A NON-PLASMA APPROACH TO THE SOFT DILEPTON PRODUCTIONIN hA and AA COLLISIONSP. Lichard^{*)}

CERN - Geneva

A B S T R A C T

We propose a method of evaluating the soft dilepton production rate in hadron-nucleus and nucleus-nucleus collisions at high energies based on summing the contributions from quark-antiquark systems (chains) created in individual hadron-nucleon interactions. The individual contributions are determined by using the square dependence of the soft dilepton production rate on hadron multiplicity predicted for hadron-hadron collisions by the soft-quark-annihilation model and observed recently in pp collisions at the CERN ISR. The possibility of additional contributions originating from the annihilation of quarks and antiquarks that belong to different chains is taken into account in a phenomenological way, which nevertheless enables to correlate different phenomena in soft dilepton production. More detailed predictions are given for proton-nucleus collisions at 200 GeV/c.

^{*)} On leave of absence from the Department of Theoretical Physics, Faculty of Mathematics and Physics, Comenius University, Bratislava, Czechoslovakia.

1. Introduction

The production of $\mu^+\mu^-$ and e^+e^- pairs with low masses (below the ρ - ω region) in both hadron-hadron and hadron-nucleus collisions with yields that cannot be explained by known processes (particle decays, hadronic bremsstrahlung, Bethe-Heitler process) is nowadays experimentally a well-established phenomenon (for a review of experiments see e.g. [1]). The excess of lepton pairs over the sum of backgrounds and known particle decay sources is higher at least by an order of magnitude than what is expected through the standard Drell-Yan model.

Another interesting phenomenon (see [2] and literature cited there) is the copious production of electrons with low transverse momenta in pp collisions, which manifests itself as a rise in the e/π ratio with decreasing p_T . This has recently [2] been confirmed down to $p_T=0.1$ GeV/c, where only about 1/6 of the electron cross section can be accounted for by "trivial" sources (including semileptonic decay of charm).

In the last few years, the renewed interest in dilepton¹ production is connected with the attempts to produce and detect the quark-gluon plasma in high-energy nucleus-nucleus collisions [3]. The thermal emission of photons and dileptons is considered as one of the tools for quark-matter diagnostics.²

The dilepton production in nucleus-nucleus collisions can be, of course, described also in a more conventional way, namely by summing the contributions from individual elementary collisions (nucleon-nucleon, quark-nucleon, ...). It is of great importance to know the predictions of dilep-

¹ Throughout this paper we shall have in mind only "anomalous" leptons and dileptons, which cannot be attributed to trivial sources (particle decays, hadronic bremsstrahlung, Bethe-Heitler process).

² The present state of the art is reviewed in [4], where also the references to earlier work can be found.

ton properties following from such an approach. The dilepton rates can be used as a probe of the formation of a qualitatively new state of matter (hadron plasma, mixed phase, quark-gluon plasma) only if their magnitudes and/or their dependence on kinematical quantities that characterize the collision (for example, transverse energy) differ significantly from the expectations of the conventional approach.

However, even if a conventional model of dilepton production underestimated the data from nucleus-nucleus collisions, this would not have to signify the presence of a new production mechanism, but simply the fact that the model is not physically sound. It is therefore of great importance to scrutinize the models of dilepton production in processes which are more complex than hadron-hadron collisions, but in which the formation of the thermalized quark-gluon plasma is generally taken for improbable, i.e. in high energy hadron-nucleus collisions. Unfortunately, the data on low-mass dilepton production in the latter are scarce and far from being systematic. Even the A -dependence of the cross section in the low mass region is not very well explored. This is caused by the notorious difficulties of low mass dilepton experiments.

The topic is interesting also from the point of view of "classical" high-energy nuclear physics, because it may serve as a decisive test of models of hadron-nucleus and nucleus-nucleus collisions. The models based on different physical assumptions may well give an equally good description of some quantities (cross section, hadron multiplicities, transverse energy distribution) but, on the other hand, may strongly differ in their predictions of the soft dilepton production.

As regards the conventional approach to the soft dilepton production in hA and AA collisions, we are not in a good position, because this process is not very well understood theoretically even in the case of hadron-hadron collisions. The only concept which has proven fruitful is that of Bjorken and Weisberg [5]. It assumes that during the course of a hadron-hadron collision quarks and antiquarks are created in addition to those that already existed in the incoming hadrons. After some finite time they materialize to final-state hadrons, but before this happens they can also

annihilate to lepton pairs. The main drawback of the Bjorken-Weisberg idea lies in the fact that it is hard to fit into most of the present-day schemes of multiparticle production in hadronic collisions.

The problem that stimulated the present investigation can be formulated as follows: It has been argued [6] that the square dependence of the dilepton production rate on associated multiplicity within a fixed rapidity bin is to be expected not only in hadron-hadron collisions in the region of applicability of so-called intermediate stage models (described in some detail in the next section), but also for photons and dileptons produced from the hot quark matter during its isentropic expansion [7]. Does the fact that such a square dependence has been observed experimentally [1,8] diminish its role as an indicator of the quark-gluon plasma formation?

In the present paper we shall try to answer this question (and other closely related ones) assuming that the dilepton yield (as well as that of hadrons) in hA and AA collisions is built up as a sum of individual contributions. For every elementary contribution we shall suppose the proportionality of the mean number of dileptons to the square of the number of charged hadrons within a fixed rapidity bin. Even though our approach is only phenomenological, we cannot avoid the question about the dynamical origin of individual contributions. With respect to what has already been said before, it is natural to connect them with the Bjorken-Weisberg excited systems of quarks and antiquarks. For the sake of brevity we shall call the latter "chains", apologizing in advance for the confusion that this may cause.

The picture of the dilepton production in hadron-nucleus and nucleus-nucleus collisions as just sketched is probably oversimplified. It does not take into account the space-time aspect of these processes and its relation to the space-time evolution of individual hadron-nucleon interactions. It is well possible that the individual chains may overlap in space-time and rapidity. In such case also the quarks and antiquarks from different chains may annihilate and produce low-mass dileptons. The quadratic law should then hold between the total contributions to both dilepton rate and hadron multiplicity from the system of two (or more) overlapping chains.

The lack of a detailed theory of the space-time evolution of hA and AA collisions forces us to treat its consequences in a rather crude phenomenological way. We introduce a parameter λ equal to the probability that two chains created in the same event overlap, and allow also n-chain overlap with the probability λ^{n-1} . It is clear that such an approach can predict only the lower ($\lambda=0$) and upper ($\lambda=1$) bounds on dilepton production rates in processes which we are interested in. But, as we shall see in what follows, if the value of the parameter λ is fixed by one piece of data (e.g. by the dependence of dilepton-to-hadron ratio on hadron multiplicity for certain projectiles and targets) we obtain unambiguous³ predictions for other phenomena (dependence of dilepton-to-hadron ratio on atomic numbers, on the number of elementary collisions, etc.).

The organization of the paper is as follows. The next section recalls some experimental results about soft dilepton production in hh collisions and their interpretation. Section 3 contains the quantitative formulation of our approach to dilepton production in hA and AA collisions together with crude but transparent estimates of what can be expected on its basis. Section 4 is devoted to the description and tests of a particular model of pA collisions, which is then used in Section 5 to get, as we hope, more realistic predictions of dilepton production in this process at 200 GeV/c. The last section contains various comments.

³ Of course, within a chosen model of pA or AA collisions.

2. Soft leptons and dileptons in hadron-hadron collisions

Recent experimental results [1,8] of the AFS (Axial Field Spectrometer) collaboration at the CERN Intersecting Storage Rings represent an important step towards the understanding of physics behind the production of soft leptons (low p_T) and soft dileptons (mass below the ρ - ω region) in hadron-hadron collisions. They have clearly shown that in the central region ($|y| < 1$) of pp collisions at $\sqrt{s}=63$ GeV

- (1) the yield of positrons with low p_T 's (< 0.4 GeV/c) is proportional to the square of the charged multiplicity [8];
- (2) the production of prompt e^+e^- pairs can explain the observed [8] rise of the e^+/π ratio with decreasing p_T [1];
- (3) the production rate of the e^+e^- pairs with masses $0.05 < M < 0.6$ GeV/c² is proportional to the square of the charged multiplicity [1].

The observation (2) confirmed what was assumed, partly on the basis of the finding [9] that prompt muons are produced predominantly in unlike pairs, in some models (see e.g. [10,11]) that aimed at explaining the behaviour of the e/π ratio at small p_T . Here, we shall take the close connection between soft leptons and soft dileptons for granted and consider the observation (1) of the AFS collaboration as a consequence of (2) and (3). In what follows, we shall speak only about soft dileptons, tacitly assuming that everything holds also for the production of soft leptons.

The correlation between the mean number of low mass dileptons per event within some (not too wide) rapidity interval and the number of hadrons in the same rapidity interval has recently been proposed [6] as an indicator of the underlying production mechanism. The quadratic behaviour found by the AFS collaboration is typical for the intermediate stage models. All these models are based on the Bjorken and Weisberg idea [5] that low-mass pairs originate mainly from the annihilation of those quarks and anti-quarks that have been created during the collision and have had a chance to annihilate before the final hadrons emerge. This idea was made quantitative first in the framework of the "Drell-Yan with enhanced sea" models

[10,12]. Later, it turned out that the original idea of Bjorken and Weisberg must be supplemented in two respects:

- (1) In order to suppress redundant production of dileptons in the high-mass region, the constraints imposed by the space-time evolution of hadron-hadron collisions [13] must be taken into account (see discussion in [14]);
- (2) The experimentally observed shape of p_T - and x_F -spectra of low-mass dileptons as well as the e/π ratio at small momenta can be reproduced only if the elementary subprocess

$$q + \bar{q} \rightarrow (l^+ l^-) + g$$

is included [11] in addition to the direct quark-antiquark annihilation into a lepton pair.⁴

The model incorporating these two effects (now called "soft annihilation model" in the literature) was successfully compared [1,11,15] to the experimental data on soft lepton and dilepton production.

It should be stressed that the difference between the soft annihilation model and the Drell-Yan one is not only quantitative, but also a matter of principle. In the approach of Drell and Yan [16] the cross section of dilepton production is given by electromagnetic interactions and by quark and antiquark distribution functions within free hadrons, as known from deep inelastic scattering (we ignore the existence of the K-factor to simplify the reasoning). On the other hand, in the soft annihilation model the cross section cannot in principle be evaluated from electromagnetic interactions and distribution functions alone. This follows from the fact that the creation of additional quark-antiquark pairs starts only after the initializa-

⁴ Some kinematically similar (Dalitz-like) processes were suggested independently by the authors of [17] to explain their data, but with a photon in the final state. These considerations were put in doubt by results of the experiment [18], which found no excess of photons in the anomalous pair events.

tion of the whole process by strong interactions. The cross section for low-mass dilepton production must therefore be proportional to the inelastic cross section.⁵ The soft annihilation model itself can only give the mean number of dileptons per inelastic event. But also here the strong interaction leaves its fingerprints by setting the scenario (distribution functions, volume and time in which the annihilations can take place). All this is accounted for in the basic formula of the soft annihilation model, which can be found e.g. in [14] or [15].

Further approaches to dilepton production in hadron-hadron collisions that can be characterized as intermediate stage models are based on thermodynamical concepts [19-21]. Various models of this class differ by the assumptions about the space-time evolution of the hadron-hadron collision and about the distribution functions of newly produced quarks and antiquarks. Pioneering work in the field of photon and dilepton production by thermal radiation of hot hadronic matter was done by Feinberg [19]. Shuryak [20] presented quantitative results assuming the existence of a thermalized quark-gluon plasma. Adachi and Yotsuyanagi [21] used the Boltzmann equation for the parton probability distributions, which enabled them to transfer smoothly from the hard (Drell-Yan) regime to the soft one. In the soft annihilation model the quark distribution functions are given by longitudinal phase space with transverse cut-off, modified by Kuti-Weisskopf [22] factors for valence quarks. Free parameters were fixed by the comparison of the quark-recombination-model [23] outcome with multiparticle production data.

We would like to add a few words about one feature of the soft annihilation model, which was not sufficiently stressed in the paper [6]. The square dependence of the dilepton rate on particle multiplicity predicted by it is intimately connected with the assumption that an excited quark-antiquark system exists in some volume for some time before it materializes into hadrons. The detailed form of the cross section of the elementary subprocess plays only a minor role. The quadratic dependence is therefore to

⁵ It implies that the "Drell-Yan with enhanced sea" models did not respect the underlying physics properly.

be expected also for the reaction caused by elementary subprocess

$$q + \bar{q} \rightarrow \gamma + g,$$

i.e. for the soft photon production in hadronic collisions. But, unfortunately, from the experimental point of view this process is even more elusive [24,25] than direct dilepton production.

3. A conventional view on dileptons in pA and AA collisions

In this section we first formulate our basic assumptions and establish the notation that will be used throughout the paper. Then we shall explore our formalism in a transparent though simplified way, postponing more precise but also more obscure and more specific calculations to the next two sections.

Let us assume that any inelastic event in hadron-nucleus or nucleus-nucleus collisions can be characterized by the number of chains consisting of quarks and antiquarks and being created in elementary interactions. The relative frequency of events with κ chains is described by the probability distribution $P(\kappa)$. Let us further choose a rapidity⁶ window $(y, y+\Delta y)$ and denote the number of charged hadrons that go into this window from the i -th chain as n_i (this does not necessary mean the assumption of the space-time ordering of interaction points). Let the joint probability distribution of random variables $n_1, n_2, \dots, n_\kappa$ in κ -chain events be denoted as $P_\kappa(n_1, n_2, \dots, n_\kappa)$.

Under all these assumptions the probability of observing N charged

⁶ The rapidity in lab system is meant, unless stated otherwise.

hadrons in our rapidity window⁷ can be written as follows⁸

$$\tilde{P}(N) = \sum_{\kappa} P(\kappa) \sum_{n_1, \dots, n_{\kappa}} P_{\kappa}(n_1, \dots, n_{\kappa}) \delta(N - \sum n_j). \quad (1)$$

The mean charged multiplicity within Δy in events with a fixed number of contributing chains is simply

$$\langle N \rangle_{\kappa} = \sum_{j=1}^{\kappa} \langle n_j \rangle_{\kappa}. \quad (2)$$

The all-event mean charged multiplicity comes out equal to

$$\langle N \rangle = \sum_{\kappa} P(\kappa) \sum_{j=1}^{\kappa} \langle n_j \rangle_{\kappa}. \quad (3)$$

If the overlap of different chains is negligible ($\lambda = 0$), the dilepton production can be estimated by using the relation

$$\langle l^+ l^- \rangle_{n_i} = \alpha n_i^2, \quad (4)$$

which is assumed to hold for the mean number⁹ of dileptons produced from the i -th chain in events, where the latter contributes to the charged hadron multiplicity by n_i . Summing (4) for all chains in a κ -chain event, then averaging over all combinations of n_i 's that have the fixed sum N , and finally averaging over the number of chains we get the following expression for the mean number of dileptons produced in events with fixed number N

⁷ From now on, all quantities refer to a fixed rapidity window.

⁸ For purely typographical reasons the Kronecker's symbol is featured as Dirac's delta function.

⁹ What is meant is the mean number per inelastic event.

of charged hadrons

$$\langle l^+ l^- \rangle_N = \frac{\alpha}{P(N)} \sum_{\kappa} P(\kappa) \sum_{n_1, \dots, n_{\kappa}} P_{\kappa}(n_1, \dots, n_{\kappa}) (\sum n_i^2) \delta(N - \sum n_j). \quad (5)$$

The mean number of dileptons produced in events with κ contributing chains is obviously

$$\langle l^+ l^- \rangle_{\kappa} = \alpha \sum_{j=1}^{\kappa} \langle n_j^2 \rangle_{\kappa}, \quad (6)$$

whereas for the mean number of dileptons produced within Δy in all inelastic events we obtain

$$\langle l^+ l^- \rangle = \alpha \sum_{\kappa} P(\kappa) \sum_{j=1}^{\kappa} \langle n_j^2 \rangle_{\kappa}. \quad (7)$$

Similar equations accounting for the possible overlap of different chains could be obtained by replacing the sum $\sum n_i^2$ in (5) by

$$\sum_G W(G, \lambda) \sum_{k=1}^{N(G)} \left(\sum_{i=1}^{g_k} n_i \right)^2. \quad (8)$$

In (8), the first summation runs over all possible groupings of chains into groups, $W(G, \lambda)$ is the weight of a specific grouping, $N(G)$ is the number of groups, and g_k is the number of chains in the k -th group.

The modified equations (5-7) would look a bit cumbersome, but we shall not need them because we are not going to treat the case $0 < \lambda < 1$ analytically, but only by means of a Monte Carlo algorithm incorporated into a computer code.

The case $\lambda=1$ is especially simple; all chains then overlap and instead of eqs. (5-7) we have

$$\langle l^+l^- \rangle_N = \alpha N^2 \quad (9)$$

$$\langle l^+l^- \rangle_K = \alpha \langle N^2 \rangle_K \quad (10)$$

$$\langle l^+l^- \rangle = \alpha \langle N^2 \rangle \quad (11)$$

The eqs. (9) and (11) are the same as in the case of pp scattering. Eq. (10), which has no analog in pp scattering, relates the dilepton production in events with a fixed number of contributing chains to another experimentally accessible quantity, the mean squared multiplicity in those events. The number of contributing chains can be inferred (unfortunately not in a model-independent way and not on an event-by-event basis) from the number of slow protons in the final state, which are called "grey tracks" since the era of nuclear emulsions.

In what follows we shall often give the results in the form of dilepton-to-hadron ratios, which we introduce in the following way:

$$R_N = \frac{\langle l^+l^- \rangle_N}{N} \quad (12)$$

$$R_K = \frac{\langle l^+l^- \rangle_K}{\langle N \rangle_K} \quad (13)$$

$$R = \frac{\langle l^+l^- \rangle}{\langle N \rangle} \quad (14)$$

We deem it instructive to calculate these ratios first in a very simple model of pA and AA collisions at least for two extreme cases of chains' overlapping, namely for those of no ($\lambda=0$) and full overlap ($\lambda=1$). We shall

suppose that the joint probability distribution of the numbers of hadrons coming from different chains [see (1) and (5)] factorizes into a product of equal single distributions that are, in addition, independent of the number of chains

$$P_K(n_1, \dots, n_K) = P_1(n_1) \dots P_1(n_K) \quad (15)$$

The results for the ratios (12)-(14), obtained from eqs. (1)-(8) and (15) after some elementary mathematics, are summarized in Table 1.

Table 1: Dilepton-to-hadron ratios in a simple model [eq.(15)]

for two extreme cases of chains' overlapping: no overlap ($\lambda=0$) and full overlap ($\lambda=1$). R_K and R [defined by eqs. (13) and (14), respectively] are scaled by $R_1 = R_{K=1}$ which has the value of $\alpha \langle n_1^2 \rangle / \langle n_1 \rangle$ in this approach.

	$\lambda = 0$	$\lambda = 1$
R_N	$< \alpha N$	$= \alpha N$
R_K/R_1	1	$1 + (\kappa - 1) \frac{\langle n_1 \rangle^2}{\langle n_1^2 \rangle}$
R/R_1	1	$1 + \left(\frac{\langle \kappa^2 \rangle}{\langle \kappa \rangle} - 1 \right) \frac{\langle n_1 \rangle^2}{\langle n_1^2 \rangle}$

We now comment on them briefly in the order in which they appear in Table 1.

(1) We have already learned [1,8] that in the proton-proton collisions the dilepton-to-hadron ratio in events with fixed charged multiplicity N is proportional to N

$$(R_N)_{pp} = \alpha N. \quad (16)$$

In the case of $\lambda = 1$ its linear dependence on hadron multiplicity is restored also in hadron-nucleus and nucleus-nucleus reactions.

On the other hand, if there is no chain overlapping ($\lambda=0$) and if at least the contributions from two chains are non-vanishing, the inequality

$$\sum n_i^2 < (\sum n_i)^2 = N^2$$

can be used together with (5) to obtain the relation

$$R_N < \alpha N, \quad (17)$$

which shows that in pA and AA collisions the ratio (12) is expected to rise with charged multiplicity more slowly than in pp collisions.

To get further insight into the problem, let us assume that if the number of contributing chains is high, then the fluctuations in the numbers of charged hadrons coming from individual chains can be neglected. In such a situation the contribution from every chain is $n_1 = N/\kappa$ and we even obtain the constant value αn_1 of the ratio (12).

This was, of course, an extreme case. Nevertheless, we can judge that the dependence of R_N on N will flatten out with increasing number of contributing chains (increasing atomic number of target or/and projectile). The dependence of R_N on N for a given process can be calculated if the probability distributions $P(\kappa)$ and $P_1(n)$ are specified.

For example, if we assume that the number of hadrons from a single chain is a Poisson distributed quantity with a mean μ ,¹⁰ we obtain from (5) after some algebra

$$R_N = \alpha \left[1 + (N - 1) \frac{\langle e^{-\kappa\mu} \kappa^{N-1} \rangle}{\langle e^{-\kappa\mu} \kappa^N \rangle} \right], \quad (18)$$

where the averaging with weight $P(\kappa)$ is understood on the right-hand side. In the collisions in which also two- and more-chain events contribute, the ratio of mean values in the square brackets is smaller than unity and the inequality (17) is satisfied, as it should be. If further $N \gg \mu$, the expressions that enter the mean values in (18) are strongly peaked at $\kappa = (N-1)/\mu$, resp. $\kappa = N/\mu$. Supposing that the probability distribution $P(\kappa)$ does not behave wildly around these values, we can estimate the ratio of mean values in (18) by dividing the corresponding maximal values. We thus obtain

$$R_N = \alpha (1 + e\mu), \quad (N \gg \mu). \quad (19)$$

This shows that the initial increase of the ratio (12) with N slows down and R_N is approaching a constant value at high N , in agreement with our previous guess.

(2) Another interesting phenomenon that can perhaps be studied experimentally is the dependence of the dilepton-to-hadron ratio on the number of contributing chains in proton-nucleus and nucleus-nucleus collisions. Let us start again with the alternative $\lambda = 1$, for which the quantity R_κ turns out to be extremely sensitive to the form of the one-chain-multiplicity distribution $P_1(n)$. If there were no fluctuations in the number of hadrons coming from single chains [$P_1(n) = \delta(n-n_1)$], we would obtain

¹⁰ The results for the Poisson distribution in the number of hadron pairs, which is closer to the experimental one, can be obtained by simple rescaling of α and μ and therefore leads to qualitatively similar results.

$$R_K = R_1 \kappa, \quad (20)$$

which implies (together with $\langle N \rangle_K = n_1 \kappa$) the square dependence of the dilepton production rate on the number of contributing chains. If we assume, as another example, a negative-binomial-distribution form of $P_1(n)$, we get

$$\frac{R_K}{R_1} = 1 + \frac{\langle n_1 \rangle}{(1 + k^{-1})\langle n_1 \rangle + 1} (\kappa - 1). \quad (21)$$

Taking just for orientation the values $\langle n_1 \rangle = 1.87$, $1/k = 0.37$ ([26], pp collisions, $2.03 < y < 3.03$) the right-hand side of (21) becomes $0.48 + 0.52\kappa$, what leads to substantially lower values of R_K/R_1 than eq. (20). It is clear, and results that will be presented in Sec. 5 confirm it, that also further effects (fluctuations of chain invariant energies, correlations among multiplicities from different chains, dependence of $\langle n_1 \rangle$ on the number of elementary interactions) work towards the flattening of the κ -dependence of R_K/R_1 .

In the case of $\lambda = 0$ (non-overlapping chains) both the mean number of dileptons within a rapidity bin and that of hadrons are proportional to the number of chains κ , so that the dilepton-to-hadron ratio R_K comes out independent of κ . The more complex model of Sec. 5 leads for $\lambda = 0$ to R_K even falling with κ . This is again a manifestation of the effects that are ignored in an approach based on eq. (15).

To get a feeling of what the influence of those effects may be, let us investigate the matter in a toy model with the mean hadron multiplicity from chains decreasing with the ordinal number of the chains (this corresponds to the spirit of the successive-collisions model). To be more specific, we assume $P_i(n_i)$ to be a Poisson distribution with μ_i forming a geometric progression with a quotient $q < 1$. Then the mean number of dileptons produced from the i -th chain is given by

$$\langle l^+ l^- \rangle = \alpha \langle n_i^2 \rangle = \alpha \mu_i (1 + \mu_i).$$

Summing the contributions from all κ chains we obtain

$$\langle N \rangle_{\kappa} = \mu_1 \frac{1 - q^{\kappa}}{1 - q},$$

$$\langle |^{+}|^{-} \rangle_{\kappa} = \alpha \left(\langle N \rangle_{\kappa} + \mu_1^2 \frac{1 - q^{2\kappa}}{1 - q^2} \right).$$

Combining these two results we finally obtain for the dilepton-to-hadron ratio in events with fixed number of chains

$$R_{\kappa} = \alpha \left(1 + \mu_1 \frac{1 + q^{\kappa}}{1 + q} \right),$$

which is a decreasing function of κ , starting from the value $1 + \mu_1$ at $\kappa = 1$ and approaching the value

$$R_{\infty} = \alpha \left(1 + \frac{\mu_1}{1 + q} \right)$$

for large κ .

(3) The dilepton-to-hadron ratio within a fixed rapidity window in all inelastic events is from the experimental point of view the best measurable quantity of those listed in Table 1. In our simple approach the properties of both projectile and target can enter only through the probability distribution for the number of elementary interactions, which is given in first approximation only by the geometry of the collision and by the inelastic cross section of elementary interactions [27,28]. We can thus set, using the expressions from Table 1, the upper ($\lambda = 1$) and lower ($\lambda = 0$) bounds on the dilepton-to-hadron ratio for various projectiles and targets. The lower bound is very simple - it should be the same for all projectiles and tar-

gets.¹¹ In section 5 we shall see that in a more realistic model of pA collisions this rule is violated as a result of fluctuating invariant energies of individual chains, and the dilepton-to-hadron ratio slightly decreases with increasing A for $\lambda = 0$.

It would, of course, be possible to calculate the ratio $\langle \kappa^2 \rangle / \langle \kappa \rangle$ (see Table 1) in order to obtain an upper bound on R in various processes. But we shall not elaborate further in this direction because the results would have anyway only a small chance to be quantitatively correct due to the simplifying assumptions of this section.

4. A model of proton-nucleus collisions

If we want to cast the conventional predictions of the soft dilepton production in pA and AA collisions into a more quantitative form, we need a model of these processes that is in agreement with the experimental data on multiplicity distributions within rapidity windows. In this work we shall concentrate on the high-energy proton-nucleus collisions, where the data from the lepton production experiment NA34 can be expected in the near future (both e 's and μ 's measured in the same apparatus).

There are many models¹² of pA collisions which are more or less successful in describing various aspects of the process. In this paper we prefer to choose as simple a model as possible, so as not to obscure the basic idea of our approach, sketched in the previous section, by technical details.

¹¹ This reproduces the conclusion drawn in [29] for pA reactions on the basis of less stringent assumptions.

¹² The references can be found in or traced back from [30-32].

For this reason¹³ we have adopted the model of successive projectile-nucleon interactions, often used in the literature [33-36], even if there are some arguments against its plausibility [32,37]. Here we summarize the basic ingredients of the model that we shall use in the following:

(1) The projectile (or after the first collision with a nucleon - a remnant of it) undergoes several inelastic collisions in the nucleus. The number ν ¹⁴ of them is governed by Glauber probabilities [27] $P(\nu)$. These we calculate using the same nuclear densities as in [33], except for light nuclei (Li, Be, C) for which we use the harmonic oscillator densities from [27] with a free parameter fixed to fit more recent values of absorption cross sections [38].

(2) In every elementary collision initiated by a projectile (or a projectile remnant) p_{i-1} except the last collision (for definiteness let us speak about the i -th collision, $i < \nu$)

$$p_{i-1} + N \rightarrow S_i \rightarrow p_i + H_i$$

an excited parton system S_i is created, from which a forward baryon-like cluster p_i of leading quarks emerges and the rest materializes to a baryonic system H_i . The properties of the system H_i are the same as those of the hadronic system produced in deep inelastic μp scattering with the same invariant energy W ,¹⁵ and these can be inferred from the data [39]. The motivation for this assumption is that both these systems contain no leading

¹³ The second, and frankly speaking, even more important reason is not to expose any of available models to the danger of being blamed if our predictions fail.

¹⁴ In the present model one chain per elementary collision is produced (see below). The number of chains κ is therefore the same as the number of collisions ν .

¹⁵ W is given here by the rapidity of the projectile before and after an elementary collision (we kept the projectile mass fixed).

baryon in the forward direction. This is clearly a further approximation, because our system H_i is not of a quark-diquark type.¹⁶ But we nevertheless consider this choice more suitable than that of pp inclusive data, which would lead to the multiple counting of the leading baryon.

(3) In the last elementary collision ($i=v$) the excited system S_i evolves into a hadronic system which has the same properties as final states in pp interactions. For the charged multiplicities in the full rapidity range we use the fit from Ref. 41 because it agreed, better than others we examined, with the multiplicities in pp interactions from the experiment [42]. It is with the latter that we shall compare our results on multiplicities in pAr and pXe collisions. The rapidity density of charged particles is parametrized by the formula taken from [34], but symmetrized with respect to $y^* \rightarrow -y^*$ (y^* being the cms rapidity in pp collisions).

(4) The degradation of the projectile rapidity can be described by the formalism [33] which stemmed from [34]. We have only modified slightly the kernel function $K(z, z_0)$ (for notations see [33]) and chosen it in the form

$$K(z, z_0) = \alpha \frac{(z - z_0)^{\alpha-1}}{1 - z_0^\alpha} \quad (22)$$

in order to obtain the distribution function $F_n(x, x_0)$ without the unaesthetic singularity at $x \rightarrow x_0$. In the high energy limit $x_0 \rightarrow 0$ (which was considered in [33]) the kernel (22) leads to the same stopping power as the original one. The value $\alpha = 3$, obtained [33] from the analysis of the data on the leading baryon distribution in pA collisions is therefore used also here for all elementary collisions.¹⁷

¹⁶ In the paper [40], the use of μp data is fully justified.

¹⁷ This is probably a source of a small inconsistency with the data in very forward rapidity spans (see tables 2 and 3), because as it has been shown in [43], the first collision is more effective in stopping the projectile than the following ones.

In order to see whether such a simple model is capable of giving a true picture of the main features of pA reactions, we compared it first to the data [42,44] on mean multiplicities of charged particles produced in interactions of 200 GeV/c protons with various targets. The results of this investigation are depicted in Fig.1. The discrepancies between the model and the data¹⁸ [42] can be ascribed, as we shall see, to the abundance of slow ($y < 0.5$) secondaries in the data. They indicate that the intranuclear cascading (not taken into account in the model) cannot be neglected. But for our study of dilepton production this does not represent an obstacle, because we shall concentrate on a rapidity bin situated at higher y .

As a further test we calculated the mean charged multiplicities in rapidity spans of various lengths extending from $y_{pp}^* = 0$ ($y = 3.03$) to forward and backward hemispheres in pAr and pXe collisions at 200 GeV/c. The results are shown in Tabs. 2 and 3 together with the experimental data [26]. The agreement is reasonable, except for the two widest spans in the backward direction, which indicates that the source of inconsistency lies in the region $y^* < -2.5$, as was already mentioned above.

The last test that we performed concerned the multiplicity distribution of charged hadrons in the rapidity window (2.03, 3.03). We restricted ourselves to this interval for several reasons. First, in connection with our forthcoming study of dilepton production, it is reasonable to choose its width equal to 1. In broader intervals the relation (4) may be violated [6]. Second, the parameters of the negative binomial distributions are given in the data [26] not for rapidity bins, but for rapidity spans covering various widths in rapidity from $y = 3.03$ towards both forward and backward hemispheres. We have chosen the span $\Delta y = 1$ spreading to the backward hemisphere, where the number of contributing chains is higher than in the forward hemisphere. This can make the effects we want to study more pronounced (see previous section). Finally, this bin is contained in the kinematic region covered by the NA34 experiment.

¹⁸ The model is below the data by 9% for pAr and by 14% for pXe collisions.

Table 2: pAr collisions at 200 GeV/c

 $\langle N \rangle$ in rapidity spans of width Δy extending from $y^* = 0$

Δy	f o r w a r d		b a c k w a r d	
	model	data [26]	model	data [26]
0.5	1.45	1.45 ± 0.08	1.53	1.60 ± 0.11
1.0	2.74	2.75 ± 0.16	3.07	3.06 ± 0.16
1.5	3.80	3.66 ± 0.14	4.53	4.53 ± 0.22
2.0	4.60	4.38 ± 0.15	5.84	5.83 ± 0.24
2.5	5.10	4.93 ± 0.15	6.90	6.56 ± 0.26
3.0	5.33	5.17 ± 0.15	7.63	8.41 ± 0.36
3.5	5.40	5.26 ± 0.14	8.01	9.12 ± 0.40

Table 3: pXe collisions at 200 GeV/c

 $\langle N \rangle$ in rapidity spans of width Δy extending from $y^* = 0$

Δy	f o r w a r d		b a c k w a r d	
	model	data [26]	model	data [26]
0.5	1.75	1.67 ± 0.07	1.91	1.76 ± 0.06
1.0	3.25	3.08 ± 0.09	3.89	3.76 ± 0.13
1.5	4.45	4.20 ± 0.10	5.83	5.80 ± 0.19
2.0	5.29	4.93 ± 0.12	7.61	7.58 ± 0.24
2.5	5.79	5.40 ± 0.11	9.10	9.01 ± 0.30
3.0	6.01	5.68 ± 0.11	10.19	12.01 ± 0.40
3.5	6.06	5.78 ± 0.10	10.77	14.00 ± 0.53

The Monte-Carlo computer code that evaluated the multiplicity distribution of charged hadrons in a fixed lab rapidity window (and which was also used to calculate the dilepton yields) proceeded event-by-event in the following steps:

(i) the number of elementary interactions v was randomly generated on the basis of Glauber probabilities;

(ii) the rapidities of the projectile after the 1st, 2nd, ..., $(v-1)$ -th elementary collisions were generated at random using the probability distribution given by kernel function (12);

(iii) the invariant energies W_i ($i=1, \dots, v$) of the hadronic systems coming from individual collisions [see points (2) and (3) above] were calculated;

(iv) for every but the last elementary interaction the lab rapidity interval (2.03, 3.03) was converted into the corresponding bin in cms rapidity of the hadronic system with invariant energy W_i and the mean charged multiplicity within it was inferred from the μp data [39];

(v) the mean charged multiplicity within the lab rapidity bin considered and for invariant energy W_v of the last hadronic system was calculated using the symmetrized version of the rapidity-density parametrization for pp collisions from Ref. 34;

(vi) the negative-binomial-distribution (NBD) random generator was used to generate the numbers of charged hadrons coming into our rapidity window from different chains using the mean multiplicities as calculated above and the NBD parameter $1/k$ fixed at 0.243 [39] for the μp systems and at 0.370 [26] for the pp one;

(vii) the total charged multiplicity within the rapidity bin was determined as a sum of individual contributions.

From the multiplicity distributions obtained in that way for pAr and pXe collisions at 200 GeV/c we calculated their parameters (Tab. 4) and com-

pared them to the data [26]. The mean multiplicities from Tab. 4 agree well with the data and with our previous results (Tabs. 2 and 3) obtained by a simpler method, which shows that the statistical errors of our Monte Carlo calculations were small.

The data on multiplicity distributions are well parametrized [26] by negative binomial distributions. We therefore tested whether also the model distributions were compatible with NBD by calculating the skewness $S = \langle (N - \langle N \rangle)^3 \rangle^{1/3}$ on the one hand directly, and on the other from the formula ($D^2 = \langle N^2 \rangle - \langle N \rangle^2$)

$$S^3 = \left(2 \frac{D^2}{\langle N \rangle} - 1 \right) D^2 ,$$

which is valid if the distribution has the NBD form. The results (Tab. 4) did not indicate any dramatic deviation of the model multiplicity distributions from the negative binomial one. Finally, we have calculated the NBD parameter $1/k$ using the formula

$$\frac{1}{k} = \frac{D^2 - \langle N \rangle}{\langle N \rangle^2} .$$

The results (table 4) showed that the model distributions were slightly narrower than the experimental ones. Knowing from the study [45] that the resulting $1/k$ is mostly influenced by the dispersion-to-mean ratio of the distribution in the number of collisions, we blame the oversimplified treatment of the nuclear densities for this defect. A detailed investigation of the matter would require a rather painful and time-consuming study of the implications of both nuclear models and data (see e.g.[46]), and of the relation between charge and matter distributions. This is probably the reason of the widespread use of the Fermi distribution formula¹⁹ with a

¹⁹ Now often called "Woods-Saxon parametrization" in contrast with what is meant under this term in nuclear physics, see e.g. [47] or [48].

simple parametrization of its parameters.

Table 4: pA collisions at 200 GeV/c

The parameters of the negative binomial distribution in rapidity interval (2.03, 3.03) and the test of satisfying NBD by the model multiplicity distribution by comparing the skewness calculated directly (S_1) with that determined from $\langle N \rangle$ and $\langle N^2 \rangle$ assuming the validity of NBD (S_2).

	mean multiplicity		NBD parameter $1/k$		skewness (model)	
	model	data [26]	model	data [26]	S_1	S_2
Ar	3.07	3.06 ± 0.16	0.37	0.555 ± 0.077	2.70	2.78
Xe	3.85	3.76 ± 0.13	0.33	0.555 ± 0.050	2.98	3.15

5. Results on dilepton production

When calculating the mean number of dileptons, the computer code described in some detail in the previous section was supplemented by the following amendments:

(i) In the case of no chain overlap ($\lambda = 0$), the expected number of dileptons in an event was determined [up to a factor of α , see (4)] as a sum of squares of the numbers of hadrons from individual chains.

(ii) The case of full overlap ($\lambda = 1$) was treated by taking the square of the total number of hadrons.

(iii) In the intermediate case $0 < \lambda < 1$ we proceeded in the spirit of the model of successive collisions. Every group of overlapping chains was allowed to contain only neighbouring chains. For every elementary collision except the first one, a random number ξ from the interval $(0, 1)$ was uniformly generated. If it was less than λ , the relevant chain was added to the previous chain (or to the group which contained it) to form a group of overlapping chains. In the opposite case the chain was considered as a first member of a new group. The expected number of dileptons was then determined by calculating the sum of the squares of the total numbers of hadrons within individual groups [see (8)].

The results of the calculations of the dilepton-to-hadron ratios within lab rapidity interval $(2.03, 3.03)$ in pA collisions at 200 GeV/c are shown in Figs. 2 - 5.

Figure 2 contains the N -dependence of the dilepton-to-hadron ratio in events with fixed charged multiplicity N (within Δy) in the case $\lambda = 0$ (no overlap of chains), where the differences among various nuclei are most pronounced. It can be seen that the original square dependence of dilepton rate on N (linear rise of R_N), assumed for every elementary collision, is smeared out by incoherent summing of the contributions from different chains. For heavy nuclei the R_N behaves very roughly as $\sqrt{N}$ with an apparent tendency to approach a constant at higher values of N , in agreement with the semi-qualitative arguments of section 3.

Figure 3 shows the influence of the chains overlap (parameter λ) on the same ratio R_N (divided by $\sqrt{N}$ to distinguish the various cases better), but only for pPb collisions, where the extreme cases ($\lambda = 0$ and $\lambda = 1$) differ most.

The integral dilepton-to-hadron ratio in Δy (divided by that for pp collisions) is depicted on Fig. 4 as a function of the atomic number of the target for different values of parameter λ . Results can again be understood on the basis of the qualitative reasoning of Sec. 3. For $\lambda = 1$ (full overlap of quark-antiquark chains from different elementary collisions) R increases by a factor of 2 when passing from pp to pPb collisions. We hope that the

comparison of Fig. 4 with data will tell us soon how important the annihilations of quarks and antiquarks from different chains are in pA collisions in this energy range.

The predicted dependence of the dilepton-to-hadron ratio on the number of elementary collisions v (normalized to $v=1$, i.e. to pp collisions) is shown in Fig. 5 for different values of the chain-overlap parameter λ . It follows from the basic assumption of our model (Sec. 4) that this dependence is universal, valid for every nucleus. The extreme cases $\lambda = 0$ and $\lambda = 1$ are again in accord with what was expected on the basis of a simplified approach in Sec. 3. The reader is urged to note the essential deviation from the $R_v = vR_1$ form, which would be expected in the case $\lambda = 1$ if there were no fluctuations in the numbers of hadrons from individual collisions, and which would imply the square dependence of the dilepton rate on the number of collisions v .

The latter phenomenon is perhaps a bit surprising because one intuitively connects the high transverse energy E_T with violent (high v) collisions. In the case of pp collisions, the square dependence of the dilepton rate on E_T was observed by the AFS collaboration [1,8], which also found the linear relation between E_T and the hadron multiplicity N . The message from the present work is that even if the square dependence of the dilepton rate on E_T is restored in pA collisions, it will not imply the same dependence on the number of collisions. The explanation of this paradox is very simple: It is true that by triggering on the high E_T one increases the content of high- v events in the sample. But these are high- v events which have also high E_T . In the calculation of the dilepton production rate at fixed v all v -collision events are included (both low- and high- E_T ones).

6. Comments

In spite of a certain phenomenological success of the soft annihilation model of dilepton production in hadron-hadron collisions the physics behind it remains vague. It is well possible that its main assumption about the existence of an excited system of quarks and antiquarks with nearby rapidities in a finite volume for a finite time would not have seen the world if the results [49] of the Monte-Carlo lattice calculations based on pure gauge theory (without quarks) and implying the need of high energy densities to produce a quark-gluon plasma had been known in 1975-76. Fortunately, there are some suggestions that the deconfinement transition may be smooth [50] and show up even in small systems as those produced in pp collisions [51]. This raises hopes that the soft annihilation model may be given a firm theoretical basis and that it can, in turn, help to indicate such phenomena by setting a phenomenological framework for using the soft dilepton production as a "densitometer" of deconfined quarks and antiquarks.

The present work can only be regarded as a step in this direction with a great deal of optimism, due to the crude parametrization of the consequences of the interplay between space-time evolution of pA collisions and that of elementary interactions. It is clear that such an approach is unable to give detailed predictions for soft dilepton production (x_F^- , M^- , and p_T -spectra and their correlations). This can be expected only from a more sophisticated approach which would combine some model of hadron-nucleus or nucleus-nucleus collisions (realized perhaps as an event generator [35,52, or this work]) with the soft annihilation model supplemented by a notion about the space-time evolution of the process (according to Refs. 13 and 53, for example).

But, unfortunately, at the present stage of the theory of strong interactions one cannot hope to have a parameter-free description of the space-time evolution even in the simplest case of hadron-hadron collision. Under such conditions it is perhaps justified to use simple phenomenological models as the one used here. The present model sets lower and upper bounds on the dilepton-to-hadron ratio and thus offers the first test of the applicabil-

ity of a conventional approach to dilepton production in pA and AA collisions. As a further test one may use the correlations between different dependences of the dilepton-to-hadron ratio.

A very important point, which we have not mentioned yet, concerns the possible dependence of our chains-overlap parameter λ on the mass of the dileptons under study.²⁰ The constraints on the space overlap required if a quark from one chain is to annihilate with an antiquark from another one are more vital for $q\bar{q}$ pairs with higher invariant masses. This can be seen from the characteristic time of the annihilation $\tau \approx \hbar/Mc^2$, which sets the limit on the distance of a q from a $\bar{q}$ in their rest frame to $r \approx \hbar/Mc$. A different chain of reasoning based on the maximal orbital momentum ($L = 2$) allowed for $q\bar{q} \rightarrow \gamma^*$ leads via the semiclassical inequality $pr \leq 2\hbar$ to the same estimate $r \leq \hbar/Mc$. At small masses also the pairs with relatively large space separation are allowed to annihilate. This weakens the restrictions imposed by the space-time aspects of the process, and leads, in our language, to values of λ in the vicinity of unity.

From this point of view it would be interesting to have experimental results on direct real soft photon production in proton-nucleus collisions, which is caused by the annihilations of $q\bar{q}$ pairs with rather low masses (see the text at the end of Sec. 2). Hence, a value of λ near to 1 is expected, together with an increase of the photon-to-hadron ratio in comparison with the pp case (see Fig. 4 with $\lambda = 1$), which can facilitate the separation of the direct signal from the background.

In addition to the results presented in the previous section we calculated the same quantities also for pA collisions at higher energy (450 GeV/c). The increase of the dilepton-to-hadron ratio was only marginal - several per cent, compared to the 200 GeV/c results at the same value of λ . If a higher increase is observed experimentally and if the data are better fitted by a higher value of the chain-overlap parameter λ , this will clearly indi-

²⁰ I am grateful to J. Pišút for discussions about similar problems in Drell-Yan production on nuclei which helped me in formulating this point.

cate an increase of the amount of matter in the deconfined state.

Our last remark concerns the (dis)similarities between soft-dilepton and Drell-Yan production. In pp collisions the former is proportional to the square of the transverse energy, whereas the latter is assumed to be roughly independent of it [6]. It was recently shown [54] that in heavy ion collisions the Drell-Yan cross section should be proportional to the transverse energy. On the other hand, the present work indicates that in these processes the soft dilepton production rate would rise with E_T more slowly than in pp collisions.

ACKNOWLEDGEMENTS

I am indebted to C. Fabjan, M. Gyulassy, W. Hedberg, L. Van Hove, K. Kajantie, H. Pirner, J. Pišút, W. Willis and K. Zalewski for stimulating discussions. Some of the ideas presented here got clearer shape when I was preparing a talk for the symposium Parton Aspects in Nuclear Physics (Heidelberg, July 2-3, 1987) and during the discussions there. Thanks are due to J. Hüfner and O. Nachtmann for organizing this useful meeting.

Finally, I would like to thank L. Van Hove for careful reading of the manuscript and for many comments which improved the text.

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8. Figure captions

- Fig.1 Comparison of the present model of pA collisions with the data on charged multiplicities in full rapidity range.
- Fig.2 Dependence of $R_N = \langle l^+l^- \rangle_N / N$ on charged hadron multiplicity N in lab rapidity range (2.03, 3.03). The normalization constant α [see (3), (8)] can be fixed by pp data.
- Fig.3 R_N divided by $\sqrt{N}$ (to separate the different curves better) as a function of charged hadron multiplicity N in lab rapidity range (2.03, 3.03) in pPb collisions for different values of parameter λ .
- Fig.4 A-dependence of $R = \langle l^+l^- \rangle / \langle N \rangle$ normalized to pp collisions, for different values of parameter λ (see Section 5)
- Fig.5 Dilepton-to-hadron ratio in the rapidity interval (2.03, 3.03) normalized to pp collisions as a function of the number of elementary collisions in pA interactions at 200 GeV/c for different values of parameter λ .

- Figure 1 -

- Figure 2 -

- Figure 3 -

- Figure 4 -

- Figure 5 -