**PION INTERFEROMETRY IN JET EVENTS AT THE
CERN INTERSECTING STORAGE RINGS**

The Axial Field Spectrometer Collaboration

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ABSTRACT

We have observed an enhanced production of pairs and triplets of identical charged particles at small momentum differences in events with two central jets produced in pp collisions at the CERN Intersecting Storage Rings. The size and structure of this 'Bose-Einstein' enhancement, shown by the particles within a jet, resembles that observed in minimum-bias events. The correlation function can be well parametrized by the single variable Q —the absolute value of the four-momentum transfer—and is more peaked than a single Gaussian.

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1. INTRODUCTION

In hadronic final states from particle collisions, the rate of production of identical bosons is enhanced when the particles have very small momentum difference. This phenomenon was first observed in $p\bar{p}$ annihilations and is attributed to the Bose-Einstein statistics valid for identical boson pairs [1]. Elementary considerations indicate that Bose-Einstein correlations between pion pairs could reveal the space-time structure of the particle source. Complications arise from the fact that the degree of enhancement depends not only on the dimensions of the source but also on other facts, such as to what extent it is chaotic or coherent [2].

The phenomenon of Bose-Einstein correlations is usually studied in terms of the ratio R of the joint probability for a pair of identical particles to be emitted, $P(p_1, p_2)$, to the product $P(p_1) \cdot P(p_2)$ of the single-particle probabilities, where p_1, p_2 are the four-momenta of the two particles. For fully chaotic sources this ratio is given by

$$R = \frac{P(p_1, p_2)}{P(p_1) \cdot P(p_2)} = 1 + \varrho^2(Q), \quad (1)$$

where $Q = p_1 - p_2$ is the four-momentum difference between the identical bosons, and $\varrho(Q)$ is the Fourier transform of the space-time source distribution, normalized to unity as $Q \rightarrow 0$.

Two different formulae have been used to fit the extracted correlations:

$$R = 1 + \lambda \exp [-(r_G \cdot Q)^2] \quad (2)$$

corresponding to an assumed Gaussian-distributed source of width r_G in space-time, and

$$R = 1 + \lambda \left[\frac{2J_1(r_B \cdot q_T)}{(r_B \cdot q_T)} \right]^2 / [1 + (Q_0 \cdot \tau)^2]. \quad (3)$$

Formula (3) corresponds to pion sources uniformly distributed over the surface of a sphere with radius r_B ; in some models τ represents the lifetime of the sources, in others it is ascribed to the depth of the emission layer [3]. Here q_T is the component of $\vec{p}_1 - \vec{p}_2$ transverse to $\vec{p}_1 + \vec{p}_2$, and Q_0 is the energy difference between the two identical bosons. The factor λ , which can take values between 0 and 1, is the so-called incoherence factor which takes into account that the interference, for various reasons, may not be complete.

Formula (3) has mostly been used to investigate the structure of the interaction region in hadron-hadron collisions, whilst formula (2) is preferred in analysis of the jet events in e^+e^- collisions.

Both formulae turn out to be nearly indistinguishable in shape, and both fit the measured correlations quite well, formula (3) giving a radius parameter r_B approximately twice as large as r_G of formula (2), as can be seen from the exponential approximation of (3):

$$R \approx 1 + \lambda \exp [-(r_B^2/4)q_T^2 - \tau^2 Q_0^2]. \quad (4)$$

In the experiment discussed here, we study the structure of the Bose-Einstein effect in detail. A large sample of events triggered on two central high- p_T jets permits us to study different kinematical regions in the events. We will first study the correlation of identical particles within a jet. To this end we will assume azimuthal symmetry around the jet axis, and study the correlation as a function of the momentum difference transverse and parallel to the jet axis, and as a function of the energy difference. We will also analyse the particles emitted perpendicularly to the jets, and compare the results with those from minimum-bias events.

2. DATA

The data were taken with the Axial Field Spectrometer (AFS) at the CERN Intersecting Storage Rings (ISR). All the data were obtained from proton–proton collisions at $\sqrt{s} = 63$ GeV.

The AFS has been described in detail elsewhere [4]. The interaction region is surrounded by an inner hodoscope, a cylindrical drift chamber, and a uranium/copper/scintillator calorimeter with full azimuthal coverage.

Charged-particle tracking was provided by the cylindrical drift chamber, operated in an axially symmetric magnetic field of 0.5 T. The chamber was azimuthally segmented into 4° sectors. The active volume extended over 2π in azimuth, except for two 16° wedges in the vertical plane, used for mechanical support. Coordinates in the transverse plane were provided by drift-time measurements with a spatial resolution of $230\ \mu\text{m}$, and those along the beam direction by charge division with 1.5 cm resolution. Pulse-height measurements also provide dE/dx information, which is used for particle identification.

The data were obtained by triggering on two separated jets anywhere in the calorimeter. Each of these jets represents transverse energy in a part of the calorimeter covering, on the average, $\Delta\phi \approx 45^\circ$, $45^\circ < \theta < 135^\circ$ in the centre-of-mass system. We required two non-overlapping jets to exceed 9 GeV each. Further details are found in Ref. [5].

Some data were taken with a minimum-bias trigger which required at least one hit in the inner hodoscope, or at least coincident hits in the two downstream scintillation counters surrounding the outgoing beams.

In the off-line analysis, the events were processed through the standard AFS cluster-finding procedure for the calorimeter, and through the track and vertex reconstruction for the central drift chamber as detailed in Ref. [5].

Quality cuts on the reconstructed charged-particle tracks in the drift chamber include a minimum of 22 time-digitization and 22 charge-division measurements. This is one more than half the number of readout wires per sector, thus ensuring that measurements due to the passage of one particle are not divided into two separate track candidates. The χ^2/NDF of the track fit is required to be less than 5 and the error on the momentum measurement $\delta p/p$ to be below $\sqrt{[(0.025p)^2 + 0.015^2]}$ (p in GeV/c), thus selecting only tracks that are well-measured throughout the full drift chamber. With these requirements the acceptance for charged particles having $p_T > 200$ MeV/c in the central region ($|y| < 0.7$) and not passing through the blind wedges is very close to 100%, as estimated by a full Monte Carlo simulation of the drift-chamber response.

When two particles pass through the same 4° sector of the drift chamber, the z -information—being measured by charge division—and also the dE/dx information of the tracks, is useless, even when the drift-time measurements can resolve the tracks. As we have stringent requirements on the track quality in order to be sure not to treat one particle track as two fictitious tracks, we will lose pairs of particles of the same charge when the azimuthal separation is lower than 4° . For pairs of particles of the opposite charge, a similar drop in efficiency is observed when the azimuthal separation is less than 7.5° , which corresponds to particles which cross each other within one sector of the drift chamber. In this study we will not try to correct for this inefficiency but will require at least 4° (7.5°) azimuthal separation of tracks of equal (opposite) charge.

The directions of jet axes were determined from calorimeter information only, as in Ref. [5]. Our jet sample contains events with two central jets with an average E_T of 10 GeV well inside the acceptance of the apparatus.

The data used in this study are: the jet-sample consisting of 161,000 pp events, and the minimum-bias sample consisting of 232,000 pp events and 90,000 $p\bar{p}$ events.

3. EVALUATION OF THE CORRELATION FUNCTION R

To evaluate the probability $P(p_1, p_2)$ of like-sign pairs within a jet, charged tracks are selected by the following criteria:

- 1) The jet direction defines a jet-related coordinate system. The jet particles are defined to be those within a 45° (half-angle) cone around the jet axis (see Ref. [5]).
- 2) The transverse momentum of the particles is restricted to be in the range $0.2 < p_T < 2.0$ GeV/c. The higher cut-off guarantees that particle correlations are not influenced by phase-space limitations. The average value of z , the fractional particle momentum in the jet, is 0.05 for the selected particles.
- 3) The dE/dx information from the tracks was used to separate most of the protons and kaons from the pions. Kaons could be identified in the momentum range 0.3 to 0.7 GeV/c, and protons in the momentum range 0.4 to 1.1 GeV/c.

To evaluate the correlation function R of Eq. (1) a reference distribution is required, corresponding to the product $P(p_1) \cdot P(p_2)$ of the single-particle probabilities. A common technique is to use unlike-sign particle combinations, i.e. $\pi^+\pi^-$ pairs. This, however, involves additional correlations due to resonances, such as K^0 , ρ^0 , ω , and to unidentified electron-positron pairs from converted photons. Our data are not charge symmetric, being produced by pp collisions, and the two-particle distributions of $++$ and $--$ combinations are slightly different. A common background distribution for both would introduce biases.

Instead, we rely on a jet-mixing technique which avoids these problems, taking a particle in one jet together with a particle from another jet. The jet axis is defined by the calorimeter only, with an angular resolution of $\sim 6^\circ$ [5]. The drift chamber has good acceptance properties, but the dead wedges between the drift-chamber halves give some problems, as the jet direction varies with respect to these wedges. This is taken care of in the calculation of the reference distribution by combining a pion from one jet with a pion from a jet in another event produced in the same angular part of the detector. The jet axes are required to coincide within 4° . This reference distribution has the desired properties: the presence of momentum correlations due to the jet structure, but absence of Bose-Einstein correlations and other dynamical correlations.

The mean error of the track momentum is estimated to be ≤ 10 MeV/c, and the resulting error in the absolute value of the four-momentum transfer Q is less than 15 MeV/c. This resolution translates into a sensitivity in r -values up to 10 fm.

In the study of the regions outside the jets, we require the jets to be opposite in azimuth within 10° , and select only those particles which are within $\pm 40^\circ$ in azimuth from the normal to the jet direction. The reference sample is obtained by combining a pion from one event with a pion from a corresponding region in another event, rotated azimuthally into the same coordinate system.

4. ANALYSIS IN TERMS OF COMPONENTS OF THE FOUR-MOMENTUM TRANSFER

We introduce a coordinate system with the z -axis given by the jet direction, and study the dependence of the Bose-Einstein effect using different pairs of variables. We define Q_T^2 as the squared three-momentum difference transverse to the jet direction, and Q_L^2 to be

$$Q_L^2 = Q_z^2 - Q_0^2, \quad (5)$$

where Q_z is the longitudinal momentum difference and Q_0 is the energy difference between the two particles. This pair of variables, Q_T^2 and Q_L^2 , is invariant to boosts along the jet direction. We thus attempt to distinguish a longitudinal development from a transversal development of the jet.

We divide our data into a two-dimensional array with 20 bins in Q_T^2 over the region 0.0–0.8 (GeV/c)², and 25 bins in Q_L^2 , $-0.25 \leq Q_L^2 \leq 0.25$ (GeV/c)². We note that Q_L^2 can take negative values. The overall kinematical condition is

$$Q^2 = -(p_1 - p_2)^2 = Q_T^2 + Q_L^2 = m_{12}^2 - 4m_\pi^2 \geq 0, \quad (6)$$

where m_{12} is the effective mass of the pair and m_π is the mass of the pion.

We first fit to our data the function

$$R = N[1 + \lambda \exp(-r_T^2 Q_T^2 - r_L^2 Q_L^2)] \quad (7)$$

and obtain a good fit with the parameters $r_T \approx r_L$ (see Table 1). In Fig. 1 we show some slices in Q_T^2 and Q_L^2 with the fit superimposed upon the data. The data given in the figures have been normalized so that $N = 1$.

Thus we have found the same dependence of the correlation function on Q_T^2 and Q_L^2 and, consequently, it is well parametrized as a Gaussian in the single variable $Q^2 = Q_T^2 + Q_L^2$. Similar results have recently been presented by the TASSO Collaboration [6].

Another way to search for the shape of the production source along the jet axis is to measure $R(Q)$ for different angles between $\Delta\vec{p} = \vec{p}_1 - \vec{p}_2$ and the jet axis. The quantity Q is equivalent to twice the momentum of a particle in the rest frame of the pair, which means that $R(Q)$ provides a measure of the source distribution as observed in the rest frame of the boson pair, in a direction given by the momentum difference $\Delta\vec{p}$ in that system. We have not found any significant evidence for such an elongation. A similar analysis by the PEP4(TPC) and EMC Collaborations [7,8] also found no significant departure from a spherical source.

We next try to fit the data using as pairs of variables (Q_T^2, Q_0^2) and (Q_T^2, Q_z^2) , i.e.

$$R = N[1 + \lambda \exp(-r_T^2 Q_T^2 - r_0^2 Q_0^2)] \quad (8)$$

or

$$R = N[1 + \lambda \exp(-r_T^2 Q_T^2 - r_z^2 Q_z^2)], \quad (9)$$

and find reasonable fits with the parameters shown in Table 1.

We see in Table 1 that $r_0 \approx r_z$ and that they are both significantly smaller than $r_T \approx r_L$, corresponding to a rather slow decrease of the correlation function R with increasing Q_z^2 or Q_0^2 . We also note that the parametrization in (Q_T^2, Q_L^2) is marginally favoured over the other parametrizations.

It is remarkable that the fit to the ratio $R(Q)$ with formula (7) works so well, as this fitted formula contains a factor $\exp(+r_L^2 Q_0^2)$, at first view contradicting the general fact that the Bose–Einstein effect decreases with Q_0 , the energy difference between the particles. When we use the variables (Q_T^2, Q_0^2) , integrating over Q_z^2 , we find, on the other hand, that $\exp(-r_0^2 Q_0^2)$ fits with $0 < r_0 < r_L$. When we use the variables (Q_T^2, Q_z^2) , integrating over Q_0^2 , we also find that we obtain a good fit using $\exp(-r_z^2 Q_z^2)$ with $r_z \approx r_0$. This is not astonishing as the Q_L^2 distribution shows a strong peak at $Q_L^2 = 0$, implying that generally we have $Q_z^2 \approx Q_0^2$. However, we call attention to the presence of a mechanism where, on an event-to-event basis, the increase of the correlation with Q_0^2 as $\exp(+r_L^2 Q_0^2)$ is counterbalanced by a decrease with Q_L^2 , giving as a resultant correlation function $\exp(-r_T^2 Q_T^2 + r_L^2 Q_0^2)$.

We want to point out that the Kopylov–Podgoretsky formulation of the Bose–Einstein effect does not contain this counterbalancing effect between Q_z and Q_0 . In a general case we have

$$R = 1 + [2J_1(qr_B)/(qr_B)]^2/[1 + (Q_0\tau)^2], \quad (10)$$

with $q = |\vec{p}_1 - \vec{p}_2|$. In most applications, q_T is used instead of q , which gives formula (3). In the exponential approximation, where we split the three-momentum difference q into Q_T and Q_L , this can be written

$$R = 1 + \exp[-(r_B/2)^2(Q_T^2 + Q_z^2) - (\tau Q_0)^2], \quad (11)$$

demonstrating that Q_z^2 and Q_0^2 enter the formula with the same sign.

Formula (3) has been widely used to describe the data. Within a jet, Q_T is a good approximation for k_T , as the jet axis is approximately given by the sum of momenta of the two particles in the pair. The fit in (Q_T^2, Q_0^2) thus corresponds to a fit to formula (3). Sometimes Q_0 is substituted in formula (3) by q_L , the Q component parallel to $\vec{p}_1 + \vec{p}_2$, which here can be approximated by Q_z , and we see that the result of the fit in (Q_T^2, Q_z^2) resembles very much the fit in (Q_T^2, Q_0^2) .

Earlier we have reported [9] that the minimum-bias events show an elongation of the measured source region along the beam axis, when seen in the centre-of-mass system, $r_z = 1.3 \pm 0.1$ fm and $r_T = 0.72 \pm 0.04$ fm. Table 1 shows that within a jet there is no similar elongation along the jet axis in the centre-of-mass system; we even find $r_T > r_z$, indicating an oblate shape of the source distribution when seen from the pp centre of mass. On the other hand, when we do the same fit in the boost-independent variables Q_T^2 and Q_L^2 for minimum-bias data as for these jet data, we find $r_T = 0.67 \pm 0.04$ fm and $r_L = 0.67 \pm 0.04$ fm, which are compatible with the results obtained for the jet particles.

In summary, the Bose-Einstein effect is well described by just the single variable Q^2 , the square of the four-momentum transfer between the identical particles.

5. ANALYSIS IN Q

In this section we study the correlation function R of the single variable Q in order to extract more information about the small- Q region. Figure 2a shows the two-particle and reference distributions as a function of Q , normalized to the tail of the two-particle distribution. Figure 2b displays the ratio of these distributions R , and it clearly shows an enhancement at low Q above the level of $R = 1$. The correlation is more peaked than a simple Gaussian (dashed line), but can be parametrized as a sum of two Gaussians. The evidence for this second Gaussian comes from the region $Q < 100$ MeV/c. We have enough statistics to exploit our Q -resolution of 10–15 MeV/c.

In the literature there are hints of such an increased strength of the correlation function at very low values of Q , e.g. the TPC Collaboration [8] discusses an enhancement in their first bin ($Q < 100$ MeV/c), and also in our earlier reported studies of minimum-bias events [9] the first bin is high.

The fit to the sum of two Gaussians,

$$R = N\{1 + \lambda_1 \exp[-(r_1 Q)^2] + \lambda_2 \exp[-(r_2 Q)^2]\} \quad (12)$$

gives

$$\begin{aligned} \lambda_1 &= 0.28 \pm 0.05, \quad r_1 = 0.66 \pm 0.08 \text{ fm}, \\ \lambda_2 &= 0.48^{+0.18}_{-0.13}, \quad r_2 = 2.1 \pm 0.5 \text{ fm and } \chi^2/\text{NDF} = 27/34. \end{aligned}$$

The radius of the first Gaussian corresponds quite well to that measured by the PEP4(TPC) and TASSO Collaborations at PEP and PETRA, respectively, with jets of approximately the same energy as in our experiment. The PEP4 Group measures $r = 0.65 \pm 0.04 \pm 0.05$ fm, and TASSO

$r = 0.80 \pm 0.05$ fm. They see no evidence for the second Gaussian, but their statistics for $Q < 0.1$ GeV/c are rather poor.

Our fit to a single Gaussian,

$$R = N\{1 + \lambda \exp[-(rQ)^2]\}, \quad (13)$$

gives

$$\lambda = 0.40 \pm 0.03 \text{ and } r = 0.82 \pm 0.05 \text{ fm, with } \chi^2/\text{NDF} = 44/36.$$

We have made a number of consistency checks of our analysis:

- 1) Two thirds of the data were taken with positive direction of the magnetic field over the drift chamber, and one third with the field switched to the opposite direction. The analysis of the two separate samples gives consistent results.
- 2) All studies have been made separately for $++$ and $--$ combinations, which have somewhat different two-particle relative momentum distributions. The correlation $R(Q)$ is consistently the same for both charge combinations.
- 3) The acceptance for particles within a jet will depend on the azimuth of the jet due to the blind wedges between the drift chamber halves. There is no significant difference in the result for different jet directions.
- 4) We have enlarged the $\Delta\phi$ cut between the tracks from 4° to 8° and found only a loss of statistics.
- 5) We have selected a subset of data where both particles were measured with the highest momentum resolution obtainable. Also in this case we did not find any significant change.
- 6) Treating all particles as pions, even the dE/dx -identified kaons and baryons, we find the parameters of the two Gaussians to be within one standard deviation of the fit with identified kaons and baryons excluded. The λ -values were lower, but not significantly so.
- 7) Another test to verify the nature of the correlation was to study equally charged $K\pi$ combinations, identified by dE/dx . Despite the fact that one of the particles is identified as a kaon, we have assigned the pion mass to both and analysed them in the same way as pion pairs, as if we had no identification. Figure 3 shows that there is no enhancement near threshold for these non-identical particles. The kaon sample contains a small background ($< 10\%$) of pions.
- 8) Also, $+-$ combinations have been analysed in the same way, showing only a weak signal from possible photon conversions. There is no evidence for K^0 or η in the jet data within the selected cones. In the minimum-bias data, however, we find a K^0 with a mass of 503 ± 1 MeV/c² and a width of 12 ± 2 MeV/c², well in accordance with an estimated mass resolution of 10 MeV/c².
- 9) To further verify that the observed signal is a consequence of Bose-Einstein correlations and not a result of acceptance effects or a reflection of particle decay kinematics, we generated a sample of Monte Carlo events without any Bose-Einstein correlations [5] and calculated $R(Q)$. The Monte Carlo events were tracked through a detector simulation which included the effects of pattern recognition and track overlap, and were analysed in the same way as the real data. It is observed that the jet-mixing technique does not introduce any artificial correlation, since the correlation function for this Monte Carlo study is constant, proving the effectiveness of the reference calculation.
- 10) A final problem is that our reference distribution contains remnants of the initial correlation between particles, as we are using the same jets in the construction of the reference sample. The effect is, of course, largest for detectors with limited acceptance. However, it might not necessarily be negligible for detectors such as ours, covering a large fraction of 4π . The solution [7,10] is to use an iterative procedure: a first approximation to $R(Q)$ is obtained following the

prescription in Section 3, and this is used to derive an improved reference density, weighting the reference combinations by $1/R(Q)$. In our case the first-order correction turned out to be less than the statistical errors.

For small ($< 1^\circ$) angular separation of equally charged particles, the Coulomb interaction between the particles will influence the correlation function. Our experimental requirement of 4° azimuthal separation will make such Coulomb corrections to the correlation function negligible.

From these tests we estimate the systematic errors of the fitted parameters to be the same as the statistical errors. There are correlations between the parameters λ and r of a Gaussian and the normalization constant N .

We attempted to interpret the presence of two Gaussians as evidence for two different length-scales. To test a possible dependence of these scales on the event structure, we used the measured energy in the calorimeter outside the jet region, E_{out} , to select events where E_{out} was small, medium, or large. As the mean E_{out} is around 1.5 GeV, we selected $E_{\text{out}} < 1$ GeV, $1 < E_{\text{out}} < 2$ GeV, and $E_{\text{out}} > 2$ GeV. The results in Table 2 show that there is no significant evidence for a change of the relative strength of the two possible components with E_{out} .

To test the presence of the second Gaussian further, we have reanalysed the Bose-Einstein correlations in the minimum-bias sample in terms of the four-momentum transfer Q . Figure 4a shows the two-particle and reference distribution as a function of Q , normalized to the tail of the two-particle distribution. Figure 4b displays the ratio R of these distributions with a binning of 25 MeV/c, and the correlation appears to be very similar to that observed in the two-jet sample. In Ref. [11] we had bins in q_T of 50 MeV/c. Even in this case we need a second Gaussian. The fit has a $\chi^2/\text{NDF} = 53/35$ and gives

$$\begin{aligned}\lambda_1 &= 0.32 \pm 0.04, & r_1 &= 0.65 \pm 0.03 \text{ fm}, \\ \lambda_2 &= 0.41 \pm 0.07, & r_2 &= 2.0 \pm 0.4 \text{ fm}.\end{aligned}$$

These values are consistent with those found within the jets. It is quite nice to see that, although the two-particle and the reference distributions are different within the jets and in minimum-bias events (compare Fig. 2a with Fig. 4a), their ratio is very similar, as can be seen from Figs. 2b and 4b. This emphasizes the Bose-Einstein nature of the correlations.

In summary, the correlation function is more peaked than a single Gaussian and can be fitted by the sum of two Gaussians. Other possible parametrizations are $1 + \lambda/[1 + (\mu Q)^2]$ or an exponential function (see Table 3). These parametrizations are, by the way, the Fourier transforms of each other. Since we see no variation of the relative strengths of the two Gaussians with the amount of energy outside the jets, these parametrizations may be preferred.

6. THREE-PARTICLE CORRELATIONS

The number of triplets of particles, all of like sign, has also been shown to exhibit an enhancement attributable to the Bose-Einstein effect [6, 12, 13]. We have studied this enhancement as a function of the single variable Q^2 , where, for three pions, $Q_{3\pi}^2$ is given by a sum of $Q_{2\pi}^2$ of the three possible two-particle combinations:

$$Q_{3\pi}^2 = Q_{12}^2 + Q_{23}^2 + Q_{31}^2 = m_{3\pi}^2 - 9m_\pi^2. \quad (14)$$

In this case the enhancement is observed in the ratio of triplets, all of the same sign, to triplets constructed of tracks from three different jet-events in a straightforward generalization from the doublet case. This ratio has been fitted with the form

$$R = N\{1 + \alpha \exp[-(R_1 Q_{3\pi})^2] + \beta \exp[-(R_2 Q_{3\pi})^2]\} . \quad (15)$$

As in the two-particle case, a second term is included. The ratio is displayed in Fig. 5 and the fitted parameters for a $\chi^2/\text{NDF} = 20/32$ are

$$\begin{aligned} \alpha &= 0.75 \pm 0.07 , & R_1 &= 0.41 \pm 0.02 \text{ fm} , \\ \beta &= 2.3 \pm 1.3 , & R_2 &= 1.4 \pm 0.3 \text{ fm} . \end{aligned}$$

The three-body correlation can be expressed in a general way in terms of two-body correlations as

$$R_3(Q_{3\pi}) = 1 + \Sigma \varrho^2(Q_{ij}) + 2 \text{Re} [\varrho(Q_{12})\varrho(Q_{23})\varrho(Q_{31})] , \quad (16)$$

where Σ is the sum over the three two-particle combinations ij , and $\varrho(Q_{ij})$ is the Fourier transform of the source distribution with respect to $Q_{ij} = p_i - p_j$, the four-momentum difference between the identical bosons i and j . We note that $R_3(Q_{3\pi})$ reaches a maximum value of six when $Q_{3\pi} = 0$.

We have seen in Section 5 that the extracted two-particle correlation can be parametrized as

$$\varrho^2(Q_{ij}) = \lambda_1 \exp(-r_1^2 Q_{ij}^2) + \lambda_2 \exp(-r_2^2 Q_{ij}^2) . \quad (17)$$

This result can be used to test experimentally whether there exists any three-particle correlation not taken into account by the two-particle correlation. To this end we introduce the two-particle correlation in the calculation of the reference distribution by weighting the uncorrelated three-particle combinations using formula (16) and the experimentally determined $\varrho^2(Q_{ij})$. The ratio $R(Q_{3\pi})$ between the observed three-particle distribution and this weighted reference distribution will show a residual three-particle correlation as a departure from the level $R(Q_{3\pi}) = 1$. We find no evidence for any specific three-body correlation (see Fig. 5). The lower set of points around $R = 1$ in the figure are calculated with this weighted reference distribution.

A similar conclusion has been reached in Refs. [6, 13].

We can also derive the fitted two-Gaussian parameters in the three-particle case in an approximate way. In formula (16) the dominant term is given by $R \approx 1 + \Sigma \varrho^2(Q_{ij})$. Taking only this term and using the approximation $Q_{ij}^2 \approx Q_{3\pi}^2/3$, the fitted form of $\varrho^2(Q_{ij})$ implies

$$R_3 \approx 1 + \alpha \exp(-R_1^2 Q_{3\pi}^2) + \beta \exp(-R_2^2 Q_{3\pi}^2) , \quad (18)$$

i.e. a sum of two exponentials with

$$\begin{aligned} \alpha &\approx 3\lambda_1 \approx 0.8 , & \beta &\approx 3\lambda_2 \approx 1.2 , \\ R_1 &\approx r_1/\sqrt{3} \approx 0.4 \text{ fm} , & R_2 &\approx r_2/\sqrt{3} \approx 1.2 \text{ fm} . \end{aligned}$$

This is, within errors, what we have found above. A similar difference in the two- and three-particle scale parameters has been reported earlier [6, 12]; however, the parametrization with two Gaussians was not used.

7. THE REGION OUTSIDE THE JETS

It is interesting to measure the Bose–Einstein effect in the regions perpendicular to the jets and to compare the result with the situation within the jet and with ordinary minimum-bias-triggered proton–proton events.

The jet events have two jets which differ in azimuth by approximately 180° . We restrict our sample to those events where this jet-jet azimuthal difference is between 170° and 190° , select 80° wedges around the azimuthal bisector to the jet directions, and calculate the correlation function between particles of the same sign within each of these wedges. We restrict ourselves to jet configurations for which the wedges are fully covered by the drift chamber halves. We find a positive correlation (Fig. 6), which can be nicely fitted by a single Gaussian, Eq. (2), with $\lambda = 0.39 \pm 0.07$ and a radius $r = 1.4 \pm 0.2$ fm. The statistics for this sample are not very large, and the measured distribution is not sensitive to the presence of a second Gaussian. The dashed curve in Fig. 6 represents the correlation function fitted by formula (12) to tracks selected to be within the jets. It is evident that the interference patterns outside and inside the jets are different.

It also follows that the correlation function outside the jets is different from the correlation measured in ordinary minimum-bias events (Section 5). However, we have reported earlier that the parameters of the Bose-Einstein effects depend on the multiplicity in the events [11]. The mean multiplicity in each of these 80° wedges, when demanding two tracks of the same charge, is 2.4, which would correspond to an event of total multiplicity 11. Such events show a value of $r \approx 2$ fm when parametrized by the Bessel function [formula (3)], which would correspond to $r \approx 1$ fm with the Gaussian parametrization, in rough agreement with what is found here.

There is evidence for a directional dependence in the minimum-bias events corresponding to an elongation parallel to the beam axis [9]. We do not find any such dependence in the regions away from the jets, neither with respect to the beam direction nor with respect to the jet axis within the events. We have studied the data both in the overall c.m. system and in the $\pi\pi$ c.m. system.

8. DISCUSSION AND CONCLUSIONS

We have analysed a large sample of events with two central jets produced in pp collisions at the ISR. An enhanced production of pairs and triplets of identical charged pions has been observed when the particles have small momentum difference. For pairs of pions this enhancement is usually parametrized with a Gaussian in Q , the four-momentum transfer. We find, however, that the sum of two Gaussians is needed to reproduce the measurements. The first Gaussian is characterized by a radius of 0.66 ± 0.08 fm. Similar radii have been reported by other experiments studying hadronic jets [6–8, 12, 14]. Our superior statistics and good momentum resolution make it possible to present evidence of a second Gaussian with a radius of 2.1 ± 0.5 fm. The form with two Gaussians is not specific to the jets. An analysis of a large sample of minimum-bias-triggered events shows a similar shape of the correlation function when studied as a function of Q . Alternatively, one may conclude that the shape of the correlation function is more accurately represented by a single exponential than by a single Gaussian.

In a search for experimental signals on a space-time structure of the development of the jet, we have analysed the enhancement in different components of Q^2 . We find that the enhancement is best parametrized in simply Q^2 , confirming the results of the TASSO Collaboration [6].

Previous studies of the Bose-Einstein effect in hadron-induced experiments have mostly used the Kopylov-Podgoretsky form to analyse data. In this way, only effects corresponding to the first Gaussian have been seen. Deutschmann et al. [15] find a radius parameter of 1.4 fm in large bubble chamber samples, corresponding to 0.7 fm in a Gaussian parametrization, for πp and $K p$ at 16 GeV/c and $p\bar{p}$ annihilations at rest. Other bubble chamber experiments with smaller data samples are compatible with these results [16]. De Marzo et al. [17] find a radius of $R \approx 1.5$ fm using the Kopylov-Podgoretsky expression (3), corresponding to $r \approx 0.7$ – 0.8 fm in the Gaussian approximation, for proton reactions with various nuclei such as proton, neon, argon, and xenon in a streamer chamber experiment at 200 GeV. Breakstone et al. measure a radius ranging between 0.7 and 1.7 fm, depending on multiplicity at the ISR [18].

All these experimental studies show the presence of an enhancement, due to symmetrization of the total wave function, for identical bosons. The smallest Gaussian radii measured are all in the region 0.6–0.7 fm for experiments on hadronization of jets, in hadron-induced reactions at low energies, and for high-energy minimum-bias events. This corresponds to what could be expected from final-state interactions between the produced particles or from the size of the pions themselves, which may be taken as $r_G \approx (\sqrt{2}m_\pi)^{-1} \approx 1$ fm.

The peaked structure parametrized with a second Gaussian could be due to resonance effects. It is well known that many of the pions in jets are produced via resonances. Short-lived resonances, such as the ρ -resonance, will have decay lengths of only a few fermi, and their decay pions will show interference—due to Bose–Einstein statistics—with the directly produced pions as well as with those produced via other short-lived resonances [19]. This effect ought to be seen as an enlargement of the source of pion production, given that the primary source is ≤ 1 fm. This might explain the observed structure of the two-particle correlation, common to hadronization within a jet and to minimum-bias events.

Another interpretation is possible using a string model. Andersson and Hofmann [20] have made an estimate of interference effects for particles produced in a jet within the Lund string model. Their approximate form resembles our measured correlation function. Bowler [20] has shown that a full inclusion of resonance production in the same string-model calculation will not change the results very much. Our results—that the correlation function is similar within jets and in minimum-bias events—might imply that these events have a simple string structure in the central region.

The incoherence factor λ , parametrizing the strength of the correlation, has been measured with typical values in the region 0.4–0.6 [6–9, 11, 12, 14–18]. We see here that our form with two Gaussians gives a total incoherence factor $\lambda = \lambda_1 + \lambda_2 = 0.74 \pm 0.19$, much larger than the value corresponding to a single Gaussian, where we found $\lambda = 0.40 \pm 0.03$. The measured value of λ can be considered a lower limit, as the sources of systematic errors discussed tend to dilute the enhancement.

For a completely incoherent source we expect $\lambda = 1$ when $Q \rightarrow 0$, but there are many effects which tend to diminish the value of λ [2]. The situation is quite complicated, and effects due to interferences between resonances, seen in their decay products, could even result in a λ -value larger than 1 [19].

Long-lived resonances such as ω and η with decay lengths much larger than 1 fm will diminish the interference effect in a way that is difficult to estimate quantitatively. The CLEO Collaboration [14] made a model-dependent estimate implying large corrections for their data sample, raising the λ -value from ≈ 0.4 to around 1.

Earlier we reported a dependence of the radius on multiplicity in hadronic events [11]. In the regions outside the jets the multiplicity is higher than in ordinary minimum-bias events, and we find a radius $r = 1.4 \pm 0.2$ fm compatible with the multiplicity dependence.

We have analysed the three-particle correlation in terms of two-particle correlations, and have shown that the three-particle effect is fully accounted for by the two-particle correlations.

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Table 1

Resulting parameters from fits to two-pion data following Eqs. (7), (8), and (9).
The parameter $r_{\parallel}$ represents a distance measure parallel to the jet axis.

Type of fit	λ	r_T (fm)	$r_{\parallel}$ (fm)	χ^2/NDF
(Q_T^2, Q_L^2)	0.37 ± 0.02	0.68 ± 0.05	$r_L = 0.66 \pm 0.05$	510/461
(Q_T^2, Q_0^2)	0.48 ± 0.03	0.72 ± 0.04	$r_0 = 0.44 \pm 0.05$	570/496
(Q_T^2, Q_z^2)	0.46 ± 0.03	0.70 ± 0.04	$r_z = 0.43 \pm 0.06$	580/496

Table 2

The parameters λ and r for the two-component fit [Eq. (12)] for three ranges of E_{out} , the calorimeter energy outside the jet region

E_{out} (GeV)	λ_1	r_1 (fm)	λ_2	r_2 (fm)	χ^2/NDF
$E_{\text{out}} < 1$	0.31 ± 0.05	0.61 ± 0.08	$0.55^{+0.87}_{-0.38}$	2.7 ± 1.1	22/34
$1 < E_{\text{out}} < 2$	0.31 ± 0.09	0.72 ± 0.13	0.54 ± 0.36	2.3 ± 1.0	26/34
$2 < E_{\text{out}}$	0.28 ± 0.04	0.68 ± 0.08	$0.68^{+0.54}_{-0.35}$	3.0 ± 0.8	37/34

Table 3

Fits to two alternative parametrization of the correlation function

Formula	λ	μ (fm)	χ^2/NDF
$1 + \lambda/[1 + (\mu Q)^2]$	0.65 ± 0.07	1.37 ± 0.16	28/36
$1 + \lambda \cdot \exp[-(\mu Q)]$	0.77 ± 0.07	1.32 ± 0.13	28/36

Figure captions

Fig. 1: Representation of a part of the fit in (Q_T^2, Q_L^2) . Figures 1a–c show the data versus Q_L^2 in slices of Q_T^2 :

a) $0.00 < Q_T^2 \leq 0.04 \text{ (GeV/c)}^2$,

b) $0.04 < Q_T^2 \leq 0.08 \text{ (GeV/c)}^2$,

c) $0.08 < Q_T^2 \leq 0.12 \text{ (GeV/c)}^2$.

Figures 1d–f show the data versus Q_T^2 in slices of Q_L^2 :

d) $-0.01 < Q_L^2 < 0.01 \text{ (GeV/c)}^2$,

e) $0.01 < Q_L^2 \leq 0.03 \text{ (GeV/c)}^2$,

f) $-0.03 < Q_L^2 \leq -0.01 \text{ (GeV/c)}^2$.

The curves superimposed on the figures are the results of the fit explained in the text.

Fig. 2a: The two-particle and reference distributions for particle combinations within a jet cone of 45° around the jet axis as a function of Q , the four-momentum transfer. The reference distribution is normalized to the upper part of the two-particle distribution.

Fig. 2b: The correlation function $R(Q)$, i.e. the ratio of the two-particle distribution to the reference distribution. The full line shows the fit with the sum of two Gaussians, whereas the dashed line is the fit with one Gaussian.

Fig. 3: The correlation function $R(Q)$ for equally charged $K\pi$ combinations. The four-momentum transfer Q is calculated using the pion mass for both particles.

Fig. 4a: The two-particle and reference distributions for minimum-bias-triggered data as a function of Q , the four-momentum transfer. The reference distribution is normalized to the upper part of the two-particle distribution.

Fig. 4b: The correlation function $R(Q)$, i.e. the ratio of the two-particle distribution to the reference distribution. The full line shows the fit with the sum of two Gaussians, whereas the dashed line is the fit with one Gaussian.

Fig. 5: The correlation function $R(Q)$ for three-particle combinations within a jet cone. The full line shows the result of the fit with two Gaussians. The dashed line indicates the level $R = 1$, corresponding to no correlation. The lower set of points are calculated with a reference distribution including the two-particle ‘Bose-Einstein’ correlations. For the upper set of points, the reference distribution contains no such correlations.

Fig. 6: The correlation function $R(Q)$ for particles emitted perpendicularly to the jet directions. The full line superimposed on the figure represents the Gaussian fit to the data. The dashed line shows the fitted correlation within the jet cones.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6