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THE LIMITED ANGULAR MOMENTUM IN STRONGLY DEFORMED ROTATIONAL BANDS

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A B S T R A C T

The moments of inertia of rotational bands in the cranked harmonic oscillator are explored. The important role played by the maximum spin is pointed out. A qualitative agreement is found with the properties of recently observed superformed bands.

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In the recently observed superdeformed band of ^{152}Dy [1], the nucleus rotates very much like a rigid body, i.e. the energy is quite well described by $(\hbar^2/2J) \times I^2$ with J constant. The superdeformed bands in Ce/Nd nuclei [2-4] on the other hand show important deviations from this simple formula. In the present letter we want to explore if these features or some major part of them come out in the simplest conceivable microscopic models, the rotating harmonic oscillator. Especially, we want to find out how the deviations from the simple I^2 dependence are related to the intrinsic deformation and the maximal spin in the rotational bands.

Both terminating rotational bands and strongly collective bands are obtained in the rotating harmonic oscillator [5]. Thus, for small deformations, all bands gradually approach the non-collective limit and terminate at angular momentum $I = I_{\text{max}}$ in a state of pure particle-hole nature (rotating around the symmetry axis in the cranking model language). For larger deformations, on the other hand, no such termination is obtained. Instead, the collective rotation continues with no formal limit on the angular momentum.

Transition probabilities measured in the superdeformed bands suggest a deformation close to $\epsilon=0.6$ (2:1 deformation) in ^{152}Dy [6] and $\epsilon \approx 0.4$ in ^{132}Ce [3]. Similar deformations also come out from calculations [7-10]. An interesting observation is that the division line between the two cases in the pure oscillator goes in between these deformations, i.e. rotational bands of the pure oscillator with a deformation of $\epsilon=0.4$ at $I=0$ terminate while this is not the case at $\epsilon=0.6$. One possibility is then that this fact might largely explain the differences between the observed bands.

The Hamiltonian for a spinless particle in a triaxial harmonic oscillator potential rotating with the frequency ω around the 1-axis is given by:

$$H = (p^2/2m) + (m/2) \times (\omega_1^2 x_1^2 + \omega_2^2 x_2^2 + \omega_3^2 x_3^2) - \omega l_1 \quad (1)$$

The rotation leads to a mixture of the quanta in the 2- and 3-directions, but by a straightforward transformation [11] the Hamiltonian can be put into the form of three independent oscillators with the "new frequencies" ω_+ and ω_- in addition to ω_1 . In the limit of no rotation, $\omega_+ = \omega_2$ and $\omega_- = \omega_3$.

With N particles filling the orbitals of the rotating oscillator, the configuration is fully determined by the the total number of quanta in the three "directions"

$$\Sigma_i = \sum_{\text{occ}} (n_i + 1/2) \quad (2)$$

where $i = 1, +$ and $-$ (and where $\Sigma_+ = \Sigma_2$ and $\Sigma_- = \Sigma_3$). The standard procedure [12] in cranking calculations is then to minimize the total energy under the constraint that the volume of the potential is kept constant, $\omega_1 \omega_2 \omega_3 = \bar{\omega}_0^3$. This is also the method employed here (cf. [5]). Note, however, that it is not necessary to perform any Strutinsky renormalisation in the harmonic oscillator case because for light and medium-heavy nuclei, the average stiffness towards deformation corresponds to a realistic liquid-drop stiffness while the average moment of inertia equals the rigid-body value (e.g. [12]).

In the cranking model, the total spin is identified with the sum of the expectation values of L_1 of the occupied orbitals. In the pure oscillator model, it reaches a maximum value of $I_{\text{max}} = \Sigma_- - \Sigma_+ (= \Sigma_3 - \Sigma_2)$ for those configurations which terminate, i.e. those configurations for which $(\Sigma_3/\Sigma_2) < 1.7478$ [5]. As the static solutions are characterized by the self-consistency condition $\Sigma_1 \omega_1 = \Sigma_2 \omega_2 = \Sigma_3 \omega_3$, the ratio (Σ_3/Σ_2) can also be expressed as a frequency ratio corresponding to $\epsilon < 0.499$ in the prolate case. However, also for larger ϵ -values one could expect some special features of the solution for $I \approx I_{\text{max}}$ and it appears that (I/I_{max}) is a natural unit for the spin. Similarly, for low spins all bands behave as $E = E_0 (\hbar^2/2J_{\text{rig}}) \times I^2$ so

$$J_{\text{rig}} = \sum_{\text{occ}} \langle |y^2 + z^2| \rangle \quad (3)$$

(evaluated at the spin zero deformation) is a natural unit for the moment of inertia. Indeed, in these units, the calculated total energy and spin will only depend on the ratio between the Σ :s, i.e. on two parameters. Furthermore, in this letter we shall only consider configurations that are prolate for $I=0$ which means that two frequencies (and two Σ :s) are equal. Thus only one parameter is left which, by help of the selfconsistency condition, can be chosen as an axis ratio or a deformation parameter, e.g. ϵ .

At large deformations, it is essentially the couplings due to the λ_1 -term which lead to important differences compared to the low-deformation case. If these couplings are neglected and if furthermore $(\omega_2\omega_3)^{1/2}$ is set equal $(\omega_2+\omega_3)/2$ in the total spin expression, it is possible to get the total energy of the N-particle system in closed form [13],

$$E = 3\hbar\omega_0(\Sigma_1(\Sigma_+\Sigma_- + I^2/4))^{1/3}. \quad (4)$$

Within this approximation, all bands terminate for $I = I_{\max}$. At the termination, where $\omega_2 = \omega_3$, the couplings between the shells disappear so the solution (4) is valid also in the general case with all couplings included. Thus, for bands which terminate, this simplified solution is valid for $I = 0$ and $I = I_{\max}$ but not for intermediate spins. Also for large deformation configurations which do not terminate when the shells are coupled, the axially symmetric solution for $I = I_{\max}$ still exists [5] but is not yrast. Similarly, for bands which terminate, higher spin states than $I=I_{\max}$ can be formed. However, these states come quite high in energy and will not be considered here.

The general features of the total energy for particles occupying the orbitals of a rotating pure oscillator potential are illustrated in figs. 1 and 2. Thus, in fig. 1 we give the total spin vs. transition energy for some configurations at different deformation. Maybe somewhat surprisingly, the configurations at small deformation come closest to the diagonal corresponding to a pure parabola, $E = (\hbar^2/2J_{\text{rig}}) \times I^2$, while at larger deformation the rotation gets more and more disturbed with increasing spin. This holds true for deformations up to $\epsilon=0.6-0.7$. Indeed, although no termination occurs, the energy of " $\epsilon=0.5-0.6$ configurations" is very strongly disturbed around $I = I_{\max}$. Furthermore, although these configurations do not terminate, they decrease their deformation with increasing spin up to $I \approx I_{\max}$. They also become non-axial approaching the limit of rotation around the symmetry axis ($\gamma=60^\circ$). Thus, the $\epsilon=0.545$ configuration has a deformation of $\epsilon=0.45$ and $\gamma=21^\circ$ for $I=I_{\max}$. Corresponding numbers for the $\epsilon=0.60$ configuration are $\epsilon=0.56$ and $\gamma=12^\circ$. One should also note that fig. 1 would look rather different if the spin values were not divided by $I_{\max}$ because $I_{\max}$ increases strongly with deformation.

In fig. 1 we have plotted one configuration at very large deformation, $\epsilon=0.857$ (3:1 deformation). In this case, the transition energies show no sign of a termination; instead they become smaller than the rigid rotation estimate long before $I_{\max}$ is reached. These small transition energies are accompanied by a substantial stretching of the system. Finally, the transition energies resulting from the simplified solution, eq. (4), are given at for an $\epsilon=0.6$ configuration in fig. 1. This solution is quite illustrative to get a general understanding of terminations but it is also evident that it fails completely if one begins to study transition energies in more detail.

An alternative way to represent the data is in the form of moments of inertia, $J^{(1)}$ and $J^{(2)}$ (where in the present application we always refer to the values within one rotational band). $J^{(1)}$ is simply $I \times (dE/dI)^{-1}$ and is thus straightforward to read out in fig. 1. The $J^{(2)}$ moment of inertia is defined as $(d^2E/dI^2)^{-1}$, i.e. it is a measure of the slopes in fig. 1 or the curvature in the E vs. I curve. It is a much more sensitive measure of differences compared to rigid rotation as illustrated in fig. 2 where the same cases as in fig. 1 are plotted. In this figure we also show one example of $J^{(1)}$ that displays typical features. Thus, for the cases where $J^{(2)}$ decreases smoothly with increasing spin, $J^{(1)}$ decreases in a similar but less pronounced way.

All the cases shown in figs. 1 and 2 correspond to magic gaps at different axis ratios, 4:3, 3:2, 2:1 and 3:1. When comparing with experiment in ^{152}Dy , it seems evident that we should consider the 2:1 configurations. Let us first point out one fact of some importance (e.g. ref. [14]), namely that it is only for every second particle number, ...40, 80, 140, ... that the total energy minimum is found at the exact 2:1 deformation, $\epsilon = 0.6$. These particle numbers just correspond to twice the value of the spherical magic numbers. However, for the other configurations with particle numbers corresponding to the sum of two adjacent spherical particle numbers, the $I=0$ deformation is considerably smaller, $\epsilon=0.508$ for $N=28$, $\epsilon=0.545$ for $N=60$ and $\epsilon=0.564$ for $N=110$. This smaller deformation means that these configurations are much closer to termination and thus also show a rather different behaviour for $I \approx I_{\max}$, cf. $N=60$ and $N=80$ in figs. 1 and 2.

In order to make a numerical comparison with the observed band in ^{152}Dy , we combine the 60 and 80 2:1 magic numbers. The frequencies $\hbar\omega_0$ for protons and neutrons are chosen in the standard

way [15]. The system is scaled to $A=152$ by the assumption that the moment of inertia varies as $A^{(5/3)}$. The resulting $J^{(2)}$ moment of inertia is compared to the observed one in fig. 3. Although we do not believe that the 3:2 axis ratio is really important to form the favoured shell effects¹ in the Ce/Nd region, it still seems reasonable to combine 3:2 magic numbers to simulate the superdeformed bands in this region. Those with particle numbers 54 and 66 both have an equilibrium deformation $\epsilon=0.38$ ($\epsilon=0.382$ and $\epsilon=0.385$) while their maximum spin is 34 and 44. The combined system, having an $I=0$ deformation of $I=0.384$, is scaled to $A=132$ for comparisons with the observed features in ^{132}Ce which is the nucleus where by far the highest spin has been observed in the Ce/Nd region (the other superdeformed bands in this region show very similar $J^{(2)}$'s for the spins where they have been observed, $I \leq 40$). We then find that in agreement with experiment, the calculated decrease of $J^{(2)}$ is much smaller in ^{152}Dy than in ^{132}Ce . This is caused by the fact that I_{max} is much larger in the former case, $I_{\text{max}} = 160$ compared to 78. Thus, in ^{152}Dy , less than 40% of the available spin is used up compared to more than 60% in ^{132}Ce . We also note that the observed decrease of $J^{(2)}$ is essentially reproduced from our simple model in ^{152}Dy while it only gives about half of the observed decrease in the case of ^{132}Ce .

Our results for ^{152}Dy are very similar to those obtained in the more realistic modified oscillator model [8]. This is not only the case for $J^{(2)}$ but also for $J^{(1)}$, which is calculated somewhat larger than $J^{(2)}$ (cf. fig. 2) while, with present spin assignments [1], experiment gives a $J^{(1)}$ which is smaller than $J^{(2)}$. In the case of ^{132}Ce , on the other hand, the observed decrease in $J^{(2)}$ is better reproduced in the modified oscillator [9] than in the pure oscillator. Indeed, the decrease of $J^{(2)}$ is essentially reproduced also in the pure oscillator if the deformation is fixed to that calculated for spin zero. Generally, it seems that deformation changes are larger in the pure oscillator than in the modified oscillator.

¹ The different disturbances introduced to make the potential realistic appear too large to leave any trace of the 3:2 shell gaps which are $0.375\hbar\omega_0$ wide. Instead the supershell effects associated with the strongly degenerate $n_3=0$ orbitals appear responsible for the overall shell effects [15, 10]

It is instructive to compare the terminating bands of the pure oscillator with those calculated and observed in neutron deficient $A \approx 154-160$ nuclei [16]. A salient feature of the latter is that the last spin units are obtained energetically very cheaply corresponding to a very large (or even infinite) moment of inertia $J^{(2)}$. This is thus in contrast to the pure oscillator bands with decreasing $J^{(2)}$ up to $I_{\max}$. Consequently, it seems evident that the special properties observed in terminating bands are caused by the presence of essentially pure high- j shells in realistic potentials. The pure oscillator bands, on the other hand, find no really favoured way to terminate and the angular momentum is thus obtained energetically more and more expensively (decreasing $J^{(2)}$) the closer they come to termination. Such bands were discussed theoretically in ref. [17] but the superdeformed bands in the Ce/Nd region appear to constitute the first clear observation of such bands.

Here we have only considered configurations that are axial for $I=0$ but it is evident that the general conclusions concerning the dependence of $J^{(2)}$ on $(I/I_{\max})$ and deformation (terminating or non-terminating region) will be valid also for configurations that are triaxial already for $I=0$.

In summary, we have found a very regular behaviour of the energy spectrum of the cranked triaxial oscillator when plotted as a function of $(I/I_{\max})$. Our solutions suggest that the much larger deviation from the simple I^2 dependence (larger decrease of $J^{(2)}$) observed in the superdeformed bands in the Nd/Ce region than in ^{152}Dy is largely due to the fact that the highest spins observed in the former case come much closer to the maximum value than in the latter case. These bands are thus in some sense the opposites of the terminating bands recently observed in $A \approx 154-160$ nuclei which obtain the last spin units energetically very cheaply, i.e. $J^{(2)}$ increases sharply when the termination is approached. Furthermore, when considering local shell effects and the importance of high- j shells, it seems more appropriate to use the harmonic oscillator solutions as a reference than the constant moment of inertia (rigid rotation) solution.

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Figure captions.

Fig. 1.

Spin vs. transition energy for rotational bands at different deformation of the cranked harmonic oscillator. The spin is given in units of $I_{\max}$ where $I_{\max}$ equals the difference in the number of quanta in the two directions perpendicular to the rotation axis. The transition energies are given in units of $\Delta E(J_{\text{rig}}, I_{\max}) = (\hbar^2/2J_{\text{rig}})(I_{\max}+1)^2 - (\hbar^2/2J_{\text{rig}})(I_{\max}-1)^2 = (\hbar^2/2J_{\text{rig}})(4I_{\max})$. As indicated in the lower left corner, each band is labelled by the deformation at $I=0$, the ratio between the symmetry axis and the perpendicular axis which gives the shell gap, the corresponding "magic" particle number used in the calculation and the maximum spin, $I_{\max}$, for this particle number. Finally, the simplified solution with no coupling between the N-shells is illustrated in the case of $\epsilon=0.6$.

Fig. 2

Moments of inertia as functions of $(I/I_{\max})$ for the cases shown in fig. 1. $J^{(2)}$ is inversely proportional to the slopes in fig. 1 while $J^{(1)}$, shown in one case, is defined as $2I/\Delta E$.

Fig. 3

Observed $J^{(2)}$ moments of inertia in the superdeformed bands of ^{152}Dy and ^{132}Ce compared with those calculated in the cranked harmonic oscillator. The latter correspond to magic proton + neutron 2:1 and 3:2 configurations scaled to $A=152$ and $A=132$ respectively. For comparison, the calculated value is also given when the deformation is kept constant at the $I=0$ value. The fact that $J^{(2)}$ decreases much less in the $A=152$ case than in the $A=132$ case is essentially caused by the much larger $I_{\max}$ in the former case, $I_{\max}=160$ compared to $I_{\max}=78$.

- Figure 1 -

- Figure 2 -

- Figure 3 -