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DECAY CONSTANTS OF THE B AND D MESONS FROM QCD-DUALITY SUM RULES

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A B S T R A C T

We show that various (non-) relativistic versions of QCD duality sum rules lead to a unique prediction $f_B \approx (1.38 \pm 0.14)f_\pi$ for the decay constant of the pseudoscalar B-meson. We also obtain $f_D \approx (1.31 \pm 0.12)f_\pi$ from relativistic Laplace (Borel) sum rules and show that the $SU(3)_F$ -breaking effects tend to increase the decay constant of the corresponding mesons containing a strange quark. In view of our results, we re-examine some qualitative arguments predicting the mass behaviour of the decay constant of heavy-light quark bound states.

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Recent interest in the $B^0-\bar{B}^0$ mixing¹⁾ has again pointed out the need to have an accurate determination of the parameters of the standard model. Among others, the main source of uncertainties for the estimate of the $\langle \bar{B}^0 | (\bar{b}\gamma_\mu (1-\gamma_5)d)^2 | B^0 \rangle$ matrix element is the decay constant f_B of the B-meson once one assumes that the vacuum saturation hypothesis is a good approximation for heavy mesons. There is not any available measurement of f_B which might come from the rare $B \rightarrow \tau\nu_\tau, \mu\nu_\mu$ leptonic decay. Therefore, the only information on f_B comes from theory. Among other QCD-inspired models, one might expect that QCD-duality sum rules provide a reliable prediction on f_B as the method has been able to explain quite successfully most dynamical properties of hadrons in terms of the QCD perturbative (Λ and quark masses) and non-perturbative (quark and gluon condensates) scales^{2),3)}. However, it is quite surprising that existing predicted values⁴⁾⁻¹⁰⁾ of f_B from various QCD-type sum rules differ by a factor of about two! What is much more disappointing is the fact that the relativistic moment^{6),7)} and Laplace⁴⁾ (Borel) sum rules already give two different predictions on f_B while non-relativistic-type sum rules^{8),10)} seem to agree to provide a low value of f_B . In view of these existing discrepancies, we try to understand the origins of the above disease by re-examining the estimates of f_B (and f_D) from these sum rules. For our analysis we shall be concerned with the renormalization group invariant pseudoscalar current^{*)}

$$J_P = (m_q + M_Q) \bar{q}(i\gamma_5)Q, \quad (1)$$

built with the light q and heavy Q quark fields. m_q (resp M_Q) are the light (resp heavy) quark masses. The associated two-point correlator is:

$$\Psi(q^2) = i \int d^4x e^{iqx} \langle 0 | T J_P(x) (J_P(0))^+ | 0 \rangle \quad (2)$$

and its QCD-expression is known in the literature^{7),11),12),6),4)}. Its asymptotic behaviour is $m^2 Q^2 \log Q^2 / v^2$ ($Q^2 = -q^2 \gg \Lambda^2$) which indicates that one should at least work with its second derivative in order to get rid of the unknown subtraction constants. In this way, one has the lowest superconvergent sum rules:

*) In the case where one works with the current $\bar{q}(i\gamma_5)Q$ it is well known that one should take into account its anomalous dimension $\gamma_1 / -\beta_1$ [$\gamma_1 = 2$, $-\beta_1 = \frac{1}{2}(11 - 2/3n_f)$ are respectively the mass anomalous dimension and the first coefficient of the β -function].

$$\frac{1}{2!} \frac{\partial^2 \Psi(q^2)}{(\partial q^2)^2} = \int_{-\infty}^{\infty} dt \cdot \frac{1}{(m_q + M_Q)^2 (t - q^2)^3} \frac{1}{\pi} \text{Im} \Psi(t) \quad (3)$$

The spectral function $1/\pi \text{Im}\Psi(t)$ can be saturated by the naïve duality ansatz: "one resonance" plus "QCD continuum" where the latter comes from the discontinuity of the QCD diagrams. In this way:

$$\frac{1}{\pi} \text{Im} \Psi(t) = 2 q_B^2 M_B^4 \delta(t - M_B^2) + \Theta(t - t_c) \cdot \text{"QCD continuum"} \quad (4)$$

The parametrization in Eq. (4) can be still improved by incorporating finite width corrections and by using a much more sophisticated form of the continuum model. However, in a region where the optimal information from the sum rule is obtained, it has been realized from various examples^{2),3)} that these corrections do not give an observable change in the results obtained from Eq. (4). We shall therefore use the QCD form of the continuum:

$$\begin{aligned} \frac{1}{\pi} \text{Im} \Psi_{\text{pert}}(t) = \frac{3}{8\pi^2} x t^2 (1-x)^2 \left\{ 1 + \frac{4}{3} \frac{Q_2}{\pi} \left[\frac{9}{4} + 2 \text{Li}_2(x) \right. \right. \\ \left. \left. + \log x \log(1-x) - \frac{3}{2} \log\left(\frac{1}{x} - 1\right) - \log(1-x) \right. \right. \\ \left. \left. + x \log\left(\frac{1}{x} - 1\right) - (x/(1-x)) \log x \right] \right\} \end{aligned} \quad (5)$$

with $x \equiv M_Q^2/t$ and we take $m_q \approx 0$.

1. - LAPLACE (BOREL) SUM RULE

This sum rule can be obtained by applying to both sides of (3) the Laplace (Borel) operator:

$$\hat{\mathcal{L}} \equiv (-Q^2)^N \frac{1}{(N-1)!} \left(\frac{\partial}{\partial Q^2} \right)^N \quad \lim_{N, Q^2 \gg} \frac{N}{Q^2} \equiv \tau \text{ fixed}, \quad (6)$$

where τ is the imaginary time sum rule variable. One obtains in this way and by using Eq. (4) the finite-energy-like sum rule (FESR):

$$2q_B^2 M_B^4 e^{-M_B^2 \tau} \approx \int_{M_b^2}^t dt e^{-t\tau} \frac{1}{\pi} \text{Im} \Psi_{\text{Pent}}(t) + [C_4 \langle O_4 \rangle + C_6 \langle O_6 \rangle \tau] e^{-\tau M_b^2} \cdot M_b^2, \quad (7)$$

where

$$C_4 \langle O_4 \rangle \approx M_b \mu_q^3 (-\log \sqrt{\tau} \Lambda)^{\delta_1 / -\beta_1} - \frac{\alpha_s \langle G^2 \rangle}{12\pi}, \quad (8a)$$

$$C_6 \langle O_6 \rangle \approx -\frac{1}{2} M_b g \langle \bar{q} \sigma^{\mu\nu} G_{\mu\nu}^a q \rangle \left(1 - \frac{M_b^2 \tau}{2}\right) - \frac{8\pi}{27} \alpha_s \langle \bar{q} q \rangle^2 \left(2 - \frac{M_b^2 \tau}{2} - \frac{M_b^4 \tau^2}{6}\right), \quad (8b)$$

with¹³⁾ $\mu_q \approx 192.4$ MeV is the renormalization group "spontaneous" light quark mass. $g \langle \bar{q} \sigma^{\mu\nu} \lambda_a / 2 G_{\mu\nu}^a q \rangle \approx -M_0^2 \langle \bar{q} q \rangle$: $M_0^2 \approx 0.4 \text{ GeV}^2$ from baryonic sum rules¹⁴⁾ while we take $\alpha_s \langle \bar{q} q \rangle^2 \approx 1.91 \times 10^{-4} \text{ GeV}^6$ from the fit of $e^+e^- \rightarrow I=1$ hadron data¹⁵⁾ and from baryonic sum rules^{14),*)}. We shall, however, see in our numerical analysis that the effects of the dimension-six condensates are not relevant and some changes of M_0^2 and of $\alpha_s \langle \bar{q} q \rangle^2$ within a factor two will not be seen in the analysis. The leading non-perturbative term is the $\langle \bar{q} q \rangle$ condensate but its value is accurately known from PCAC and from the value of the quark mass. The heavy quark mass to be used in the spectral function is the pole mass, which is known to be^{**) M_b(t=M_b^2) \approx (4.71 \pm 0.13) \text{ GeV}}. We study in Fig. 1a the behaviour of f_B for given values of $\Lambda \approx 0.18 \text{ GeV}$ and $M_b \approx 4.71 \text{ GeV}$ and for the changes in the sum rule variable τ for three values of the continuum threshold. Minimum sensitivity in the changes of τ is obtained at the minimum of the curves.

*) Gluon condensate $\alpha_s \langle G^2 \rangle \approx 0.04 \text{ GeV}^4$ is a small correction to the quark condensate contribution.

**) This value has been obtained by using the fitted^{3a)} value of 4.23 GeV of the Euclidean mass in the Landau gauge and the relation $M_b(t=M_b^2) = M_b(t=-M_b^2) (1+2\log 2\tilde{\alpha}_s/\pi)$ with $\Lambda = (180 \pm 80) \text{ MeV}$.

In Fig. 1b, we take each minimum and study its behaviour by changing t_c . One notes that a stability is obtained for $t_c > 40 \text{ GeV}^2$, i.e., beyond this value the result is insensitive to the form of the continuum model. In contrast, the result of Ref. 4) obtained at $t_c \approx 33 \text{ GeV}^2$ is not yet in this stability region and is therefore sensitive to the way of parametrizing the continuum. The curve AE shown in Fig. 1b would correspond to the favoured value of M_b and $\langle \bar{q}q \rangle$ used in Ref. 4). We show in Fig. 1c the effects of M_b given the values of t_c and Λ , while we show in Fig. 1d the effects of Λ at fixed t_c and M_b . Taking into account the changes of various parameters, the Laplace sum rule provides the stable result:

$$f_B \approx (132 \pm 17) \text{ MeV} \equiv (1.41 \pm 0.18) f_\pi, \quad (9)$$

where the main uncertainties come from M_b . One might consider the too high value obtained in Ref. 5) as due to too low a value of M_b used in their analysis and to the non-inclusion of radiative corrections which feels the correct mass definition to be used in the analysis. We repeat the above method for the charm quark. We use the value of the pole mass $M_c(p^2=M_c^2) = (1.46 \pm 0.06) \text{ GeV}$ deduced from the Euclidean mass obtained in Refs. 2) and 3a). Our results are shown in Fig. 1e, from which we deduce the estimate:

$$f_D \approx (122 \pm 11) \text{ MeV}, \quad (10)$$

i.e., $f_D \approx f_B^*$). The result in Eq. (10) is comparable with previous results of Refs. 4) and 7b).

*) We shall comment on this point later on. One should remember that Laplace sum rules applied to the pseudoscalar light quark current have been able to estimate accurately the values of the running quark masses for given values of f_π and f_K^{13}).

2. - $Q^2 = 0$ MOMENT SUM RULE

A standard method used for the heavy quark systems is the $Q^2 = 0$ moment²⁾:

$$\left. \frac{(-1)^n}{n!} \frac{\partial^n \psi}{(\partial Q^2)^n} \right|_{Q^2=0} = \int_{M_Q^2}^{\infty} \frac{dt}{t^{n+1}} \frac{1}{\pi} \text{Im } \psi(t), \quad (11)$$

where one exploits the fact that $M_Q \gg \Lambda$ in such a way that an expansion in $1/M_Q^2$ is expected to work. The lowest moment in Eq. (3) has already been used to get a useful bound on f_D once radiative and non-perturbative corrections are included. This bound is^{16), 7)}:

$$f_D \leq 2.14 f_{\pi}, \quad (12)$$

which is stronger than the new charmed data upper limit of $2.6 f_{\pi}$ ¹⁷⁾. However, the extension of this result to heavier systems such as the B meson might give a very weak bound because it increases linearly with the B-mass. Uses of higher moments become necessary in this latter case. From the model in Eq. (4), the sum rule in Eq. (11) provides:

$$2 f_B^2 \approx (M_b^2)^{n-1} \cdot \left\{ \int_{M_b^2}^{t_c} \frac{dt}{t^{n+1}} \frac{1}{\pi} \text{Im } \psi_{\text{pert}}(t) + \right. \\ \left. \frac{1}{(M_b^2)^n} \left[\frac{g_2 \langle G^2 \rangle}{12\pi} - M_b \langle \bar{q}q \rangle \right] + \dots \right\}, \quad (13a)$$

where $\langle \bar{q}q \rangle = -\mu_q^3 (\log M_b/\Lambda)^{\delta_1/\beta_1};$

We give in Fig. 2a the behaviour of f_B versus n and t_c by giving the values of M_b and Λ . The bound $5 f_{\pi}$ obtained from the analogue of (12) could be compared with the estimate of $f_B \approx 2.4 f_{\pi}$ from the lowest moment ($n=2$). The continuum effect is the main cause of such differences. This effect can be decreased when working with higher moments. However, we cannot go to very high n as QCD corrections become too important^{*)}. The compromise values of n appear to be about

*) One should note that for the D-meson, non-perturbative corrections are already large for $n = 3$.

8 ~ 10. We show in Fig. 2b the effects of t_c on a different minimum of Fig. 2a by giving M_b and Λ . In Fig. 2c we show the effects of M_b . In Fig. 2d, we show the effects of Λ on the value of the minimum. The estimate from the $Q^2 = 0$ sum rule for $\Lambda = (180 \pm 80)$ MeV; $M_b = (4.71 \pm 0.13)$ GeV is:

$$\mathcal{J}_B = (130 \pm 22) \text{ MeV} \quad (13b)$$

3. - $Q^2 = Q_0^2$ MOMENT SUM RULE

A variant of Eq. (11) which, in principle, is more convergent than the QCD series in (11) is the sum rule⁶⁾:

$$\frac{(-1)^n}{n} \left. \frac{\partial^n \psi}{(\partial Q^2)^n} \right|_{Q^2=Q_0^2} = \int_{M_b^2}^{\infty} \frac{dt}{(t+Q_0^2)^{n+1}} \frac{1}{\pi} \text{Im} \psi(t), \quad (14a)$$

which gives

$$\begin{aligned} 2Q_B^2 \approx (M_B^2 + Q_0^2)^{n+1} \frac{1}{M_b^4} \left\{ \int_{M_b^2}^t \frac{dt}{(t+Q_0^2)^{n+1}} \frac{1}{\pi} \text{Im} \psi_{\text{pert}}(t) \right. \\ \left. + \frac{1}{(Q_0^2 + M_b^2)^n} \left[\frac{\alpha_s \langle G^2 \rangle}{12\pi} - M_b \langle \bar{q}q \rangle \right] \right\}. \end{aligned} \quad (14b)$$

In order to be less dependent on the deviation of M_Q from the pole mass we work at quite small values of Q_0^2 , say $\frac{1}{2}M_Q^2$ and M_Q^{2*}). We give the effects of Q_0^2 in Fig. 2e for given M_Q , Λ and t_c . The values of the minimum are almost unaffected by this change of Q_0^2 . Our result in Eq. (13) agrees within the errors with the one obtained previously from the same method^{6),7)}.

*) For higher values of Q_0^2 one should take care about the exact definition of M_Q^2 which is, however, gauge dependent²⁾.

4. - NON-RELATIVISTIC SUM RULES

These are based on a Laplace sum rule analysis of the B-meson wave function from the study of the two-point correlator⁹⁾:

$$T_n^{\mu\nu} = i \int d^4x e^{iqx} \langle 0 | \bar{\psi}(x) \gamma^\mu \gamma_5 \psi(x) \bar{q}(0) \gamma^\nu (i \overleftrightarrow{D})^n q(0) | 0 \rangle$$

$$\equiv (3q)^n \left\{ g^\mu g^\nu T_{||}(q^2) + (g^\mu g^\nu - g^{\mu\nu} q^2) T_{\perp}(q^2) \right\}$$

(15)

(D being the covariant derivative) or of $\psi(q^2)$ in Eq. (1)⁸⁾ which is related to the longitudinal part $T_{||}$ by the Ward identity¹²⁾:

$$\psi(q^2) - \psi(0) = (q^2)^2 T_{||}(q^2).$$

(16)

Equation (16) shows that the Laplace transform of $T_{||}(q^2)$ should depend on the value of the two-point correlator at $q^2 = 0^{*})$. Therefore a sum rule which does not depend on $\psi(0)$ is the Laplace transform of e.g. $\psi(q^2)$. Like Ref. 8), we work in the "infinite mass" limit and we therefore neglect non-leading mass corrections. Then, we get the sum rule^{8),**)}:

$$2q_B^2 M_B e^{-E_c/m} \simeq \left\{ 6 \frac{m^3}{\pi^2} (1 + 0.75 \bar{\alpha}_0(M_B)) \cdot \right.$$

$$\cdot \left[1 - e^{-E_c/m} \cdot \left(1 + \frac{E_c}{m} + \left(\frac{E_c}{m} \right)^2 \frac{1}{2} \right) \right] +$$

$$\mu_q^3 \left(1 - \frac{M_0^2}{32 m^2} \right) \left. \right\} \left(\log \frac{M_B}{\Lambda} \right)^{\delta_1 / -\beta_1} +$$

$$\frac{\pi}{62 m^3} \alpha_0 \langle \bar{q}q \rangle^2,$$

(17)

where we have taken into account the log-dependence due to the anomalous dimension of the $\bar{q}\gamma_5 q$ current and to $\langle \bar{q}q \rangle$; m is the Laplace sum rule variable.

*) This contribution has not been considered in Ref. 9).

**) The radiative correction for the perturbative part has been obtained by using a $1/M_Q$ expansion. This result agrees with the previous one^{7),16)} but disagrees with Ref. 8).

E_r is the energy of the lowest ground state which is expected to be around 400 MeV^{8),9)}. The continuum threshold E_c can be estimated from the FESR deduced from Eq. (17) for $m \rightarrow \infty$

$$2f_B^2 M_B \approx \frac{E_c^3}{\pi^2} - \langle \bar{q}q \rangle, \quad (18)$$

which would correspond to $E_c \approx 1.2$ GeV if one uses our previous estimate of f_B . We give the results from Eq. (17) in Fig. 3. Figure 3a shows the effect of Λ and the sum rule scale m at given (M_Q, E_c, E_r) . In Fig. 3b, we show the effects of E_c given the other set of parameters. Stability of the prediction is obtained at very low m of about 0.25 ~ 0.4 GeV. At these values, the perturbative term dominates the $\langle \bar{q}q \rangle$ ones while the effects of the $\langle \bar{q}q \rangle^2$ contribution are negligible. However, it is not clear if high dimension condensates will be important at such a scale. We show in Fig. 3c the effects of E_c on f_B . The result is insensitive to the choice of E_c for $E_c > 1.3$ GeV. Previous results obtained at very low E_c should be very sensitive to the form of the continuum. This explains why the result of Refs. 8) and 9) is too low. However, we fail to understand the normalization of f_B in Ref. 9) when they convert their result in order to compare it with the one of Ref. 8). We have checked that Refs. 8) and 9) have the same definition of f_B . The effect of E_r on f_B is also obvious. Taking $E_r \approx (0.4 \pm 0.1)$ GeV will introduce an additional 18% uncertainty on f_B . Combining various effects, we have from the non-relativistic sum rule:

$$f_B \approx (125 \pm 28) \text{ MeV}. \quad (19)$$

If we take the mean value of f_B obtained from different forms of the sum rules and if we add the error quadratically, we obtain:

$$f_B \approx (129 \pm 13) \text{ MeV} \quad (20)$$

which we consider as the best estimate of f_B from the sum rules.

5. - MASS DEPENDENCE OF f_B

One might expect that the non-relativistic sum rule in Eq. (17) provides a clear mass dependence of f_B . This could have been the case if the $\langle \bar{q}q \rangle$ condensate had given the dominant effect at the optimization scale of this sum rule, as in this way one would have obtained:

$$f_B^2 \approx \frac{e}{2M_B} \mu_q^3 \left(\log \frac{M_b}{\Lambda} \right)^{\gamma_1 / -\beta_1}, \quad (21)$$

as expected from analogous arguments^{10),18)}. However, our analysis has shown that the rôle of the perturbative term is more important than the $\langle \bar{q}q \rangle$ one. We do not have a clear statement on how the sum rule scale behaves with M_B . If it is flavour-independent then the behaviour in $M_b^{-\frac{1}{2}}$ of f_B is still maintained except that the main effect on the numerical value is due to the perturbative term but not by Eq. (21). An alternative qualitative way which can help to understand the important rôle of the perturbative effect on f_B is the estimate of the value of the two-point correlator $\psi(q^2)$ at zero momentum transfer. It reads, for zero values of m_q and $\langle \bar{Q}Q \rangle$:

$$\psi(0) = -M_Q \langle \bar{q}q \rangle + \frac{3}{8\pi^2} M_Q^4 \log \frac{M_Q^2}{v^2}, \quad (22)$$

where the non-perturbative term comes from the fact that the axial charge of the light quark does not annihilate the vacuum¹⁹⁾ while the second term comes from the analysis in Ref. 12). If one uses now a pole dominance of the spectral function, one would get a "low-energy theorem" for pseudoscalar mesons made with light and heavy quarks:

$$2f_B^2 M_B^2 \approx -M_b \langle \bar{q}q \rangle + \frac{3}{8\pi^2} M_b^4 \log \frac{M_b^2}{v^2}, \quad (23)$$

where v is the $\overline{MS}$ subtraction scale. For light quark systems, Eq. (23) represents the successful PCAC relation for the pion where a perturbative term can be neglected in the chiral limit ($m_q \rightarrow 0$) and where the pion mass squared vanishes like m_q . Here, Eq. (23) shows that the perturbative term cannot be neglected as indicated from the sum rule analysis. We want to point out from the above analysis that the M_b dependence on f_B is not quite obvious contrary to the claims in Refs. 8)-10) and 18).

6. - $SU(3)_F$ BREAKINGS AND ESTIMATE OF f_{D_s} AND f_{B_s}

Estimate of the decay constant for the meson having a strange quark can be approximately deduced from the non-strange one by using the expression of the $n = 2$ moments evaluated at $Q^2 = 0^{16)}$. One has:

$$f_{H_2} \approx f_H \cdot \frac{M_{H_2}}{M_H} \left\{ 1 + 3 \frac{\bar{m}_D}{M_Q} \left(1 - \frac{M_Q^2}{t_c} \right) - \frac{8\pi^2}{M_Q^3} \langle \bar{D}D - \bar{u}u \rangle \right\}^{1/2} \quad (24)$$

where t_c is the continuum threshold, $H \equiv D, B$ mesons and $Q \equiv c, b$ quarks.

Using¹³⁾_c: $\langle \bar{D}D \rangle / \langle \bar{u}u \rangle \approx 0.6$, $\langle \bar{u}u \rangle = -\mu_q^3 (\log M_Q/\Lambda)^{1/\beta_1}$,
 $m_c/M_c \approx 0.12$, $m_b/M_b \approx 0.04$,

one deduces:

$$f_{D_s} \approx 1.26 f_D \quad \text{and} \quad f_{B_s} \approx 1.08 f_B \left(\frac{M_{B_s}}{M_B} \right), \quad (25)$$

i.e., due to $SU(3)_F$ -breaking, the decay constant of the meson containing strange quarks is slightly larger than the one built with up quarks.

CONCLUSIONS

Motivated by the apparent discrepancies of the values of the decay constants of mesons built with light and heavy quarks, we have re-examined various predictions from QCD-duality sum rules. We have shown that in the region where the results are quite insensitive to the continuum threshold values, various (non-) relativistic sum rules lead to a unique prediction [Eq. (20)] for f_B . However, we might mention that non-relativistic-type sum rules are much affected by the anomalous dimension of the pseudoscalar current as well as by the value of the energy of the lowest ground state. We have also shown that the value of f_D is almost equal to that of f_B [Eq. (10)] while $SU(3)_F$ breaking effects tend to increase the analogous decay constant by 26% for the charm and about 10% for the bottom [Eq. (25)]. Our previous results still raise the question of the exact dependence of the decay constant of heavy-light quark bound states when one moves

the heavy quark mass. Due to the importance of the perturbative contributions to the sum rule analysis, it is not clear to see the argued $M_Q^{-\frac{1}{2}}$ behaviour^{10),18)} of such a decay constant. At present $f_D \approx f_B$ is very similar to the $f_\pi \approx f_K$ situation²⁰⁾. A future measurement of f_T will certainly shed light on the mass behaviour of the decay constant of the "infinitely" heavy meson.

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FIGURE CAPTIONS

Fig. 1 Laplace (Borel) sum rules

- a) Behaviour of f_B versus the sum rule scale τ for different values of the continuum threshold t_c at given values of (Λ, M_b) .
- b) Behaviour of f_B obtained at the minimum of a) versus t_c . AE represents the curve which one would obtain using the set of parameters in Ref. 4).
- c) Behaviour of f_B versus τ for various values of M_b .
- d) Λ effects on f_B .
- e) The same as for f_B but for f_D .

Fig. 2 Moment sum rules

- a) Behaviour of f_B versus the moment number n for different values of t_c .
- b) Behaviour of f_B obtained at the minimum of a) versus t_c .
- c) Behaviour of f_B versus n for different values of M_b .
- d) Λ effects on f_B .
- e) Effects of Q_0^2 on f_B versus n .

Fig. 3 Non-relativistic Laplace sum rules

- a) Behaviour of f_B versus the sum rule scale m for two values of Λ .
- b) Continuum threshold E_c effects on f_B .
- c) Plot of the minimum from Fig. 3b versus E_c .

- Figure 1a-d -

- Figure 1e -

- Figure 2a-d -

- Figure 2e -

- Figure 3 -