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NEW MINIMAL HIGHER DERIVATIVE SUPERGRAVITY COUPLED TO MATTER^{*)}

S. Cecotti
CERN -- Geneva

S. Ferrara
CERN -- Geneva
and
Department of Physics, UC Los Angeles, CA 90024

M. Porrati⁺⁾ and S. Sabharwal
Department of Physics, UC Los Angeles, CA 90024

A B S T R A C T

We consider quadratic invariants in the new minimal formulation of $N = 1$, four-dimensional supergravity, and obtain a simpler form of the Gauss-Bonnet combination. When added to the Einstein Lagrangian, these new multiplets propagate different states than those in the old minimal formulation. In particular, the $R + \alpha R^2$ theory is shown to be equivalent to standard supergravity coupled to a massive vector multiplet. Our result shows the non-uniqueness of minimal supersymmetrizations of higher derivative theories.

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⁺⁾ Address after October 1, 1987: University of California, Berkeley, CA 94720.

I. INTRODUCTION

In a superstring theory,¹ the integration over the massive string modes results into an effective Lagrangian for the massless states which contains higher curvature and higher derivative terms of any order. More technically, the requirement of conformal invariance² for the nonlinear σ -model underlying the motion of the superstring in a background with non-trivial massless fields, leads to supersymmetric equations of motion of the form² $\beta_i = 0$ which contain higher derivative terms.

This potential application has motivated new efforts to understand higher derivative supergravities,^{3,4,5} their nonlinear structure and dynamics. The greatest progress in this program was obtained,^{5,6,7} of course, in the simplest case: namely, in $N = 1$ supergravity in four dimensions, where we have full kinematical control of the possible higher invariants and their symmetries, thanks to the existence of an explicit tensor calculus^{8,9,10} or, equivalently, of a superspace geometrical formalism.^{11,12} Anyhow, this last case is the relevant one for the low-energy Lagrangian of a superstring theory compactified^{13,14} to 4D while preserving $N = 1$ supersymmetry, as well as for $N = 1$ superstring theories directly formulated in four dimensions,^{15,16} which are generally believed to be those with brightest phenomenological perspectives. However, here we are confronted with a problem. The construction of the higher curvature SUSY invariants relies on the off-shell field representation of supersymmetry (i.e. superfields) which drastically differ from the on-shell ones. What is worse, is that the off-shell field representation is not uniquely determined by the on-shell one, so in constructing higher invariants we may have some ambiguities.

This non-uniqueness is already present in the case of rigid supersymmetry. A chiral multiplet (with spin content $1/2, 0^+, 0^-$) needs an "F" auxiliary field, whereas the linear multiplet¹⁷ which has the same "on-shell" particle content (the pseudoscalar particle being described by the dual 2-form) does not need any "auxiliary field" since a 4D antisymmetric tensor contains more off-shell degrees of freedom than a scalar field.

The same problem occurs for the supergravity multiplet itself, for which many different off-shell formulations exist.^{8,18,19,20} Here we shall focus only on the so-called "minimal"^{8,19} off-shell formulations of $N = 1$ supergravity. These formulations contain just six auxiliary "degrees of freedom." The other "non-minimal" formulations^{18,20} which contain additional fermionic fields are reducible^{21,22} in the sense that they can always be seen as a "minimal" $N = 1$ Sugra coupled to some additional matter multiplets. All these different formulations get unified in the framework of superconformal supergravity.^{9,10,23} In the superconformal case,²⁴ the gravitational multiplet needs no auxiliary fields to close the algebra off-shell, so no ambiguity arises. In order to write a superconformal theory which is gauge-equivalent²³ to a given Poincaré supergravity, we add some compensating multiplets which are ordinary "matter" superconformal multiplets. As a choice of gauge,^{9,23} we fix some components of the compensators; this procedure breaks the superconformal gauge-invariance down to the Poincaré one.²³ The remaining components of the compensators as well as the surviving connections of the coset [superconformal]/[super-Poincaré] can then be seen as the auxiliary fields of the given Poincaré theory. In the minimal formulations of $N = 1$ supergravity, they are a chiral multiplet for the "old-minimal"^{8,23} and a real linear multiplet for the "new-minimal."^{8,23}

At the level of standard supergravity^{25,26} (i.e. no higher derivative terms and no higher-spin matter fields) all the off-shell formulations are equivalent.²⁷ For the minimal case this can be proven simply by a duality transformation^{17,27,28,29} mapping the chiral compensator of the old-minimal formulation into the linear compensator of the new minimal, and vice versa. This transformation is just the supersymmetric version of the usual duality transformation between a scalar and a 2-form field in 4 dimensions.³⁰

The Lagrangian of pure (Poincaré) supergravity in the old-minimal formulation is^{8,9,23}

$$\mathcal{L} = -[S_0 \bar{S}_0]_D \quad (1.1)$$

where S_0 is the chiral compensator (which has Weyl and chiral weights^{9,23} $w = n = 1$) and $[...]_D$ denotes the D-type superconformal invariant action density^{8,9,23} for a real vector multiplet with weights $w = 2, n = 0$. The right-hand side of Eq. (1.1) can also be seen as the volume form of Poincaré superspace, an old result due to Wess and Zumino.³¹

The Lagrangian (1.1) can be rewritten in an equivalent form by performing a duality transformation. Consider the superconformal Lagrangian²⁷

$$\mathcal{L} = -[S_0 \bar{S}_0 e^U]_D + [LU]_D \quad (1.2)$$

where U is an arbitrary real multiplet with $w = 0$ and L is a real linear multiplet^{9,23} ($w = 2, n = 0$) satisfying

$$\Sigma(L) = \bar{\Sigma}(L) = 0, \quad (1.3)$$

Σ being the superconformal chiral projection.^{9,23} Later on we shall find chiral projections which are not superconformal invariant in the present sense. Varying Eq. (1.2) with respect to U , we get:

$$\Sigma(D_\alpha U) = 0 \quad \Rightarrow \quad \begin{cases} U = \phi + \bar{\phi} \\ \bar{D}_\alpha \phi = 0, \quad \text{i.e. } \phi \text{ chiral} \end{cases} \quad (1.4)$$

Inserting back Eq. (1.4) into the Lagrangian Eq. (1.2) and making the field redefinition $S_0 \exp \phi \rightarrow S_0$, we get back Eq. (1.1). Eqs. (1.1) and (1.2) are thus equivalent.²⁷

The dual formulation (corresponding to the new-minimal case) is obtained by varying Eq. (1.2) with respect to U . In this case, one gets the standard supergravity Lagrangian in the new-minimal form,^{19,12} i.e.

$$\mathcal{L} = [L \ln(L/S_0 \bar{S}_0)]_D. \quad (1.5)$$

Notice that Eq. (1.5) is S_0 independent since as $S_0 \rightarrow S_0 \exp(\phi)$ (with ϕ chiral), the variation of $\mathcal{L}$, $-[L(\phi + \bar{\phi})]$ vanishes. Thus the old and new minimal formulations of pure SUGRA are equivalent.²⁷ This equivalence can be immediately generalized²⁷ to any standard supergravity²⁶ which does not contain a superpotential term*

$$\mathcal{L} = -[S_0 \bar{S}_0 \Phi(S_1, \bar{S}_1)]_D + 2 \operatorname{Re}[f_{AB} W^A W^B]_F. \quad (1.6)$$

The "new" minimal version of Eq. (1.6) is²⁷

* $[...]_F$ is the F-type superconformal action density ([9,23]) for a chiral multiplet with $w = n = 3$.

$$\begin{aligned}\mathcal{L} &= [L \ln(L/S_0 \bar{S}_0) - L \ln \Phi]_D + 2 \operatorname{Re}[f_{AB} W^A W^B]_F \\ &= [L \ln(L/S_0 \bar{S}_0) + \frac{1}{3} LJ]_D + 2 \operatorname{Re}[f_{AB} W^A W^B]_F\end{aligned}\quad (1.7)$$

where J is the Kähler potential of the matter σ -model.²⁶

The situation changes if we add a superpotential term²⁶

$$2 \operatorname{Re}[g(S_1) S_0^3]_F. \quad (1.8)$$

In this case the above duality transformation can be performed only when a field redefinition exists, which cancels the dependence of the superpotential term on the compensator, and does not destroy the needed properties of the D-term piece. This is possible²⁷ if and only if $\Phi(S_1, \bar{S}_1)$ is invariant under an R-type symmetry under which $g(S_1)$ has weight 3. Under this restriction, the equivalence of the two formulations is easily proven. The above restrictions are necessary since in the new-minimal case only R-invariant models can be formulated.^{9,23} All the previous equivalence theorems between the two minimal formulations rely essentially on the fact that the compensator S_0 (resp. L) enters in the superconformal Lagrangian algebraically and hence--after adding some Lagrange multiplier to enforce the chiral (resp. linear) restriction--can be eliminated from $\mathcal{L}$, in favor of the other, using its algebraic equations of motion.

Contrary to the standard case, in the presence of higher curvature terms, the Lagrangian is no longer algebraic in the compensator superfield. This is already clear from the fact that the Einstein multiplet $G_{\alpha\dot{\alpha}}$ and the scalar curvature multiplet R ^{11,32,8} of the old-minimal formulation can be written as expressions involving the derivatives of S_0 ³³

$$R = S_0^{-1} \Sigma(\bar{S}_0) \quad (1.9a)$$

$$G_{\alpha\dot{\alpha}} = -\frac{1}{2} S_0^{-1} D_{\alpha\dot{\alpha}}(S_0) S_0 \quad (1.9b)$$

the same is true for the new-minimal formulation as will be discussed below.

In higher derivative supergravity then, it is not obvious that the ambiguities in the off-shell formulation do not lead to physically inequivalent theories. In the present paper we show that, indeed, the two formulations lead to different higher derivative theories. This will be shown directly by comparing the physical spectrum of the SUSY completion of "R + αR^2 " gravity in the two schemes. In the old minimal case this theory turns out to be equivalent³⁴ to standard supergravity coupled to two Wess-Zumino multiplets (which are physical for $\alpha > 0$). This can be shown at the full nonlinear level.⁶

Instead, as we shall see below, the corresponding Lagrangian in the new minimal case propagates--besides the usual massless gravitational multiplet--a massive vector multiplet (physical for $\alpha > 0$). In Section III this will be proven to the complete nonlinear level and we will also describe the self-coupling of the massive vector multiplet in the equivalent standard theory.

In this paper we also discuss all the other gravitational multiplets of the new-minimal scheme (at the linearized level), their properties and the Bianchi identities they satisfy.¹² The supersymmetric versions of the Gauss-Bonnet and Hirzebruch signature theorems are also proven (in the linearized case) for the new-minimal curvatures. These topological theorems turn out to have a more geometrical structure than in the old minimal case:^{35,32} This is related to the fact that in this formulation there exists a chiral multiplet (or-better-an L-chiral multiplet, in the sense of Kugo and Uehara¹⁰) $R_{\mu\nu\alpha}$

which contains the full Riemann tensor. The existence of such a multiplet can be easily proven in the nonlinear case using superconformal techniques. In terms of $R_{\mu\nu\alpha}$ the super-topological theorems have a form which is reminiscent of the usual Weil homomorphism of the bosonic case.

The paper is organized as follows: In Section II, we discuss the linearized gravitational multiplets in the new scheme, as well as their properties, Bianchi identities and component form. Then we prove the supersymmetric version of the above topological theorems, write down the explicit component form (at the linearized level) of the general SUSY Lagrangian quadratic in the gravitational curvatures, and discuss their physical spectrum at the linear level. The "new-minimal" formulation of the free massive spin-2 multiplet is also discussed.

In Section III it is shown at the nonlinear level that the new-minimal " $R + \alpha R^2$ " supergravity is equivalent to a standard supergravity coupled to a massive vector, with specified self-couplings.

II. LINEARIZED NEW MINIMAL SUPERGRAVITY

In this section we consider the new minimal formulation at the linearized level. The Bianchi identities of superspace^{11,12} lead to the solution of all torsions and curvatures in terms of three superfields¹² $E_{\alpha\dot{\alpha}}$, W_{α} and $W_{\alpha\beta\gamma}$, which satisfy certain constraints. We give explicit expressions for these fields at the linearized level, in analogy with the treatment for the old minimal formulation done by Ferrara and Zumino.³⁵ We then discuss the free massive spin-2 multiplet and higher curvature Lagrangians for supergravity.

The graviton, $h_{\mu\nu}$, is contained in a real vector superfield ϕ_{μ} , whose component expansion is given by

$$\begin{aligned} \phi_{\mu} = & C_{\mu} + \theta\chi_{\mu} + \bar{\theta}\bar{\chi}_{\mu} + \theta^2 F_{\mu} + \bar{\theta}^2 \bar{F}_{\mu} + \bar{\theta}\sigma_{\nu}\theta(h_{\nu\mu} + A_{\nu\mu}) \\ & + \frac{1}{2}\bar{\theta}^2\theta^2\left(A_{\mu} + \frac{1}{2}\partial^2 C_{\mu}\right) - i\theta^2\bar{\theta}\left(\bar{\psi}_{\mu} - \frac{1}{2}\sigma\cdot\partial\chi_{\mu}\right) \\ & + i\bar{\theta}^2\theta\left(\psi_{\mu} + \frac{1}{2}\sigma\cdot\partial\chi_{\mu}\right) \end{aligned} \quad (2.1)$$

where $A_{\mu\nu} = -A_{\nu\mu}$ and ψ_{μ} corresponds to the gravitino.

We choose Einstein transformations as

$$\delta\phi_{\mu} = \partial_{\mu}\phi + S_{\mu} + \bar{S}_{\mu} \quad (2.2)$$

where S_{μ} is chiral, i.e. $\bar{D}_{\dot{\alpha}}S_{\mu} = 0$. These transformations allow us to choose a gauge where $C_{\mu} = \chi = F_{\mu} = 0$, after which the remaining transformations are

$$\begin{aligned}
 \delta h_{\mu\nu} &= \partial_\mu \xi_\nu + \partial_\nu \xi_\mu \\
 \delta \psi_\mu &= \partial_\mu \epsilon \\
 \delta A_\mu &= \partial_\mu \theta \\
 \delta A_{\mu\nu} &= \partial_\mu \theta_\nu - \partial_\nu \theta_\mu .
 \end{aligned} \tag{2.3}$$

The first two are just the usual Einstein transformations, and the remaining two are needed in order to give a multiplet with the minimal 12 + 12 degrees of freedom. This is the new minimal formulation with auxiliary fields A_μ and $A_{\mu\nu}$.

The supersymmetry transformation rules in this gauge are derived by adding the compensating gauge transformations. They are

$$\begin{aligned}
 \delta A_\mu &= \epsilon \sigma_\mu \bar{\partial} \bar{\psi} - \bar{\epsilon} \sigma_\mu \partial \psi \\
 \delta \psi_\mu &= i \sigma_{\rho\lambda} \partial_\rho h_{\lambda\mu} \epsilon - i A_\mu \epsilon + \sigma_{\mu\lambda} v_\lambda \epsilon \\
 \delta (h_{\mu\nu} + A_{\mu\nu}) &= -i \epsilon \sigma_\mu \bar{\psi}_\nu - i \bar{\epsilon} \sigma_\mu \psi_\nu .
 \end{aligned} \tag{2.4}$$

We next construct various multiplets invariant under the Einstein transformations. The Einstein tensor is contained in the following real linear superfield,

$$E_\mu = \frac{1}{2} \epsilon_{\mu\rho\lambda\nu} \bar{D}\sigma_\rho D \partial_\lambda \phi_\nu \tag{2.5}$$

is easily seen to be invariant under (2.2) using the identity $\bar{D}\sigma_\mu D + D\sigma_\mu \bar{D} = -4i\partial_\mu$. The component expansion is given by

$$E_\mu = -2v_\mu - 2i\theta R_\mu + 2i\bar{\theta}\bar{R}_\mu + \bar{\theta}\sigma_\nu\theta\{E_{\nu\mu} + \partial_\lambda F_{\lambda\nu\mu} + {}^*F_{\nu\mu}\} +$$

$$- \theta^2 \bar{\theta} \sigma \cdot \partial R_\mu + \bar{\theta}^2 \theta \sigma \cdot \partial \bar{R}_\mu + \frac{1}{2} \bar{\theta}^2 \theta^2 \partial^2 v_\mu \quad (2.6)$$

where

$$\begin{aligned} F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu, & {}^*F_{\mu\nu} &= \frac{1}{2} \epsilon_{\mu\nu\rho\lambda} F_{\rho\lambda} \\ F_{\mu\nu\lambda} &= \partial_\mu A_{\nu\lambda} + \partial_\nu A_{\lambda\mu} + \partial_\lambda A_{\mu\nu}, & v_\mu &= \frac{1}{3!} \epsilon_{\mu\nu\rho\lambda} F_{\nu\rho\lambda} \\ R_\mu &= \frac{1}{4} \epsilon_{\mu\nu\rho\lambda} \sigma_\nu \psi_{\rho\lambda}, & \psi_{\mu\nu} &= \partial_\mu \psi_\nu - \partial_\nu \psi_\mu \\ R_{\mu\nu\rho\lambda} &= -\partial_\mu \partial_\rho h_{\nu\lambda} + \partial_\nu \partial_\rho h_{\mu\lambda} + \partial_\mu \partial_\lambda h_{\nu\rho} - \partial_\nu \partial_\lambda h_{\mu\rho} \\ E_{\mu\nu} &= R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R. \end{aligned} \quad (2.7)$$

In terms of the multiplet E_μ , the Lagrangian for Einstein supergravity is

$$\begin{aligned} \mathcal{L}_R &= \int d^4\theta \phi \cdot E \\ &= \frac{1}{2} R + \bar{\psi} \cdot R - (2A+V) \cdot V + \text{surface terms}. \end{aligned} \quad (2.8)$$

The transformations (2.2) are reminiscent of those for Abelian gauge theories. The invariant multiplet corresponding to the field strength is

$$R_{\mu\nu\alpha} = -\frac{1}{8} \bar{D}^2 D_\alpha (\partial_\mu \phi_\nu - \partial_\nu \phi_\mu). \quad (2.9)$$

We call this the Riemann multiplet. Its component's expansion is given by

$$R_{\mu\nu} = \frac{1}{2} \psi_{\mu\nu} + \frac{1}{2} \theta^2 \sigma \cdot i \partial \bar{\psi}_{\mu\nu} - \frac{1}{2} \theta F_{\mu\nu} - \frac{1}{4} \sigma_{\rho\lambda} \theta R(A)_{\rho\lambda\mu\nu} \quad (2.10)$$

where

$$R(A)_{\rho\lambda\mu\nu} = R_{\rho\lambda\mu\nu} - \partial_\mu F_{\nu\rho\lambda} + \partial_\nu F_{\mu\rho\lambda} \quad (2.11)$$

is the linearized Riemann tensor with torsion $T_{\mu\nu\lambda} = 2F_{\mu\nu\lambda}$.

The Riemann multiplet allows us to give a simple form for the supersymmetric generalization of the Gauss-Bonnet theorem.^{35,32} From (2.9), it is obvious that the chiral multiplet $R_{\mu\nu}^* R_{\mu\nu}$ is a total divergence. The real and imaginary parts of its θ^2 component correspond to the Hirzebruch and Gauss-Bonnet densities. The complete expression is

$$\begin{aligned} R_{\mu\nu}^* R_{\mu\nu} = & \frac{1}{2} \psi_{\mu\nu}^* \psi_{\mu\nu} + \frac{1}{4} \theta \sigma_{\rho\lambda} \psi_{\mu\nu} R(A)_{\rho\lambda\mu\nu} - \frac{1}{2} \theta \psi_{\mu\nu}^* F_{\mu\nu} + \theta^2 \left\{ \frac{1}{2} \psi_{\mu\nu} \sigma \cdot i \partial^* \bar{\psi}_{\mu\nu} \right. \\ & \left. - \frac{1}{4} F_{\mu\nu}^* F_{\mu\nu} + \frac{1}{8} R(A)_{\mu\nu\rho\lambda}^* R(A)_{\mu\nu\rho\lambda} + \frac{1}{8} R(A)_{\mu\nu\rho\lambda}^* R(A)^*_{\mu\nu\rho\lambda} \right\} \quad (2.12) \end{aligned}$$

where

$$\begin{aligned} -R_{\mu\nu\rho\lambda}^* R_{\mu\nu\rho\lambda}^* &= R_{\mu\nu\rho\lambda}^2 - 4R_{\mu\nu}^2 + R^2 \\ &= W_{\mu\nu\rho\lambda}^2 - 2E_{\mu\nu}^2 + \frac{2}{3} R^2 \quad (2.13) \end{aligned}$$

is the usual Gauss-Bonnet integrand.

From the Riemann multiplet, we can construct multiplets containing only the scalar curvature, the Einstein or Weyl tensors. The scalar curvature is contained in

$$W = -\sigma_{\mu\nu} R_{\mu\nu} . \quad (2.14)$$

In terms of the superfield ϕ_μ it can be expressed as

$$W_\alpha = \frac{1}{16} \bar{D}^2 D_\alpha (\bar{D}\sigma_\mu D - D\sigma_\mu \bar{D}) \phi_\mu . \quad (2.15)$$

Using (2.10) and (2.14), the component expression is

$$W = -\sigma \cdot \bar{R} + \theta^2 \sigma \cdot i\partial\sigma \cdot R + \frac{1}{2} \sigma_{\mu\nu} \theta F(A+2V)_{\mu\nu} + \frac{1}{2} \theta R . \quad (2.16)$$

The Einstein tensor is also contained in a chiral superfield, which however, is not independent of E_μ , i.e.

$$2\sigma_\lambda^* R_{\mu\lambda} = \bar{D}E_\mu . \quad (2.17)$$

Finally, the Weyl tensor appears in

$$\begin{aligned} W_{\mu\nu} &= \frac{1}{24} (3\sigma_{\rho\lambda} \sigma_{\mu\nu} + \sigma_{\mu\nu} \sigma_{\rho\lambda}) R_{\rho\lambda} \\ &= \frac{1}{2} \left(R_{\mu\nu} + i^* R_{\mu\nu} + \frac{1}{6} \sigma_{\mu\nu} W \right) . \end{aligned} \quad (2.18)$$

The matrix multiplying $R_{\rho\lambda}$ in the above expression is just the projection operator for the σ_μ traces, i.e.

$$\sigma_\mu W_{\mu\nu} = 0 . \quad (2.19)$$

In spinor notations, by defining

$$W_{\alpha\beta\gamma} = \frac{1}{4} (\sigma^{\mu\nu})_{\alpha\beta} W_{\mu\nu\gamma} \quad (2.20)$$

which is symmetric in $\alpha\beta\gamma$ due to (2.19), the expression for the Weyl multiplet in terms of ϕ_μ is

$$W_{\alpha\beta\gamma} = \frac{1}{16 \cdot 3!} \bar{D}^2 \{ D_\alpha \partial_\beta \dot{\phi}_{\dot{\alpha}\gamma} + \text{symmetrizations in } \alpha, \beta \text{ and } \gamma \} . \quad (2.21)$$

Using (2.10) and (2.18), we find the component expansion of $W_{\mu\nu}$ to be

$$\begin{aligned} W_{\mu\nu} = & \frac{1}{2} \frac{1}{24} (3\sigma_{\rho\lambda} \sigma_{\mu\nu} + \sigma_{\mu\nu} \sigma_{\rho\lambda}) \psi_{\rho\lambda} \\ & + \frac{1}{2} \theta^2 \left\{ \partial_\mu \left(R_\nu + \frac{1}{3} \sigma_\nu \sigma \cdot R \right) - \partial_\nu \left(R_\mu + \frac{1}{3} \sigma_\mu \sigma \cdot R \right) \right. \\ & + i \epsilon_{\mu\nu\rho\lambda} \partial_\rho \left(R_\lambda + \frac{1}{3} \sigma_\lambda \sigma \cdot R \right) \Big\} \\ & - \frac{1}{6} \left\{ F(A-V)_{\mu\nu} + i {}^*F(A-V)_{\mu\nu} - \frac{1}{2} \sigma_{\mu\lambda} F(A-V)_{\lambda\nu} + \frac{1}{2} \sigma_{\nu\lambda} F(A-V)_{\lambda\mu} \right\} \theta \\ & - \frac{1}{4} \sigma_{\rho\lambda} \theta W_{\rho\lambda\mu\nu} \end{aligned} \quad (2.22)$$

where $W_{\rho\lambda\mu\nu} = \frac{1}{2} (R_{\rho\lambda\mu\nu} - {}^*R^*_{\rho\lambda\mu\nu}) - \frac{1}{12} (g_{\rho\mu} g_{\lambda\nu} - g_{\rho\nu} g_{\lambda\mu})$ R is the Weyl tensor.

From the above multiplets we can reconstruct the Riemann multiplet as

$$R_{\mu\nu} = W_{\mu\nu} - \frac{1}{4} \epsilon_{\mu\nu\rho\lambda} \sigma_\rho \bar{D} E_\lambda + \frac{1}{6} \sigma_{\mu\nu} W$$

or $R_{\dot{\alpha}\dot{\beta}\dot{\gamma}} = -2\epsilon_{\dot{\alpha}\dot{\beta}} R_{\dot{\alpha}\dot{\gamma}} + 2\epsilon_{\dot{\alpha}\dot{\gamma}} R_{\dot{\alpha}\dot{\beta}}$ where

$$R_{\alpha\beta\gamma} = \frac{1}{4} \sigma_{\alpha\beta}^{\mu\nu} R_{\mu\nu\gamma} = W_{\alpha\beta\gamma} + \frac{1}{12} (\epsilon_{\alpha\gamma} W_\beta + \epsilon_{\beta\gamma} W_\alpha)$$

$$R_{\dot{\alpha}\dot{\beta}\gamma} = \frac{1}{4} \sigma_{\dot{\alpha}\dot{\beta}}^{\mu\nu} R_{\mu\nu\gamma} = -\frac{1}{8} (\bar{D}_{\dot{\alpha}} \dot{E}_{\dot{\beta}\gamma} + \bar{D}_{\dot{\beta}} \dot{E}_{\dot{\alpha}\gamma}) . \quad (2.23)$$

This generalizes to the full nonlinear level, when the Riemann multiplet is identified with the superspace torsion $T_{ab}^{\alpha}{}^{12}$.

The supersymmetric Gauss-Bonnet theorem (2.12) can be expressed in terms of the multiplets E_{μ} , W and $W_{\mu\nu}$ as

$$\begin{aligned} i R_{\mu\nu}^* R_{\mu\nu} &= W_{\mu\nu} W_{\mu\nu} + \frac{1}{8} \bar{D}^2 (E_{\mu} E_{\mu}) - \frac{1}{3} W W \\ &= -2 R_{\alpha\beta\gamma} R^{\alpha\beta\gamma} + 2 R_{\dot{\alpha}\dot{\beta}\gamma} R^{\dot{\alpha}\dot{\beta}\gamma} . \end{aligned} \quad (2.24)$$

Let us now consider the identities satisfied by the above multiplets.

From (2.15) and (2.17), we get

$$\bar{D}_{\dot{\alpha}} W_{\dot{\beta}} = DW + \bar{D}\bar{W} = 0$$

$$W = \frac{1}{2} \sigma_{\mu}^{\dot{\alpha}\dot{\beta}} \bar{D}_{\dot{\alpha}} E_{\dot{\beta}\mu}$$

or

$$W_{\dot{\alpha}} = -\frac{1}{2} \bar{D}^{\dot{\alpha}} E_{\dot{\alpha}\alpha} . \quad (2.25)$$

The Riemann multiplet can be shown to satisfy

$$DR_{\mu\nu} = -\bar{D}\bar{R}_{\mu\nu} = -\frac{1}{2} (\partial_{\mu} E_{\nu} - \partial_{\nu} E_{\mu}) + \frac{1}{8} \epsilon_{\mu\nu\rho\lambda} (\bar{D}\sigma_{\rho} D - D\sigma_{\rho} \bar{D}) E_{\lambda} . \quad (2.26)$$

This then implies the following identities for $W_{\mu\nu}$,

$$\bar{D}_{\dot{\alpha}} W_{\alpha\beta\gamma} = 0$$

$$DW_{\mu\nu} = -\frac{1}{2} (\partial_{\mu} E_{\nu} - \partial_{\nu} E_{\mu}) - \frac{1}{2} * (\partial_{\mu} E_{\nu} - \partial_{\nu} E_{\mu}) - \frac{1}{6} D\sigma_{\mu\nu} W$$

or

$$D_{\alpha} W^{\alpha}_{\beta\gamma} = \frac{1}{4} \left(\partial_{\beta} \dot{E}_{\alpha\gamma} + \partial_{\gamma} \dot{E}_{\alpha\beta} \right) + i \frac{1}{6} (D_{\beta} W_{\gamma} + D_{\gamma} W_{\beta}) . \quad (2.27)$$

The identities (2.25) and (2.27) are precisely the conditions on $E_{\alpha\alpha}, W_{\alpha}$ and $W_{\alpha\beta\gamma}$ obtained by analyzing the Bianchi identities of superspace.¹² The equations (2.5), (2.15), and (2.21) represent their solutions in terms of the unconstrained superfield ϕ_{μ} .

The Weyl multiplet is, of course, invariant under conformal transformations, which are given by,³⁵

$$\delta\phi_{\mu} = \bar{D}\sigma_{\mu}\lambda + D\sigma_{\mu}\bar{\lambda} . \quad (2.28)$$

For

$$\bar{D}\bar{D}^2\lambda - \bar{D}\bar{D}^2\bar{\lambda} = 0 , \quad (2.29)$$

these are just the Einstein transformations (2.2). To get the conformally invariant analogues of E_{μ} and W , let us introduce a compensator, ψ , which transforms as

$$\delta\psi = \lambda . \quad (2.30)$$

Then

$$\hat{\phi}_\mu = \phi_\mu - \bar{D}\sigma_\mu\psi - D\sigma_\mu\bar{\psi} \quad (2.31)$$

is invariant under conformal transformations. By simply replacing ϕ_μ by $\hat{\phi}_\mu$ in (2.5) and (2.15), we get the invariant multiplets $\hat{E}_\mu$ and $\hat{W}$,

$$\hat{E}_\mu = E_\mu + \frac{1}{8} (\bar{D}\sigma_\mu D - D\sigma_\mu \bar{D}) L \quad (2.32)$$

and

$$\begin{aligned} \hat{W} &= W + \frac{3}{16} \bar{D}^2 D L \\ &= \frac{1}{16} \bar{D}^2 D \{ (\bar{D}\sigma_\mu D - D\sigma_\mu \bar{D}) \phi_\mu + 3L \} \end{aligned} \quad (2.33)$$

where

$$L = D\bar{D}^2\psi - \bar{D}D^2\bar{\psi} \quad (2.34)$$

is the linear multiplet associated with ψ . Similarly, $\hat{R}_{\mu\nu} = R_{\mu\nu} + \frac{1}{8} (\sigma_\mu\partial_\nu - \sigma_\nu\partial_\mu) \bar{D}L$ and $\hat{W}_{\mu\nu} = W_{\mu\nu}$.

The Lagrangian for Einstein supergravity can then be written as

$$\begin{aligned} \mathcal{L} &= \int d^4\theta \hat{\phi} \cdot \hat{E} \\ &= \int d^4\theta \hat{\phi} \cdot \hat{E} - 2\hat{W}\psi - 2\hat{\bar{W}}\bar{\psi} + \text{s.t.} \\ &= \int d^4\theta \hat{\phi} \cdot E + \frac{1}{4} (\bar{D}\sigma_\mu D - D\sigma_\mu \bar{D}) \phi_\mu L + \frac{3}{8} L^2 + \text{s.t.} \end{aligned} \quad (2.35)$$

We remark en passant that if we perform a duality transformation on the above Lagrangian (by adding the Lagrange multiplier $L(\bar{D}^2 u + D^2 \bar{u})$, and then eliminating L) we recover the old minimal formulation. In fact, treating L as an arbitrary real superfield, Eqs. (2.21), (2.32), and (2.33) along with the chiral superfield $\hat{R} = -\frac{1}{8} \bar{D}^2 L$, represent the most general solution of the Bianchi identities¹² (the identity (2.25) is now modified to $\hat{W}_\alpha = -\frac{1}{2} \bar{D}^{\dot{\alpha}} \hat{E}_{\dot{\alpha}\alpha} + \frac{1}{2} D_\alpha \hat{R}$). In particular, for $L = -\frac{1}{3} (\bar{D}\sigma_\mu D - D\sigma_\mu \bar{D}) \phi_\mu$, we get the multiplets of the old minimal formulation.³⁵

Let us now briefly describe the free massive spin-2 multiplet. By adding the mass term $m^2(\phi^2 - 3\psi\bar{D}^2\psi - 3\bar{\psi}D^2\psi)$ to the Lagrangian (2.35) we get

$$\mathcal{L}_m = \int d^4\theta \left\{ \phi \cdot E + m^2 \phi^2 + \frac{3}{8} L^2 \right\} - \text{Re} \int d^2\theta \left\{ 2\bar{W}D^2\psi + \frac{3}{2} m^2 \bar{D}^2\psi D^2\psi \right\}. \quad (2.36)$$

The equations of motion following from (2.36) are

$$\begin{aligned} \hat{E}_\mu + m^2 \phi_\mu &= 0 \\ \hat{W} + \frac{3}{2} m^2 \bar{D}^2\psi &= 0. \end{aligned} \quad (2.37)$$

The Bianchi identity (2.25) then implies

$$m^2 \sigma_\mu \bar{D} \phi_\mu = m^2 3\bar{D}^2\psi. \quad (2.38)$$

Using this in (2.33), we get $\hat{W} = 0$. Then (2.37) implies $\bar{D}^2\psi = 0$, and we are left with

$$\sigma_\mu \bar{D} \phi_\mu = 0$$

$$E_\mu + m^2 \phi_\mu = 0 \quad \text{or} \quad (-\partial^2 + m^2) \phi_\mu = 0 \quad (2.39)$$

which are precisely the equations for a massive spin-2 multiplet.³⁵

As a final remark, we analyze higher curvature supergravity Lagrangian at the linearized level. Using Eqs. (2.16) and (2.22), we can write the supersymmetric R^2 and conformal Lagrangians as

$$\begin{aligned} \mathcal{L}_{R^2} &= -4 \operatorname{Re} \int d^2\theta \, WW \\ &= R^2 - 2F(A+2V)_{\mu\nu}^2 - 4i\overline{\gamma \cdot R} \not{\partial} \gamma \cdot R \end{aligned} \quad (2.40)$$

and

$$\begin{aligned} \mathcal{L}_c &= 4 \operatorname{Re} \int d^2\theta \, W_{\mu\nu} W_{\mu\nu} = \frac{1}{2} W_{\mu\nu\rho\lambda}^2 - \frac{1}{3} F(A-V)_{\mu\nu}^2 \\ &\quad - 4 \epsilon_{\mu\rho\lambda\nu} \left(\overline{R_\mu + \frac{1}{3} \gamma_\mu \gamma \cdot R} \right) \gamma_5 \gamma_\rho \partial_\lambda \left(R_\nu + \frac{1}{3} \gamma_\nu \gamma \cdot R \right). \end{aligned} \quad (2.41)$$

The general supersymmetric Lagrangian including terms quadratic in the curvatures, is a linear combination of (2.8), (2.40), and (2.41), (see (2.13))

$$\mathcal{L} = \mathcal{L}_R + \alpha \mathcal{L}_{R^2} + \beta \mathcal{L}_c. \quad (2.42)$$

To analyze the propagating degrees of freedom for such a Lagrangian, it is easiest to work directly in terms of superfields. First consider the $\mathcal{L}_R + \alpha \mathcal{L}_{R^2}$ Lagrangian. Using an auxiliary field $\bar{D}^2\psi$, it can be written as

$$\mathcal{L}_R + \alpha \mathcal{L}_{R^2} = \int d^4\theta \, \phi \cdot E - 4\alpha \operatorname{Re} \int d^2\theta \, W^2$$

$$= \int d^4\theta \phi \cdot E - \text{Re} \int d^2\theta \left\{ 2W\bar{D}^2\psi - \frac{1}{4\alpha} \bar{D}^2\psi \cdot \bar{D}^2\psi \right\} . \quad (2.43)$$

Next perform the conformal transformation

$$\phi_\mu \rightarrow \phi_\mu + \bar{D}\sigma_\mu\psi + D\sigma_\mu\bar{\psi} . \quad (2.44)$$

This leads to (see (2.35))

$$\mathcal{L}_R + \alpha\mathcal{L}_R \rightarrow \int d^4\theta \left\{ \phi \cdot E - \frac{3}{8} L^2 \right\} + \frac{1}{4\alpha} \text{Re} \int d^2\theta \bar{D}^2\psi \cdot \bar{D}^2\psi + \text{s.t.} \quad (2.45)$$

which is precisely the Lagrangian for a massless spin-2 and a massive spin-1 (with $m^2 = 1/12\alpha$) multiplets.

Now consider the $\mathcal{L}_R + \alpha\mathcal{L}_C$ Lagrangian. Using the Gauss-Bonnet theorem (2.24) and introducing the Lagrange multipliers ϕ'_μ and $\bar{D}^2\psi'$, it can be written as

$$\begin{aligned} \mathcal{L}_R + \beta\mathcal{L}_C &= \int d^4\theta \phi \cdot E + 4\beta \text{Re} \int d^2\theta W_{\mu\nu}^2 \\ &= \int d^4\theta \{ \phi \cdot E - 2\beta E^2 \} + \frac{4\beta}{3} \text{Re} \int d^2\theta W^2 + \text{s.t.} \\ &= \int d^4\theta \left\{ \phi \cdot E + 2\phi' \cdot E + \frac{1}{2\beta} \phi'^2 \right\} \\ &\quad - \text{Re} \int d^2\theta \left\{ 2W\bar{D}^2\psi' + \frac{3}{4\beta} \bar{D}^2\psi' \cdot \bar{D}^2\psi' \right\} + \text{s.t.} \end{aligned} \quad (2.46)$$

The field redefinition $\phi_\mu \rightarrow \phi_\mu - \phi'_\mu$ leads to

$$\mathcal{L}_R + \beta\mathcal{L}_C \rightarrow \int d^4\theta \phi \cdot E - \int d^4\theta \left\{ \phi' \cdot E' - \frac{1}{2\beta} \phi'^2 \right\} +$$

$$- \text{Re} \int d^2\theta \left\{ 2W\bar{D}^2\psi' - 2W'\bar{D}^2\psi' + \frac{3}{4\beta} \bar{D}^2\psi' \bar{D}^2\psi' \right\} + \text{s.t.} \quad (2.47)$$

Next, by performing a conformal transformation $\phi_\mu \rightarrow \phi_\mu + \bar{D}\sigma_\mu\psi' + D\sigma_\mu\bar{\psi}'$, we finally get

$$\begin{aligned} \mathcal{L}_R + \beta\mathcal{L}_C \rightarrow \int d^4\theta \phi \cdot E - \int d^4\theta \left\{ \phi' \cdot E' - \frac{1}{2\beta} \phi'^2 + \frac{3}{8} L'^2 \right\} + \\ + \text{Re} \int d^2\theta \left\{ 2W'\bar{D}^2\psi' - \frac{3}{4\beta} \bar{D}^2\psi' \bar{D}^2\psi' \right\} + \text{s.t.} \end{aligned} \quad (2.48)$$

Hence we have a physical massless spin-2 and a ghost massive spin-2 (with $m^2 = -1/2\beta$) multiplet (2.36).

In the presence of both $\mathcal{L}_{R2}$ and $\mathcal{L}_C$ we can just perform the above analysis sequentially, as $\mathcal{L}_C$ is invariant under the transformation (2.44). To summarize, the propagating degrees of freedom for the supersymmetric $\mathcal{L}_R + \alpha\mathcal{L}_{R2} + \beta\mathcal{L}_C$ Lagrangian in the new minimal formulation are:

1. A physical massless (2,3/2) multiplet.
2. A ghost massive (2,3/2,3/2,1) multiplet with $m^2 = -1/2\beta$.
3. A physical massive (1,1/2,1/2,0) multiplet with $m^2 = 1/12\alpha$.

The above results have to be contrasted with the analysis for the old minimal formulation done by Ferrara, Grisaru, and van Nieuwenhuizen.³⁴ Instead of (3), they found two physical (1/2,0,0) multiplets with $m^2 = 1/12\alpha$. The massive vector in (3) being replaced by the three scalar modes (S, P and $\partial \cdot A$) of the old minimal formulation.

As the only difference in the analysis for the two minimal formulations is in the auxiliary field parts, it is interesting to analyze it directly in

terms of components. Using (2.42), (2.8), (2.40), and (2.41), the auxiliary field part of the Lagrangian is

$$\begin{aligned}\mathcal{L}_{\text{aux}} &= -V \cdot (2A+V) - 2\alpha F(A+2V)_{\mu\nu}^2 - \frac{\beta}{3} F(A-V)_{\mu\nu}^2 \\ &= -\frac{1}{3} A_1^2 + \frac{1}{3} A_2^2 - 2\alpha F(A_1)_{\mu\nu}^2 - \frac{\beta}{3} F(A_2)_{\mu\nu}^2\end{aligned}\quad (2.49)$$

where

$$A_1 = A + 2V \quad \text{and} \quad A_2 = A - V.$$

Hence, we directly see two vectors of masses $1/12\alpha$ and $-1/2\beta$.

Notice that for $\alpha < 0$, the states in the tachyonic vector multiplet have different norms. Namely, a ghost 1, $1/2$ and physical $1/2$, 0 . This has to be contrasted to the common lore which wants all states in a supermultiplet to be either all ghosts or all physical. A similar phenomenon also occurs for $\beta > 0$ in the massive spin-2 multiplet.

We conclude this section by a remark on the nonlinear multiplets. Even in this case the three curvature multiplets can be combined into a single "chiral" multiplet $R_{ab\alpha}$ which contains the full Riemann tensor in its components. Thus the supersymmetric topological densities continue to have the nice suggestive form in Eq. (2.12).

Since $R_{ab\alpha}$ carries also dotted spinor indices, it cannot be a superconformal chiral multiplet.¹⁰ However, since the compensator L is linear, we can define L -chiral multiplets with arbitrary external Lorentz indices.¹⁰ An L -chiral multiplet $S_{\alpha_1 \dots \alpha_n \dot{\beta}_1 \dots \dot{\beta}_n}$ is defined by the equation

$$\bar{D}_{\dot{\gamma}}^{(L)} S_{\alpha_1 \dots \alpha_n \dot{\beta}_1 \dots \dot{\beta}_m} = 0 \quad (2.50)$$

where $\bar{D}_{\dot{\gamma}}^{(L)}$ is the L-associated superconformal derivative, which is explicitly defined in Ref. 10. $\bar{D}_{\dot{\gamma}}^{(L)}$ has four properties which are useful here¹⁰

1. $L^{-1/4} \bar{D}_{\dot{\gamma}}^{(L)}$ can be identified with the spinorial derivative $\bar{\mathcal{D}}_{\dot{\gamma}}$ of the flexible superspace geometry corresponding to the new-minimal case.
2. $\bar{D}_{\dot{\gamma}}^{(L)} L = D_{\gamma}^{(L)} L = 0$. (2.51)
3. Acting on a superconformal multiplet with just undotted spinor indices and with weights $w = n$, $\bar{D}_{\dot{\gamma}}^{(L)}$ is the same as the usual conformal spinor derivative. (Thus any superconformal chiral multiplet is also an L-associated chiral multiplet.)

$$4. \{ \bar{D}_{\dot{\alpha}}^{(L)}, \bar{D}_{\dot{\beta}}^{(L)} \} = \{ D_{\alpha}^{(L)}, D_{\beta}^{(L)} \} = 0. \quad (2.52)$$

Because of Eq. (2.23), $R_{ab\alpha}$ --if it exists at all--contains the following irreducible multiplets $W_{\alpha\beta\gamma}$, W_{γ} and $G_{\alpha\beta\gamma} = G_{\beta\alpha\gamma}$. The first two are the usual Weyl and "scalar curvature" multiplets whose existence is well known.^{12*} What remains to be shown is that there exists an L-chiral superconformal multiplet $G_{\alpha\beta\gamma}$, containing the Ricci part of the Riemann tensor, with weights $w = n = 3/2$, such that its square is an ordinary superconformal chiral multiplet ($w = n = 3$) as is required by the superconformal action formula

$$[\epsilon^{abcd} R_{ab}{}^{\alpha}{}_{cd\alpha}]_F \quad (2.53)$$

*They are not just L-chiral but also superconformally chiral.

to make sense. Defining the superconformal curvatures $E_{\alpha\alpha}^{\bullet}$ and W_{α} to have the weights (1,0) and (3/2,3/2), respectively, the Bianchi identities are

$$\begin{aligned}\bar{D}^{(L)\dot{\alpha}} E_{\alpha\alpha}^{\bullet} &= -2W_{\alpha} \\ \bar{D}_{\dot{\alpha}}^{(L)} W_{\alpha} &= 0.\end{aligned}\tag{2.54}$$

From Eqs. (2.54) and (2.52), we get

$$\begin{aligned}\bar{D}^{(L)\dot{\beta}} \bar{D}^{(L)\dot{\alpha}} E_{\alpha\alpha}^{\bullet} &= 0 \\ \Rightarrow \bar{\Sigma}^{(L)} E_{\alpha\alpha}^{\bullet} &= \Sigma^{(L)} E_{\alpha\alpha}^{\bullet} = 0\end{aligned}\tag{2.55}$$

where $\Sigma^{(L)}$ is the L-associated chiral projection¹⁰

$$\Sigma^{(L)} = \bar{D}^{(L)\dot{\alpha}} \bar{D}_{\dot{\alpha}}^{(L)}.\tag{2.56}$$

Then $E_{\alpha\alpha}^{\bullet}$ is a real L-linear conformal multiplet.*

Consider the superconformal multiplet

$$G_{\alpha\beta\gamma}^{\bullet} = -\frac{1}{8} (\bar{D}_{\dot{\alpha}}^{(L)} E_{\beta\gamma}^{\bullet} + \bar{D}_{\dot{\beta}}^{(L)} E_{\alpha\gamma}^{\bullet}).\tag{2.57}$$

Then

*But not linear.

$$\bar{D}_{\dot{\gamma}}^{(L)} G_{\dot{\alpha}\dot{\beta}\gamma} = \frac{1}{16} \left\{ \epsilon_{\dot{\gamma}\dot{\alpha}} \Sigma^{(L)}(E_{\dot{\beta}\gamma}) + \epsilon_{\dot{\gamma}\dot{\beta}} \Sigma^{(L)}(E_{\dot{\alpha}\gamma}) \right\} = 0 \quad (2.58)$$

by Eq. (2.55). Thus $G_{\dot{\alpha}\dot{\beta}\gamma}$ is L-chiral and so it is $R_{ab\alpha}$. Now, $G_{\dot{\alpha}\dot{\beta}\gamma} G^{\dot{\alpha}\dot{\beta}\gamma}$ is a scalar superconformal multiplet with $w = n = 3$. By property (3) and Eq. (2.58)

$$\bar{D}_{\dot{\delta}} (G_{\dot{\alpha}\dot{\beta}\gamma} G^{\dot{\alpha}\dot{\beta}\gamma}) = \bar{D}_{\dot{\delta}}^{(L)} (G_{\dot{\alpha}\dot{\beta}\gamma} G^{\dot{\alpha}\dot{\beta}\gamma}) = 0 \quad (2.59)$$

and thus

$$[G_{\dot{\alpha}\dot{\beta}\gamma} G^{\dot{\alpha}\dot{\beta}\gamma}]_F \quad \text{and} \quad [R_{ab\alpha} R_{cd}^{\alpha}]_F \epsilon^{abcd} \quad (2.60)$$

are both superconformal invariant action densities at the full nonlinear level.

III. THE EQUIVALENCE OF NEW MINIMAL HIGHER DERIVATIVE SUPERGRAVITY AND STANDARD SUPERGRAVITY COUPLED TO MATTER

In the previous section we saw that at the linearized level the new minimal " $R + \alpha R^2$ " supergravity propagates, besides the usual massless fields of the gravitational multiplet a massive vector multiplet which is physical for $\alpha > 0$. In this section we show that this result holds at the full nonlinear level and explicitly construct the self-coupling of the vector multiplet in the equivalent theory.

The analog theorem for the old minimal formulation of $N = 1$ supergravity can be found in Ref. 6. Similar arguments were also used in Ref. 29 to rewrite in the standard supergravity form the higher derivative theory involving linear multiplets. Here the situation is exactly the same when the linear multiplet of Ref. 29 is identified with the superconformal compensator L .

First of all we need a superconformal expression for the chiral multiplet W_α in terms of the compensating linear multiplet L which replaces the equation of the old minimal scheme

$$R = S_0^{-1} \Sigma(\bar{S}_0) \quad (3.1)$$

which was exploited in Ref. 6 in order to prove the equivalence. The superconformal expression of W_α is

$$W_\alpha = \Sigma D_\alpha \ln(L/S_0 \bar{S}_0) \quad (3.2)$$

where Σ and D_α are the superconformal operations defined by Kugo and Uehara.¹⁰ Notice that W_α , as given by Eq. (3.2) does not depend on S_0 , as it should.

The right-hand side of Eq. (2) is obviously chiral and has the correct Weyl and chiral weights, $w = n = 3/2$. The simplest way to see that Eq. (3.2) is correct, is to look for the equation we obtain by varying the Lagrangian of pure supergravity

$$[L \ell n(L/S_0 \bar{S}_0)]_D \quad (3.3)$$

with respect to the compensator L . This equation is, by definition, the "trace" of the supersymmetric Einstein equations, i.e. the "scalar curvature multiplet." In fact the variation gives

$$\Sigma D_\alpha \ell n(L/S_0 \bar{S}_0) = 0 . \quad (3.4)$$

Thus the " $R + \alpha R^2$ " supergravity in the new minimal formulation is

$$\mathcal{L} = [L \ell n(L/S_0 \bar{S}_0)]_D + \frac{\alpha}{2} ([W^\alpha_\alpha (\ell n L)]_F + \text{h.c.}) . \quad (3.5)$$

We can rewrite Eq. (3.5) in the equivalent form

$$\mathcal{L} = [L \ell n(L/S_0 \bar{S}_0)]_D - \frac{\alpha}{2} ([M^\alpha_\alpha - 2M^\alpha_\alpha (\ell n L)]_F + \text{h.c.}) \quad (3.6)$$

where M_α is a chiral-spinor multiplet with weights $w = n = 3/2$. Eq. (3.5) is recovered by eliminating M_α from Eq. (3.6) by using its equations of motion

$M_\alpha = W_\alpha (\ell n L)$. Now,

$$\left[M^\alpha_\alpha \left(\ell n \frac{L}{S_0 \bar{S}_0} \right) \right]_F + \text{h.c.} =$$

$$= [(D_{\alpha} M^{\alpha}) \ln(L/S_0 \bar{S}_0)]_D + \text{h.c.} + \text{surface terms} \quad (3.7)$$

$$= [M \ln(L/S_0 \bar{S}_0)]_D + \text{s.t.} ,$$

where the linear multiplet M is defined to be

$$M = D_{\alpha} M^{\alpha} + \bar{D}^{\dot{\alpha}} \bar{M}_{\dot{\alpha}} . \quad (3.8)$$

Then Eq. (3.6) becomes

$$\mathcal{L} = [(L + \alpha M) \ln(L/S_0 \bar{S}_0)]_D - \frac{\alpha}{2} ([M^{\alpha} M_{\alpha}]_F + \text{h.c.}) \quad (3.9)$$

which is a standard supergravity coupled to a massive vector multiplet.²⁹

Indeed, by a duality transformation, Eq. (3.9) can be recast in the old minimal formulation

$$\mathcal{L} = \left[F \left(\frac{\alpha M}{S_0 \bar{S}_0} \right) S_0 \bar{S}_0 \right]_D - \frac{\alpha}{2} ([M^{\alpha} M_{\alpha}]_F + \text{h.c.}) \quad (3.10)$$

where the function $F(x)$ is defined by the Legendre-like transformation

$$F(x) = (y+x) \ln y - y \quad \text{at} \quad y \ln y + x = 0 . \quad (3.11)$$

Now we perform a further duality transformation [for details, see Ref. 29, Section IV, Eqs. (33)-(38) and Section III, Eqs. (25) and (26)]. According to Ref. 29, the Lagrangian in Eq. (3.10) is equivalent to

$$\mathcal{L} = -[\Phi(V) S_0 \bar{S}_0]_D + \left\{ \frac{\alpha}{2} [W^{\alpha}(V) W_{\alpha}(V)]_F + \text{h.c.} \right\} \quad (3.12)$$

where

$$-\Phi(z) = F(x) + zx \quad \text{at} \quad F'(x) + z = 0 . \quad (3.13)$$

Comparing Eqs. (3.13) and using well known properties of the Legendre transform, we get

$$\Phi(z) = (1+z) e^{-z} . \quad (3.14)$$

By making the field redefinition

$$V \Big|_{\text{old}} = -\ln S - \ln \bar{S} - V \Big|_{\text{new}} , \quad (3.15)$$

then Eq. (3.12) can be rewritten as

$$\mathcal{L} = -[S e^V \bar{S} \{1 - \ln(S e^V \bar{S})\}]_D + \left(\frac{\alpha}{2} [W^\alpha(V) W_\alpha(V)]_F + \text{h.c.} \right) \quad (3.16)$$

which is exactly in the standard supergravity form, with Kähler G functions

$$G(S, e^{V-\bar{S}}) = -3 \ln S - 3 \ln(e^{V-\bar{S}}) - 3 \ln[1 - \ln S - \ln(e^{V-\bar{S}})] \quad (3.17)$$

$$f_{AB} = \frac{\alpha}{2} \delta_{AB} . \quad (3.18)$$

Notice that whereas the vector's kinetic term depends linearly on α , the chiral interactions do not, in agreement with our discussion in the previous section.

As in the old minimal case, Ref. 6, this result can be generalized in various ways, namely (i) by adding couplings to matter; (ii) allowing for general functions of $W_\alpha(\ell n L)$, (iii) by adding still higher derivative terms. Here we limit ourselves to the first case. The other two generalizations can also be easily obtained by using in a recursive way the methods and results of Ref. 6 and Ref. 29, especially those of Section V.

The most general Lagrangian in which the only higher derivative term is quadratic in $W_\alpha(\ell n L)$ is²⁷

$$\begin{aligned} \mathcal{L} = & [L \ell n \{F_1(L, S_1, e^{g_A V_A \bar{S}_1}) / S_0 \bar{S}_0\} + L F_2(L, S_1, e^{g_A V_A \bar{S}_1}) \\ & + q_A^L V_A]_D + \left([g(S_1)]_F - [f_{AB}(S_1) W^A W^B]_F - [h(S_1) L^\alpha L_\alpha]_F \right. \\ & \left. - \frac{1}{2} [\eta(S_1) W^\alpha(\ell n L) W_\alpha(\ell n L)]_F + \text{h.c.} \right). \end{aligned} \quad (3.19)$$

We can put $h(S_1) = 0$ without any loss of generality, as follows from the equivalence of the Lagrangians in Eq. (3.3) and Eq. (3.7) of Ref. 29. In Eq. (3.19), F_1 , F_2 , g , f_{AB} and η are gauge invariant functions with weights (2,0), (0,0), (3,3), (0,0) and (0,0) respectively. By the same argument as in Eqs. (3.6) through (3.9), Eq. (3.9) can be rewritten as (up to a surface term)

$$\begin{aligned} & [L \ell n (F_1 / S_0 \bar{S}_0) + L F_2 + q_A^L V_A]_D + [M \ell n (L / S_0 \bar{S}_0)]_D \\ & + \left(\frac{1}{2} [\eta(S_1)^{-1} M^\alpha M_\alpha]_F + [g(S_1)]_F - [f_{AB}(S_1) W^A W^B]_F + \text{h.c.} \right) \end{aligned} \quad (3.20)$$

where the linear multiplet M is defined by Eq. (3.8). Again, eliminating L from Eq. (3.20) by a duality transformation,²⁹ we get a Lagrangian of the form

$$\begin{aligned} \mathcal{L} = & [\psi(M S_0^{-1} e^{-q_A V_A} \bar{S}_0^{-1}, S_1 S_0^{-W_1}, e^{(g_A - W_1 q_A) V_A} \bar{S}_1 \bar{S}_0^{-W_1}) S_0 e^{q_A V_A} \bar{S}_0]_D \\ & + \left(\frac{1}{2} [\eta(S_1)^{-1} M^\alpha_{\alpha}]_F + [g(S_1)]_F - [f_{AB}(S_1) W^{A\alpha} W^B_{\alpha}]_F + \text{h.c.} \right) \end{aligned} \quad (3.21)$$

where W_1 is the Weyl weight of the chiral multiplet S_1 .

By a further duality transformation, this is rewritten as (see Ref. 29, Sections IV and V for details)

$$\begin{aligned} \mathcal{L} = & -[\phi(\Sigma e^{V_{\bar{E}}}, S_1 S_0^{-W_1}, e^{(g_A - W_1 q_A) V_A} \bar{S}_1 \bar{S}_0^{-W_1}) S_0 e^{q_A V_A} \bar{S}_0]_D \\ & + ([g(S_1)]_F - \frac{1}{2} [\eta(S_1) W^\alpha(U) W_A(U)]_F - [f_{AB}(S_1) W^{A\alpha} W^B_{\alpha}]_F + \text{h.c.}) \end{aligned} \quad (3.22)$$

where the massive vector multiplet $U + \ell n \Sigma + \ell n \bar{\Sigma}$ is dual to the massive linear multiplet M .²⁹ Eq. (22) is manifestly in the standard supergravity form, with Kähler-invariant functions

$$G = -3 \ell n \phi \quad \text{and} \quad f_{\alpha\beta} = \left(f_{AB}, \frac{1}{2} \eta(S_1) \right).$$

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