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WHAT WE LEARN ON THE HETEROTIC STRING VACUA FROM ANOMALY FREE SUPERGRAVITY^{*)}

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A B S T R A C T

The recently constructed $D = 10$ anomaly free supergravity (AFS) has been argued to contain the full effective theory of the heterotic string. The solutions of the effective theory must be solutions of AFS, while the converse is not necessarily true since string theory might specify the boundary conditions for the AFS torsion equation.

We show that Calabi-Yau spaces are exact solutions of AFS, while compact group and coset manifolds are not. This is due to a positivity argument, which is the extension to anomaly free supergravity of the "ten into four won't go" theorem of Freedman, Gibbons and West for the Chapline-Manton theory.

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Superstrings¹⁾ and in particular the heterotic superstring²⁾ live in ten dimensions: this raises the problem of compactifying six of them, since we observe a four-dimensional world.

The compactification is obtained by choosing appropriate background fields for the metric $g_{\mu\nu}(x)$, for the two-index "photon" $B_{\mu\nu}(x)$, for the dilaton $\phi(x)$ and for the gauge fields $A_\mu^I(x)$. These background fields are required to describe the geometry of a direct product manifold $M_{10} = M_4 \otimes M_6$ where M_4 is Minkowski spacetime and M_6 is compact. It is furthermore desirable that the manifold M_6 admits one Killing spinor to maintain $N = 1$ supersymmetry in $D = 4$ and that a nonvanishing gauge field $A_\mu^I(x)$ exists on M_6 in order to produce four-dimensional chiral fermions³⁾.

The background fields $g_{\mu\nu}(x)$, $B_{\mu\nu}(x)$, $\phi(x)$, $A_\mu^I(x)$ cannot be chosen at random. They must:

- i) define a conformal invariant 2-dimensional σ -model, namely guarantee that the β -functions are identically zero.
- ii) fulfil the field equations of the effective theory of the massless modes associated to the heterotic string.

Conditions i) and ii) are believed³⁾ and perturbatively proved⁴⁾ to be equivalent. A set of background fields with the above characteristics defines what we name a "vacuum" of the string theory.

The first proposal for such a vacuum³⁾, which aroused a great interest three years ago because of its phenomenological prospects, was based on a very special type of 6-dimensional manifolds, deprived of any isometry and characterized by a vanishing Ricci curvature. Requiring $N = 1$ $D = 4$ supersymmetry, implemented via the SUSY rules of the low energy approximation to the string ($D=10, N=1$ supergravity⁵⁾), and assuming a zero v.e.v. for $H_{\mu\nu\rho}$, one was led to consider as candidate vacua Ricci flat Kähler manifolds with $SU(3)$ holonomy, i.e. Calabi-Yau manifolds.

This approach had some weak points:

- a) the definition used for Killing spinors was suited to a theory known to be incomplete.
- b) the equations of motion were amended by some higher curvature terms, but still missed other terms, discovered later⁶⁾, which do not vanish on Ricci-flat spaces.
- c) The choice $H_{\mu\nu\rho} = 0$, thought to be necessary in a first time, was later recognized to be somewhat arbitrary⁷⁾.

The difficulties related to point b) have led to dismiss the smooth Calabi-Yau manifolds in favour of orbifolds (algebraic varieties with singular points) which, being Riemann-flat, are certainly solutions of whatever Einstein equation. Also, because of their nontrivial holonomy, they do provide mechanisms to break supersymmetry and produce chiral fermions. Although string models live comfortably on orbifolds, to consider these spaces as solutions of the effective field theory is still problematic.

Point c) has always suggested that much simpler and more convincing solutions, such as group or coset manifolds, could exist and provide the same nice features ($N = 1$ supersymmetry and chiral fermions) once the degree of freedom associated to $H_{\mu\nu\rho}$ is fully exploited. Such a viewpoint could be supported by the following facts:

- i) the existence of conformal invariant 2-dimensional G -models on group manifolds with a parallelizing torsion (Wess-Zumino-Witten models⁸⁾);
- ii) the proof that there exist 6-dimensional cosets G/H for which, thanks to special symmetry properties of the root diagram, any higher curvature contribution to the β -function has the same structure as the lowest term $R_{\mu\nu}$ ⁹⁾. These cosets can, in principle, define conformal invariant G -models if the constant parameters are suitably fine tuned to yield $\beta = 0$.
- iii) the very existence of 4-dimensional superstring models based on free fermions¹⁰⁾. By means of Witten bosonization^{8),11)} they could possibly be reinterpreted as $D = 10$ models compactified on suitable coset manifolds G/H .

Whether solutions of this type do exist cannot be decided until one has a closed and complete form for the heterotic string effective theory.

In our opinion this possibility is now available. Indeed Anomaly Free $D = 10$ Supergravity has been by now constructed¹²⁻¹⁵⁾, and in ref. 14) two of us (P.F., R.D'A.) have argued that it is the complete effective theory of the heterotic string including all the possible higher order correction terms.

The argument, which disclaims the previous interpretation of our results given in ref. 13), is essentially based on the following observations:

A) Once the underlying free differential algebra is chosen, the on-shell structure of $D = 10$ supersymmetry implies the uniqueness of the form of the SUSY rules, up to field redefinitions. In other words, the superspace Bianchi identities have a unique solution. Furthermore, each algebra yields a unique set of field equations implied by the corresponding Bianchi identities.

B) The anomaly cancellation mechanism of the heterotic string requires to change the Chapline-Manton free differential algebra into a new one which includes the Lorentz Chern Simons term. The unique solution of the corresponding Bianchi identities is Anomaly Free Supergravity (AFS). This latter theory is the only anomaly free supersymmetric theory with the same massless spectrum as the heterotic string. The conclusion seems unavoidable that it actually is the complete effective theory of the heterotic string.

C) The apparent contradiction between this uniqueness and the partial results¹⁶⁾ indicating the existence of various independent higher curvature SUSY invariants is resolved by noticing that AFS is unique only in first order formalism. The transition to second order formalism is obtained by solving the torsion equation which relates the torsion to the $H_{\hat{a}\hat{b}\hat{c}}$ field strength (hatted indices run from 1 to 10):

$$H_{\hat{a}\hat{b}\hat{c}} = -\frac{1}{3} e^{\frac{4}{3}\sigma} (T_{\hat{a}\hat{b}\hat{c}} - 4i \bar{\lambda} \Gamma_{\hat{a}\hat{b}\hat{c}} \lambda) - \frac{2}{3} \chi_1 W_{\hat{a}\hat{b}\hat{c}}(R, T, \varrho) \quad (1)$$

In this equation χ_1 is a parameter with the dimension of α' , λ is the gaugino, σ the dilaton and the antisymmetric tensor $W_{\hat{a}\hat{b}\hat{c}}$ is a complicated differential expression involving $T_{\hat{a}\hat{b}\hat{c}}$, the curvature $R^{\hat{a}\hat{b}}_{\hat{c}\hat{d}}$ and the gravitino field strength $\varrho_{\hat{a}\hat{b}} = D_{[\hat{a}} \psi_{\hat{b}]}$. Its schematic structure is

(see eq.s (5.11a) and (9.10a) of ref. 14)):

$$W_{\hat{a}\hat{b}\hat{c}} = \square T_{\hat{a}\hat{b}\hat{c}} + (RT)_{\hat{a}\hat{b}\hat{c}} + (\partial TT)_{\hat{a}\hat{b}\hat{c}} + (T^3)_{\hat{a}\hat{b}\hat{c}} + (\rho\rho)_{\hat{a}\hat{b}\hat{c}} \quad (2)$$

At $\alpha' = 0$ $H_{\hat{a}\hat{b}\hat{c}}$ and $T_{\hat{a}\hat{b}\hat{c}}$ are simply identified. When $\alpha' \neq 0$ the expansion of $T_{\hat{a}\hat{b}\hat{c}}$ in powers of α' can be regarded as a slow field expansion. In this case the torsion $T_{\hat{a}\hat{b}\hat{c}}$ can include in principle, besides H-terms, also pure graviton terms constructed out of the curvature and its derivatives. The number of curvatures increases with the power of α' . For instance we may have something of the type :

$$T_{\hat{a}\hat{b}\hat{c}} = (\alpha')^3 (\partial RRR) + (\alpha')^5 (\partial RRRR) + \dots \text{H and F terms} \quad (3)$$

Clearly this series must satisfy eq. (1). However equation (1) alone cannot completely determine (3), since we must also give boundary conditions. This, in our opinion, is the source of the freedom we apparently have of choosing different SUSY invariants when we adopt second order formalism.

This being clarified, we can use the field equations of AFS to decide whether group or coset manifold compactifications of the heterotic string do exist. Also, the transformation rules of AFS enable us to determine in a rigorous way what is the residual supersymmetry of any solution.

Remarkably, our conclusion is that no group or coset compactifications do exist. To prove this, we recall the explicit form of the bosonic field equations of AFS :

$$R = \frac{2}{3} T^2 \equiv \frac{2}{3} T^{\hat{a}\hat{b}\hat{c}} T_{\hat{a}\hat{b}\hat{c}} \quad \begin{array}{l} \text{(dilaton} \\ \text{field eq.)} \end{array} \quad (4.a)$$

$$D_{\hat{m}} T^{\hat{m}\hat{a}\hat{b}} = 0 \quad \begin{array}{l} \text{(B-field eq.)} \end{array} \quad (4.b)$$

$$d\mathcal{H} = -4\alpha' (F \wedge F) - \gamma_2 R^{\hat{a}\hat{b}} \wedge R_{\hat{a}\hat{b}} \quad \begin{array}{l} \text{(Bianchi identity} \\ \text{for } \mathcal{H}) \end{array} \quad (4.c)$$

$$D^{\hat{m}} F^I_{\hat{m}\hat{a}} = 0 \quad (\text{Yang-Mills field eq.}) \quad (4.d)$$

$$R^{\hat{a}}_{\hat{b}} = t^{\hat{a}}_{\hat{b}}(F, T, R) \quad (\text{Einstein eq.}) \quad (4.e)$$

The curvature tensor $R^{\hat{a}\hat{b}}_{\hat{c}\hat{d}}$, from which we obtain the curvature scalar $R = R^{\hat{a}\hat{b}}_{\hat{a}\hat{b}}$ and the Ricci tensor $R^{\hat{a}}_{\hat{b}} = R^{\hat{a}\hat{c}}_{\hat{b}\hat{c}}$, is given by the intrinsic components of the 2-form $R^{\hat{a}\hat{b}}$:

$$R^{\hat{a}\hat{b}} \equiv d\omega^{\hat{a}\hat{b}} - \omega^{\hat{a}}_{\hat{c}} \wedge \omega^{\hat{c}\hat{b}} = R^{\hat{a}\hat{b}}_{\hat{c}\hat{d}} V^{\hat{c}} \wedge V^{\hat{d}} \quad (5)$$

The spin connection $\omega^{\hat{a}\hat{b}}$ is not in general the metric one and is determined by the equation

$$DV^{\hat{a}} \equiv dV^{\hat{a}} - \omega^{\hat{a}\hat{b}}_{\hat{c}} V^{\hat{b}} \wedge V^{\hat{c}} = T^{\hat{a}\hat{b}\hat{c}} V_{\hat{b}} \wedge V_{\hat{c}} \quad (6)$$

$V^{\hat{a}}$ being the 10-dimensional vielbein. $\mathcal{F} = dA + A \wedge A$ is the Yang-Mills curvature and

$$\begin{aligned} \mathcal{H} \equiv & dB - 4 \operatorname{tr}(\mathcal{F} \wedge A - \frac{1}{3} A \wedge A \wedge A) \\ & - \gamma_1 \operatorname{tr}(R \wedge \omega - \frac{1}{3} \omega \wedge \omega \wedge \omega) = H_{\hat{a}\hat{b}\hat{c}} V^{\hat{a}} \wedge V^{\hat{b}} \wedge V^{\hat{c}} \end{aligned} \quad (7)$$

is the field strength of the 2-index photon, whose components $H_{\hat{a}\hat{b}\hat{c}}$ are related to $T_{\hat{a}\hat{b}\hat{c}}$ by equation (1).

The explicit form of the energy momentum tensor $t^{\hat{a}}_{\hat{b}}$ is calculable but has not been worked out yet. It will be irrelevant for our purposes.

To prepare the stage for a discussion of the residual supersymmetries

we write also the equations defining a 10-dimensional Killing spinor Θ :

$$\mathcal{D}_{\hat{m}} \Theta + \frac{1}{36} \Gamma_{\hat{m}} \Gamma_{\hat{a}\hat{b}\hat{c}} \Theta T^{\hat{a}\hat{b}\hat{c}} = 0 \quad (8.a)$$

$$Z_{\hat{a}\hat{b}\hat{c}} \Gamma^{\hat{a}\hat{b}\hat{c}} \Theta = 0 \quad (8.b)$$

$$F_{\hat{a}\hat{b}}^I \Gamma^{\hat{a}\hat{b}} \Theta = 0. \quad (8.c)$$

These equations are obtained by requiring that the supersymmetry variation of the three spinor fields vanishes on a purely bosonic background. They are completely exact to all orders in α' as long as $T^{\hat{a}\hat{b}\hat{c}}$ is a solution of eq.(1) and the antisymmetric tensor $Z_{\hat{a}\hat{b}\hat{c}}$ is given in terms of $T_{\hat{a}\hat{b}\hat{c}}$ by a suitable equation (see eq. (5.11b) of ref. 14)).

On a compactifying background $M_{10} = M_4 \otimes M_6$ the 10-dimensional spinor Θ decomposes as:

$$\Theta = \sum_A \varepsilon^A \eta^A \quad (9)$$

where ε^A is a 4-dimensional anticommuting spinor and η^A a 6-dimensional commuting spinor. Using the convention $\hat{a} = (a, \alpha)$ where a, b, c run from one to four and α, β, γ run from five to ten, and assuming $M_4 =$ flat Minkowski space, we must set:

$$\mathcal{D}_a \varepsilon^A = 0 \quad (10)$$

which is the correct definition of a supersymmetry parameter in four dimensions. Furthermore all the tensors, T, R, Z, F have components only in the M_6 "greek" directions. Hence from eq. (8a), utilizing (10), we get:

$$\Gamma_{\alpha\beta\gamma} T^{\alpha\beta\gamma} \eta = 0 \quad ; \quad \mathcal{D}_a \eta = 0 \quad (11)$$

which must be coupled with

$$Z_{\alpha\beta\gamma} \Gamma^{\alpha\beta\gamma} \eta = 0 \quad ; \quad F_{\alpha\beta}^I \Gamma^{\alpha\beta} \eta = 0. \quad (12)$$

These equations should be compared with eq.s (15.1.1) and (15.1.3) of ref. 1) where the definition of Killing spinor is obtained using the approximate SUSY rules of the Chapline-Manton theory.

If we set $T_{\alpha\beta\gamma} = 0$, from eq. (1) we get $H_{\alpha\beta\gamma} = 0$ and from eq.(5.11b) of ref. 14) we also obtain $Z_{\alpha\beta\gamma} = 0$. Hence eq.s (11) and (12) reduce to

$$\overset{\circ}{D}_{\alpha} \eta = 0 \quad ; \quad F_{\alpha\beta}^I \Gamma^{\alpha\beta} \eta = 0. \quad (13)$$

$\overset{\circ}{D}$ denoting the covariant derivative with respect to the metric connection. Eq. (13) coincides with eq. (15.1.3) of ref. 1) and it has the same consequences. We deduce that the six-dimensional manifold is a Ricci-flat Kähler manifold. Requiring $N = 1$ residual supersymmetry, namely just one solution of eqs. (13), fixes the holonomy of M_6 to be $SU(3)$ ($\rightarrow$ Calabi-Yau spaces) and implies the Yang-Mills connection A_{β}^I to be identified with the spin connection, in order to solve eq. (4.c) at $\mathcal{H} = 0$. The dilaton equation (4.a) is trivially satisfied since both R and T^2 are zero. The graviton equation (4.e) is satisfied as well since, from the expressions given in the Appendix of ref. 14), it is easy to see that at $T_{\alpha\beta\gamma} = 0$ it reduces to

$$R^{\alpha}_{\beta} = (\text{differential operator on}) R^{\alpha}_{\beta}$$

so that $R^{\alpha}_{\beta} = 0$ is a solution*.

* We remind the reader that extra (Riemann)² terms appear in general on the r.h.s. of eq. (14). However, when A_{β}^I is identified with the spin connection, so that $R = F$, these terms are exactly cancelled by the Yang-Mills energy-momentum tensor F^2 (cf. ref. 3)).

We can conclude that Calabi-Yau spaces are exact solutions of Anomaly Free Supergravity. This, in our opinion, suggests that they might be exact solutions of the string effective theory, in spite of doubts raised by the perturbative σ -model calculations (Grisaru-Zanon-Van de Ven, Gross-Witten $\zeta(3)$ terms⁶⁾). The only way this could fail is if string theory prescribes boundary conditions for eq. (1) which rule out the simplest solution $T_{\alpha\beta\gamma} = H_{\alpha\beta\gamma} = 0$. In this case Calabi-Yau spaces would be solutions of AFS which violate the string theory boundary conditions.

Let us now consider the case of group manifolds G or of coset manifolds G/H . If the torsion $T_{\alpha\beta\gamma}$ is zero we immediately find a contradiction with the dilaton equation (4.a). Indeed, in the conventions we use, compact group and coset manifolds have a metric scalar curvature which is strictly positive, while eq. (4.a) requires $\overset{\circ}{R} = 0$. The contradiction, however, is present also for nonvanishing torsion. If one separates the spin connection $\omega^{\alpha\beta}$ in the metric plus the torsion part, one has

$$\omega^{\alpha\beta} = \overset{\circ}{\omega}^{\alpha\beta} - T^{\alpha\beta\gamma} V_{\gamma} \quad (14)$$

and

$$R(\omega) = R(\overset{\circ}{\omega}) + \frac{1}{2} T^2 \quad (15)$$

Hence the dilaton equation (4.a) implies

$$R(\overset{\circ}{\omega}) = -\frac{1}{6} T^2 \quad (16)$$

which has no solutions if $R(\overset{\circ}{\omega}) > 0$ since

$$T^2 \equiv T^{\alpha\beta\gamma} T_{\alpha\beta\gamma} = - T^{\alpha\beta\gamma} T^{\alpha\beta\gamma} < 0 \quad (17)$$

due to the negative signs in the 10-dimensional metric we use $(+,-,\dots,-)$.

This very simple observation is sufficient to rule out any compact group or compact coset manifold from the solutions of AFS.

To gain a better understanding of this result it is worth to notice the following. The dilaton equation maintains the same form (4.a) in the $\chi_1 = 0$ and $\chi_1 \neq 0$ cases, namely in the Chapline-Manton theory and in AFS. On the other hand eq.(4.a) is obtained from the Chapline-Manton form of the dilaton field equation through the field redefinition (Weyl transformation) described by eq.s (10.2) of ref. 14). In view of this, the impossibility of solving eq. (16) with group or coset manifolds is nothing else but the celebrated no go theorem of Freedman, Gibbons and West¹⁷⁾. Another way of stating our result is that "Ten into four won't go" extends from the Chapline-Manton theory to the complete effective theory of the heterotic string.

Finally we verify our theorem in a specific example. We choose the coset manifold $G_2 / SU(3)$ which is one of the two proposed in ref. 9). Introducing a parameter r (the length scale of the coset) the Cartan-Maurer equations can be written as follows :

$$dV^\alpha + \Omega^{\alpha\beta} \wedge V^\beta = \frac{1}{2r} t^{\alpha\beta\gamma} V^\beta \wedge V^\gamma \quad (18.a)$$

$$d\Omega^{\alpha\beta} + \Omega^{\alpha\gamma} \wedge \Omega^{\gamma\beta} = \frac{3}{4r^2} K^{\alpha\beta} = \frac{3}{4r^2} K^{\alpha\beta\gamma\delta} V^\gamma \wedge V^\delta \quad (18.b)$$

where the indices $\alpha, \beta, \dots$ run on six values and are summed with plus signs if repeated. Dividing their range into two subsets $\alpha = (i, i^*)$ ($i=1,2,3$; $i^*=1^*,2^*,3^*$) the matrix $\Omega^{\alpha\beta}$, which describes the 1-forms of the $SU(3)$ subalgebra, has the structure

$$\Omega = \begin{pmatrix} A & -S \\ S & A \end{pmatrix} \quad (19)$$

$A^T = -A$, $S^T = S$ ($\text{tr } S = 0$) being the real and imaginary parts of an antihermitean 1-form $\Omega = A + iS$. The fully antisymmetric tensor $t^{\alpha\beta\gamma}$ has the following nonzero components:

$$t^{ijk} = \varepsilon^{ijk} \quad ; \quad t^{ij^*k^*} = \varepsilon^{ij^*k^*} \quad (20)$$

and the 2-form $K^{\alpha\beta}$ reads

$$K^{ij} = V^i \wedge V^j + V^{i*} \wedge V^{j*} = K^{i*j*} \quad (21.a)$$

$$K^{ij*} = V^i \wedge V^{j*} - V^{i*} \wedge V^j - \frac{2}{3} \delta^{ij*} V^m \wedge V^{m*} \quad (21.b)$$

Comparison of eqs. (18) with the structural equations (5) and (6) leads to setting

$$T^{\alpha\beta\gamma} = \frac{k}{2\kappa} t^{\alpha\beta\gamma} \quad ; \quad \omega^{\alpha\beta} = \Omega^{\alpha\beta} + \frac{1-k}{2\kappa} t^{\alpha\beta\gamma} V^\gamma \quad (22)$$

where k is a constant free parameter. The corresponding curvature 2-form is

$$R^{ij} = \frac{1}{4\kappa^2} \left[(4-k^2) V^i \wedge V^j + k(4-k) V^{i*} \wedge V^{j*} \right] \quad (23.a)$$

$$R^{i*j*} = \frac{1}{4\kappa^2} \left[k(4-k) V^i \wedge V^j + (4-k^2) V^{i*} \wedge V^{j*} \right] \quad (23.b)$$

$$R^{ij*} = \frac{1}{4\kappa^2} \left[(4-k^2) V^i \wedge V^{j*} - k(4-k) V^{i*} \wedge V^j - 2k(k-2) \delta^{ij*} V^m \wedge V^{m*} \right] \quad (23.c)$$

from which we obtain the Ricci tensor and curvature scalar

$$R^\alpha{}_\beta = \frac{1}{2\kappa^2} (5-k^2) \delta^\alpha{}_\beta \quad ; \quad R = \frac{3}{\kappa^2} (5-k^2) \quad (24)$$

while we have

$$T^2 \equiv T^{\alpha\beta\gamma} T_{\alpha\beta\gamma} = - \frac{k^2}{4\kappa^2} t^{\alpha\beta\gamma} t^{\alpha\beta\gamma} = -6 \frac{k^2}{\kappa^2} \quad (25)$$

Inserting (25) into (4.a) yields the impossible equation

$$\frac{15}{\kappa^2} = - \frac{k^2}{\kappa^2} \quad (26)$$

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