CERN-TH.4755/87

TRUNCATED COMPOUND POISSON (TCP) ALTERNATIVE TO
MULTIPLICITY DISTRIBUTION IN HADRON-HADRON COLLISIONS

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A B S T R A C T

On using the Poisson distribution truncated at zero for cluster decay in a compound Poisson process, we obtain TCP distributions which describe quite well the hadronic multiplicity data in hadron-hadron collisions. A comparison is made between TCP and negative binomial distributions for UA5 data.

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The hadronic multiplicity distributions arising from several types of colliding particles over a wide energy range seem to be well described^{1,2)} by a Negative Binomial (NB) distribution³⁾ which is determined in terms of two parameters, the mean multiplicity $\bar{n}$ and the shape parameter k where $D^2/\bar{n}^2 = 1/\bar{n} + 1/k$, $\langle n^2 \rangle = \bar{n}^2$. This is true for the full phase space data as well as for the data with symmetric rapidity windows of varying rapidity gaps^{1,2)}. For example, UA5 group used this distribution to fit the experimental data on all charged particles from 10 GeV up to 900 GeV with average multiplicity increasing from about 6 to 34 and k having the unexpected property of decreasing¹⁾ from about 19 to 3. The NB distribution may be looked upon as arising from a two-step process⁴⁾. At the first stage individual independent clusters are produced which in the second stage decay via a logarithmic distribution. The average number of clusters is found to increase with energy⁴⁾ and the decrease of k is not well understood.

We present here an alternative called Truncated Compound Poisson (TCP) distribution which appears to be as good on phenomenological considerations. The average number of clusters for this distribution, however, is found to decrease with energy from about 6 to 3 for the UA5 data reminding us the cosmic rays energy data in contrast to the value of about 8 for the previous case. For TCP distribution the decrease of k with energy follows as a consequence and the clusters grow wider. The TCP distribution arises if we require that aside from the Poisson-distributed independent clusters they decay subsequently also according to a Poisson distribution except for, as is clear on physical considerations, that the latter must be truncated at zero. The resulting distribution is quite similar to the NB one apart from some marked differences at high and low multiplicities where, however, the experimental uncertainties are larger. The plots of the two distributions are compared at several energies from UA5 data in Fig. 1, while the moments $C_q = \langle n^q \rangle / \langle n \rangle^q$ computed from TCP distribution are compared with the experimental data in the Table. The agreement in the latter case is quite good.

The TCP distribution may conveniently be described in terms of the probability generating function of a compound Poisson process^{*)}

$$G(t) = \text{Exp}(\bar{N}(g(t)-1))$$

corresponding to the multiplicity distribution

$$P(n) = \sum_N e^{-\bar{N}} (\bar{N}^N / N!) (1/n!) (d^n(g(t)^N) / dt^n) |_{t=0}$$

with $g(t) = (e^{Bt} - 1) / (e^B - 1)$. Here $\bar{N}$ indicates the average number of clusters produced and $P(0) = \text{Exp}(-\bar{N})$. We find

*) The same holds for NB distribution as well.

$$P(n) = e^{-\bar{N}} (B^n/n!) A_n(k e^{-B}),$$

$$\bar{N} = k(1 - e^{-B}) \quad (1)$$

where $B = \bar{n}/k$, $A_n(x)$ are Bell polynomials defined by the recurrence relation $A_{n+1}(x) = x(A_n(x) + A'_n(x))$, $A_1(x) = x$ and $\bar{n}/\bar{N}$ gives the mean multiplicity of a single cluster. For UA5 data $\bar{N} \approx k$ for energies about 200 GeV and $\bar{N}$ decreases like k does¹⁾ above this energy. The moments C_q are found to be

$$C_3 = L^2 + (3 + (1/k))L + 1 - 1/k^2$$

$$C_4 = L^3 + (7 + (4/k))L^2 + (6 + 4/k - 5/k^2)L + 1 - 4/k^2 + 1/k^3$$

$$C_5 = L^4 + (15 + 11/k)L^3 + (25 + 30/k - 14/k^2)L^2 + (10 + 10/k - 35/k^2 + 1/k^3)L + 1 - 10/k^2 + 5/k^3 + 2/k^4$$

(2)

where $L = C_2 - 1 = 1/\bar{n} + 1/k$.

The charge conservation constraint must be taken into account for fitting the full phase space all charged distributions. From (1) we may derive a modified distribution of the same form which allows for only the even multiplicities. We ignore the odd multiplicity points and rescale the even ones by a renormalization factor 2α where $\alpha^{-1} = \text{Exp}(-\bar{N}(1 - g(-1)) + 1)$. The theoretical moments $\langle n^q \rangle$ for the modified distribution in practice may be computed quite accurately from the expressions (2), with the same parameters $\bar{N}$ and B . We simply rescale these values by the renormalization factor^{*)} α . The values obtained by following this procedure are shown in the Table. We remark that we may use other distributions, for example, the one corresponding to $g(t) = (e^{Bt^2} - 1)/(e^B - 1)$ with good results. However, the resulting multiplicity distribution is found not to be defined for $\bar{n} < k$, e.g., for energies below about 27 GeV for UA5 data.

For the compound Poisson distribution the inclusive correlation integrals^{5,6)} f_q as well as the cumulants measure essentially the moments of the cluster multiplicity distribution^{**)} . The experimental errors, however, do not allow us to conclude in favour of one distribution or the other under discussion here.

*) The result may be arrived at by simple considerations or checked by exact computation.

**) $f_q = 0$, $q \geq 2$ for pure Poisson, $f_q/f_1 = (\bar{n}/k)^{q-1}$ for TCP and $(q-1)!(\bar{n}/k)^{q-1}$ for NB. Here $f_2 = \langle (n-\bar{n})^2 \rangle - \bar{n}$, $f_3 = \langle (n-\bar{n})^3 \rangle - 3\langle (n-\bar{n})^2 \rangle + 2\bar{n}$ etc.

Finally, we remark that for the full phase space data if we assume that the clusters are neutral carrying oppositely charged particles we derive $\bar{n} = 2n_-$ and $C_q = C_q^-$ where the minus sign refers to the negatively charged particles and $\bar{n}$, C_q to the all charged ones. It follows^{*)} that $k/k_- = 1 - (k/\bar{n})$. The UA5 data suggest that k/k_- approaches unity as the energy increases.

ACKNOWLEDGEMENTS

The author acknowledges with thanks several constructive suggestions from Professors L. Van Hove and M. Jacob during the progress of the work. Acknowledgements with thanks are due to Dr. J. Dias de Deus for explaining us the experimental data and useful comments, and to Dr. Renato P. Santos for very valuable help in handling the computer. The grant from CERN-CNPq agreement which permitted my visit at Theory Division is also acknowledged.

^{*)} $C_2 = C_2^-$ implies $1/k + 1/\bar{n} = 1/k_- + 1/\bar{n}_-$.

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FIGURE CAPTION

Figs. 1 (a),(b),(c),(d): Comparison of the multiplicity distributions arising from TCP and NB distributions at c.m. energies 19.7 GeV, 44.5 GeV, 200 GeV and 546 GeV for UA5 data. $z = n/\bar{n}$.

$E_{c.m.}$ GeV	$\bar{n}$ (input)	$B = \bar{n}/k$	$\bar{N}$	$\langle n \rangle$	C_2	C_3	C_4	C_5
900	34.6	10.76	3.21	33.26 34.6±1.2	1.39 1.34±0.033	2.32 2.22±0.13	4.43 4.30±0.40	9.59 9.3±1.1
546	29.1	8.02	3.63	28.35 29.1±0.9	1.34 1.31±0.03	2.15 2.12±0.11	3.91 4.05±0.32	7.92 8.8±1.0
200	21.4	4.56	4.64	21.20 21.4±0.8	1.27 1.26±0.03	1.89 1.91±0.12	3.19 3.30±0.30	5.94 6.6±0.9
62.6	13.63	1.73	6.49	13.62 13.63±0.16	1.20 1.20±0.01	1.65 1.67±0.03	2.53 2.60±0.08	4.28 4.4±0.2
52.6	12.76	1.68	6.18	12.75 12.76±0.14	1.21 1.21±0.01	1.69 1.70±0.03	2.63 2.70±0.09	4.52 4.8±0.3
44.5	12.08	1.42	6.46	12.08 12.08±0.13	1.20 1.20±0.01	1.65 1.67±0.03	2.53 2.63±0.10	4.27 4.6±0.3
27.6	9.77	1.05	6.04	9.77 9.77±0.16	1.21 1.21±0.01	1.69 1.72±0.05	2.63 2.76±0.13	4.52 5.0±0.4
19.7	8.56	0.489	6.77	8.56 8.56±0.11	1.17 1.174±0.010	1.56 1.57±0.03	2.29 2.34±0.08	3.68 3.8±0.2
11.5	6.35	0.219	5.70	6.35 6.35±0.08	1.19 1.192±0.009	1.62 1.63±0.03	2.44 2.49±0.08	4.04 4.2±0.2

TABLE

Comparison of moments C_q obtained from TCP distribution with experimental data¹⁾ indicated below each computed value. $\bar{N}$ indicates the average number of clusters for each energy.