T(y') U(y')]_{\alpha\beta} = \delta_{\alpha\beta} \end{aligned} \quad (11)$$

since from Eq. (6) U is an orthogonal matrix due to the antisymmetry of M .

The Riemann tensor clearly vanishes and the background (8), as a result, is a solution of the conformal invariance equations (5) to all orders. Therefore, it defines an exact solution of the classical closed superstring equations. Besides the fundamental Planck scale, new arbitrary mass scales are introduced in the theory. These supersymmetry breaking scales give masses to the gravitini and are presumably fixed by radiative corrections. It is believed that type II closed superstrings with spontaneously broken supersymmetry lead to a gauging of $N = 8$ supergravity with as many as four Z_2 truncations, while $N = 4$ with up to two Z_2 truncations is obtained from the heterotic superstring. In the maximal case of $N = 8$ broken supergravity without truncation, the matrix M would introduce four mass parameters.

The construction described here only establishes the existence of these very interesting closed superstring vacuum solutions and does not provide their direct construction on the string level. From the compactification point of view, the internal manifold is $T^5 \times T(y^1)$, i.e. a "co-ordinate dependent" torus. It would be most interesting to obtain such theories in the purely fermionic language⁴⁾, where the partition function of each four-dimensional closed superstring can be explicitly evaluated. Ultimately, of course, a string theory leading to a broken chiral $N = 1$ supergravity with zero cosmological constant should emerge.

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