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SUPERMEMBRANES: THE FIRST FIFTEEN WEEKS*)

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ABSTRACT

There are four fundamental extended objects with $d > 2$ world-volume dimensions and D -dimensional space-time supersymmetry: $(d=3, D=11)$, $(d=6, D=10)$, $(d=4, D=6)$ and $(d=3, D=4)$. A simultaneous dimensional reduction of the world-volume and the space-time then yields four sequences of super extended object which include the four classical Green-Schwarz superstrings $(d=2, D=10, 6, 4 \text{ and } 3)$ as special cases. We discuss which of these models is likely to be quantum consistent and finish with some speculations.

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1. - INTRODUCTION

Many of the supergravity theories that we used to study a few years ago are now known to be merely the field theory limit of an underlying string theory¹⁾. For example, $N = 2a$ supergravity in ten dimensions is just the field theory limit of the Type IIA superstring. What are we to make, therefore, of supergravity theories which cannot be obtained from strings such as $N = 1$ supergravity in eleven dimensions^{2),3)}? This is a particularly puzzling example since it is well known that upon dimensional reduction to ten dimensions, it yields the above-mentioned $N = 2a$ theory. Indeed, if supersymmetry allows $D \leq 11$, why do strings stop at $D = 10$? Another mystery concerns the

dual formulation of $N = 1$ supergravity in ten dimensions in which the three-form field strength $H_{\mu\nu\rho}$ is swapped for a seven-form field strength $\tilde{H}_{\mu\nu\rho\sigma\lambda\kappa\tau}$ ⁴⁾. As a field theory it seems every bit as good. In particular, provided we couple it to $E_8 \times E_8$ or $SO(32)$ Yang-Mills, it is just as anomaly free as its three-form counterpart⁵⁾. Why then does only the $H_{\mu\nu\rho}$ version correspond to the field theory limit of a string?

In the last few weeks we have begun to see a glimpse of an answer to some of these questions, thanks to the work of Bergshoeff, Sezgin and Townsend⁶⁾ on "supermembranes". In particular they have constructed a theory which describes an extended object with two space and one time dimensions moving in an eleven-dimensional supergravity background. We describe this theory in Section 2 and then show in Section 3 how to derive from it the Type IIA superstring in ten dimensions by a simultaneous dimensional reduction of the world-volume and the space-time [Duff, Howe, Inami and Stelle⁷⁾].

Inspired by earlier work of Hughes, Liu and Polchinski⁸⁾, Bergshoeff, Sezgin and Townsend⁶⁾ pointed out that a covariant Green-Schwarz action for other extended objects with one temporal and p spatial dimensions is possible whenever there is a closed $(p+2)$ -form in superspace. As described in Section 4, the four fundamental "p-branes" are then given by $p = 2$ in $D = 11$, $p = 5$ in $D = 10$, $p = 3$ in $D = 6$ and $p = 2$ in $D = 4$ [Achúcarro, Evans, Townsend and Wiltshire⁹⁾]. Applying the above-mentioned simultaneous dimensional reduction k times, we find four sequences of $(p-k)$ -branes in $(D-k)$ dimensions which include the well-known Green-Schwarz superstrings in $D = 10, 6, 4$ and 3 .

Of course, of these four superstrings, only the $D = 10$ case is consistent at the quantum level and this raises the crucial question of which of the super p -branes, if any, can be quantum-mechanically consistent. We do not yet know an answer to this question, but in Section 5 we discuss some of the objections which have been raised in

$$\begin{aligned}\hat{T}_{\hat{\alpha}\hat{\beta}}^{\hat{c}} &= -i (\hat{\Gamma}^{\hat{c}})_{\hat{\alpha}\hat{\beta}} \quad ; \quad \hat{F}_{\hat{\alpha}\hat{\beta}}^{\hat{c}\hat{d}} = i (\hat{\Gamma}^{\hat{c}\hat{d}})_{\hat{\alpha}\hat{\beta}} \quad (2.6) \\ \hat{F}_{\hat{\alpha}\hat{\beta}\hat{\gamma}\hat{\delta}} &= \hat{F}_{\hat{\alpha}\hat{\beta}\hat{\gamma}\hat{d}} = 0\end{aligned}$$

$$\hat{T}_{\hat{\alpha}\hat{b}}^{\hat{c}} = \eta_{\hat{b}\hat{c}} \hat{\Lambda}_{\hat{\alpha}} \quad ; \quad \hat{F}_{\hat{\alpha}\hat{b}}^{\hat{c}\hat{d}} = (\hat{\Gamma}^{\hat{c}\hat{d}})_{\hat{\alpha}}^{\hat{\beta}} \hat{\Lambda}_{\hat{\beta}} \quad (2.7)$$

Although these equations are not the standard equations of on-shell $D = 11$ supergravity in superspace^{14),15)}, they are equivalent to them. That is to say, by suitable redefinitions of the superconnections and parts of the supervielbein, we may set $\hat{\Lambda}_{\hat{\alpha}}$, $\hat{T}_{\hat{\alpha}\hat{\beta}}^{\hat{\gamma}}$ and $\hat{T}_{\hat{a}\hat{b}}^{\hat{c}}$ to zero⁷⁾,

$$\hat{T}_{\hat{\alpha}\hat{b}}^{\hat{c}} = \hat{T}_{\hat{\alpha}\hat{\beta}}^{\hat{\gamma}} = \hat{F}_{\hat{\alpha}\hat{b}}^{\hat{c}\hat{d}} = 0 \quad (2.8)$$

Equations (2.8) are just the field equations of $D = 11$ supergravity as may be checked using the Bianchi identities. At this classical level, at least, all solutions of $D = 11$ supergravity are possible backgrounds for the supermembrane. We shall return to this point in Section 6.

The other important point to make about the Siegel invariance, is that

$$\hat{\Gamma}^2 = \mathbb{1} \quad (2.9)$$

on using (2.5), and hence $\frac{1}{2}(1 \pm \hat{\Gamma})$ act as projection operators. This will allow us to gauge away one half of the fermionic degrees of freedom, i.e., there is a total of $16 \div 2 = 8$ on-shell. The number of bosonic degrees of freedom is also $8 = 11 - 3$ owing to the diffeomorphisms on the three-dimensional world-sheet.

3. - SUPERSTRINGS FROM SUPERMEMBRANES BY SIMULTANEOUS DIMENSIONAL REDUCTION

It is well known that $N = 2a$ supergravity in ten dimensions $(g_{mn}, A_m, \Phi; \psi_m, \chi; A_{mnp}, A_{mn})$ may be obtained by dimensional reduction from $N = 1$ supergravity in eleven dimensions $(\hat{g}_{\hat{m}\hat{n}}, \hat{\psi}_{\hat{m}}, \hat{A}_{\hat{m}\hat{n}\hat{p}})$. On the other hand, $N = 2a$ supergravity is also the field-theory limit of the Type IIA superstring. The purpose of this section is to derive the Type IIA superstring from the supermembrane of Section 2 by a dimensional reduction of the world-volume from three to two dimensions and, simultaneously, a dimensional reduction of the space-time from eleven to ten⁷⁾.

To see how the dimensional reduction works, let us first focus our attention on the purely bosonic sector for which the action (2.1) reduces to

$$S = \int d^3 \xi \left[\frac{1}{2} \sqrt{-\hat{g}} \hat{g}^{\hat{i}\hat{j}} \partial_{\hat{i}} \hat{x}^{\hat{m}} \partial_{\hat{j}} \hat{x}^{\hat{n}} \hat{g}_{\hat{m}\hat{n}}(\hat{x}) - \frac{1}{2} \sqrt{-\hat{g}} + \frac{1}{6} \epsilon^{\hat{i}\hat{j}\hat{k}} \partial_{\hat{i}} \hat{x}^{\hat{m}} \partial_{\hat{j}} \hat{x}^{\hat{n}} \partial_{\hat{k}} \hat{x}^{\hat{p}} \hat{A}_{\hat{m}\hat{n}\hat{p}}(\hat{x}) \right] \quad (3.1)$$

Varying with the respect to the metric $\hat{g}_{\hat{i}\hat{j}}$ yields the embedding equation

$$\hat{g}_{\hat{i}\hat{j}} = \hat{g}_{\hat{i}\hat{j}} \equiv \partial_{\hat{i}} \hat{x}^{\hat{m}} \partial_{\hat{j}} \hat{x}^{\hat{n}} \hat{g}_{\hat{m}\hat{n}}(\hat{x}) \quad (3.2)$$

while varying with respect to $\hat{x}^{\hat{m}}$ yields the equation of motion

the literature against membranes and, whenever possible, provide a refutation. In particular, we discuss the cancellation of the vacuum energy between the bosons and fermions in the eleven-dimensional supermembrane [Duff, Inami, Pope, Sezgin and Stelle¹⁰⁾ *).

Section 6 is reserved for some wilder speculations concerning the heterotic five-brane, supermembranes on the seven-sphere and supermembranes as "singletons".

2. - THE D = 11 SUPERMEMBRANE

To describe the coupling of a closed $d = 3$ membrane to a $D = 11$ supergravity background, let us introduce world-volume co-ordinates $\hat{\xi}^{\hat{i}}$ ($\hat{i} = 1, 2, 3$) and a world-volume metric $\hat{\gamma}_{\hat{i}\hat{j}}(\hat{\xi})$ with signature $(-, +, +)$. The target space is a supermanifold with superspace co-ordinates $\hat{z}^{\hat{M}} = (\hat{x}^{\hat{m}}, \hat{\theta}^{\hat{\mu}})$ where $\hat{m} = 1, \dots, 11$ and $\hat{\mu} = 1, \dots, 32$ with space-time signature $(-, +, \dots, +)$. We also define $\hat{E}_{\hat{i}}^{\hat{A}} = (\partial_{\hat{i}} \hat{z}^{\hat{M}}) \hat{E}_{\hat{M}}^{\hat{A}}(\hat{z})$ where $\hat{E}_{\hat{M}}^{\hat{A}}$ is the supervielbein and $\hat{A} = (\hat{a}, \hat{\alpha})$ is the tangent space index ($\hat{a} = 1, \dots, 11$ and $\hat{\alpha} = 1, \dots, 32$). The action is then given by⁶⁾

$$S = \int d^3 \hat{\xi} \left\{ \frac{1}{2} \sqrt{-\hat{\gamma}} \hat{\gamma}^{\hat{i}\hat{j}} \hat{E}_{\hat{i}}^{\hat{a}} \hat{E}_{\hat{j}}^{\hat{b}} \eta_{\hat{a}\hat{b}} - \frac{1}{2} \sqrt{-\hat{\gamma}} \right. \\ \left. - \frac{1}{6} \varepsilon^{\hat{i}\hat{j}\hat{k}} \hat{E}_{\hat{i}}^{\hat{A}} \hat{E}_{\hat{j}}^{\hat{B}} \hat{E}_{\hat{k}}^{\hat{C}} \hat{A}_{\hat{C}\hat{B}\hat{A}} \right\} \quad (2.1)$$

Note that there is a Wess-Zumino term involving the super three-form $\hat{A}_{\hat{A}\hat{B}\hat{C}}(\hat{z})$ and also a world-volume cosmological term. In addition to world-volume diffeomorphisms, target space superdiffeomorphisms, Lorentz invariance and three-form gauge invariance, the action (2.1) is invariant under a fermionic gauge transformation⁶⁾ whose parameter

*) For further detailed discussion, see the conference talk by K.S. Stelle¹¹⁾.

$\hat{\kappa}^{\hat{\alpha}}(\hat{\xi})$ is a 32-component space-time Majorana spinor and a world-volume scalar. This is a generalization to the case of membranes of the symmetry discovered by Siegel¹²⁾ for the superparticle and Green and Schwarz¹³⁾ for the superstring in the form given by Hughes, Liu and Polchinski⁸⁾.

To facilitate a discussion of this fermionic symmetry, it is convenient to eliminate the world-volume metric as an independent variable. In this way we avoid having to discuss the rather complicated transformation rule for the metric. The action (2.1) then takes on its Nambu-Goto form

$$S = \int d^3\hat{g} \left[\sqrt{-\det \hat{E}_{\hat{i}}^{\hat{a}} \hat{E}_{\hat{j}}^{\hat{b}} \eta_{\hat{a}\hat{b}}} - \frac{1}{6} \epsilon^{\hat{i}\hat{j}\hat{k}} \hat{E}_{\hat{i}}^{\hat{a}} \hat{E}_{\hat{j}}^{\hat{b}} \hat{E}_{\hat{k}}^{\hat{c}} \hat{A}_{\hat{c}\hat{b}\hat{a}} \right] \quad (2.2)$$

It is invariant under the transformation⁶⁾

$$\begin{aligned} \delta \hat{z}^{\hat{a}} &\equiv \delta \hat{z}^{\hat{M}} \hat{E}_{\hat{M}}^{\hat{a}} = 0 \\ \delta \hat{z}^{\hat{\alpha}} &\equiv \delta \hat{z}^{\hat{M}} \hat{E}_{\hat{M}}^{\hat{\alpha}} = \hat{\kappa}^{\hat{\beta}} (1 + \hat{\Gamma})_{\hat{\beta}}^{\hat{\alpha}} \end{aligned} \quad (2.3)$$

where

$$\hat{\Gamma}_{\hat{\beta}}^{\hat{\alpha}} = \frac{1}{6\sqrt{-\hat{g}}} \epsilon^{\hat{i}\hat{j}\hat{k}} \hat{E}_{\hat{i}}^{\hat{a}} \hat{E}_{\hat{j}}^{\hat{b}} \hat{E}_{\hat{k}}^{\hat{c}} (\hat{\Gamma}_{\hat{a}\hat{b}\hat{c}})_{\hat{\beta}}^{\hat{\alpha}} \quad (2.4)$$

In (2.4) $\hat{g}_{\hat{i}\hat{j}}$ is the metric on the world-volume induced from the bosonic metric on superspace,

$$\hat{g}_{\hat{i}\hat{j}} = \hat{E}_{\hat{i}}^{\hat{a}} \hat{E}_{\hat{j}}^{\hat{b}} \eta_{\hat{a}\hat{b}} \quad (2.5)$$

In order for (2.2) to be invariant under this transformation, it is necessary that the background supergeometry be constrained. The constraints found in Ref. 6) are

$$\begin{aligned} \frac{1}{\sqrt{-\hat{g}}} \partial_{\hat{i}} (\sqrt{-\hat{g}} \hat{g}^{\hat{i}\hat{j}} \partial_{\hat{j}} \hat{x}^{\hat{m}}) + \hat{\Gamma}_{\hat{n}\hat{p}}^{\hat{m}} \partial_{\hat{i}} \hat{x}^{\hat{n}} \partial_{\hat{j}} \hat{x}^{\hat{p}} \hat{g}^{\hat{i}\hat{j}} \\ = \frac{1}{6} \hat{F}_{\hat{n}\hat{p}\hat{q}}^{\hat{m}} \partial_{\hat{i}} \hat{x}^{\hat{n}} \partial_{\hat{j}} \hat{x}^{\hat{p}} \partial_{\hat{k}} \hat{x}^{\hat{q}} \frac{\epsilon^{\hat{i}\hat{j}\hat{k}}}{\sqrt{-\hat{g}}} \end{aligned} \quad (3.3)$$

where $\hat{F}_{\hat{m}\hat{n}\hat{p}\hat{q}}$ is the field-strength of $\hat{A}_{\hat{m}\hat{n}\hat{p}}$

$$\hat{F}_{\hat{m}\hat{n}\hat{p}\hat{q}} \equiv 4 \partial_{[\hat{m}} \hat{A}_{\hat{n}\hat{p}\hat{q}]} \quad (3.4)$$

We now make a two-one split of the world-volume co-ordinates

$$\hat{\xi}^{\hat{i}} = (\xi^i, s) \quad i = 1, 2 \quad (3.5)$$

and a ten-one split of the space-time co-ordinates

$$\hat{x}^{\hat{m}} = (x^m, y) \quad m = 1, \dots, 10 \quad (3.6)$$

in order to make the partial gauge choice

$$s = y \quad (3.7)$$

which identifies the eleventh space-time dimension with the third dimension of the world-volume. The dimensional reduction is then effected by demanding that

$$\partial_s x^m = 0 \quad (3.8)$$

and

$$\partial_y \hat{g}_{\hat{m}\hat{n}} = 0 = \partial_y \hat{A}_{\hat{m}\hat{n}\hat{p}} \quad (3.9)$$

A suitable choice of ten-dimensional variables is now given by

$$\hat{g}_{\hat{m}\hat{n}} = \Phi^{-2/3} \begin{bmatrix} g_{mn} + \Phi^2 A_m A_n & \Phi^2 A_m \\ \Phi^2 A_n & \Phi^2 \end{bmatrix} \quad (3.10)$$

$$\begin{aligned} \hat{A}_{\hat{m}\hat{n}\hat{p}} &= (\hat{A}_{mnp}, \hat{A}_{mny}) \\ &= (A_{mnp}, A_{mn}) \end{aligned}$$

From (3.2), the induced metric on the world-sheet is now given by

$$\hat{g}_{\hat{i}\hat{j}} = \Phi^{-2/3} \begin{bmatrix} g_{ij} + \Phi^2 A_i A_j & \Phi^2 A_i \\ \Phi^2 A_j & \Phi^2 \end{bmatrix} \quad (3.11)$$

where

$$g_{ij} \equiv \partial_i x^m \partial_j x^n g_{mn}, \quad A_i \equiv \partial_i x^m A_m. \quad (3.12)$$

Note that

$$\sqrt{-\hat{g}} = \sqrt{-g} \quad (3.13)$$

Substituting these expressions into the field equations (3.3) yields in the case $\hat{x}^{\hat{m}} = x^m$

$$\begin{aligned} \frac{1}{\sqrt{-g}} \partial_i (\sqrt{-g} g^{ij} \partial_j x^m) + \Gamma_{np}^m \partial_i x^n \partial_j x^p g^{ij} \\ = \frac{1}{2} F_{np}^m \partial_i x^n \partial_j x^p \frac{\epsilon^{ij}}{\sqrt{-\hat{g}}} \end{aligned} \quad (3.14)$$

where F_{mnp} is the field-strength of A_{mn}

$$F_{mnp} \equiv 3 \partial_{[m} A_{np]} = \hat{F}_{mnp} y \quad (3.15)$$

In the case $\hat{x}^{\hat{m}} = y$, (3.3) is an identity, as it must be for consistency. But (3.14) is just the ten-dimensional string equation of motion derivable from the action

$$S = \int d^2 y \left[\frac{1}{2} \sqrt{-\gamma} \gamma^{ij} \partial_i x^m \partial_j x^n g_{mn} + \frac{1}{2} \epsilon^{ij} \partial_i x^m \partial_j x^n A_{mn} \right] \quad (3.16)$$

Comparing with (3.1), we see that the overall effect is to reduce the eleven-dimensional membrane to a ten-dimensional string, to replace the three-form by a two-form in the Wess-Zumino term and to eliminate the world-volume cosmological constant. Note that the other ten-dimensional bosonic fields A_{mnp} , A_m and Φ have all decoupled. They have not disappeared from the theory, however, since their coupling still survives in the fermionic θ sector, to which we shall turn shortly. First, we make some remarks.

As is well known, the dimensional reduction (3.9) corresponds to a Kaluza-Klein compactification of space-time on a circle in which one discards all the massive modes. The difference from conventional Kaluza-Klein is that by identifying the eleventh space-time dimension with the third dimension on the world-volume as in (3.7), the world-

volume is also compactified on the same circle. The condition (3.8) means that we are discarding the massive world-sheet modes at the same time. By retaining all the U(1) singlets but only the U(1) singlets, these truncations are guaranteed to be consistent³⁾ with the membrane equations of motion and, as we shall see, with the equations of motion of the background fields. As an extra check on consistency, we have been careful to substitute the Kaluza-Klein ansatz into the equations of motion rather than directly into the action. The signal for consistency is that the $\hat{x}^{\hat{m}} = y$ component of the field equations (3.3) is an identity. Having established consistency one may then, if so desired, substitute directly into the action (3.1) and integrate over ρ .

To include the fermions, it is now straightforward to repeat the above procedure in superspace. The Kaluza-Klein ansatz for the $N = 1$, $D = 11$ supervielbein is

$$\hat{E}_{\hat{M}}^{\hat{A}} = \begin{bmatrix} \hat{E}_M^a & \hat{E}_M^\alpha & \hat{E}_M'' \\ \hat{E}_y^a & \hat{E}_y^\alpha & \hat{E}_y'' \end{bmatrix} \quad (3.17)$$

$$= \begin{bmatrix} E_M^a & E_M^\alpha + A_M \chi^\alpha & \Phi A_M \\ 0 & \chi^\alpha & \Phi \end{bmatrix} \quad (3.18)$$

where $E_M^A = (E_M^a, E_M^\alpha)$ is the $N = 2a$, $D = 10$ supervielbein, A_M the superspace U(1) gauge field and Φ and χ^a are superfields whose leading

components are the dilaton and the dilatino respectively. In writing (3.18), we have made a partial $D = 11$ local Lorentz gauge choice to set $\hat{E}_y^a = 0$. For the superspace three-index potential $\hat{\hat{A}}_{MNP}$ we have

$$\hat{\hat{A}}_{MNP} = A_{MNP} \quad (3.19)$$

$$\hat{\hat{A}}_{MNP} = A_{MN} \quad (3.20)$$

All of the $D = 10$ superfields E_M^A , χ^α , A_M , Φ , A_{MN} , A_{MNP} are taken to be independent of y . Note also that ten-dimensional spinor indices run from 1 to 32 so that α and α can be identified. With $\hat{z}^{\hat{M}} = (z^M, y)$, we also impose $\partial_\rho z^M = 0$ and $y = \rho$ as before. Substituting the ansätze (3.18), (3.19) and (3.20) into (2.2) yields the action for a Type IIA superstring coupled to a supergravity background

$$S = \int d^2\zeta \left[\frac{1}{2} \sqrt{-\det E_i^a E_j^b \eta_{ab}} - \frac{1}{2} \epsilon^{ij} \partial_i z^M \partial_j z^N A_{MN} \right] \quad (3.21)$$

Purely for convenience in superspace calculations, we have omitted an overall factor of $\Phi^{-2/3}$ in the ansatz (3.17); the factor of Φ in (3.21) can be removed by a suitable rescaling of the supervielbein. To find the fermionic symmetry of the dimensionally-reduced action (3.21), one substitutes the Kaluza-Klein ansätze into (2.3). It is straightforward to show that

$$\delta \hat{z}^\alpha = \delta z^\alpha \equiv \delta z^M E_M^\alpha = \kappa^\beta (1 + \Gamma)_{\beta}{}^\alpha \quad (3.22)$$

and that

$$\hat{\Gamma}_{\beta}{}^{\alpha} = \Gamma_{\beta}{}^{\alpha} = \frac{1}{2\sqrt{-g}} \epsilon^{ij} E_i^a E_j^b (\Gamma_{ab} \Gamma_{11})_{\beta}{}^{\alpha} \quad (3.23)$$

Since (2.2) is invariant under (2.3) when the $D = 11$ field equations are satisfied, it follows that the reduced action (3.21) will be invariant under (3.23) if the $N = 2a$, $D = 10$ supergravity field equations are satisfied. This is because the compactification of the $N = 1$, $D = 11$ field theory on a circle is known to yield the $N = 2a$, $D = 10$ field theory. Note that all of the $N = 2a$ supergravity fields are now coupled, including A_{mnp} , A_m and Φ which decoupled from the purely bosonic sector. The transformation (3.22) can be recast into the Green-Schwarz¹³⁾ form by introducing

$$\lambda^{i\alpha} = \frac{1}{2} \frac{\varepsilon^{ij}}{\sqrt{-g}} E_j^a \kappa^\beta (\Gamma_a \Gamma_\beta)^\alpha \quad (3.24)$$

In conclusion, we have succeeded in deriving [for the first time^{*)}] the action of the Type IIA superstring coupled to an $N = 2a$, $D = 10$ supergravity background starting from the action of the supermembrane coupled to the background of $N = 1$ supergravity in $D = 11$. The dimensional reduction corresponds to a compactification of both the space-time and the world-volume on the same circle and then discarding the massive modes. Classically, this is equivalent to letting the membrane tension κ_2 tend to infinity and the radius of the circle R tend to zero in such a way that the string tension $\kappa_1 = [\alpha']^{-1}$

$$\kappa_1 = 2\pi R \kappa_2 \quad (3.25)$$

remains finite.

^{*)} The Type IIB action is given in Ref. 16).

4. - OTHER DIMENSIONS, OTHER BRANES

So far we have discussed a $p = 2$ extended object in $D = 11$ dimensions, but super p -extended objects can exist in D dimensions whenever there is a closed $(p+2)$ -form in superspace^{*}). This follows from the Siegel invariance of the Green-Schwarz action⁶⁾:

$$S = \int d^{p+1} \xi \left\{ \frac{1}{2} \sqrt{-g} g^{ij} E_i^a E_j^b \eta_{ab} - \frac{(p-1)\sqrt{-g}}{2} + \frac{1}{(p+1)!} \varepsilon^{i_1 i_2 \dots i_{p+1}} E_{i_1}^{A_1} E_{i_2}^{A_2} \dots E_{i_{p+1}}^{A_{p+1}} A_{A_{p+1} \dots A_2 A_1} \right\} \quad (4.1)$$

This closure is equivalent to the identity

$$(d\bar{\theta} \Gamma_a d\theta) (d\bar{\theta} \Gamma^{a b_1 \dots b_{p-1}} d\theta) = 0 \quad (4.2)$$

for a commuting spinor $d\theta$.

Achúcarro, Evans, Townsend and Wiltshire⁹⁾ have recently classified those values of p and D , and the number of space-time supersymmetries N for which this identity is valid. They find that this is possible only when the number of on-shell p -dimensional Bose and Fermi degrees of freedom are equal. There are thus four "fundamental" super p -branes as shown in the table. All four have the minimal $N = 1$ supersymmetry and the $D = 10$ and $D = 6$ cases are chiral. The $D = 10$ case couples to the dual form of $D = 10$ supergravity discussed in Section 1.

^{*}) We confine our attention to closed p -branes. This is required in $D = 11$. The existence of open supermembranes in $D < 11$ is less clear.

p	D	Bose+Fermi	Algebra
2	11	8 + 8	$\mathcal{O}$
5	10	4 + 4	$\mathcal{H}$
3	6	2 + 2	$\mathcal{C}$
2	4	1 + 1	$\mathcal{R}$

Table

The fundamental super p-branes.

However, starting from these fundamental p-branes in D dimensions other (p-k)-branes in (D-k) dimensions may be obtained by repeating k times the simultaneous dimensional reduction of Section 3. Thus we make a (D-k)-k split of the target space co-ordinates

$$\hat{x}^{\hat{m}} = (x^m, y^\alpha) \quad \alpha = 1 \dots k \quad (4.2)$$

and a (p+1-k)-k split of the world-volume co-ordinates

$$\hat{\xi}^{\hat{i}} = (\xi^i, g^\alpha) \quad \alpha = 1 \dots k \quad (4.3)$$

in order to make the partial gauge choice

$$g^\alpha = y^\alpha \quad (4.4)$$

which identifies the extra k space-time dimensions with the extra k dimensions of the world-volume. The dimensional reduction is then effected by demanding that the remaining co-ordinates x^m be independent of ρ^α and the target-space background fields be independent of y^α . This procedure now results in four sequences of super p-branes as shown in the figure.

The brane-scan

Remarkably, for $p = 1$ we recover the well-known four classical Green-Schwarz superstrings in $D = 10, 6, 4$ and 3 and these are now seen to be but special cases of a more general framework of four sequences of super p -brane with $(8+8)$, $(4+4)$, $(2+2)$ and $(1+1)$ degrees of freedom respectively. This immediately suggests a connection with the four-division algebras $\mathbb{R}$, $\mathbb{C}$, $\mathbb{H}$ and $\mathbb{O}$, a connection which can in fact be made more precise^{9),17)}. In the $p = 1$ case, of course, we know that only the $D = 10$ superstring is quantum consistent. Note that the only other member of this $\mathbb{O}$ sequence is the eleven-dimensional supermembrane.

All the points on the scan represent $N = 1$ theories except for the $p = 1$ superstring case where we find $N = 2$. Two world-sheet dimensions are also special in that one can separate modes into left- and right-movers. This permits a truncation to $N = 1$. The horizontal arrows in the Table represent a "duality transformation" in which each of these $N = 1$ string theories is swapped for its dual theory in which the three-form field strength is replaced by a $(p+2)$ -form field strength. Note that there is no dual version of $N = 1$, $D = 11$ supergravity in which the four-form field strength is replaced by a seven-form¹⁸⁾. (This means that one cannot simultaneously dualize the four-form and three-form field strengths of $N = 2a$, $D = 10$ supergravity, but one can do either separately^{*)}. There appear to be no corresponding $N = 2$ membranes, however.)

Finally, we note that one may travel vertically downwards in the figure by making an ordinary dimensional reduction of the space-time only. The theories so obtained need not have a closed $(p+2)$ -form since Chern-Simons terms will arise from the dimensional reduction¹⁰⁾.

*) H. Nishino, private communication.

5. - OBJECTIONS TO QUANTUM MEMBRANES

In the literature one may find statements to the effect that strings provide the only consistent quantum theories of extended objects. Below we list some of these objections and, whenever possible, provide the refutation.

Fermionic Symmetry

The Siegel symmetry of the Green-Schwarz superstring is essential for ensuring the correct count of boson and fermion degrees of freedom and hence its consistency. It has been claimed that this fermionic symmetry cannot be generalized to membranes. But, as we have already seen in Section 2, it can! This first objection is thus rather easy to refute.

Triality and $SO(8)$

Another argument in favour of superstrings and, in particular, ten space-time dimensions, invokes the appearance of the group $SO(8)$, with its unique triality properties, as the invariance group of the space of physical variables. Thus the eight bosonic variables corresponding to the transverse $10-2 = 8$ dimensions are matched by the $8 = 16 \div 2$ fermionic variables remaining after the Siegel symmetry has been taken into account. But as we have seen in Section 2, the same balance occurs for the supermembrane in eleven dimensions: here we have $11-3 = 8$ physical bosonic variables, and again $16 \div 2 = 8$ fermionic variables after taking account of the Siegel symmetry.

Ghosts and Lorentz Invariance

Clearly, any consistent theory must be free from ghosts. Both strings and membranes apparently suffer from the existence of an infinity of ghosts, owing to the negative sign of the kinetic term of

the time co-ordinate x^0 . In the case of strings, this problem is known to be solved in the critical dimension. The proof^{19),20),21)} invokes the conformal symmetry, which in two dimensions is infinite dimensional. In more than two dimensions the conformal group is only finite dimensional, and is in any case unlikely to be relevant for membranes²²⁾. To address the ghost problem for membranes, one can proceed in one of two ways. In the first, covariant, approach one looks for an infinite dimensional symmetry that will substitute for the Virasoro algebra. Little has been achieved in this direction so far, although it is interesting to note that Hoppe²³⁾ has demonstrated the existence of infinite parameter symmetries of the membrane Hamiltonian. An alternative approach is to go to a physical gauge, where ghost-freedom is manifest but Lorentz invariance is not. The problem now is to demonstrate that Lorentz invariance is nevertheless preserved in the quantization. For strings, this is a problem owing to the existence of conformal anomalies which spoil the closure of the Lorentz algebra. Hence the need for a critical dimension, in which the conformal anomalies cancel. For membranes, it is difficult to see how a critical dimension could arise in this sense, when there is no obvious candidate for an anomalous symmetry at least in the case of the bosonic membrane. In particular, there is no classical conformal invariance. Moreover, there are no diffeomorphism anomalies in three dimensions. In fact it seems that many of the special features of string theory are needed in order to cure the problems of living in two dimensions. For a Hamiltonian formulation of the bosonic membrane, see Refs. 24)-27), and of the supermembrane Refs. 10) and 28).

Massless Particles

An important question for any membrane theory is whether massless particles are present in the spectrum. In the case of the supermembrane, consistency presumably requires the occurrence of the graviton, gravitino and antisymmetric tensor of eleven-dimensional

supergravity in the spectrum^{*)}. For the purely bosonic membrane, Kikkawa and Yamasaki have concluded that the open membrane does not admit massless spin-one or spin-two particles in any integer dimension²⁹⁾. Their argument involved computing the intercept of the classical mass/spin relation induced by one-loop vacuum energy effects by analogy with the string calculations of Brink and Nielsen³⁰⁾.

Consider a bosonic p-brane with Nambu-Goto action

$$S = \kappa_p \int d\tau d^p\sigma \sqrt{\det \partial_i x^m \partial_j x^n \eta_{mn}} \quad (5.1)$$

Then the membrane tension κ_p has dimensions

$$[\kappa_p] = M L^{1-p} T^{-1} \quad (5.2)$$

To obtain a classical ($\hbar$ independent) relation between the angular momentum J

$$[J] = M L^2 T^{-1} \quad (5.3)$$

of a rotating p-brane and its rest mass, using only κ_p and the velocity of light c

$$[c] = L T^{-1} \quad (5.4)$$

Kikkawa and Yamasaki²⁹⁾ argue that we must have

$$J = A \kappa_p^{-1/p} (cm)^{(p+1)/p} \quad (5.5)$$

and that this gives rise in the quantum theory to a spin-mass relation

^{*)}Or, perhaps, the massless particles to which they give rise after spontaneous compactification of the space-time. See Section 6.

$$J = A K_p^{-1/p} (cm)^{(p+1)/p} + B \hbar \quad (5.6)$$

where A and B are dimensionless numbers.

This certainly reduces in the case of the bosonic string ($p=1$), to the familiar Regge formula ($A=1$, $\kappa_1^{-1}=\alpha'$)

$$J = \alpha' m^2 + \alpha_0 \quad (5.7)$$

To compute α_0 , Brink and Nielsen³⁰⁾ noted that the intercept α_0 arises from the Casimir energy and that the mass squared is shifted by an amount

$$\Delta m^2 = \frac{1}{\alpha'} \frac{D-2}{2} \sum_{n=1}^{\infty} n \quad (5.8)$$

$$(5.9)$$

$$= -\frac{1}{\alpha'} \frac{D-2}{24}$$

on using ξ function regularization for which $\zeta(-1) = -1/12$ where $\zeta(s) = \sum_{n=1}^{\infty} n^{-s}$. Thus we obtain from (5.7) a massless ($m=0$) spin one ($J=1$) particle when $D = 26$. As noted by Kikkawa and Yamasaki²⁹⁾, it is a miracle that the Casimir term happens to be a rational number so that J is made an integer by adjusting the dimension of space-time. In the case of the closed bosonic string, the Casimir energy is doubled (one considers two open strings back-to-back) and we find a massless spin two particle

$$\Delta m^2 = K_1 (D-2) \sum_n n \quad (5.10)$$

Similar miracles occur for the Neveu-Schwarz formulation of the open and closed superstrings where we find massless particles when $D = 10$.

To examine the case for membranes, Kikkawa and Yamasaki consider rigid rotator solutions since these maximize the angular momentum for a given energy. Assuming that the Casimir energy is independent of the membrane tension κ , they argue that the one-loop result will be exact since this corresponds to the $\kappa \rightarrow \infty$ limit. Perhaps not surprisingly they find that there are no miracles and that the open membrane ($p=2$) does not admit massless spin one or spin two particles in any integer dimension.

The question naturally arises as to whether the inclusion of fermions would cure this problem. However, in order to follow the method of Ref. 30), it would be necessary to have a Neveu-Schwarz-Ramond (NSR) formalism for the supermembrane; i.e., a formalism which exhibits world-volume supersymmetry. The supermembrane theory of Ref. 6) is written in a Green-Schwarz formalism, in which space-time supersymmetry is manifest. It is not clear at present whether an NSR formalism exists for the supermembrane, but we draw attention to the following problem. As discussed in Section 3, the supermembrane, by contrast to the superstring, requires a cosmological term in the action. In three-dimensional supergravity, the cosmological term arises from eliminating an auxiliary field S which appears both linearly and quadratically in the action. However, there is no superinvariant quadratic in S that does not also involve the Einstein-Hilbert gravitational action. This means that the Nambu-Goto action cannot be supersymmetrized. This problem was implicit in the paper of Howe and Tucker³¹⁾ (we are grateful to P. Howe for drawing this to our attention), and is further discussed in Ref. 32).

Although an NSR calculation of the shift in the intercept is thus not possible, it is gratifying to find that the vacuum energy can be zero in the Green-Schwarz formalism [Duff, Inami, Pope, Sezgin and Stelle¹⁰⁾]. Owing to the intrinsic non-linearity of the membrane equations, there is no clever gauge which linearizes the equations of motion for $p > 1$. Consequently, in order to analyze the self-energy

question, the best we can do at the moment is to pick a stable classical solution and to quantize the fluctuations about this background. Stable solutions are not easy to find, and instead of considering a rotating membrane in Minkowski space as in Ref. 29), we considered in Ref. 10) a space-time with topology $R^9 \times S^1 \times S^1$. With such a topology there is a stable classical solution with a closed toroidal membrane stretched around the torus. In this case, the mode expansions are particularly simple. For the bosonic membrane, one finds a $(\text{mass})^2$ shift arising from the normal ordering of the oscillators which is proportional to $(D-3)$, and is given by an Epstein ζ -function

$$\Delta m^2 = K_2 (D-3) \sum_{n,m} \left[(m \ell_2 R_2)^2 + (n \ell_1 R_1)^2 \right]^{1/2} \quad (5.11)$$

where R_1 and R_2 are the radii of the two circles and ℓ_1 and ℓ_2 are integers characterizing the winding numbers of the membrane around the two circles. The beauty of the supermembrane is that this vacuum energy is cancelled mode by mode by an equal and opposite contribution from the fermions, just as in the Green-Schwarz superstring. This is encouraging for the prospect of the theory's spectrum containing massless states, but one would not expect to find them in the toroidal background considered here but rather in the classical background that corresponds to the expected leading Regge trajectory.

Linear Trajectories?

On the subject of the Regge trajectory, an apparent paradox arises. According to the dimensional argument (5.6) of Kikkawa and Yamasaki²⁹⁾, the Casimir energy of the membrane ($p=2$) induces a quantum shift, not in m^2 as for the string ($p=1$) but rather in $m^{3/2}$. Whereas our analysis¹⁰⁾ yielded a shift in m^2 for the same reason as for the string, namely the operator ordering of terms quadratic in oscillators.

The resolution of the paradox is that the toroidal compactification introduces a length scale R just as in Section 3, where the membrane tension κ_2 is related to the string tension κ_1 by

$$K_1 = 2\pi R K_2 \quad (5.12)$$

The introduction of a length scale means that the dimensional arguments of Kikkawa and Yamasaki no longer apply, and linear trajectories are possible even for membranes. Thus, we may replace (5.6) by

$$J = A \kappa_p^{-1} c^2 m^2 R^{1-p} + B \hbar \quad (5.13)$$

which also reduces to the familiar formula when $p = 1$. In fact it is tempting to speculate that consistency may require space-time compactification when $p > 1$. See Section 6.

Non-Renormalizability

The Green-Schwarz action for super p -branes is power-counting non-renormalizable except in the case of strings ($p=1$)^{*}). Of all the objections to membranes, this is in our opinion the most difficult to refute. One suggestion is to follow Weinberg³⁴⁾ and appeal to asymptotic safety. Another suggestion is to include couplings to all the modes of the membrane in the non-linear σ -model and not just the massless ones. In the case of strings this would also render the σ -model non-renormalizable (since higher rank tensors would require more derivative to form an invariant) and yet presumably leads to a perfectly sensible theory. The hope of this suggestion is that the problem resides not with the membrane per se but only in the truncation to the massless modes which is consistent for strings but perhaps not for membranes (in analogy with the consistency problems that arise in

^{*}) One thus expects to generate curvature corrections to the Nambu-Goto action. These are typically of the form $\kappa^{1-\frac{2}{p+1}} R^{\frac{2}{p+1}}$ at ℓ loop order with only ℓ even for even p . Note that we never induce an Einstein-Hilbert term, however. For this reason, we do not follow recent attempts to interpret space-time itself as the world-volume of a membrane³³⁾.

Kaluza-Klein theories). A third suggestion is based on the observation that certain three-dimensional $O(N)$ σ -models do become renormalizable in the large N limit³⁵⁾.

A fourth (wild?) idea which I favour personally is that although the theory is non-renormalizable and non-finite for generic choices of the background supergravity fields, there may exist a very special choice of vacuum which renders the σ -model finite. This idea is further discussed in Section 6. Even if this problem is solved, we still have to worry about constructing membrane scattering amplitudes and whether such amplitudes are finite.

6. - WILD SPECULATIONS

Looking at the brane-scan of the figure, the most interesting p -branes seem to be the supermembrane in $D = 11$ and the super five-brane in $D = 10$. In this section we make some speculations concerning their properties, beginning with the five-brane.

A Heterotic Five-Brane?

In Section 1 we remarked that there are two anomaly-free ten-dimensional $N = 1$ supergravity theories coupled to either $E_8 \times E_8$ or $SO(32)$: the version with the three-form field strength and the version with the seven-form field strength. Since the former arises as the field-theory limit of the superstring, it is natural to conjecture that the latter is related to the super five-brane. However, this would require a coupling of the super five-brane to background Yang-Mills fields which, to my knowledge, has not yet been achieved. Of course, the heterotic string construction³⁶⁾ is peculiar to a two-dimensional world-sheet which permits a separation into left- and right-moving modes, one corresponding to the 26-dimensional bosonic co-ordinates and the other to the ten-dimensional super co-ordinates. For the five-brane we have no such split and no conformal invariance to guide us.

However, a world-volume with one time and $(4k+1)$ space co-ordinates does have something in common with one time and one space in that both permit a real self-duality condition. In the case of the five-brane it is left- and right-handed three-forms rather than the left- and right-handed one-form of the world-sheet. Perhaps this construction or else one involving chiral fermions on the world-volume may permit the required Yang-Mills coupling. If such a five-brane were quantum consistent, it would be every bit as appealing from the point of view of chirality and gauge groups as the heterotic string.

Of course, this five-brane lies on the same (quaternionic) sequence as the superstring in six dimensions which is known to be quantum inconsistent. From this point of view a more promising candidate for a consistent quantum theory is the supermembrane in $D = 11$ which is the only other member of the (octonionic) sequence containing the superstring in ten dimensions, which is known to be quantum consistent.

Supermembrane in $D = 11$: Hidden Symmetries?

It is interesting to note that the three-eight split

$$so(1,10) \supset so(1,2) \times so(8) \quad (6.1)$$

implied by the embedding of the three-dimensional world-volume of the supermembrane in eleven-dimensional space-time had previously been invoked in Refs. 37) and 38) to exhibit the hidden $SO(16)$ symmetry of $D = 11$ supergravity, where the 128 bosonic degrees of freedom may be assigned to the coset $E_{8(+8)}/SO(16)$. One cannot help wondering what rôle these symmetries will play for the supermembrane, not to mention the Kac-Moody extension E_9 and the Lorentzian algebra E_{10} ³⁷⁾.

Another mathematical curiosity, already noted by string theorists, is that the number of disconnected components of the $D = 10$ diffeomorphism group is 992, which is just the dimension of $G \times G$ for

$G = SO(32)$ or $E_8 \times E_8$. But this, in turn, is given by the number of exotic eleven-spheres S^{11} ³⁹⁾.

Spontaneous compactification: $11 = 4+7$

One of the most appealing features of $D = 11$ supergravity, which was swept aside in the wake of the superstring revolution, was the rather natural way in which four space-time dimensions were singled out. This is because the field equations admit a ground state of $(D=4$ non-compact Einstein space-time) $\times$ $(D=7$ compact Einstein space) in which the four-index field strength F_{mnpq} of Section 2 acquires a non-zero VEV proportional to ϵ_{mnpq} when the indices range over 1, 2, 3, 4 and zero otherwise⁴⁰⁾. This introduces a length scale into the theory. Since, at the classical level, all solutions of $D = 11$ supergravity are possible backgrounds for the supermembrane, this four-dimensional feature persists for the supermembrane. The classical field equations of $D = 11$ supergravity also permit $D = 11$ Minkowski space, of course, but we have already seen in Section 5 that the quantum membrane may somehow prefer space-time compactification and the accompanying length scale. Fujikawa and Kubo⁴¹⁾ have also suggested, for different reasons, that space-time compactification is a natural phenomenon in the membrane theory. It is worth recalling that the reason why we live in four dimensions remains one of the greatest unsolved mysteries of string theory.

Of course, $11 = 7+4$ too⁴²⁾ and an equally acceptable solution of the $D = 11$ field equations is to set the dual of F_{mnpq} proportional to $\epsilon_{mnpqrst}$ when the indices range over 1, 2, 3, 4, 5, 6, 7 and zero otherwise⁴⁰⁾. This yields a ground state of $(D=7$ non-compact Einstein space-time) $\times$ $(D=4$ compact Einstein space). Although locally both solutions seem to be on an equal footing, globally they are quite different³⁾ since the volume form on a compact manifold is closed but not exact, i.e., the four-index field strength cannot be written as the curl of any non-singular three-form potential. This has an interesting

consequence for the supermembrane when we recall that we may rewrite the Wess-Zumino term as an integral over a four-manifold whose boundary is the three-dimensional world-sheet. Thus (dropping hats), the bosonic part of the action is

$$S = \int_{\partial M^4} d^3\zeta \sqrt{-g} + \int_{M^4} d^4\zeta \varepsilon^{ijkl} \partial_i x^m \partial_j x^n \partial_k x^p \partial_l x^q F_{mnpq} \quad (6.2)$$

In the case $(D=7 \text{ space-time}) \times (D=4 \text{ compact space})$ the VEV of F_{mnpq} and hence the curvature of the compact four-manifold will be quantized by virtue of the non-trivial topology (non-vanishing fourth Betti number).

It remains to be seen what the implications are, but it is curious to note that it is the four-dimensional space-time choice which does not give rise to this quantization. A possible resolution of this old dilemma is given below.

Supermembranes on the Seven-Sphere

An important special case of the above compactifications corresponds to $(d=4 \text{ anti-de Sitter space-time}) \times S^7$ since this gives rise to a four-dimensional supergravity with maximum $(N=8)$ supersymmetry and local $SO(8)$ invariance³⁾. The symmetry of the vacuum is given by the supergroup $OSp(4/8)$. Thus the supermembrane provides a niche for the gauged extended supergravities⁴³⁾ which had no place in superstring theory^{*)}.

*) For better or worse, this also means a fond "hello again" to anti-de Sitter space (AdS)⁴⁴⁾.

Apart from group manifolds, S^7 is the only compact manifold which admits an absolute parallelism^{*)} and consequently one is tempted to draw an analogy between supermembranes on S^7 and strings on group manifolds^{**) .}

Now the non-linear σ -models describing strings on group manifolds are known to be conformally invariant and finite in $D > 26$ provided we satisfy the correct critical dimension formula. The observation that the theories are not finite in $D > 26$ "even" for flat backgrounds is true but irrelevant. In fact, one can show that the σ -models on space-time $\times G$ correspond to a free field theory. Thus, as discussed in Section 5, might the observation that the supermembrane σ -model is not renormalizable or finite "even" for flat backgrounds be equally irrelevant? Might the theory be finite on a very special background? Might that background be $(d=4 \text{ AdS}) \times S^7$?

Is the Supermembrane a Singleton?

Owing to its anti de Sitter subgroup, the supergroup $OSp(4/8)$ admits the strange "singleton" representations which have no analogue in the Poincaré group and no immediate field theory interpretation. Owing to the $N = 8$ supersymmetry they form an ultrashort $N = 8$ supermultiplet with maximum spin- $\frac{1}{2}$ ⁴⁶⁾. The multiplet consists of eight spin- $\frac{1}{2}$ fermions and eight spin-0 bosons which transform according to the 8_s and 8_v representations of $SO(8)$. (Thus the fermions have only half the naive number of degrees of freedom.) Although we are dealing with the four-dimensional anti de Sitter group $SO(2,3)$, one cannot write down a four-dimensional action for these singletons. However, as discussed by Fronsdal⁴⁷⁾, we can write down a three-dimensional action

*) A totally antisymmetric three-index torsion for which the corresponding Riemann tensor vanishes.

**) We note in this connection the recent discovery of loop algebras and superalgebras based on S^7 45).

where the three-dimensional integration is over the boundary of anti-de Sitter space, which has signature $(-,+,+)$ and topology $S^1 \times S^2$. The $N = 8$ case is discussed by Sezgin and Nicolai⁴⁸⁾.

But 8_s fermions and 8_v scalars on a three-dimensional world-volume with signature $(-,+,+)$ is just what we obtain after gauge-fixing the supermembrane action! Just as for the singleton, the fermions have only half the naive number of degrees of freedom owing to the Siegel symmetry. Moreover, we have already seen from (6.2) how the world-volume of the membrane is naturally regarded as the boundary of a four-dimensional manifold. Can the supermembrane be regarded as the singleton of $OSp(4/8)$ whose world-volume is the boundary of anti-de Sitter space? If so, this would definitely favour $11 = 4+7$ and not $11 = 7+4$ too!

In fact, it will not have escaped the reader's attention that $OSp(4/8)$ is also the vacuum symmetry of the $(d=4 \text{ AdS}) \times S^7$ background of $D = 11$ supergravity. Indeed, the above singleton representations appear in the harmonic expansion of the $D = 11$ fields on S^7 , but can be gauged away^{49),3)}. This is suggestive of a form of bootstrap in which the field theory gives rise to the membrane which, in turn, yields the field theory. [Singleton representations also occur for the supergroups $OSp(6,2/4)$ and $SU(2,2/4)$ whose anti-de Sitter groups are $SO(2,6)$ and $SO(2,4)$ ⁵⁰⁾. This is discussed in Ref. 50) where an attempt is made to relate singletons to superstrings. The dimensions of the boundaries of the corresponding space-times are $d = 6$ and $d = 4$ which, from the table, are also the world-volume dimensions of the quaternionic and complex supermembranes. In these cases, however, the bootstrap idea seems no longer to apply.]

Finally, there is one other feature which membranes and singletons have in common: they were both invented by Dirac^{51),52)} at about the same time!

Supermembrane-Inspired Phenomenology?

No concrete experimental predictions so far (much like superstrings).

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