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DYNAMICAL ESTIMATES AND BOUNDS FOR THE LOCAL DENSITY OF DARK MATTER

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ABSTRACT

Bounds on the density of halo dark matter (DM) in the solar neighbourhood, $\rho_{\odot}^{\text{DM}}$, are discussed using a model that assumes that the radial distribution of DM in protogalaxies changes adiabatically as the gas falls toward the centre to form the visible cores. If protogalaxies have the structure expected in the cold DM cosmology, we argue that a variety of observational and theoretical constraints imply $0.2 \lesssim \rho_{\odot}^{\text{DM}}/\text{GeV cm}^{-3} \lesssim 0.43$ and that the luminous matter mass fraction $F \lesssim 0.12$. We argue that more likely bounds are $0.3 \lesssim \rho_{\odot}^{\text{DM}}/\text{GeV cm}^{-3} \lesssim 0.43$ and $F \lesssim 0.1$ and that an estimate is possible for $F \approx 0.1$. It appears that similar bounds could apply in the cosmic string cosmology with hot DM.

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The nature of the DM that dominates the dynamics of galaxies and larger structures on scales larger than their inner cores¹⁾ is a question of fundamental importance to both particle physics and astrophysics. A variety of arguments suggest that the DM is not baryonic²⁾ [although it may be argued that it is "little Jupiters"³⁾, low luminosity objects of planetary masses], and it could well be a sea of elementary particles: a variety of particle physics models⁴⁾ allow very weakly interacting elementary particles to be left in sufficiently high numbers after the Big Bang to be gravitationally dominant on large scales. We are very fortunate that relatively inexpensive particle-DM detectors (for modern particle physics and astronomy standards) that have been proposed or are under construction⁵⁾ will shed much light on this most fundamental question just a few years from now; many implications are obvious, although nothing escapes human imagination more than the future.

Estimates of detection rates for the various particle-DM candidates depend, of course, on the local density of DM and, therefore, it is important to determine this quantity as carefully as possible. We discuss in this paper the local density of DM that is implied by the infall model of the rotation curves of spiral galaxies that has been extensively discussed by Blumenthal et al.^{6),7)}. In view of the many uncertainties that plague an analysis of this kind (due to uncertainties in the observational parameters, on the one hand, and in the detailed dynamics of the various components of our Galaxy on the other), we will not attempt to argue for a specific value of $\rho_{\odot}^{\text{DM}}$, but will instead show that the model can be used to obtain reasonable and, we hope, useful bounds on $\rho_{\odot}^{\text{DM}}$ that are in agreement with detailed mass models of the Galaxy^{8),9)}. We find it convenient to discuss $\rho_{\odot}^{\text{DM}}$ in units of a scale density $\rho_s \equiv v_{\infty}^2 / 4\pi G r_{\odot}^2$, so as to separate the uncertainties in $\rho_{\odot}^{\text{DM}}$ that come from our imprecise knowledge of the solar distance from the centre of the Galaxy, $r_{\odot}$, and the asymptotic circular velocity, v_{∞} , from the uncertainties that come from assuming intrinsically different DM distributions. The usefulness of this is illustrated by the following example. Detailed mass models for the Galaxy have been considered by Caldwell and Ostriker⁸⁾ (hereafter CO) and Bahcall, Schmidt and Soneira⁹⁾ (hereafter BSS) from which they obtain $\rho_{\odot}^{\text{DM}}(\text{CO}) = 0.006 M_{\odot} \text{pc}^{-3}$ and $\rho_{\odot}^{\text{DM}}(\text{BSS}) = 0.009 M_{\odot} \text{pc}^{-3}$. The circular velocity at large distances is very nearly equal in both models and if we take $v_{\infty} \approx v(60 \text{ kpc})$ (see the discussion for $F = 0.1$ below), $v_{\infty}^{\text{CO}} \approx v_{\infty}^{\text{BSS}} \approx 213 \text{ km sec}^{-1}$ [the circular velocity is gently falling (rising) at this radius in the model of CO (BSS)]. Although $\rho_{\odot}^{\text{DM}}(\text{BSS})$ is 50% higher than $\rho_{\odot}^{\text{DM}}(\text{CO})$, $(\rho_{\odot}^{\text{DM}}/\rho_s)_{\text{BSS}} \approx 0.69$ is only about 20% higher than $(\rho_{\odot}^{\text{DM}}/\rho_s)_{\text{CO}} \approx 0.57$. This is because approximately 60% of the difference is due to the different

values they take for $r_\odot$: $r_\odot = 9$ kpc (8 kpc) for CO (BSS). The remaining difference is due to the different "core radii" of the assumed DM distributions, a , and the distribution themselves, $\rho^{\text{DM}}(r) \propto [1+(r/a)^\gamma]^{-1}$ with $(\gamma, a) = (2, 7.8 \text{ kpc})$ ($(=1.8, 2 \text{ kpc})$) for CO (BSS), [Bahcall and Soneira¹⁰⁾ originally considered $\gamma = 2$ also]. Turner¹¹⁾ obtains $\rho_\odot^{\text{DM}} = 7.5 \times 10^{-3} M_\odot \text{pc}^{-3}$, 20% lower than BSS; however, $(\rho_\odot^{\text{DM}}/\rho_s)_T \approx 0.67 \approx (\rho_\odot^{\text{DM}}/\rho_s)_{\text{BSS}}$ because although he takes $v_\infty = 220 \text{ km sec}^{-1}$, slightly higher than BSS, he takes $r_\odot = 9$ kpc as CO (he takes from BSS that half of the circular velocity squared at the solar radius is due to DM to obtain $a \approx 6.4$ kpc, hence the difference with CO). The fact that the values of $\rho_\odot^{\text{DM}}/\rho_s$ are similar reflects the fact that the models choose similar forms of the DM distribution. A consequence of assuming $\rho^{\text{DM}}(r) \propto [1+(r/a)^2]^{-1}$ is that $\rho_\odot^{\text{DM}}/\rho_s < 1$; large scale N-body simulations^{12), 13)} show that this profile is reasonable after virialization of DM halos, but the formation of the baryonic cores (the visible galaxies) will significantly distort the distribution of the DM^{6), 14)} and, thus, the halo distribution will not be of the form assumed in Refs. 8)-11). We find that $\rho_\odot^{\text{DM}}/\rho_s > 1$ is possible in the model, but the rotation curves fall by more than 10% from $r = r_\odot$ to $r \approx 60$ kpc for $\rho_\odot^{\text{DM}}/\rho_s \gtrsim 1.2$; this will be our upper bound. We argue that for cold DM $\rho_\odot^{\text{DM}}/\rho_s \gtrsim 0.43$. This lower bound is somewhat sensitive to the ratio $r_\odot/b$, where b is the scale size of the exponential disk of the Galaxy, due to the constraint that the circular velocity $v_c(r = 1.6 r_\odot)$ not fall below $v_c(r = r_\odot) \equiv v_\odot$ (see below), and was obtained for $r_\odot/b = 2.0$; for more likely values, $r_\odot/b \gtrsim 2.3$, $\rho_\odot^{\text{DM}}/\rho_s \gtrsim 0.5$. The constraints advocated here also imply $F \lesssim 0.12$, but for $r_\odot/b \gtrsim 2.3$, $F \lesssim 0.1$. For $r_\odot/b \approx 2.3$, the parameter space shrinks to a point for the typical value $F \approx 0.1$ and an estimate of $\rho_\odot^{\text{DM}} \approx 0.3 \text{ GeV cm}^{-3}$ results. We discuss the implied rotation curve in this case.

We begin by describing very briefly the model used to calculate $\rho_\odot^{\text{DM}}/\rho_s$; for a more detailed discussion see Refs. 6) and 7). The model assumes that while the gas of a protogalaxy dissipatively falls toward the centre, the orbits of DM-particles adiabatically shrink. This is a reasonable assumption for DM-particles not too close to the centre of the galaxy because the mass fraction of "baryonic" (i.e., ordinary matter: gas, stars, stellar remnants, etc.) matter $F \equiv M_b/M \ll 1$; here $M(M_b)$ is the total (baryonic) mass of the protogalaxy. Thus, a DM particle in circular orbit initially at radius r_i moves inwards to radius r , such that:

$$r [M_b(r) + M_{DM}(r)] = r_i M_i(r_i) \quad (1)$$

where $M_i(r_i)$ is the total mass inside the particle's orbit. Assuming that shells of DM-particles do not cross and that the baryons and the DM are initially well mixed, $M_{DM}(r) = (1-F)M_i(r_i)$ and Eq. (1) can be used to find r and, thus, $M_{DM}(r)$ provided that $M_i(r_i)$ and the final mass distribution of the baryons, $M_b(r)$, are given. The surface brightness of the disk-like component of spiral galaxies is observed to be well fitted by an exponential fall-off from the centre; assuming constant mass to light ratios (as indicated by the absence of significant colour gradients), the mass surface density of the disk is $\mu(r) \propto \exp(-r/b)$. Thus, for a disk-dominated spiral $M_b(r) \propto 1 - (1+r/b)\exp(-r/b)$ [assumed to be spherically distributed when using Eq. (1); see discussion below]. N-body simulations^{12),13)} show that protogalaxies have density profiles that are well approximated by isothermal spheres if the fluctuation power spectrum is sufficiently shallow on galaxy scales [as is the case for cold DM¹⁵⁾]. We shall take a protogalaxy to be an isothermal sphere truncated at radius R ; it will be assumed that no gas falls in from beyond R ⁷⁾. If structure in the Universe forms by hierarchical clustering from smaller to larger scales [this is the case for cold DM¹⁵⁾], then galactic angular momentum can be understood as a result of the tidal torquing exerted by nearby protogalaxies which subsequently move away, leaving a protogalaxy with a net angular momentum. In the tidal torque theory, protogalaxies have a mean angular momentum^{16),17)} $\langle \lambda \rangle \sim 0.07$ where λ is the dimensionless spin parameter, $\lambda \equiv |\vec{J}| |E|^{1/2} / GM^{5/2}$, [here $\vec{J}(E, M)$ is the total angular momentum (energy, mass) of the protogalaxy]. Thus, we take protogalaxies to be characterized by three dimensionless quantities^{6),7)}: its spin λ , the core radius of the initial isothermal distribution in units of the truncation radius, r_{core}/R , and the baryonic mass fraction, F . Assuming no angular momentum exchange, the disk scale length can be calculated from λ ⁷⁾: for $\lambda \sim 0.4 - 0.1$, $b/R \propto \lambda$. Typical values that lead to flat rotation curves are⁷⁾ $F \sim 0.1$, $r_{core}/R \sim 0.4$ and $\lambda \sim 0.07$. As such, the protogalaxy structure assumed here might be similar in a cosmic string cosmology with hot DM; halos are expected to have r^{-2} density profiles¹⁸⁾ and the phase space density constraint¹⁹⁾ would imply large values $r_{core} > 10$ kpc (for a neutrino mass of ~ 25 eV) in this case. However, it is not clear what the corresponding range for λ would be in this case.

The DM density can be calculated at a given point from Eq. (3) in Ref. 6):

$$\rho_{\odot}^{\text{DM}}/\rho_s = (b/R)^{-1} d\tilde{M}(x)/dx \Big|_{x=r_{\odot}/b} \quad (2)$$

where $\tilde{M}_{\text{DM}}(x = r/b)$ is the DM mass distribution normalized to unity at R , and $\rho_s \equiv v_{\infty}^2/4\pi G r_{\odot}^2$ with $v_{\infty}^2 \equiv GM/R$. We are now ready to calculate $\rho_{\odot}^{\text{DM}}/\rho_s$ in this model, which will depend on F , r_{core}/R , λ and $r_{\odot}/b$. Since none of these parameters is known with precision, we begin by discussing the various constraints that do exist on them. These, plus constraints on the Galaxy rotation curve, will lead to the bounds claimed here.

A variety of observations imply¹⁵⁾ $0.05 \lesssim F \lesssim 0.2$ while a rough estimate is $F \approx 0.1$ (assuming that F is a universal constant, as expected in any model in which the formation of structure proceeds hierarchically and the DM is non-baryonic). We shall take $0.05 < F < 0.2$. Protogalaxies are not expected to have a universal λ ; N-body simulations¹⁷⁾ find that the dispersion about the mean λ would be $\Delta\lambda \sim 0.03$. We shall take $0.03 < \lambda < 0.1$ (for the main result it will not matter to consider λ outside this range). Core radii are expected to be large in a Universe dominated by cold DM^{12),13)}, but it seems likely that $r_{\text{core}}/R \lesssim 0.5$. The numerical simulations^{12),13)} do not include dissipational infall and, therefore, one cannot calculate r_{core}/R ; however, it seems reasonable that $R \lesssim$ the maximum radius within which the gas can quasi-statistically contract in less than a Hubble expansion time $\approx 75(F/0.1)^{\frac{1}{2}} h^{-\frac{1}{2}}$ kpc (here h is the Hubble constant in units of $100 \text{ km sec}^{-1} \text{ Mpc}^{-1}$). Assuming that R will not be significantly less than this value, the structure of the halos in Fig. 2 of Ref. 12) indicates that $r_{\text{core}}/R \lesssim 0.5$ (taking r_{core} $\approx$ radius at which the circular velocity reaches half of its asymptotic value). Thus, it seems reasonable to us to take $r_{\text{core}}/R \leq 0.5$ for cold DM. For cosmic strings with hot DM it is not clear how large r_{core}/R could be; however, relaxing the constraint $r_{\text{core}}/R \leq 0.5$ will weaken our lower bound on $\rho_{\odot}^{\text{DM}}$ by only $\approx 10\%$.

The currently IAU recommended value for $r_{\odot}$ ²⁰⁾ is $8.5(\pm 1.1)$ kpc [it is fair to mention that the previously recommended value was 10 kpc; for a review of the implications of a smaller $r_{\odot}$ see Ref. 21)]. CO conclude $r_{\odot} = 8.8 \pm 0.6$, based on an older subset of the observations considered in Ref. 18), and their more recent model solution is²²⁾ $r_{\odot} = 8.2$ kpc; BSS take $r_{\odot} = 8$ kpc. We shall take $8 \leq r_{\odot}/\text{kpc} \leq 10$. On the other hand, it seems that²³⁾ $b_G \approx 3.5$ kpc for the Galaxy, Knapp et al.²⁴⁾ obtain $b_G = 3.2 \pm 0.6$ kpc. The mean for Sbc galaxies (the

generally assumed Hubble type of the Galaxy) in the Freeman²⁵⁾ sample is $b = 3.4 \pm 0.5$ kpc for $h = 1$ (3.0 ± 0.7 for $h = \frac{1}{2}$). Bahcall and Soneira²⁶⁾ obtain $b_G \gtrsim 2.5$ kpc. We shall take $2.5 < b_G/\text{kpc} < 4$ and, therefore, $2 < r_\odot/b_G < 4$; it is this latter constraint that really enters the calculation (thus, even a value as low as $r_\odot = 7$ kpc is considered here if $b_G < 3.5$ kpc).

The local escape velocity, $v_{E,\odot}$, provides a further constraint on the model. CO have analyzed the high velocity star data to conclude that $v_{E,\odot} = 640(\pm 15\%)$ km sec⁻¹; assuming an exact $\rho \propto r^{-2}$ mass distribution outside the sun, $v_{E,\odot} > 540$ km sec⁻¹ implies that the halo extends out to a radius > 60 kpc for $r_\odot > 8$ kpc. Modelling of the Magellanic stream^{27),28)} indicates large velocities out to at least the Magellanic Clouds. Lin and Lynden-Bell²⁸⁾ argue for a high velocity, $v_c(r \sim 70 \text{ kpc}) = 240 \pm 20$ km sec⁻¹, out to at least 70 kpc assuming a flat rotation curve from the sun outwards (a lower $v_\odot$ requires even higher velocities at large distances since one needs the halo to be sufficiently massive to produce tidal stripping). Murai and Fujimoto²⁷⁾ argue for large velocities only out to 50 kpc. Visvanathan²⁹⁾ finds a distance modulus for the SMC of 19.11 ± 0.07 , thus a distance from the centre of at least 60 kpc [taking two standard deviations, his distance modulus for the LMC is, however, three standard deviations larger than the value obtained by Schommer et al.³⁰⁾]. Assuming that R is not significantly less than the quasi-static contraction limit, it is consistent with observations then to take $R > 60$ kpc, thus $b_G/R \lesssim 0.06$ for $b_G = 3.5$ kpc. The bounds on $\rho_\odot^{\text{DM}}/\rho_s$ do not depend on this constraint. However, it will be important when discussing $\rho_\odot^{\text{DM}}/\rho_s$ for $F = 0.1$. We shall take $b_G/R < 0.06$ for illustrative purposes and allow for up to $b_G/R = 0.067$ ($b_G = 4$ kpc) wherever necessary.

The kinematics of CO-H_{II} regions outside $r_\odot$ provide a constraint on the rotation curve immediately outside $r_\odot$. CO⁸⁾ take $\delta v/v \equiv (v_c(1.6r_\odot) - v_\odot)/v_\odot = 0.08 \pm (10 \text{ km sec}^{-1}/v_\odot) = 0.080 \pm 0.045$ for $v_\odot = 220$ km sec⁻¹ as representing the data of Blitz³¹⁾ [their⁸⁾ constraint from the data of Jackson et al.³²⁾, $(v_\odot/r_\odot) - (v_c(r_\odot + 7 \text{ kpc})/(r_\odot + 7 \text{ kpc})) = 10.4 \pm 2.6 \text{ km sec}^{-1} \text{ kpc}^{-1}$, becomes weaker due to the uncertainty in $r_\odot$]. We shall take $\delta v/v > 0$ as our constraint. Constant circular velocity outside $r_\odot$ seems allowed by the data within the (large) error bars³³⁾. Also, a flat or slightly rising rotation curve at $r_\odot$ is indicated by the H_I data³⁴⁾.

The rotation curve of the Galaxy beyond ~ 18 kpc is very poorly known, but

there are constraints on it. Frenk and White³⁵⁾ argue for a flat or rising rotation curve out to at least 33 kpc (the limit of their sample) in order to understand the observed increase with galactocentric distance of the peculiar velocity dispersion of globular clusters. Sancisi and van Albada³⁶⁾ have discussed a sample of six spiral galaxies with measured H_I circular velocities out to quite large distances; while some rotation curves may gently fall beyond $\sim 2b$, no rotation curve shows fall off beyond $\sim 5b$. We shall consider the constraint of requiring the Galaxy rotation curve not to fall between $5b$ and $10b$, since the rotation curve extending the farthest is flat out to $11b$ (also, the data of Ref. 35) extend out to $\sim 10b_G$ for $b_G \sim 3.5$ kpc). We cannot argue for a statistically significant constraint from such a small sample and having to extrapolate a behaviour for four of the rotation curves. It is, however, consistent with the requirement of Ref. 35) and will be shown for illustrative purposes. Our constraint on the exterior rotation curve of the Galaxy will be at large radii. CO take the logarithmic average of the two masses [depending on whether a purely isotropic (radial) velocity dispersion is assumed] obtained for the Galaxy interior to $6.7 r_\odot$ by Hartwick and Sargent³⁷⁾ and obtain $v_c(6.2r_\odot) = 200 \text{ km sec}^{-1} (v_\odot / r_\odot)^{1/2} \pm 20\%$, with $v_\odot(r_\odot)$ in units of 220 km sec^{-1} (8.5 kpc). The modelling of the Magellanic stream^{27), 28)}, however, requires much higher velocities $v_c = 240 \pm 20 \text{ km sec}^{-1}$ at these distances. We shall take $200 < v_c(60 \text{ kpc}) / \text{km sec}^{-1} < 260$ and require that v_c not fall by more than 10% from $r_\odot$ to 60 kpc (since H_I velocities are typically determined $\pm 5\%$, this seems a limiting possibility).

We are now ready to discuss the bounds on $\rho_\odot^{\text{DM}} / \rho_s$. Figure 1 is a plot of the various constraints on $\rho_\odot^{\text{DM}} / \rho_s$, as a function of λ , for $F = 0.05$. The bottom solid line is $\rho_\odot^{\text{DM}} / \rho_s$ in the limit $r_{\text{core}} / R \rightarrow \infty$ (i.e., the protogalaxy is initially a sphere of uniform density) for $r_\odot / b_G = 2$; a higher value of $r_\odot / b_G$ increases $\rho_\odot^{\text{DM}} / \rho_s$ because $\rho_s \propto r^{-2}$, whereas $\rho_\odot^{\text{DM}}$ falls off more slowly. The lowest value of $\rho_\odot^{\text{DM}} / \rho_s$ for $r_{\text{core}} / R = 0.5$, however, is the long-dash curve ($r_\odot / b = 2$). In the region to the right of the dash-dotted line, $b/R > 0.06$ (the curve is for $r_\odot / b = 2$), while to the left and upwards of the upper dotted line, the rotation curve falls by more than 5% between $5b$ and $10b$; to the left of the short dash-long dash curve $\delta v/v < 0$. Finally, outside the upper solid curve the circular velocity falls by more than 5% between $5b$ and $17b$ (≈ 60 kpc for $b_G = 3.5$ kpc). The rotation curve is strictly flat or rising out to $17b$ ($10b$) inside the dashed (lower dotted) curve. The shaded region is the region allowed by the constraints advocated here (see discussion below).

The size of the region of parameter space, where all the constraints are satisfied, rapidly decreases for larger F . Figure 2 is as Fig. 1, but for $F = 0.1$ [the curve corresponding to the dashed and lower dotted curves of Fig. 1 are not drawn]. We see that the size of the region where all the constraints are satisfied is rather small. In fact, the available parameter space reduces to zero for $F \sim 0.12$. For a typical value $F = 0.1$ the parameter space available reduces to zero for $r_{\odot}/b \approx 2.3$ because the boundary $\delta v/v = 0$ moves rapidly to the right for increasing $r_{\odot}/b$ [the arrows indicate the effect of changing $r_{\odot}/b$ from 2 to 2.4 ($= r_{\odot}/b_G \approx 8.5 \text{ kpc}/3.5 \text{ kpc}$) on the $\delta v/v = 0$ and $b_G/R = 0.06$ boundaries]. This is because the rotation curve falls rapidly outside $\sim 2b$; thus, if we put the Sun farther out $\delta v/v$ becomes more negative.

Figures 1 and 2 show that $F > 0.05$ implies bounds on $\rho_{\odot}^{\text{DM}}/\rho_s$. An upper bound occurs at $\lambda \sim 0.05$ for the circular velocity not to fall too sharply; the upper solid line of Fig. 1 was obtained for $r_{\odot}/b = 3.5$ and corresponds to a fall off of 5% between $5b$ and $17b$: this was done just to illustrate the nature of the upper bound. In order to have $\rho_{\odot}^{\text{DM}}/\rho_s$ as high as possible we should take $r_{\odot}/b = 4$; this forces $r_{\odot} = 10 \text{ kpc}$ and $b_G = 2.5 \text{ kpc}$. That the velocity not fall by more than 10% from $r_{\odot}$ out to 60 kpc implies $\rho_{\odot}^{\text{DM}}/\rho_s \lesssim 1.2$; $r_{\odot} = 10 \text{ kpc}$ implies high $v_{\odot} \approx 250 \text{ km sec}^{-1}$ due to constraints on the Oort constants²⁰⁾, thus $v_c(60 \text{ kpc}) \approx 225 \text{ km sec}^{-1}$, about the limit of what is acceptable to have sufficient mass inside $r = 60 \text{ kpc}$ so that gravity alone can explain the high velocities in the Magellanic stream^{27), 28)}. Taking smaller $r_{\odot}/b$ decreases $\rho_{\odot}^{\text{DM}}/\rho_s$, but the constraint is easier to satisfy and the bound remains nearly constant (it decreases slightly) for $2 < r_{\odot}/b_G < 4$. The effect of adding the inner spherical components of the Galaxy to the analysis decreases $\rho_{\odot}^{\text{DM}}/\rho_s$ (keeping fixed R , the flattening of the rotation curve in the inner parts causes smaller b , and ρ_s increases faster than $\rho_{\odot}^{\text{DM}}$). For the inner components advocated by BSS and CO, $\rho_{\odot}^{\text{DM}}/\rho_s$ decreases by ~ 0.04 . We conclude that $\rho_{\odot}^{\text{DM}}/\rho_s \lesssim 1.2$.

Likewise, a lower bound on $\rho_{\odot}^{\text{DM}}/\rho_s$, arises for $r_{\text{core}}/R \lesssim 0.5$. As Fig. 1 shows, a lower bound $\rho_{\odot}^{\text{DM}}/\rho_s \gtrsim 0.43$ occurs for $\lambda \sim 0.03$; the effect of inner components is in this case to lower $\rho_{\odot}^{\text{DM}}/\rho_s$ only by ~ 0.02 . Thus, $\rho_{\odot}^{\text{DM}}/\rho_s \gtrsim 0.4$ for $r_{\odot}/b = 2$. For more realistic values $r_{\odot}/b \gtrsim 2.3$ [Knapp et al.²⁴⁾ argue that the Burton and Gordon³⁷⁾ data for $r < r_{\odot}$ imply $r_{\odot}/b_G$ slightly greater than 2.3,

in agreement with their value from data for $r > r_{\odot}$; BSS also have $r_{\odot}/b_G \sim 2.3$; $r_{\odot}/b_G \sim 2.4$ for $r_{\odot} = 8.5$ kpc and $b = 3.5$ kpc], we find $\rho_{\odot}^{\text{DM}}/\rho_s \gtrsim 0.5$ because the $\delta v/v > 0$ constraint becomes more restrictive.

Larger values of F , however, considerably narrow down the range for $\rho_{\odot}^{\text{DM}}/\rho_s$. In fact, for $r_{\odot}/b_G \sim 2.3$, the constraints advocated here imply $F \lesssim 0.1$. Thus, for $F = 0.1$ the rotation curve is completely determined for $r_{\odot}/b \sim 2.3$ if one tries to find a rotation curve that best respects all constraints. For $r_{\odot} = 8.5$ kpc and $v_{\odot} = 220$ km sec $^{-1}$, $\rho_{\odot}^{\text{DM}} \approx 0.3$ GeV cm $^{-3} \approx 0.008$ M pc $^{-3} \approx 5.4 \times 10^{-25}$ gm cm $^{-3}$ ($\rho_{\odot}^{\text{DM}}/\rho_s = 0.7$ and $v_{\infty} = v(R) \approx 210$ km sec $^{-1}$). The rotation curve is marginally acceptable ($R = 60$ kpc, $\delta v/v = -0.015$) but occurs for typical⁷⁾ values $\lambda \sim 0.08$ and $r_{\text{core}}/R \sim 0.42$. The value of $\rho_{\odot}^{\text{DM}}$ is in fairly good agreement with that of BSS, but higher than that of CO. This is understandable because assuming the baryonic mass in their models fell in from within 60 kpc implies effective values of $F \sim 0.11$ (BSS) and 0.18 (CO). The rotation curve falls monotonically outside the solar radius. Flat or rising rotation curves require $F \lesssim 0.08$; we obtain a flat rotation curve with $F = 0.08$, $\lambda = 0.07$, $r_{\text{core}}/R = 0.4$, $\rho_{\odot}^{\text{DM}} = 0.32$ GeV cm $^{-3}$ ($v_{\odot} = v_{\infty} = 220$ km sec $^{-1}$, $r_{\odot} = 8.5$ kpc, $\delta v/v > 0$ and $R > 60$ kpc); F somewhat smaller than 0.1 might be required by the observed rising optical rotation curves of some spiral galaxies as well³⁸⁾.

The bounds and estimates of $\rho_{\odot}^{\text{DM}}/\rho_s$ presented here rely on a variety of simplifying assumptions and approximations; it is thus mandatory that we discuss their effect on $\rho_{\odot}^{\text{DM}}/\rho_s$. By far the weakest assumption is that of adiabaticity in the change of the DM distribution if we live well inside the disk, $r_{\odot}/b_G \sim 2$. Barnes³⁹⁾ and Ho et al.⁴⁰⁾ find that the adiabatic approximation significantly overestimates $M_{\text{DM}}(r)$ for $r \lesssim 4b$; from a $M_{\text{DM}}(r)$ vs. r plot⁴⁰⁾, we estimate that $\rho_{\odot}^{\text{DM}}$ would be overestimated by 50% (20%) at $r = 2b$ ($4b$). We believe, however, that our lower bound on $\rho_{\odot}^{\text{DM}}/\rho_s$ is not likely to be wrong by as much as 50% for the following reason. The requirement of a non-falling rotation curve at $r_{\odot}$ implies $\rho_{\odot}^{\text{DM}}/\rho_s > (v_h(r_{\odot})/v_{\infty})^2$, where $v_h^2(r_{\odot})$ is the halo contribution to v^2 at $r_{\odot}$. If $r_{\odot}/b = 2$, the ellipticity of the globular cluster distribution constrains⁴¹⁾ $v_h^2(r_{\odot})/v_{\odot}^2 \gtrsim 0.4$ at the 2- σ level [the axial ratio of the equipotential that intersects the disk at $2b$ ($4b$) is ~ 0.5 (0.75); thus the analysis⁴¹⁾ is applicable here] if one assumes a negligible baryonic spheroid. CO were led to a heavy bulge, $M_{\text{Bulge}}/M_{\text{Disk}} \sim 1$, because they tried to fit the

high velocities seen at $r \sim 0.5$ kpc with a single component; a small central component is enough⁹⁾, however, to reproduce the inner high velocities and $(M_{\text{Central}} + M_{\text{Bulge}})/M_{\text{Disk}} \sim 0.2$; a small $M_{\text{Bulge}} \sim 0.05 M_{\text{Disk}}$ is in good agreement with high velocity star and star count data⁹⁾. A small $M_{\text{Central}} + M_{\text{Bulge}} \sim 0.2 M_{\text{Disk}}$ is also consistent with the generally assumed Galaxy morphology. The rotation curve for the smallest $\rho_{\odot}^{\text{DM}}/\rho_s \sim 0.43$ has $(v_{\odot}/v_c(R))^2 \approx 1$. Thus, one would expect $\rho_{\odot}^{\text{DM}}/\rho_s \gtrsim 0.4$. For high $r_{\odot}/b_G$ one does not have any useful constraint⁴¹⁾ because it is entirely consistent to assume $v_h(r_{\odot})/v_{\odot} \ll 1$. One reason to expect the model to give a reasonable approximation to $\rho_{\odot}^{\text{DM}}$ even at small radii, $r_{\odot}/b_G \sim 2$, is that the Galaxy, being large, would lie close to the boundary of the Rees-Ostriker cooling diagram^{15),42)} and, thus, would have a cooling time of the order of the dynamical time of the entire system. Thus, the ratio of orbital to cooling time can be rather small at small radii.

Relaxing the assumption of spherical symmetry would imply higher values of $\rho_{\odot}^{\text{DM}}$. There is very good indirect evidence that the halo of our galaxy is quite round: polar ring galaxies⁴³⁾ exhibit disk-rotation curves that match very well (within the uncertainties) those of their rings, which are at 90° with respect to the disk. We would not expect halos to be flattened by rotation because $\lambda \ll 1$ and, thus, the ratio of minor (c) to major (a) axis $c/a \sim (1-6\lambda^2)^{\frac{1}{2}} \gtrsim 0.97$ for $\lambda \lesssim 0.1$. The globular cluster system³⁵⁾ and the spheroid of the Galaxy⁴⁴⁾ are observed to rotate rather rapidly with^{26),44)} $v_{\text{rot}} \sim 70 \text{ km sec}^{-1}$, thus $c/a \sim (1-6(v_{\text{rot}}/\sigma_{3-d})^2)^{\frac{1}{2}} \sim 0.8$, in agreement with the data²⁶⁾; $\sigma_{3-d} \sim 300 \text{ km sec}^{-1}$ is the three-dimensional velocity dispersion of the Galaxy. This goes in the direction expected as a result of baryonic contraction. Thus, we do not expect the halo to rotate as rapidly as the globular cluster system, as was assumed in Ref. 45). We would expect some enhancement of the DM density in the plane of the disk above the model values due to the formation of a really flat disk [it was assumed^{6),7)} that the baryonic matter was spherically distributed when using Eq. (1)]; this effect, however, appears to be small⁴⁰⁾.

We conclude, then, that $0.4 \lesssim \rho_{\odot}^{\text{DM}}/\rho_s \lesssim 1.2$ are reasonable bounds on this quantity. A brute force range for $\rho_{\odot}^{\text{DM}}$ would then be $0.11 (v_{\infty} \approx v_c(60 \text{ kpc}) \approx 200 \text{ km sec}^{-1}, r_{\odot} = 10 \text{ kpc}) \lesssim \rho_{\odot}^{\text{DM}}/\text{GeV cm}^{-3} \lesssim 0.75 (v_{\infty} \approx v_c(60 \text{ kpc}) \approx 260 \text{ km sec}^{-1}, r_{\odot} = 8 \text{ kpc})$. However, this is not self-consistent with the model rotation curves. The rotation curves with $\rho_{\odot}^{\text{DM}}/\rho_s \sim 1.2$ are falling with galacto-

centric distance. Since the upper bound occurs for $r_{\odot} = 10$ kpc, $v_{\odot} \approx 250$ km sec⁻¹ and $v_{\infty} \approx v_c(60 \text{ kpc}) \approx 200$ km sec⁻¹; the rotation curves at the bottom of the shaded region in Fig. 1 rise in the outer parts or monotonically and $v_{\infty} \approx 260$ km sec⁻¹, but $\rho_{\odot}^{\text{DM}}/\rho_s$ is much smaller. Thus, the upper bound occurs for $\rho_{\odot}^{\text{DM}}/\rho_s \sim 1.2$ and we conclude that $\rho_{\odot}^{\text{DM}} \lesssim 0.43 \text{ GeV cm}^{-3}$. The lower bound occurs for $\rho_{\odot}^{\text{DM}}/\rho_s \sim 0.4$ and $\lambda \sim 0.03$. The rotation curve falls outside $r_{\odot}$ to $v_c(60 \text{ kpc}) \approx 200$ km sec⁻¹, but then rises to $v_{\infty} \sim v_{\odot} \sim 220$ km sec⁻¹ (since $r_{\odot} = 8$ kpc in this case, we cannot take $v_{\odot} = 250$ km sec⁻¹); the rotation curves for $F \sim 0.12$ monotonically fall, but $\rho_{\odot}^{\text{DM}}/\rho_s$ is higher. We conclude that $\rho_{\odot}^{\text{DM}}/\rho_s \gtrsim 0.4$ and that $\rho_{\odot}^{\text{DM}} \gtrsim 0.2 \text{ GeV cm}^{-3}$. For cosmic strings with hot DM it is not clear how large r_{core}/R could be. Letting $r_{\text{core}}/R \rightarrow \infty$ weakens the lower bound to $\rho_{\odot}^{\text{DM}}/\rho_s \gtrsim 0.17 \text{ GeV cm}^{-3}$. It is not clear, however, that λ would be in the appropriate range in this case. For more likely values $r_{\odot}/b_G \gtrsim 2.3$, however, the $\delta v/v > 0$ constraint becomes more restrictive and the minimum $\rho_{\odot}^{\text{DM}}/\rho_s$ occurs for $\lambda \sim 0.05$: $\rho_{\odot}^{\text{DM}}/\rho_s \gtrsim 0.5$. The rotation curve rises to $v_c(R) \sim 1.2v_{\odot}$ in this case and the lower bound on $\rho_{\odot}^{\text{DM}}$ occurs for $F = 0.1$ instead: although $\rho_{\odot}^{\text{DM}}/\rho_s \sim 0.7$ in this case, the rotation curve monotonically falls and a smaller $\rho_{\odot}^{\text{DM}} \sim 0.3 \text{ GeV cm}^{-3}$ results. Thus, we conclude that for more likely values $r_{\odot}/b_G \gtrsim 2.3$, the lower bound is $\rho_{\odot}^{\text{DM}} \gtrsim 0.3 \text{ GeV cm}^{-3}$.

We have presented dynamical bounds on the density of halo DM in our own galaxy at the solar radius, based on a set of constraints that we believe to be reasonable and reliable if galaxies formed in the hierarchical fashion of the cold DM cosmology. We have presented an estimate of this halo density for a typical value $F = 0.1$ of the fraction of luminous matter in the Galaxy, and have discussed the characteristics of the implied rotation curve. We believe that our discussion adds some dynamical weight to the BSS value, $\rho_{\odot}^{\text{DM}} = 0.009 M_{\odot} \text{ pc}^{-3}$. Finally, we hope to have separated various sources of uncertainty in such a way that the reader can best judge the interconnection between various constraints and how the model changes under them.

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FIGURE CAPTIONS

Fig. 1 : Parameter space available (shaded region) in the $(\rho_{\odot}^{\text{DM}}/\rho_s, \lambda)$ plane for $F = 0.05$. The arrows indicate the direction in which the constraints discussed here are satisfied. A $r_{\text{core}}/R < \infty$ ($r_{\odot}/b = 2$); B $r_{\text{core}}/R < 0.5$ (cold DM; $r_{\odot}/b = 2$); C the rotation curve does not fall by more than 5% between $5b$ and $10b$ ($r_{\odot}/b = 3.5$); D strictly flat or rising rotation curve between $5b$ and $10b$ ($r_{\odot}/b = 3.5$); E sufficient infall, $b/R < 0.06$ ($r_{\odot}/b = 2$); F $\delta v/v > 0$ ($r_{\odot}/b = 3.5$); G the rotation curve does not fall by more than 5% out to $r = 60$ kpc ($r_{\odot}/b = 3.5$) and H rotation curve strictly flat or rising out to $r = 60$ kpc ($r_{\odot}/b = 3.5$). The value of $r_{\odot}/b$ indicated was used to calculate the corresponding boundary. See discussion of the shaded region in the text.

Fig. 2 : Same as Fig. 1 for $F = 0.1$. Curves A, D and H are not drawn. The arrows indicate how much the boundaries $\delta v/v = 0$ and $b/R = 0.06$ move when $r_{\odot}/b = 2 \rightarrow r_{\odot}/b = 2.4$. To the right of the double dot-dash curve, $\Delta v/v \equiv [v_c(4.5b) - v_c(3.5b)]/v_c(3.5b) > 0$. Thus, the galactic rotation curve inevitably falls, if $F \sim 0.1$, beyond $\sim 3.5b$ for the halo to extend sufficiently far (b/R sufficiently small). See discussion in the text.

Fig. 1

Fig. 2