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C. NGO, R. BOISGARD
Service de Physique Nucléaire - Métrologie Fondamentale
CEN Saclay, 91191 Gif-sur-Yvette Cedex, France

J. DESBOIS
Institut de Physique Nucléaire, Division de Physique Théorique*,
91406 Orsay, France

J. NEMETH
Institute for Theoretical Physics, Eötvös Universität, Budapest, Hungary

M. BARRANCO
Universitat de Barcelona, Facultat de Física, Barcelona, Spain

J.F. MATHIOT**
CERN, Division TH, CH-1211 Genève 28

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C. NGO, R. BOISGARD
Service de Physique Nucléaire - Métrologie Fondamentale
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J. DESBOIS
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91406 Orsay, France

J. NEMETH
Institute for Theoretical Physics, Eötvös Universität, Budapest, Hungary

M. BARRANCO
Universitat de Barcelona, Facultat de Física, Barcelona, Spain

and

J.F. MATHIOT**
CERN, Division TH, CH-1211 Genève 28

We apply a simple model, based on irrotational hydrodynamics and percolation, to study hot and compressed nuclei disassembly. We study the different conditions under which an excited nucleus breaks up into several pieces (multifragmentation). A simple percolation-aggregation model is also developed in order to calculate the conditions under which one can form a quark-gluon plasma.

One of the present interest in heavy ion physics is to look at critical phenomena, to study in which conditions they may occur, what are their properties, and what will be their experimental signature. There is an interest in this field from moderate up to extremely large bombarding energies. In order to observe a critical phenomenon one has to vary the conditions of temperature and density of a piece of nuclear matter by choosing properly the projectile, target and bombarding energy. There are two kinds of critical phenomena which one wants to observe in nuclear physics. They correspond to conditions which are schematically shown in figure 1 in the temperature-density plane :

The first one is associated to densities smaller than normal and moderate temperatures. When these conditions are fulfilled the nucleus under consideration breaks up into several pieces. This process is called multifragmentation. Many experiments seem to indicate that this mechanism exists¹.

*Laboratoire associé au CNRS.

**On leave from Division de Physique Théorique, Orsay, France.

FIGURE 1

Schematic drawing in the density-temperature (ρ - T) plane of the expected phases of nuclear matter.

The second critical phenomenon is a challenge for nuclear and particle physicists. It has not yet been discovered but it has been predicted theoretically : at large densities and temperatures one expects to observe the deconfinement of quarks, a situation which has probably occurred in the early stages of the universe. When occurring, there will be a phase transition from a hadronic plasma to a quark-gluon plasma².

A theoretical understanding of both critical phenomena is very difficult if one stays at a microscopic basis. However, one may ask oneself what are the simplest assumptions which are necessary to get them. This contribution is an attempt in this direction. We shall show that with very simple hypotheses one can calculate under which conditions a nucleus breaks up, or a hadronic plasma becomes color conductor.

1. MULTIFRAGMENTATION OF NUCLEI

When a high energetic proton (> 2 GeV) goes through a nucleus it can break it up³. This also occurs in the collision of two heavy ions but at lower bombarding energy per nucleon ($E/A > 30$ -50 MeV/u) [ref.⁴]. In view of these observations one would like to know why and how nuclei break up.

Emulsion data give a direct evidence of nuclei that undergo multifragmentation⁵. The large number of medium mass fragments which are produced in such events could not be explained by a standard evaporation of a very excited nucleus. Most of the experiments are however inclusive. One of their important outcome is the mass distribution of the multifragmentation products which fol-

lows a $A^{-\tau}$ law, where A is the fragment mass and τ an exponent which takes values around 2-3.

Several models try to describe multifragmentation but a full microscopic theory is not yet available because the problem is very difficult. One has to go beyond a mean field approach because the correlations between the nucleons become of outmost importance for a proper evolution of the system which undergoes multifragmentation. These correlations will be referred to as the fluctuations of the mean field. Most of the models start with an initial excited nucleus in global statistical equilibrium which is assumed to be formed in the first stage of the collision¹. We shall describe such a model^{6,7} and particularly try to answer the following question : what happens to a hot and compressed nucleus in global statistical equilibrium? This problem is interesting for heavy ion collisions at intermediate bombarding energy where one can form such species. The model we shall describe is however less appropriate for a multifragmentation induced by high energy protons because the mechanism might be a bit different there : the proton will cascade through the nucleus, eject nucleons and the nucleus will eventually break up if it is too much destabilized.

1.1 Fluctuations of the mean field

Nuclear forces are like Van der Waals forces : they have an attractive part and a repulsive core. Their range is much shorter than the Coulomb forces which are repulsive. The fact that in a nuclear medium one has short range attractive forces and long range repulsive ones implies that the correlations between the nucleons will be important when the density of the medium decreases. In this latter case it means that a pure mean field approach will break down. In order to understand the importance of nucleon correlations in nuclei let us consider a volume V in which we put A nucleons, neutrons and protons.

If V is of the order of the volume of an ordinary nucleus, each nucleon can interact by nuclear forces with all its neighbours. This is schematically shown in figure 2a. In such a situation the physics of the system is dominated by the mean field created by the whole set of nucleons and one says that the fluctuations of the mean field are small. Clusters might be present in the nuclear medium but their probability is small. The nucleus can be viewed as a big cluster whose mean field is practically identical to the one of the total system. A nuclear signal can easily propagate from any part of the nucleus to another one. Therefore one can say that the correlation distance is large, of the order of the size of the nucleus.

If the volume V is bigger (figure 2b), each nucleon can no longer interact with all its neighbours by nuclear forces because of the short range of this interaction. Clusters of nucleons can be formed and the physics of each of them is dominated by their own mean field. However, the physics of the whole system

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FIGURE 2

Schematic behaviour of nuclear matter when the volume of a nucleus increases. In figure 2b the clusters can "talk" to each other by Coulomb forces but not by nuclear forces.

is no longer dominated by the mean field of the largest cluster. Furthermore the Coulomb field, which has a long range, will push the clusters apart from each other. This will lead to a break up of the initial system (multifragmentation). In the above situation one says that the fluctuations of the mean field are large. When this occurs, instabilities develop exponentially and the system breaks up. When the fluctuations are large the correlation distance suddenly decreases and the information about the nuclear forces remains trapped within the clusters. Clusters can "talk" to other clusters only through the Coulomb field which has a tendency to pull them apart.

In a given nucleus clusters are formed and destroyed as a function of time. The probability to obtain a given cluster could be obtained by making a time average of the concentration of this cluster. Alternatively, one can consider an ensemble of nuclei, which are identical at the macroscopic level, and evaluate the cluster probability at a given instant. In this case one makes an ensemble average and one can postulate that it is equal to the time average (ergodic hypothesis). If one proceeds like this, the evaluation of the fluctuations of the mean field is identical to a percolation problem.

1.2 Physical picture

Central and medium impact parameters heavy ion collisions at intermediate bombarding energies can be roughly decomposed in two main phases :

- In the first one, the projectile and target interact very strongly. The interaction zone is quickly excited but a statistical equilibrium is not achieved in the early stages, and energetic particles are emitted. A preequilibrium model developed in ref.⁸ shows that one needs about 10^{-22} s to reach global thermal equilibrium.
- In the second phase, the excited system formed previously expands and cools down. Two situations may occur depending on the amount of excitation energy of the nucleus. If the excitation energy is not too large, there will be a succession of oscillations around a mean configuration while the system evaporates particles. In this case the nucleus de-excites normally by particle emission and/or fission. Above a certain excitation energy value, the nucleus expands so much that the mean field fluctuations become suddenly large. In this case the nucleus will undergo multifragmentation.

The way we want to describe multifragmentation is equivalent to the problem schematically described in figure 3. Initially one has a certain number of balls which occupy an equal number of boxes. The balls play the role of the nucleons and the ensemble of boxes corresponds to the nucleus. An expansion of

FIGURE 3

Pictorial representation of the physical picture of an expanding nucleus which breaks up.

such a system corresponds to an increase of the number of boxes available while the number of balls remains constant. In the case of nuclei this expansion will be described by irrotational hydrodynamics. In this picture one can define a cluster of balls as a set of balls located in neighbouring boxes. If the expansion is not too large, one has always a big cluster which extends almost over the whole system. However, above a critical value of the expansion this will no longer be the case and one will have medium or small size clusters only. This corresponds to multifragmentation in the nuclear case. The existence or not of a big cluster is related to the fluctuations of the mean field and will be evaluated within a percolation approach.

1.3 Hydrodynamical expansion

We have used a time dependent Thomas Fermi formalism developed in ref.⁹ to describe the dynamical evolution of spherical hot and compressed nuclei. This approach is just irrotational hydrodynamics in which the internal energy is evaluated within the Thomas Fermi approximation using a Skyrme interaction. Since no dissipation is included, the expansion of the nucleus follows an isentrope. This assumption is supported by cascade calculations¹⁰ which indicate that entropy is created only in the first phase of the collision and that one has an isentropic expansion of the excited system. Using a change of variable due to Madelung¹¹ the hydrodynamical equations for neutrons and protons are transformed into non linear Schrödinger equations which are solved by the methods developed for time dependent Hartree-Fock.

The initial conditions are obtained by a static self consistent Thomas Fermi calculation. This allows two different types of excitation energy to be studied : thermal energy which corresponds to desorganized excitation, and compressional energy which is of collective nature. The initial compression of the system is fixed by constraining the mean square radius $\langle r^2 \rangle$. If λ is the Lagrange parameter associated to this constraint, and T the temperature, the total excitation energy of the nucleus is equal to :

$$E^*(\lambda, T) = E(\lambda, T) - E(0, 0) \quad (1)$$

where $E(\lambda, T)$ is the total energy of the nucleus associated with the parameters λ and T . $E^*(\lambda, T)$ can be separated in a thermal part E_T^* , and a compressional part E_C^* :

$$E^*(\lambda, T) = E_T^* + E_C^* \quad (2)$$

which are respectively defined by :

$$E_T^* = E(\lambda, T) - E(\lambda, 0) \quad (3)$$

and

$$E_C^* = E(\lambda, 0) - E(0, 0) \quad (4)$$

From the above quantities we define the associated values per nucleon :

$$\epsilon^* = \frac{E^*}{A} ; \epsilon_T^* = \frac{E_T^*}{A} \quad \text{and} \quad \epsilon_C^* = \frac{E_C^*}{A} \quad (5)$$

where A is the mass of the nucleus under consideration. At each stage of the evolution of the system, we check the importance of the fluctuations of the mean field using the percolation approach to be described below. When they become large we stop the hydrodynamical calculation because it is no longer applicable in this context : one has multifragmentation. In this latter situation one calculates the mass distribution of the multifragmentation products by means of the percolation model.

1.4 Percolation

Percolation is one of the simplest statistical approach which allows critical phenomena to be described in various areas. It has been applied to several problems in solid state physics like binary alloys, gelation, polymerisation, etc... It has also been used in other fields, for example studying the extension of forest fires or the propagation of diseases. Let us briefly recall the main characteristics of this process.

A percolation system is essentially defined by two ingredients : a collection of points and a rule to connect them. The percolation model that we have used to evaluate the fluctuations of the mean field is a 3-dimensional site-bond percolation model based on a cubic lattice¹³. We shall describe briefly the basis of the model on the 7×7 two dimensional cubic lattice displayed in figure 4. Initially (figure 4a) all the sites of the lattice are occupied by a particle and one has $N_0=49$ occupied sites. Each particle is connected to its close neighbours by a bond which is represented in the figure by a thick piece of line. For the present example there are $L_0=84$ bonds. If we pick up at random N_0-N particles and break L_0-L bonds, there remains N occupied sites and L unbroken bonds. One introduces two parameters which are : the concentration of occupied sites $p = N/N_0$ and the concentration of unbroken bonds $q = L/L_0$. These two variables take values in the interval $[0,1]$. A cluster is defined as a set of connected points. Let us consider a typical example, displayed in figure 4b, where $N=15$ and $L=10$. For this particular case, one sees that one has a cluster of size 4, 3 clusters of size 3 and 2 single particles; If $N_{\max}$ is the size of the largest cluster, it is convenient to introduce the dimensionless mass $P(p,q) = N_{\max}/N_0$. This quantity varies between 0 and 1. It turns out that $P(p,q)$ has a strong variation in the p - q plane. This is illustrated for a 5×5

FIGURE 4

Principle of a site-bond percolation approach shown on a 7×7 lattice (see text).

3-dimensional cubic lattice in figure 5 where $P(p,q)$ is plotted for different values of q . One observes that when p and q are close to unity, $P(p,q)$ is also close to unity, while it suddenly decreases as p and q decrease. It means that when p and q are close to unity, one has always a large size cluster which is called percolation cluster. When p and q are small the percolation cluster disappears and one has only small or medium-size clusters. This defines two regions in the p - q plane which are shown in figure 6. In the multifragmentation region the percolation cluster has disappeared and one has only several small or medium-size clusters. When applied to the nuclear case, region I corresponds to the case where the fluctuations of the mean field are small while region II corresponds to the case where they are large. In this latter situation the system will undergo multifragmentation whereas, in the first situation, it will

FIGURE 5

Dimensionless size of the largest cluster displayed as a function of p and q .

FIGURE 6

Percolation cluster region (I) and multifragmentation regions (II) in the p - q plane. The hatched area is a transition region due to the finite size of the system.

de-excite normally, by particle evaporation and/or fission. It is interesting to notice the existence of a transition region between I and II (hatched part in figure 6) which occurs because we have a finite system. For an infinite system the transition is very sharp and the percolation threshold is well defined.

In the case of nuclei, which we are interested in, instead of picking up particles in a fixed lattice, we increase the size of the lattice and keep the total number of particles fixed. However, this does not change the above conclusions. Since we have to deal with particles which may carry a charge (protons), it is interesting to discuss more precisely the respective role of the fluctuations of the mean field and the Coulomb repulsion. The average size of the largest cluster is analogous to the spontaneous magnetization in an Ising model. Therefore it is a measure of the importance of the mean field. It has a large value in region I of figure 6 and drops very rapidly when we go through the transition region (hatched area). It is interesting to define the "susceptibility" of the system by performing the sum :

$$S_2(p,q) = \sum_A' A^2 n(A,p,q) \quad (6)$$

where $n(A,p,q)$ is the normalized mass distribution of the products and A is the cluster mass. The prime superscript of the summation symbol means that the largest cluster occurring in each simulation event is not included in the sum. The function S_2 is, in some way, a measure of the fluctuations of the mean field and has a rather sharp maximum, as a function of p , in the transition region (see figure 4 in ref.⁷). For an infinite system, S_2 diverges at the percolation threshold but, for a finite system, it does not. Let us now estimate the mean Coulomb energy between the clusters defined by :

$$V_C(p,q) = \left\langle \sum_{i<j} e^2 \frac{Z_i Z_j}{r_{ij}} \right\rangle \quad (7)$$

where e is the elementary charge and r_{ij} the distance separating the center of mass of two clusters of size A_i and A_j . For the atomic numbers Z_i we have taken $Z_i = A_i/2$. The sum is performed over all the cluster pairs occurring in one event and the bracket means that we have averaged over the ensemble of all simulation events. The variation of $V_C(p,q)$ as a function of p (for different q values) is shown in figure 7. The arrow (for $q > 0.4$) indicates the location of the maximum of $S_2(p,q)$. For p and q values corresponding to region I we see that $V_C(p,q)$ is small. Consequently, the fluctuations of the mean field are

FIGURE 7

Coulomb energy of the fragments as a function of p for different values of q . The arrow indicates the region where the susceptibility is maximum.

small and a standard mean field theory can be used to describe the evolution of the system. In the transition region the fluctuations grow enormously. This is also the case of $V_C(p,q)$ which induces repulsive forces which will play a major role in the break up of the system. In such a situation a mean field theory can no longer be used to describe the evolution of the system : one has to stop the hydrodynamical calculation and use the percolation approach to predict the mass distribution of the resulting clusters.

1.5. Results

We have now all the tools to study the evolution of hot and compressed nuclei. One can vary independently the thermal and compressional excitation energies and compare their influence on the stability of nuclei towards multifragmentation. The hydrodynamical model is used to describe the expansion of the system. At each stage of the evolution one checks the fluctuations of the mean field with the percolation model. If they grow large the system breaks up. The connection between hydrodynamics and percolation is done through the parameters p and q which are function of time. We have defined them by :

$$p(t) = \frac{\langle \rho \rangle}{\langle \rho_0 \rangle} \quad \text{and} \quad q(t) = 1 - \frac{\epsilon_T^*}{B(t=0)} \quad (8)$$

where $\langle \rho \rangle$ and $\langle \rho_0 \rangle$ are the average density at time t and $t=0$, respectively, and $B(t=0)$ is the binding energy per nucleon of the nucleus at time $t=0$. Let us now discuss a few results :

- In figure 8 we display a kind of phase diagram, for ^{208}Pb , showing the region of multifragmentation instability as a function of ϵ_T^* and ϵ_C^* . If $\epsilon_C^* = 0$ one needs $\epsilon_T^* = 5$ MeV in order that the nucleus breaks up. However if $\epsilon_T^* = 0$ MeV one needs $\epsilon_C^* = 1.5$ MeV only. This shows that, for the same amount of excitation energy, one breaks up a nucleus more easily by compression than by thermal excitation.

FIGURE 8
Multifragmentation and normal de-excitation regions calculated for the ^{208}Pb nucleus as a function of ϵ_T^* and ϵ_C^* .

- For two values of ϵ_C^* we have calculated the ratio between the excitation energy per nucleon ($\epsilon^* = \epsilon_T^* + \epsilon_C^*$), and the binding energy per nucleon, B , of the nuclei in their ground state, above which a nucleus undergoes multifragmentation (figure 9). It there is no compression one observes that the maximum ϵ_T^* value a nucleus can sustain without breaking up, is equal to 70 % of B . This value is notably decreased when some compressional energy is involved. For instance if $\epsilon_C^* = 0.5$ MeV the ratio decreases to about 40-50 %.

- At the point where a nucleus breaks up it is interesting to note that the nucleus has not expanded too much. This is illustrated for a typical example in figure 10 which shows the initial density profile (full line) and the one where

FIGURE 9
 ϵ^*/B , where B is the binding energy per nucleon of the ground state nucleus and ϵ^* the total excitation energy, is plotted as a function of A for $\epsilon_C^* = 0$ MeV (no compression) and $\epsilon_C^* = 0.5$ MeV.

FIGURE 10
Initial density profile (full line) and density profile at the multifragmentation point (dashed curve) for a typical example.

multifragmentation takes place (dashed line). One sees that, in this latter case, the central density is still 0.1 nucleon/fm³. The mean square radius $\langle r^2 \rangle^{1/2}$ has changed from = 4.94 fm, for the initial density, to 6.2 fm at the point of instability. For this particular example the volume of the system has doubled. It is also worth to note that the temperature of the system, which was initially equal to 6 MeV has decreased to 3.4 MeV at the multifragmentation point due to isentropic expansion. The time needed for a nucleus to undergo multifragmentation is also very short : it takes about $1-1.5 \times 10^{-22}$ s from the initial configuration to reach the multifragmentation stage.

2. FROM HADRONIC MATTER TO THE QUARK-GLUON PLASMA

One of the present challenge of nuclear and particle physicists is to know if quarks can be deconfined. One way to look into this problem is to study heavy ion collisions at ultrarelativistic energies. Indeed, if the temperature and the compression of a piece of nuclear matter is large enough, one may think that a phase transition between a hadronic and a quark-gluon plasma can occur². Such extreme conditions might be achieved in ultrarelativistic heavy ion collisions. The two simplest questions one may ask are the following :

- Under which conditions can we get this transition?
- How will the quark-gluon plasma "hadronize" during its expansion and cooling, and what are the experimental signals to detect it?

The second question is the most difficult but its answer is needed by the experimentalists in their hunting for the quark-gluon plasma. One can try to answer the first one within the framework of a very simple model based on percolation-aggregation¹⁴. Indeed in quarks deconfinement one has a transition from a color insulator to a color conductor medium which could be described as a percolation process¹⁵. Following this idea, and using our experience in the description of nuclei disassembly, we have developed a very simple geometrical approach to quark deconfinement.

2.1 Physical picture

A nucleon contains 3 quarks enclosed in a bag of radius R_{bag} . This latter quantity is the free parameter of the problem. Let us for the moment consider a nucleus of radius R (volume V) and imagine a process which compresses the nucleus at temperature $T=0$. Then R (and V) decreases as the compression increases. If the compression is large enough, all the nucleon bags will overlap to form a color conductor : one will have reached quark deconfinement. In order to evaluate the critical density above which one reaches a color conductor we propose the following picture :

- We construct the nucleus by putting nucleons at random inside the volume V . Each time we put a new nucleon we check if it overlaps with a bag already present in the volume V . If it is so, we form a bigger bag containing all the quarks which were in the overlapping bags (we assume that the volume of the new bag is proportional to the number of quarks it contains). Depending on the volume V (which is related to the compression of the system) and on R_{bag} , one will get either a percolation cluster (in which case the system becomes color conductor and we have formed the quark-gluon plasma), or one will have several small-and medium-size bags (in which case one remains color insulator). R_{bag} is the key parameter of this model : if R_{bag} increases, the density at which the quark-gluon plasma is formed decreases and vice-versa.

- At finite temperature the above picture has to be modified because many pions will be created. This can be done very simply if one notices that the pion concentration can be evaluated in the same way as the number of photons in the black-body problem, apart from the fact that pions have a finite mass. Such a prescription was already proposed by Fermi¹⁶. Therefore one adds to the nucleons new bags associated to the pions. For the sake of simplicity we have assumed that the volume of a pion bag is 2/3 of the volume of a nucleon. Since extra bags coming from the pions are to be taken into account, it follows that the critical density above which quarks are deconfined decreases at finite temperature compared to the $T=0$ case.

2.2. Results

The process described in the preceding section for a nucleus at zero or finite temperature has been repeated several times to generate an ensemble of events. This gives a distribution of bags of different sizes (in the case of multifragmentation, described in I, this corresponds to the mass distribution of the fragments). The conditions where the phase transition takes place are taken at the points, in the T - ρ plane (ρ is the density of baryons), where the "susceptibility" defined in section 1.4 is maximum. However, it is important to keep in mind that, since the system is finite, the transition from hadronic matter to the quark-gluon plasma is not sharp. This might have important consequences experimentally because it means that deconfinement can be reached, with some smaller probability, in less severe conditions than expected from the maximum of the susceptibility.

As we have stressed above, R_{bag} is the free parameter of the model. Depending on its choice one will change the conditions under which one reaches quark deconfinement. In figure 11 we present the results obtained with the value $R_{\text{bag}} = 0.5$ fm. At $T=0$ one needs to reach $\rho_C = 0.65$ nucleon/fm³ in order to form the plasma. At normal density one needs to heat the system up to $T_C = 240$ MeV.

FIGURE 11

The full line shows the separation between hadronic matter and the quark-gluon plasma in the T - ρ plane.

These values are in agreement with those obtained previously with more involved models². If R_{bag} increases, ρ_C and T_C decrease and vice-versa. It should be noted that our schematic model assumes that two bags fuse if they just touch. In the case where one would require that they have at least one quark in their overlap region in order to fuse, one would get the same conditions as those displayed in figure 11 with a larger value of R_{bag} . Finally one can parametrize the transition limit, shown in figure 11, by the following expression :

$$T = 270 (\rho_C - \rho)^{1/4} \quad (9)$$

5. CONCLUSIONS

In this contribution we have applied very simple ideas to the understanding of critical phenomena in nuclear physics. As far as the disassembly of hot and compressed nuclei is concerned we found that :

- Multifragmentation is a result of the fluctuations of the mean field which become large.
- Compression is more efficient to break nuclei than thermal excitation.
- For a system at normal densities one needs a thermal excitation larger than 70 % of the binding energy in order that it breaks up.

- At the instability point, the density is of the order of 0.1 nucleon/fm³ in the case of the $^{208}_{82}\text{Pb}$ nucleus.

Similar ideas can be applied to calculate the conditions under which one can form a quark-gluon plasma. Despite the simplicity of the approach, one finds similar conditions as more refined models. This very likely indicates that geometry may govern the process to a large extent.

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