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FLIPPED SU(5) FROM MANIFOLD COMPACTIFICATION OF THE
TEN-DIMENSIONAL HETEROTIC STRING

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ABSTRACT

We show that a recently proposed flipped $SU(5) \times U(1)$ GUT cannot be obtained from a conventional $SO(10)$ GUT, nor from the heterotic string by Hosotani gauge symmetry breaking of E_6 on a manifold with $(2,2)$ world-sheet supersymmetry. It can in principle be obtained by Hosotani gauge symmetry breaking of $SO(10)$ on a manifold with $(1,2)$ world-sheet supersymmetry. We identify the topological conditions under which the required chiral matter generations, Higgs multiplets, and gauge singlet fields can be light. In particular we show that nonperturbative world-sheet instanton effects neither destabilize the manifold nor give masses to the gauge singlets. The required Yukawa couplings are not forbidden by any known selection rules.

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Once upon a time, the ten-dimensional heterotic string¹⁾ was the only phenomenologically viable candidate to be a Theory of Everything (TOE). However, enthusiasm for its uniqueness was always tempered by the realization that there were many different ways of compactifying the heterotic string to four dimensions while maintaining space-time supersymmetry. These have included manifolds²⁾ and orbifolds³⁾ with gauge groups of rank 5 or 6 (in the manifold case)⁴⁾ or some other number (in the orbifold case),⁵⁾ and (2,2), (1,2), or (0,2) world-sheet supersymmetries.⁶⁾ More recently, it has been realized how to formulate string theories directly in four dimensions.⁷⁾ These presumably include as particular cases the aforementioned compactifications of the ten-dimensional heterotic string, but may also include other models. With all these possibilities, there is apparently a high degree of ambiguity in the four-dimensional effective field theory.

However, none of the models found so far has yet emerged as clearly superior. At the time of writing, no model is known which has just three generations and the Standard Model $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group, nor is any other phenomenologically acceptable rank-4 model known. The virtues of the rank-5 gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_E$ have been advocated,⁸⁾ and manifolds are known which yield this gauge group. However, none of the known examples has three generations or avoids severe problems with baryon decay, Cabibbo mixing and neutrino masses.⁹⁾ No other phenomenologically acceptable rank-5 model has yet been derived from any string theory. Manifold compactifications leading to models with rank 6 have also been studied extensively,¹⁰⁾ but these have several potential problems including that of excess entropy generation when the gauge symmetry is broken to a rank-4 gauge group by a large Higgs v.e.v. appearing at a low temperature $T \sim m_W$.¹¹⁾ The only known way to avoid this generic difficulty is by strong coupling effects at some scale $\Lambda_c \gg m_W$, and these appear only if the gauge group contains an $SU(5)$ factor.¹²⁾ The minimal such model is based on $SU(5) \times U(1)$, which has many attractive features: a very economical Higgs structure, natural doublet-triplet Higgs splitting by a simple missing-partner mechanism, etc.^{13)*} However, this model has not yet been derived from any string theory.

*Indeed, it is easy to show that this is the only viable GUT group that can be obtained by manifold compactification of the ten-dimensional heterotic string. The only alternative GUT for which there might exist Higgs representations suitable for breaking to the Standard Model is $SU(6) \times U(1)$. However, the gauge invariant couplings $\underline{15}_H \cdot \underline{15}_i \cdot \underline{15}_j$ and $\underline{6}_H \cdot \underline{6}_i \cdot \underline{15}_j$ are both needed to

In this paper we show how one might obtain such an $SU(5) \times U(1)$ model by manifold compactification of the ten-dimensional heterotic string. It cannot be obtained from a conventional $SO(10)$ GUT, nor from Hosotani breaking of E_6 on a manifold with $SU(3)$ holonomy and $(2,2)$ world-sheet supersymmetry. It can in principle be obtained by Hosotani breaking of $SO(10)$ on a manifold with $SO(6)$ holonomy and $(1,2)$ world-sheet supersymmetry. We exhibit the conditions on the vector bundles on the manifold which are needed to give an $SU(5) \times U(1)$ gauge group with the required N_g $(\underline{10} + \bar{\underline{5}} + \underline{1})$ chiral matter fields as well as light $(\underline{10} + \bar{\underline{10}})$ and $(\underline{5} + \bar{\underline{5}})$ Higgs fields. The required light gauge singlet fields φ_m can also be obtained if $H_1(\text{End } T) \neq 0$.⁶⁾ We demonstrate that known non-perturbative world-sheet instanton effects in $(1,2)$ models neither give masses to these gauge singlets φ_m (as would be the case in generic $(2,2)$ models),⁶⁾ nor drive the manifold toward the orbifold limit (as would be the case in generic $(0,2)$ models).¹⁴⁾ Moreover, all the Yukawa couplings required in the $SU(5) \times U(1)$ model are consistent with the underlying gauge symmetry.

We start by reminding the reader of the structure of the flipped $SU(5) \times U(1)$ model.¹³⁾ It contains N_g generations of matter fields with the transformation properties

$$F_i = (\underline{10}, 1), \quad \bar{F}_i = (\bar{\underline{5}}, -3), \quad 1_i^c = (\underline{1}, 5) \quad (1)$$

two conjugate pairs of Higgs representations

$$H = (\underline{10}, 1), \quad \bar{H} = (\bar{\underline{10}}, -1); \quad h = (\underline{5}, -2), \quad \bar{h} = (\bar{\underline{5}}, 2) \quad (2)$$

and $(N_g + 1)$ gauge singlets φ_m . The matter multiplets F_i contain (d_i^c, d_i, u_i, v_i^c) , while the $\bar{F}_i$ contain (u_i^c, v_i, l_i) . The Higgs multiplet h contains an $SU(3)$ $\underline{3}$ of charge $-1/3$ and an $SU(2)$ doublet (h^-, h^0) while $\bar{h}$ contains an $SU(3)$ $\bar{\underline{3}}$ of charge $+1/3$ and an $SU(2)$ doublet (h^+, h^0) . The superpotential is

give masses to the d and to the u, e respectively, but both couplings lead to catastrophic proton decay unless the D, D^c in the $\underline{15}_H, \bar{\underline{6}}_{\bar{H}}$ acquire large masses. Although the former is possible to arrange through a $\underline{15}_H^3$ coupling once $\langle v_H^c \rangle \neq 0$, there is no way to give a mass to the D^c except through a $\underline{6}_H \cdot \bar{\underline{6}}_{\bar{H}} \cdot \underline{1}$ coupling, which would simultaneously give mass to the (h^+, h^0) Higgs doublet. Hence the natural doublet-triplet mass splitting mechanism of our $SU(5) \times U(1)$ model is unique among GUT groups obtainable from the heterotic string in ten dimensions.

$$\begin{aligned}
W = & \lambda_1^{ij} F_i F_j h + \lambda_2^{ij} F_i \bar{F}_j \bar{h} + \lambda_3^{ij} \bar{F}_i 1_j^c h + \lambda_4 H H h + \lambda_5 \bar{H} \bar{H} \bar{h} \\
& + \lambda_6^{im} F_i \bar{H} \varphi_m + \lambda_7 h \bar{h} \varphi_m + \lambda_8^{mnp} \varphi_m \varphi_n \varphi_p
\end{aligned} \tag{3}$$

The roles of the terms are as follows: λ_1 gives m_d , λ_2 gives m_u , λ_3 gives m_l , λ_4 and λ_5 split the Higgs triplets and doublets, λ_6 provides the large entries in a seesaw neutrino mass matrix, λ_7 mixes the Higgs doublets and λ_8 stabilizes the desired vacuum in which $SU(5) \times U(1) \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y$ via a v.e.v. for the ν_H^c and $\bar{\nu}_H^c$ components of the $\underline{10}, \bar{\underline{10}}$.¹³⁾

It is easy to see that this model cannot be embedded in a traditional $SO(10)$ GUT, which would require building the $\underline{10}$ H and $\bar{\underline{10}}$ $\bar{H}$ up to $\underline{16} = H + \bar{\underline{5}}_H + \underline{1}_H$ and $\bar{\underline{16}} = \bar{H} + \underline{\bar{5}}_H + \bar{\underline{1}}_H$ respectively. The couplings λ_4 and λ_5 would be related by $SO(10)$ Clebsch-Gordan coefficients to $H \cdot \bar{\underline{5}}_H \cdot \bar{h}$ and $\bar{H} \cdot \underline{\bar{5}}_H \cdot h$ couplings, which would provide large mass terms removing the Higgs doublet components of h and $\bar{h}$ from the light spectrum.

Neither can the model be embedded in E_6 . The superpotential term λ_2 which gives m_u can only come from a $\underline{27}^3$ coupling, which implies that $(F, \bar{F}, \bar{h}) \in \underline{27}$. However, the doublet-triplet splitting mechanism requires the λ_5 term, which can only come from a $\underline{\bar{27}}^3$ coupling and therefore imposes $(\bar{H}, \bar{h}) \in \underline{\bar{27}}$. Hence there is no consistent assignment for the $\bar{h}$ multiplet. This argument applies equally to the case of flux-broken E_6 , and thereby prevents one from obtaining $SU(5) \times U(1)$ from $(2,2)$ compactifications of the ten-dimensional heterotic string.

Now we show that the model can, in principle, be obtained from a manifold compactification of the heterotic string. In order to retain one space-time supersymmetry, the world-sheet σ -model must have $(2,2)$, $(1,2)$ or $(0,2)$ supersymmetry on the world-sheet.⁶⁾ We will assume the compactified ten-dimensional space-time $(M, g_{\mu\nu})$ is a direct product $M = M_2 \times L$, of two-dimensional Minkowski space with an eight space L with coordinates X^i ($i=1, \dots, 8$), matrix g_{ij} , and background fields $b_{ij}(X^k)$ (antisymmetric term), $\varphi(X^k)$ (dilaton) and $A_i(X^k)$ (Yang-Mills gauge multiplet).^{15,16)} This includes the case of particular interest to us, where $M = M_4 \times K$ or $L = R^2 \times K$ giving compactification to four-dimensional Minkowski space. With this decomposition one has for the heterotic superstring the light-cone gauge action¹⁶⁾

$$S = \frac{1}{4\pi\alpha'} \int d\sigma d\tau [(g_{ij} + b_{ij}) \partial_+ X^i \partial_- X^j + i \lambda^a (\nabla_+^{(-)} \lambda)^a + i \Psi^A (\partial_- \Psi^A + A_i^{AB} (\partial_- X^i) \Psi^B) + \frac{1}{2} F_{ab}^{AB} \lambda^a \lambda^b \Psi^A \Psi^B] \quad (4)$$

where $\partial_{\pm} = \partial/\partial\tau \pm \partial/\partial\sigma$, and the covariant derivative (with torsion)

$$(\nabla_{\pm}^{(\pm)} \lambda)^a = \partial_{\pm} \lambda^a + (\partial_{\pm} X^i) \omega_i^{(\pm)ab} \lambda_b \quad (5)$$

for a set of orthonormal frames on L : $e_i^a(X)$, $a = 1, \dots, 8$, such that $\lambda^a = e_i^a \lambda^i$. The ten-dimensional spin connection is defined as

$$\omega_{\mu}^{(\pm)ab} = \omega_{\mu}^{ab}(e) \mp H_{\mu\nu\rho} e^{\nu a} e^{\rho b} \quad (6)$$

where in forms $H = 1/2 db - 1/8 \alpha' (\omega_{3Y} - \omega_{3L})$, ω_{3Y} is the Chern-Simons form constructed from the Yang-Mills connection and ω_{3L} is the Chern-Simons form constructed from the spin connection $\tilde{\omega}$. As Hull has emphasized,¹⁷⁾ the Lorentz and gauge anomalies can be cancelled for such a ω_{3L} for arbitrary $\tilde{\omega}$ by an appropriate variation in the b field.

By construction this action has a $(0,1)$ world-sheet supersymmetry under the transformations

$$\begin{aligned} \delta X^i &= \varepsilon \lambda^i \\ \delta \Psi^A + (\delta X^i) A_i^{AB} \Psi^B &= 0 \\ \delta \lambda^i &= e_a^i (\delta \lambda^a + (\delta X^j) \omega_j^{ab} \lambda_b) = i \varepsilon \partial_- X^i \end{aligned} \quad (7)$$

Under certain restrictions the action may possess larger world-sheet supersymmetries.^{15,16)} If L is a complex manifold with a complex structure $J_j^i(X)$ such that

$$\begin{aligned} J_j^i J_k^i &= \delta_k^i ; & J_{ij} &= -J_{ji} ; & \nabla_{(-)i} J_k^j &= 0 ; \\ J_{[j}^i F_{k]i} &= 0 ; & \text{and} & N_{ij}^k &= J_i^m J_{[j,m]}^k - J_j^m J_{[i,m]}^k = 0 \end{aligned} \quad (8)$$

then the action is invariant under a second global world-sheet supersymmetry

$$\begin{aligned}\delta' X^i &= \epsilon' J_j^i \lambda^j ; & \delta' \psi^A + (\delta' X^i) A_i^{AB} \psi^B &= 0 ; \\ \delta' (J_j^i \lambda^i) &= i \epsilon' \partial_- X^i\end{aligned}\tag{9}$$

and the theory has (0,2) world-sheet supersymmetry. If, on the other hand, the Yang-Mills gauge potential A is set $A \approx \omega^{(+)}$, then the action has (1,1) world-sheet supersymmetry. The combination of these conditions (complex structure and $A \approx \omega^{(+)}$) will result in a theory with (1,2) world-sheet supersymmetry. If in addition the torsion $H_{ijk} = 0$, so that $\omega^{(+)} = \omega^{(-)}$, then $A \approx \omega^{(-)}$ and the theory has (2,2) world-sheet supersymmetry.

Space-time supersymmetry requires that the spin connection $\omega^{(-)}$ has $SU(3)$ holonomy and that $\tilde{\omega} = \omega^{(+)}$. Thus in (2,2) models of the $E_8 \times E_8'$ heterotic string, the embedding $A \approx \omega^{(-)}$ implies that the gauge connection and field strength on K have values in an $SU(3)$ subgroup of E_8 . This leaves an observable sector E_6 gauge group before Wilson loop breaking. In (1,2) models, $A \approx \omega^{(+)} \neq \omega^{(-)}$, where in general $\omega^{(+)}$ will have $SO(6)$ holonomy for six real compactified dimensions. Then the $A \approx \omega^{(+)}$ embedding means that the gauge connection and field strength on K have values in an $SO(6)$ subgroup of E_8 , leading to an $SO(10)$ gauge group (which could be further broken by Wilson loops). In a general (0,2) model there would be no relation between A and $\omega^{(+)}$, and so if A is identified with a general holomorphic bundle over K with $SO(6) \sim SU(4)$ or $SU(5)$ holonomy, it would break the observable sector E_8 to $SO(10)$ or $SU(5)$ respectively. Thus, in principle, an $SO(10)$ model can arise from a (1,2) (or (0,2)) compactification of the $E_8 \times E_8'$ heterotic string.

Before we consider questions of non-perturbative stability and the existence of light singlets in these compactifications, let us verify that Wilson loop breaking can produce the required $SO(10) \rightarrow SU(5) \times U(1)$ gauge symmetry breaking as well as the required spectrum of light, gauge non-singlet multiplets. Let us assume the manifold K of compactification has $\pi_1(K) = G$. Let K_0 be the simply connected covering space of K . Pull the vector bundles and connections from K back to K_0 . Now G is a discrete symmetry of K_0 together with the pullback bundles and connections. The massless spectrum before quotienting by G includes $n(\underline{16}) + \delta(\underline{16} + \overline{\underline{16}}) + \epsilon(\underline{10})$, where n , δ , and ϵ are determined by the topology of the vector bundles over K_0 . In what follows, we first construct an

embedding $\bar{G}$ of G in $\text{Spin}(10)$, and then derive the conditions on the action of G on the $\text{SO}(10)$ multiplets over K_0 which leave invariant under the diagonal sum $G \Delta \bar{G}$ exactly those fields required for the flipped $\text{SU}(5)$ theory.

Up to $\text{SO}(10)$ automorphisms, the only generator of $\text{SO}(10)$ that leaves $\text{SU}(5)$ unbroken is

$$-iH = \begin{pmatrix} 0 & -1 & & & & \\ 1 & 0 & & & & \\ & & \ddots & & & \\ & & & \ddots & & \\ & & & & 0 & -1 \\ & & & & 1 & 0 \end{pmatrix} \quad (10)$$

Writing the $\underline{16}$ of $\text{SO}(10)$ as $\text{span}\{|s_1 \dots s_5\rangle | s_r = \pm 1, \prod s_r = -1\}$ and defining σ_r^2 by $\sigma_r^2 |s_1 \dots s_5\rangle = s_r |s_1 \dots s_5\rangle$, we see that H acts as $\frac{1}{2} \sum_r \sigma_r^2$ on $\underline{16}$. Since H is unique up to $\text{SO}(10)$ automorphisms, $\bar{G}$ is Abelian, hence we begin by analyzing the simplest Abelian case $G = \mathbb{Z}_n$ with $\bar{G}$ generated by $\bar{g} = \exp[-(2\pi i/n) \sum_r \sigma_r^2] \in \text{Spin}(10)$, the image of $1 \in G$. Under the action of G , $\underline{16} = \underline{10}(\alpha) + \underline{\bar{5}}(\alpha^{-3}) + \underline{1}(\alpha^5)$, where $\alpha = \exp(2\pi i/n)$. $\bar{G}$ acts on the $\underline{10}$ of $\text{SO}(10)$ through the projection of $\text{Spin}(10)$ onto $\text{SO}(10)$. Under this projection $\bar{g}$ maps to $\exp[-(4\pi i/n)H] \in \text{SO}(10)$. Thus under the action of $\bar{G}$ we find that $\underline{10} = \underline{5}(\alpha^2) + \underline{\bar{5}}(\alpha^{-2})$.

The field content of the flipped $\text{SU}(5)$ theory is specified in equations (1) and (2). To account for the excess of $\underline{10}$'s of $\text{SU}(5)$ coming from $\underline{16}$'s of $\text{SO}(10)$, it is necessary to assume that the $\underline{10}$, $\underline{\bar{5}}$, and $\underline{1}$ in a $\underline{16}$ transform differently under $\bar{G}$. This is equivalent to requiring either $n = 3$ or $n \geq 5$. In this case there can be at most one $\text{SU}(5)$ multiplet in each $\text{SO}(10)$ multiplet that is a $G \Delta \bar{G}$ singlet.

To find the constraints on the action of G , let l_k , $\bar{l}_k$, and m_k be the number of $\underline{16}$'s, $\underline{\bar{16}}$'s, and $\underline{10}$'s which transform as α^k under the action of $1 \in G$. Now, for example, since the $\underline{\bar{5}}$ in a $\underline{16}$ transforms as α^{-3} under the action of $\bar{g}$, this $\underline{\bar{5}}$ will be a $G \Delta \bar{G}$ singlet if and only if the $\underline{16}$ transforms as α^3 under the action of $1 \in G$. In this way we see that the $\underline{16}$'s, $\underline{\bar{16}}$'s, and $\underline{10}$'s give rise to $l_{n-1}(\underline{10}) + l_3(\underline{\bar{5}}) + l_0(\underline{1})$, $\bar{l}_1(\underline{10}) + \bar{l}_{n-3}(\underline{\bar{5}}) + \bar{l}_0(\underline{1})$ and $m_{n-2}(\underline{5}) + m_2(\underline{\bar{5}})$ respectively. This coincides with the fields of the minimal flipped $\text{SU}(5)$ theory if and only if

$$\left. \begin{aligned} l_{n-1} &= 4, & l_3 &= 3, & l_0 &= 3 \\ \bar{l}_1 &= 1, & \bar{l}_{n-3} &= 0, & \bar{l}_0 &= 0 \\ m_{n-2} &= 1, & m_2 &= 1 \end{aligned} \right\} \quad (11)$$

In the general case, a one-dimensional representation of G corresponds to a homomorphism Φ from G to $U(1)$. Since $\ker(\Phi)$ must contain the commutator subgroup of C of G , the action of G on its one-dimensional representations factors through the maximal Abelian quotient G/C of G . Since G/C is isomorphic to a product of cyclic groups, and since representations of G with dimension greater than one cannot give rise to any $G \Delta \bar{G}$ singlets, the analysis of the action of G presented above for the case $G = Z_n$ can easily be extended to cover arbitrary finite groups G .

Finally we turn to the required light gauge singlet fields ϕ_m . Since they are required to have Yukawa couplings to the gauge non-singlet matter and Higgs fields, they cannot be identified with moduli of the manifold or with deformations of its complex structure. However, they can be identified with non-trivial elements of $H_1(\text{End } T)$. For a manifold with $SU(3)$ holonomy, these transform as $(\underline{8}, \underline{1})$ of $SU(3) \times E_6$ and have in general non-zero couplings to the $(\underline{3}, \underline{27}) \times (\bar{\underline{3}}, \bar{\underline{27}})$ matter fields.⁶⁾ For a manifold with $SO(6)$ holonomy, they transform as $(\underline{15}, \underline{1})$ of $SO(6) \times SO(10)$. Therefore the λ_6 coupling to $(F_1 \sim (\underline{4}, \underline{10})) \times (\bar{H} \sim (\bar{\underline{4}}, \bar{\underline{10}}))$ and the λ_7 coupling to $(h \sim (\underline{6}, \underline{5})) \times (\bar{h} \sim (\bar{\underline{6}}, \bar{\underline{5}}))$ are consistent with the gauge symmetry, as are the $\lambda_8 \phi^3$ couplings.

The obvious danger is that the gauge singlets ϕ might all acquire large masses $O(m_p)$, for example from the type of non-perturbative world-sheet instanton effect which gives large masses to elements of $H_1(\text{End } T)$ on manifolds with $SU(3)$ holonomy.⁶⁾ In that case each ϕ vertex operator contains two left-moving fermion fields, while the world-sheet σ -model has $(2,2)$ supersymmetry so that there are four left-moving zero modes for a holomorphic instanton. Therefore a superpotential mass term $\propto \phi^2$ matches the number of left-moving zero modes and can be generated non-perturbatively. In the case of a manifold with $SO(6)$ holonomy and $(1,2)$ world-sheet supersymmetry, each ϕ vertex operator still contains two left-moving fermion fields, but one expects there to be only two left-moving zero modes, so no superpotential term $\propto \phi^2$ can be generated.*

*The same argument a fortiori excludes the generation of superpotential terms $\propto \phi^n$: $n > 2$, and a term $\propto \phi$ would be forbidden by gauge symmetry.

This is not the only interesting class of possible non-perturbative superpotential terms. As has been pointed out previously,¹⁴⁾ since the vertex operators for the gauge singlet fields associated with moduli of the manifold only contain right-moving fermions, a non-trivial potential can be generated for them when the world-sheet σ -model has $(0,2)$ supersymmetry.* This potential is believed to drive the generic $(0,2)$ model towards some orbifold limit. It has been commented before¹⁴⁾ that such terms are absent for models with $(2,2)$ supersymmetry, which therefore are not destabilized by these world-sheet instanton effects. Clearly the $(1,2)$ models of interest here are also stable against similar non-perturbative effects, by simple zero mode counting.

The main conclusions of this paper are as follows. The economical and natural flipped $SU(5)$ GUT advocated previously¹³⁾ cannot be obtained from a conventional $SO(10)$ or E_6 GUT, nor from compactification on a complex manifold with $SU(3)$ holonomy and $(2,2)$ world-sheet supersymmetry. The world-sheet instanton effects which have the generic effect of giving masses to four-dimensional gauge singlet fields in models based on $(2,2)$ compactifications⁶⁾ are absent in $(1,2)$ compactifications, as are the world-sheet instanton effects which tend to collapse the moduli of manifolds yielding $(0,2)$ compactifications.¹⁴⁾ Compactification on a manifold K with $SO(6)$ holonomy and $(1,2)$ world-sheet supersymmetry avoids all these pitfalls. We have identified in equation (11) the conditions on the action of the group $G = \pi_1(K)$ on vector bundles over K_0 , the covering space of K , which give the spectrum of light chiral superfields required in the minimal $SU(5) \times U(1)$ model.¹³⁾ A search for manifolds K with $SO(6)$ holonomy and $(1,2)$ supersymmetry which obey the conditions (11) is now underway.

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*This argument may be avoided in certain circumstances: see ref. 18.

References

1. D.J. Gross, J. Harvey, E. Martinec and R. Rohm, Phys. Rev. Lett. 54 (1985) 502; Nucl. Phys. B256 (1985) 253 and B267 (1986) 75.
2. P. Candelas, G.T. Horowitz, A Strominger and E. Witten, Nucl. Phys. B258 (1985) 46.
3. L. Dixon, J. Harvey, C. Vafa and E. Witten, Nucl. Phys. B261 (1985) 678 and B274 (1986) 285.
4. E. Witten, Nucl. Phys. B258 (1985) 75;
M. Dine, V. Kaplunovsky, M. Mangano, C. Nappi and N. Seiberg, Nucl. Phys. B259 (1986) 519;
S. Cecotti, J.-P. Derendinger, S. Ferrara, L. Giradello and M. Roncadelli, Phys. Lett. 156B (1985) 318;
J.D. Breit, B.A. Ovrut and G. Segre, Phys. Lett. 158B (1985) 33.
5. L. Ibanez, H.-P. Nilles and F. Quevedo, Phys. Lett. 187B (1987) 25 and CERN preprint TH.4673/87.
6. E. Witten, Nucl. Phys. B268 (1986) 79.
7. I. Antoniadis and C. Bachas, CERN preprint TH.4767/87 (1987) and references therein.
8. E. Cohen, J. Ellis, K. Enqvist and D.V. Nanopoulos, Phys. Lett. 161B (1985) 85;
J. Ellis, K. Enqvist, D.V. Nanopoulos and F. Zwirner, Nucl. Phys. B276 (1986) 14 and Mod. Phys. Lett. A1 (1986) 57.
9. J. Ellis, K. Enqvist, S. Kalara, D.V. Nanopoulos and K.A. Olive, CERN preprint TH.4770/87 (1987).
10. B. Greene, K.H. Kirklin, P.J. Miron and G.G. Ross, Nucl. Phys. B274 (1986) 574, Nucl. Phys. B278 (1986) 667, Phys. Lett. 180B (1986) 69, Harvard University preprint HUTP-87/A083, and Oxford preprint ORTP-309/87;
S. Kalara and R.N. Mohapatra, Univ. of Minnesota preprint UMN/TH/601 (1987);
J. Ellis, K. Enqvist, D.V. Nanopoulos and K.A. Olive, CERN preprint TH.4710/87 (1987).
11. J. Ellis, K. Enqvist, D.V. Nanopoulos and K.A. Olive, Phys. Lett. 188B (1987) 415.
12. B.A. Campbell, J. Ellis, J.S. Hagelin, D.V. Nanopoulos and K.A. Olive, CERN preprint TH.4764/87 (1987).
13. I. Antoniadis, J. Ellis, J.S. Hagelin and D.V. Nanopoulos, CERN preprint TH.4710/87 (1987).
14. J. Ellis, C. Gomez, D.V. Nanopoulos and M. Quiros, Phys. Lett. 173B (1986) 59;
M. Dine, N. Seiberg, X.-G. Wen and E. Witten, Nucl. Phys. B278 (1986) 769, and Princeton preprint 87-0027.
15. C. Hull and E. Witten, Phys. Lett. 160B (1985) 398.
16. C. Hull, Phys. Lett. 178B (1986) 357.
17. C. Hull, Phys. Lett. 167B (1986) 51.
18. J. Distler, Phys. Lett. 188B (1987) 431.